Aggregation of Gamma-Signatures and Applications to Bitcoin

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Abstract. Aggregate signature allows non-interactively condensing multiple individual signatures into a compact one. Besides the faster verification, it is useful to reduce storage and bandwidth, and is especially attractive for blockchain and cryptocurrency. For example, aggregate signature can mitigate some bottlenecks emerged with the Bitcoin systems (and actually almost all blockchain-based systems): low throughput, capacity, scalability, high transaction fee, etc. Unfortunately, achieving aggregate signature from general elliptic curve group (*without bilinear maps*) is a long-standing open question. Recently, there is also renewed interest in deploying Schnorr's signature in Bitcoin, for its efficiency and flexibility.

In this work, we investigate the applicability of the Γ-signature scheme proposed by Yao and Zhao. Akin to Schnorr's, Γ-signature is generated with linear combination of ephemeral secret-key and static secret-key, and enjoys almost all the advantages of Schnorr's signature. Besides, Γ-signature has salient features in online/offline performance, stronger provable security, and deployment flexibility with interactive protocols like IKE. In this work, we identify one more key advantage of Γ-signature in signature aggregation, which is particularly crucial for applications to blockchain and cryptocurrency.

Specifically, we first observe the incapability of Schnorr's for aggregating signatures in the Bitcoin system. This is demonstrated by concrete attacks. Then, we show that aggregate signature can be derived from the Γ-signature scheme. To the best of our knowledge, this is the first aggregate signature scheme from general groups without bilinear maps. The security of aggregate Γ-signature is proved based on a new assumption proposed and justified in this work, referred to as non-malleable discrete-logarithm (NMDL), which might be of independent interest and could find more cryptographic applications in the future. When applying the resultant aggregate Γ-signature to Bitcoin, the storage volume of signatures reduces about 50%, and the signature verification time can even reduce about 80%. Finally, we specify in detail the implementation of aggregate Γ-signature in Bitcoin, with minimal modifications that are in turn more friendly to segregated witness (SegWit) and provide better protection against transaction malleability attacks.

1 Introduction

Bitcoin [37], with the introduction of the blockchain technology, was originally proposed by Nakamoto Satoshi in 2008. The key characteristics of blockchain consist in decentralization, openness, unforgeability, and anonymity. After about ten years of rapid development, blockchain has been more and more popular, and more applications are advocated into finance, healthcare, storage, education industries, etc. Nevertheless, there are still quite a lot of deficiencies to overcome. With Bitcoin as an example, below we review some deficiencies or bottlenecks it faces now.

Currently, due to the 1M-byte limitation of block size, about 7 transactions are conducted per second in the Bitcoin system. This leads to, in particular, longer confirmation latency, relatively high transaction fees, and easier target of spam attacks [38].[1]

As the crucial elements of a global consensus system, kept in check by the ability for every participant to validate all updates to the ledger, the size of signatures and the computational cost for verifying them are the primary limiting factors for its scalability [35]. Bitcoin uses the ECDSA signature scheme [29] over secp256k1 curve [17]. According to Bitcoin Stack Exchange, in a standard "*pay to public key hash*" (P2PKH) transaction or a "*pay to script hash*" (P2SH) transaction, the signatures occupy about 40% of transcript size.[2] In addition, an ECDSA signature involves non-linear combination of ephemeral secret-key and static secret-key, which is the source for relative inefficiency and for the cumbersome in extensions to multi-signatures [6, 35], scriptless scripts [44], etc. As a consequence, recently there is also renewed interest in deploying Schnorr's signature with Bitcoin in the future.

Aggregate signature [12] can essentially mitigate the above deficiencies or bottlenecks faced by Bitcoin (and actually almost all blockchain-based systems). An aggregate signature (AS) scheme is a digital signature scheme with the following additional property: multiple individual signatures $\{\sigma_1, \cdots, \sigma_n\}$, where σ_i is a signature on message m_i under public-key pk_i, $1 \leq i \leq n$ and $n \geq 2$, can be *non-interactively* collected and condensed into a compact aggregate signature σ. Here, in general, for any i, j such that $1 \leq i \neq j \leq n$, it is assumed that $(pk_i, m_i) \neq (pk_j, m_j)$; but it might be the case that $pk_i = pk_j$ or $m_i = m_j$. There is a corresponding aggregate verification process that takes input $\{(pk_1, m_1), \cdots, (pk_n, m_n), \sigma\}$, and accepts if and only if all the individual signatures are valid. Aggregate signature is useful to reduce bandwidth and storage volume, and is especially attractive for blockchain where communication and storage are more expensive than computation.

[1] As for spam attacks, if we can put more transactions into a block, the spammer has to send more transactions with more transaction fee to congest the network, which increases the attack cost.

[2] In more detail, for a standard P2PKH or P2SH transaction with n inputs and m outputs, its size is about $146n + 33m + 10$ bytes where the signatures occupy $72n$ bytes. For P2SH multi-signature transactions, the size of signatures may further scale up.

The differences between aggregate signature and multi-signature should be noted. With a multi-signature scheme, multiple signers sign the same message, and more importantly they need cooperation. A multi-signature scheme can also be turned into an aggregate signature scheme, but the signers need to cooperate and interact for such a task, which is, in general, unrealistic for Bitcoin or for most blockchain-based distributed systems. Though practical multi-signature schemes were built from general groups on which the discrete logarithm problem is hard [6, 35], known efficient aggregate signature schemes were all built from gap groups with *bilinear maps* [12, 5]. It is commonly believed impossible to build aggregate signature schemes from general groups, at least with a black-box reduction to discrete logarithm [12, 5].

1.1 Contributions

In this work, we investigate the applicability of the Γ-signature scheme proposed by Yao and Zhao [46]. Akin to Schnorr's, Γ-signature is generated with linear combination of ephemeral secret-key and static secret-key, and enjoys almost all the advantages of Schnorr's signature. Besides, Γ-signature has salient features in online/offline performance, stronger provable security, and deployment flexibility with interactive protocols like IKE. In this work, we identify one more key advantage of Γ-signature in signature aggregation, which is particularly crucial for applications to blockchain and cryptocurrency.

Specifically, we first observe the incapability of Schnorr's for aggregating signatures in the Bitcoin system. This is demonstrated by concrete attacks. Then, we show that aggregate signature can be derived from the Γ-signature scheme. To the best of our knowledge, this is the first aggregate signature scheme from general groups without bilinear maps. The security of aggregate Γ-signature is proved based on a new assumption proposed and justified in this work, referred to as non-malleable discrete-logarithm (NMDL), which might be of independent interest and could find more cryptographic applications in the future.

When applying the resultant aggregate Γ-signature to Bitcoin, the storage volume of signatures reduces about 50%, and the signature verification time can even reduce about 80%. Finally, we specify in detail the implementation of aggregate Γ-signature in Bitcoin, with minimal modifications that are in turn more friendly to segregated witness (SegWit) and provide better protection against transaction malleability attacks [14].

2 Preliminaries

For prime number q, denote by Z_q the additive group of integers modulo q, by Z_q^* the multiplicative group of integers modulo q. If $\mathcal{S}$ is a finite set then $|\mathcal{S}|$ is its cardinality, and $x \leftarrow S$ is the operation of picking an element uniformly at random from S. If α is neither an algorithm nor a set then $x \leftarrow \alpha$ is a simple assignment statement. A string or value α means a binary one, and $|\alpha|$ is its binary length. If α and β are two strings, $\alpha||\beta$ is their concatenation. If $\mathcal{A}$ is

a probabilistic algorithm, $\mathcal{A}(x_1, x_2, \cdots; \rho)$ is the result of running $\mathcal{A}$ on inputs $x_1, x_2, \cdots$ and coins ρ. We let $y \leftarrow \mathcal{A}(x_1, x_2, \cdots; \rho)$ denote the experiment of picking ρ at random and letting y be $A(x_1, x_2, \cdots; \rho)$. By $\Pr[E : R_1; \cdots; R_n]$ we denote the probability of event E, after the *ordered* execution of random processes $R_1, \cdots, R_n$. A function $\varepsilon(l)$ is *negligible* if for every $c > 0$ there exists an l_c such that $\varepsilon(l) < \frac{1}{l^c}$ for all $l > l_c$. Let PPT stand for probabilistic polynomial-time.

A digital signature scheme consists of three algorithms $KeyGen$, $Sign$ and $Verify$, where the key generation algorithm $KeyGen$ takes a security parameter l as its input and randomly outputs a key pair (sk, pk). The signature algorithm $Sign$ takes sk, m as its input and outputs a signature σ. And the signature verification algorithm $Verify$ takes pk, m, σ as its input and outputs $ACCEPT$ or $REJECT$. Usually, the algorithms $KeyGen$ and $Sign$ are probabilistic, while the algorithm $Verify$ is deterministic. The completeness of a signature scheme requires that $Verify(pk, m, Sign(sk, m)) = ACCEPT$ always holds for any $m \in \{0,1\}^*$, as long as (sk, pk) is a valid key pair generated by running $KeyGen$.

2.1 Elliptic Curve for Bitcoin

We consider signature implementations over elliptic curve groups. Let $E(F)$ be the underlying elliptic curve group defined over finite field F, and the point P generates a cyclic group of prime order q on which the discrete logarithm problem is assumed to be hard, where $|q| = l$ is the security parameter. The order of $E(F)$ is tq, where t is called the cofactor that is usually a small constant. Denote by ∞ the identity element in $E(F)$.

Bitcoin uses the secp256k1 curve [17], which is of the form $y^2 = x^3 + 7$ defined over F_p for prime number $p = 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1$. For the secp256k1 curve, both p and q have the same length of 256 bits, i.e., $l = \log q = 256$, and the cofactor $t = 1$. For a point on the secp256k1 curve, it can be represented with 257 bits as (x, b), where $x \in Z_p$ is its x-coordinate and $b \in \{0,1\}$ indicates the sign of its y-coordinate. Thanks to the fact that $p = 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1 = 7 \mod 8$, recovering y from (x, b) is very efficient for the secp256k1 curve [28, 32]. We remark that compact representation of public keys has already been being employed in the Bitcoin system.

2.2 Schnorr Signature

The Schnorr signature scheme is proposed in [43], and is proven secure in the random oracle model based on the discrete logarithm assumption [40]. At a high level, Schnorr's signature is an instantiation of the Fiat-Shamir transformation [22] being applied to Σ-protocols (i.e., three-round public-coin honest-verifier zero-knowledge protocols) in the random oracle model. Let $H : \{0,1\}^* \to Z_q$ be a cryptographic hash function, and $m \in \{0,1\}^*$ be the message to be signed, Schnorr's signature scheme is briefly reviewed in Table 1.

KeyGen(1^l)	**Sign**(X, x, m)	**Verify**$(X, m, \sigma = (e, z))$
$x \leftarrow Z_q^*$	$r \leftarrow Z_q$	$R := zP - eX$
$X := xP$	$R := rP$	**if** $H(R, m) \neq e$ **then**
return(x, X)	$e := H(R, m)$	**return** $REJECT$
	$z := r + ex \mod q$	**else**
	return $\sigma = (e, z)$	**return** $ACCEPT$

Table 1. Schnorr's signature

2.3 Γ-Signature

Under the motivation for achieving signature schemes of better online/offline performance, flexible and easy deployments (particularly with interactive protocols like IKE), and stronger security, Yao and Zhao introduced a new paradigm in [47]. Specifically, they proposed a special case of Σ-protocol, which is referred to as Γ-protocol, and a transformation called Γ-transformation that transforms any Γ-protocol into a signature scheme in the random oracle model. The resultant signature is named Γ-signature. Below, we briefly review the Γ-signature scheme based on discrete logarithm problem (DLP), and its security result. The reader is referred to [47] for more details.

Let $H_d, H_e : \{0,1\}^* \rightarrow Z_q^*$ be two cryptographic hash functions, and $m \in \{0,1\}^*$ be the message to be signed, the DLP-based Γ-signature scheme is briefly reviewed in Table 2. Here, for presentation simplicity, checking $z \neq 0$ in signature generation, and checking $d, z \in Z_q^*$ and $A \neq \infty$ in signature verification, are not explicitly specified. In the actual implementation, it is also suggested in [47] that $d = H_d(A)$ is replaced with $d = x_A \mod q$, where x_A is the x-coordinate of A. To ease signature verification, we can replace d in σ with d^{-1}. In this case, d^{-1} is not needed to computed in signature verification, and the signature is rejected if $H_d(A)d^{-1} \neq \infty$. Another variant of the Γ-signature scheme for the Ethereum system (where no public key is available in a transaction) is presented in Table 3, where $e = H_e(X, m)$ can also be replaced with $e = H_e(H_e(X), m)$.

KeyGen(1^l)	**Sign**(X, x, m)	**Verify**$(X, m, \sigma = (d, z))$
$x \leftarrow Z_q^*$	$r \leftarrow Z_q^*$	$e := H_e(X, m)$
$X := xP$	$A := rP$	$A := zd^{-1}P + ed^{-1}X$
return(x, X)	$d := H_d(A)$	**if** $H_d(A) \neq d$ **then**
	$e := H_e(X, m)$	**return** $REJECT$
	$z := rd - ex \mod q$	**else**
	return $\sigma = (d, z)$	**return** $ACCEPT$

Table 2. Γ-signature

Security of Γ-Signature. Strong existential unforgeability under concurrent interactive attacks for a signature scheme $\prod = (KeyGen, Sign, Verify)$,

KeyGen(1^l)	**Sign**(X, x, m)	**Verify**$(m, \sigma = (A, e^{-1}, z))$
$x \leftarrow Z_q^*$	$r \leftarrow Z_q^*$	$d := H_d(A)$
$X := xP$	$A := rP$	$X := ze^{-1}P - de^{-1}A$
return(x, X)	$d := H_d(A)$	**if** $H_e(X, m)e^{-1} \neq \infty$ **then**
	$e := H_e(X, m)$	**return** $REJECT$
	$z := rd + ex \mod q$	**else**
	return $\sigma = (A, e^{-1}, z)$	**return** $ACCEPT$

Table 3. Variant of Γ-signature for Ethereum

where a signature can be divided into two parts (d, z), is defined using the following game between a challenger and a forger adversary $\mathcal{F}$.

- Setup. On the security parameter l, the challenger runs $(PK, SK) \leftarrow KeyGen(1^l)$. The public-key PK is given to adversary $\mathcal{F}$ (while the secret-key SK is kept private).
- Suppose $\mathcal{F}$ makes at most q_s signature queries. Each signature query consists of the following steps: (1) $\mathcal{F}$ sends "Initialize" to the signer. The i-th initialization query is denoted as I_i, $1 \leq i \leq q_s$. (2) Upon the i-th initialization query, the signer responds back d_i. (3) $\mathcal{F}$ adaptively chooses the message m_i to be signed, and sends m_i to the signer. (4) The signer sends back z_i, where (d_i, z_i) is the signature on message m_i. $\mathcal{F}$ is allowed to adaptively and concurrently interact with the signer in arbitrary interleaved order. As a special case, $\mathcal{F}$ can first make q_s initialization queries, and get all the values in $\bar{D} = \{d_1, \cdots, d_{q_s}\}$ before presenting any message to be signed.
- Output. Finally, $\mathcal{F}$ outputs a pair of m and (d, z), and wins the game if (1) $Verify(PK, m, d, z) = 1$ and (2) $(m, d, z) \notin \{(m_1, d_1, z_1), \cdots, (m_{q_s}, d_{q_s}, z_{q_s})\}$.

We define $\mathsf{AdvSig}^{\text{suf-cia}}_{\prod, \mathcal{F}}(1^l)$ to be the probability that $\mathcal{F}$ wins in the above game, taken over the coin tosses of $KeyGen$ and of $\mathcal{F}$ and the signer (and the random choice of the random oracle). We say the signature scheme $\prod$ is strongly existential unforgeable, if $\mathsf{AdvSig}^{\text{suf-cia}}_{\prod, \mathcal{F}}(\cdot)$ is a negligible function for every PPT forger $\mathcal{F}$.

It is proved in [47] that the above Γ-signature scheme is *strongly existential unforgeable* under the DLP assumption, assuming H_d is a random oracle while H_e is target one-way as defined in [47].

3 Aggregate Signature and Related Work

An aggregate signature (AS) signature scheme is a tuple $(KeyGen, Sign, Verify, Agg, AggVerify)$, where the last three are deterministic, while the first three algorithms constitute a standard signature scheme. Given multiple individual signatures $\{\sigma_1, \cdots, \sigma_n\}$, where σ_i is a signature on message m_i under public-key pk_i, $1 \leq i \leq n$ and $n \geq 2$, the aggregation algorithm Agg condenses them into a compact aggregate signature sig. Here, in general, for any i, j such that

$1 \leq i \neq j \leq n$, it is assumed that $(pk_i, m_i) \neq (pk_j, m_j)$; but it might be the case that $pk_i = pk_j$ or $m_i = m_j$. The completeness of an AS scheme says that $AggVerify(\{(pk_1, m_1), \cdots, (pk_n, m_n)\}, sig)$ returns "ACCEPT", whenever $Verify(pk_i, m_i, \sigma_i)$ outputs "ACCEPT" for any i, $1 \leq i \leq n$. Roughly speaking, the security of an AS scheme says that it is infeasible for any PPT adversary $\mathcal{A}$ to produce a valid forged aggregate signature involving an honest signer, even when it can play the role of all the other signers (in particular choosing their public keys), and can mount a chosen-message attack on the target honest signer.

Definition 1 (security of aggregate signature).
Let $(pk, sk) \leftarrow KeyGen(1^l)$ be the public and secret key pair of the target honest signer. The advantage of the attacker $\mathcal{A}$ against the AS scheme is defined as $Adv^{\mathcal{A}}_{AS}(1^l) = Pr[AggVerify(\{(pk_1, m_1), \cdots, (pk_n, m_n)\}, sig) = ACCEPT]$, where n is polynomial in l and $pk \in \{pk_1, \cdots, pk_n\}$. The probability is taken over the random coins used by $KeyGen$ and $\mathcal{A}$ in the following experiment:

$$(pk, sk) \leftarrow KeyGen(1^l); (pk_1, ..., pk_n, m_1, ..., m_n, sig) \leftarrow \mathcal{A}^{Sign(sk,)}(1^l, pk) \ .$$

To make the security definition meaningful, we only consider adversaries that are legitimate in the sense that, supposing $pk = pk_i$ for some i, $1 \leq i \leq n$, must never have queried m_i to its signing oracle. Then, an AS scheme is said to be secure if for any PPT adversary $\mathcal{A}$ its advantage $Adv^{\mathcal{A}}_{AS}(1^l)$ is negligible in l. Note that $\mathcal{A}$ can choose $pk_1, ..., pk_n$ as it wishes, in particular as a function of the target public key pk. There is also no requirement that the adversary "know" the secret key corresponding to a public key it produces.

Practical aggregate signature schemes were proposed in [12, 5], which are derived from the BLS short signature [13] based on the coCDH assumption in groups with bilinear maps. Let G_1, G_2, G_T be groups of the same prime order q, g be a generator of G_2, and $e : G_1 \times G_2 \rightarrow G_T$ be a non-degenerate, efficiently computable bilinear map (i.e., a pairing). Roughly speaking, the coCDH assumption says that, given $X \leftarrow G_1$ and $Y = g^y \in G_2$ where $y \leftarrow Z_q$, no efficient algorithm can output $Z = X^y$ with non-negligible probability. Note that when $G_1 = G_2$, the coCDH problem becomes the standard CDH problem in G_1. There have been some discussions on deploying the pairing-based AS schemes [12, 5] in the Bitcoin system [34], which are briefly summarized below.

- **System complexity.** Deploying pairing-based aggregate signature schemes requires the replacement of not only the ECDSA algorithm but also the underlying elliptic curve.
- **Bilinear group vs. general group.** Intractability problems in groups with bilinear maps are weaker than the discrete logarithm problem in general ECC groups.
- **Verification speed.** More importantly, as an individual signature scheme, the verification of the pairing-based BLS signature is significantly slower than that of ECDSA. Note that the miners still need to verify the correctness of individual BLS signatures before aggregating them into a block. Some survey indicates that on a concrete hardware it can verify 70,000 secp256k1

signatures per second, while it could only process about 8,000 BLS signature per second.

It is thus highly desirable to develop aggregate signatures just based on the secp256k1 curve. Nevertheless, it is a long-standing open question to develop AS schemes based on general ECC groups without pairings, and is commonly suggested impossible (at least with a *black-box* reduction to the discrete logarithm assumption).

4 On the Insecurity of Aggregation of Schnorr's Signatures

Recently, there is renewed interest in deploying Schnorr's signature in the Bitcoin system, for its efficiency and flexibility. In comparison with EC-DSA used in Bitcoin, the linear combination of ephemeral secret-key and static secret-key with Schnorr's signature brings more desirable advantages, e.g., multi-signature, scriptless scripts (specifically, privacy-preserving smart contracts). However, we show the insecurity of aggregation of Schnorr's signatures. This is demonstrated by a concrete fatal attack.

We first present the aggregate signature based on Schnorr's scheme. Suppose there are n signers, $n \geq 2$, and each has the public and secret key pair (X_i, x_i) where $X_i = x_i P$ and $x_i \leftarrow Z_q^*$, $1 \leq i \leq n$. Denote by $\sigma_i = (e_i, z_i)$ the signature by user i on message $m_i \in \{0,1\}^*$. After receiving $\{(X_1, m_1, \sigma_1), \cdots, (X_n, m_n, \sigma_n)\}$, the miner first verifies the correctness of each individual signature (X_i, m_i, σ_i), during which it gets $R_i = z_i P - e_i X_i$. If all the individual signatures are correct, the miner finally outputs $\hat{R} = \{R_1, \cdots, R_n\}$ and $z = \sum_{i=1}^{n} z_i$ as the resultant aggregate signature. On input $(X_1, \cdots, X_n, m_1, \cdots, m_n, \hat{R}, z)$, $AggVerify$ works as follows: computes $e_i = H(X_i, R_i, m_i)$, and accepts if $zP = \sum_{i=1}^{n} R_i + \sum_{i=1}^{n} e_i X_i$.

The above AS scheme looks fine, and was just what we originally proposed. But a deep speculation divulges the following subtle yet fatal attack. Without loss of generality, suppose the index of the attacker is 1, who possesses the public and secret key pair (X_1, x_1) and acts as follows.

- For any j, $2 \leq j \leq n$, the attacker selects m_j and arbitrary R_j (from the underlying ECC group) on behalf of X_j, and computes $e_j = H(X_j, R_j, m_j)$. Note that the attacker does not know the discrete logarithm of either X_j or R_j.
- The attacker chooses its own message m_1, sets $R_1 = (-\sum_{j=2}^{n} R_j - \sum_{j=2}^{n} e_j X_j)$, and computes $e_1 = H(X_1, R_1, m_1)$ and $z = e_1 x_1$.
- Finally, the attacker outputs $(R_1, \cdots, R_n, z)$ as the forged aggregate signature.

Note that $zP = e_1 x_1 P = e_1 X_1 = e_1 X_1 + R_1 + (\sum_{j=2}^{n} R_i + \sum_{j=2}^{n} e_i X_i) = \sum_{i=1}^{n} R_i + \sum_{i=1}^{n} e_i X_i$. Thus, the forged aggregate signature is valid, the attacker

can sign arbitrary messages on behalf of the victim users $(X_2, \cdots, X_n)$. There is no doubt that such an attack is really fatal, particularly in a cryptocurrency system like Bitcoin.

5 Aggregate Γ-Signature

The aggregate Γ-signature scheme is described in Table 4, and a variant is presented in Appendix A. Here, the algorithms $(KeyGen, Sign, Verify)$ just constitute the Γ-signature scheme presented in Section 2.3. For presentation simplicity, we use a single cryptographic hash function $H : \{0,1\}^* \to Z_q$, and checking $d, z \in Z_q^*$ and $m_i \neq \lambda$ and $A \neq \infty$ is omitted in the specification of verification algorithms, where λ represents the empty string.

KeyGen(1^l)
$x \leftarrow Z_q^*$
$X := xP$
return (x, X)

Sign(X, x, m)
$r \leftarrow Z_q$
$A := rP$
$d := H(A)$
$e := H(X, m)$
$z := rd - ex \mod q$
return $\sigma = (d, z)$

Verify$(X, m, \sigma = (d, z))$
$e := H(X, m)$
$A := zd^{-1}P + ed^{-1}X$
if $H(A) \neq d$
return $REJECT$
else
return $ACCEPT$

Agg$(\{(X_1, m_1, \sigma_1), ..., (X_n, m_n, \sigma_n)\})$
$\hat{T} := \emptyset, \hat{A} := \emptyset, z := 0$
for $i = 1$ *to* n
if $Verify(X_i, m_i, \sigma_i) = ACCEPT \wedge (X_i, m_i) \notin \hat{T} \wedge A_i \notin \hat{A}$
$\hat{T} := \hat{T} \cup \{(X_i, m_i)\}$
$\hat{A} := \hat{A} \cup \{A_i\}$
$z := z + z_i \mod q$
return $(\hat{T}, \hat{A}, z)$

AggVerify$(\hat{T}, \hat{A}, z)$
if *elements in* $\hat{T}$ *are not distinct*
return $REJECT$
if *elements in* $\hat{A}$ *are not distinct*
return $REJECT$
if $|\hat{T}| \neq |\hat{A}|$
return $REJECT$
$n' := |\hat{T}| = |\hat{A}|$
for $j = 1$ *to* n'
$d_j := H(A_j), e_j := H(X_j, m_j)$
if $(zP + \sum_{j=1}^{n'}(e_j X_j - d_j A_j)) \neq \infty$
return $REJECT$
return $ACCEPT$

Table 4. Aggregate Γ-signature

Given a list of individual signatures $\{(X_1, m_1, \sigma_1 = (d_1, z_1)), \cdots, (X_n, m_n, \sigma_n = (d_n, z_n))\}$, where $n \geq 2$, the aggregation algorithm discards (X_i, m_i, σ_i) if the signature verification fails, or any one of (X_i, m_i) or A_i is repeated. The latter checking is for provable security, as we shall see. But it still might be the case that, for some $i \neq j$, $X_i = X_j$ or $m_i = m_j$ (this case occurs with Bitcoin P2SH multi-signature transactions). We assume that the elements in $\hat{T}$ and those in $\hat{A}$ output by Agg are sorted to ease verification of aggregate signature. More details about the implementations are discussed in the next subsection. Observe that $\hat{T}$ and $\hat{A}$ are output and treated *separately*, and $AggVerifier$ actually does

not care about the correspondence between the elements in $\hat{A}$ and those in $\hat{T}$. This flexibility allows for implementations more friendly to SegWit and to being resistant to transaction malleability attacks, as we shall discuss in Section 6.

The total size of the aggregate signature $(\hat{A}, z)$ is $n'(l+1)+l$ bits, where each A_i is represented with $\log p + 1 = l + 1$ bits, while that of the n' individual signatures is $2n'l$ bits. We use the simultaneous point multiplication techniques [28, 25, 21] in computing $zP + \sum_{j=1}^{n'} e_j X_j - \sum_{j=1}^{n'} d_j A_j$. Specifically, we divide the $2n'+1$ point multiplications into $\lceil (2n'+1)/8 \rceil$ groups, and then apply the simultaneous multiplication technique to each group of at most 8 point multiplications. Usually, simultaneous multiplication of k ECC points needs to prepare a table of size at most 2^k. For simultaneous multiplication of 8 points over the secp256k1 curve, the cost amounts to about 2 point multiplications, and thus verifying the aggregate signature performs about $2\frac{n'}{8} = \frac{n'}{4}$ point multiplications. In comparison, with simultaneous multiplication, the verification of an individual Schnorr or Γ signature needs about 1.2 point multiplications, and $1.2n'$ in total.

5.1 NMDL Assumption, and Justification

Motivated to break some impossibility barriers of black-box cryptography and to achieve cryptographic schemes of conceptually simple structure and analysis, the research community has been paying more attention to achieving cryptographic schemes based on non-black-box assumptions or primitives in recent years [2, 15, 16, 24, 19, 11]. As a popular non-black-box assumption, the knowledge-of-exponent assumption (KEA) and its variants have been shown to be successful and powerful in solving a large number of (at times notoriously open) cryptographic problems, (see, e.g., [18, 27, 7, 8, 41, 30, 20, 42, 1, 36, 15, 16, 26, 23, 24, 45, 19, 11, 46, 48, 9, 10]). In particular, a type of KEA assumption on pairing groups is used in Zcash [9, 10].

Yao and Zhao introduced and justified a variant of the KEA assumption, referred to as *joint* KEA (JKEA) assumption [46]. Let $H_1, \cdots, H_\kappa : \{0,1\}^* \rightarrow Z_q$ be cryptographic hash functions that are modelled as random oracles (RO). Roughly speaking, the JKEA assumption says that, given $X = xP$ for $x \leftarrow Z_q$, the ability of an efficient algorithm $\mathcal{A}$ to output $\{(Y_1, m_1), \cdots, (Y_\kappa, m_\kappa), Z\}$ such that $Z = (\sum_{i=1}^{\kappa} e_i Y_i)^x$, where $Y_i \in E(F)$ and $m_i \in \{0,1\}^*$ and $e_i = H_i(Y_i, m_i)$ for $1 \leq i \leq \kappa$, implies knowing $(y_1, \cdots, y_\kappa)$ simultaneously where y_i is the discrete logarithm of Y_i. Here, "knowing" implies that $(y_1, \cdots, y_\kappa)$ can be efficiently extracted by an extractor algorithm $\mathcal{E}$ from the input and the random type of $\mathcal{A}$. The JKEA assumption is justified in [46] by the fact that, assuming H_i's are random oracles, no efficient algorithm can make the values in $\{e_1 Y_1, \cdots, e_\kappa Y_\kappa\}$ correlated. That is, no matter how the PPT algorithm $\mathcal{A}$ does, the values $\{H_1(Y_1, m_1)y_1, \cdots, H_\kappa(Y_\kappa, m_\kappa)y_\kappa\}$ are computationally independent as defined in [46].

The JKEA assumption implies the following weaker assumption, referred to as *explicit* knowledge-of-exponent assumption (EKEA). Specifically, the ability

of outputting $\{(Y_1, m_1), \cdots, (Y_\kappa, m_\kappa), z\}$, satisfying $z \in Z_q$ and $zP = \sum_{i=1}^{\kappa} e_i Y_i$, implies knowing $(y_1, \cdots, y_\kappa)$ simultaneously. That is, $(y_1, \cdots, y_\kappa)$ can be efficiently extracted. Unlike the JKEA assumption where the algorithm $\mathcal{A}$ only outputs $CDH(X, \sum_{i=1}^{\kappa} e_i Y_i)$, here $\mathcal{A}$ *explicitly* outputs the discrete logarithm $z = \log(\sum_{i=1}^{\kappa} e_i Y_i)$. Clearly, the EKEA assumption is implied by, and weaker than, the JKEA assumption. It is easy to check that the security of aggregate Γ-signature can be derived from the EKEA assumption and the discrete logarithm assumption. But we would like to have a further weaker *black-box* assumption, which is proposed below.

Definition 2 (non-malleable discrete logarithm (NMDL) assumption). *Let $G = (E(F_p), P, q)$ define a cyclic group over $E(F_p)$ generated by P of order q, where p and q are prime numbers, and $l = \log q$ be the security parameter. Let $H_1, \cdots, H_\kappa : \{0,1\}^* \to Z_q$ be cryptographic hash functions, which may not be distinct and are modelled as random oracles. On input (G, X) where $X = xP$ for $x \leftarrow Z_q$, a PPT algorithm $\mathcal{A}$ (called an NMDL-solver) succeeds in solving the NMDL problem, if it could output $\{(b_1, Y_1, m_1) \cdots, (b_\kappa, Y_\kappa, m_\kappa), z\}$, satisfying:*

- *$z \in Z_q$, and for any i, $1 \le i \le \kappa$, $Y_i \in G$, $m_i \in \{0,1\}^*$ that can be the empty string, and $b_i \in \{0,1\}$.*
- *For any $1 \le i \ne j \le \kappa$, it holds that $(Y_i, m_i) \ne (Y_j, m_j)$. But it might be the case that $Y_i = Y_j$ or $m_i = m_j$.*
- *$X \in \{Y_1, \cdots, Y_\kappa\}$, and $zP = \sum_{i=1}^{\kappa} (-1)^{b_i} e_i Y_i$ where $e_i = H_i(Y_i, m_i)$.*

Then, the NMDL assumption says that, for any PPT algorithm $\mathcal{A}$, the probability that it succeeds in solving the NMDL problem is negligible in l. The probability is taken over the random coins used to generate (G, x), the random coins used by $\mathcal{A}$ (and the choices of the random functions $H_1, \cdots, H_\kappa$ in the random oracle model).

It is easy to see that the NMDL assumption is implied by the standard discrete logarithm assumption and the EKEA assumption. Note also that the NMDL assumption is itself black-box in nature. We suggest the NMDL assumption should be of independent interests, and could find more cryptographic applications in the future. In the next subsection, we conduct the security analysis of aggregate Γ-signature based on it.

5.2 Security Analysis

Theorem 1. *The aggregate Γ-signature scheme presented in Table 4 is secure under the NMDL assumption.*

Proof. According to the security definition of aggregate signature presented in Section 3, supposing there exists a PPT forger $\mathcal{A}$ who breaks the security of the aggregate Γ-signature with non-negligible probability, we present another PPT algorithm $\mathcal{B}$ who can solve the NMDL problem also with non-negligible probability. Denote by $(X = xP, x)$ the public and secret key pair of the target

honest user, where $x \leftarrow Z_q^*$. The algorithm $\mathcal{B}$ takes (G, X) as input (where G is the underlying cyclic group defined in the elliptic curve), runs $\mathcal{A}$ as a subroutine, and works as follows.

$\mathcal{B}$ controls and programs the random oracle H. Whenever $\mathcal{A}$ asks the target user to sign a message m, $\mathcal{B}$ answers the signing query by running the Γ-signature simulator as described in [47]. As analyzed in [47], the simulation is statistically indistinguishable from what $\mathcal{A}$ gets in reality. Finally, suppose that $\mathcal{A}$ outputs a valid aggregate Γ-signature denoted $(\hat{T}, \hat{A}, z)$, where $\hat{T} = \{(X_1, m_1), \cdots, (X_{n'}, m_{n'})\}$, $\hat{A} = \{A_1, \cdots, A_{n'}\}$. Assume that there are n'' distinct elements $\bar{X} = \{X_{i_1}, \cdots, X_{i_{n''}}\}$ in $\hat{X} = \{X_1, \cdots, X_{n'}\}$, where X_{i_j} appears t_j times in $\hat{X}$ and $\sum_{j=1}^{n''} t_j = n'$. For each j, $1 \leq j \leq n''$, denote by $I_j = \{j_1, \cdots, j_{t_j}\}$ the set of indices that X_{i_j} appears in $\hat{X}$ where $1 \leq j_\alpha \leq n'$ for $1 \leq \alpha \leq t_j$; specifically, $X_{i_j} = X_{j_1} = \cdots = X_{j_{t_j}}$. $\mathcal{B}$ outputs $\{\bar{T}, \bar{A}, z\}$, which are specified below:

- $\bar{T} = \{(b_1, X_{i_1}, m_1), \cdots, (b_{n''}, X_{i_{n''}}, m_{n''})\}$, where for each j, $1 \leq j \leq n''$, $b_j = -1$ and $m_j = m_{j_1} || \cdots || m_{j_{k_j}}$.
- $\bar{A} = \{(b'_1, A_1, \lambda) \cdots, (b'_{n'}, A_{n'}, \lambda)\}$, where for each i, $1 \leq i \leq n'$, $b'_i = 1$, and λ represents the empty string.

According to the security analysis of Γ-signature in [47], what seen by $\mathcal{A}$ under the run of $\mathcal{B}$ is statistically indistinguishable from what seen in reality. Thus, with also non-negligible probability, $\mathcal{A}$ will output a *valid* aggregate Γ-signature $(\hat{T}, \hat{A}, z)$ under the simulation of $\mathcal{B}$. Consequently, $\mathcal{B}$ outputs $(\bar{T}, \bar{A}, z)$ with the same probability. Define $H' : G \times (\{0,1\}^*)^\beta \rightarrow Z_q$ as follows: $H'(X, m_1, \cdots, m_\beta) = H(X, m_1) + \cdots + H(X, m_\beta) \mod q$ for any β, $1 \leq \beta \leq n'$. It is easy to see that, assuming $H\{0,1\}^* \rightarrow Z_q$ is a random oracle, so is H'. Finally, we show that the output $\{\bar{T}, \bar{A}, z\}$ by $\mathcal{B}$ is a correct solution to the NMDL problem, by the following observations:

- All the tuples in $\bar{T} \bigcup \bar{A}$ are distinct. This is from the facts that: (1) the tuples in $\bar{T}$ are distinct and $m_j \neq \lambda$, $1 \leq j \leq n''$; (2) the tuples in $\bar{A}$ are also distinct with the same empty string as the third element in each tuple.
- As we assume the aggregate signature $(\hat{T}, \hat{A}, z)$ output by $\mathcal{A}$ is valid, we have that $X \in \bar{X} = \{X_{i_1}, \cdots, X_{i_{n''}}\}$, and $zP = \sum_{i=1}^{n'} d_i A_i - \sum_{j=1}^{n''} e'_j X_{i_j}$, where $d_i = H(A_i)$ and $e'_j = H'(X_{i_j}, m_j)$.

6 Applications to Bitcoin

In this section, we specify how to apply our work to Bitcoin. The key point is to maximize compatibility with the existing Bitcoin system, while with the least modifications. Our modifications involve: txid, unlocking script, locking script, Merkle tree, block construction, block mining and block verification. We try to describe our implementation in a self-contained manner in the following subsections.

6.1 Inheritances: Keys, Addresses and Network

Bitcoin uses a specific elliptic curve, as defined in a standard called secp256k1, established by NIST. Our aggregate Γ-signature scheme also works on the secp256k1 curve. As for new key pair generation, algorithm $KeyGen(1^l)$ is the same as in the existing Bitcoin system.

As for Bitcoin addresses, we inherit the existing design in Bitcoin. Specifically, this is the process of generating address from public key through the use of one-way hash algorithms SHA256 and RIPEMD160,

$$\text{A=RIPEMD160(SHA256(X))},$$

where X is the public key and A is the Bitcoin address. The above address is called P2PKH address. There is another type of address called P2SH address, which is generated by the following equation:

$$\text{A=RIPEMD160(SHA256(script))}.$$

We also use the Base58 [3] and Base58Check [4, 33] formats for unambiguously and compactly encoding Bitcoin data such as addresses, etc.

We adopt the existing Bitcoin network which is structured as a peer-to-peer (P2P) network on top of the internet. And the Bitcoin network refers to the collection of nodes running the Bitcoin protocol. When a peer receives data, it will broadcast the data to its neighbouring peers after some necessary verification. With the usage of P2P network, in a very short period of time, the data such as transactions and blocks can be efficiently spread all over the network.

6.2 Transactions

Transactions are the most important part of the Bitcoin system. Everything else in Bitcoin is designed to ensure that transactions can be created, propagated on P2P network, validated, and finally added to the global ledger of transactions (i.e., the blockchain).

The Bitcoin transaction consists of fields such as version, in-counter, inputs list, out-counter, outputs list and locktime, which is shown in Table 5.

Field	Description	Size
version	Transaction version number	4 bytes
in-counter	Counter of inputs	1-9 bytes
inputs-list	List of transaction inputs	variable
out-counter	Counter of outputs	1-9 bytes
outputs-list	List of transaction outputs	variable
locktime	Earliest time that a transaction is valid	4 bytes

Table 5. Structure of Bitcoin transaction

Within the inputs list field of transaction, it consists of

- **txid**: a pointer to the transaction containing the UTXO to be spent.
- **vout**: the index number of the UTXO to be spent.
- **unlocking script**: a script that fulfills the conditions of the UTXO locking script.
- **sequence**: the block number where the UTXO is recorded in the blockchain.

In the Bitcoin system, txid is the double SHA256 hash of the transaction, including the witness. It is impossible to retrieve the txid after our modifications, as our work condenses multiple individual signatures into a compact one and signature is a type of witness. So, we modify the txid to be the double SHA256 hash of the transaction without witness. Note that tampering with the witness data is the source for launching transaction malleability attacks [14]. Removing it from the hash input in generating txid also removes the opportunity for *transaction malleability attacks*. This can also greatly improve the implementations for many other protocols, such as payment channels, chained transactions, and lightning networks.

Unlocking script of P2PKH is in the format of <sig><PubK>, where PubK is a public key and sig is a signature signed by the private key corresponding to PubK; The unlocking script of P2SH has a basic format of <sig I><sig J>, mainly for multi-signature. In our modifications, the sig is generated by the $Sign(X, x, m)$ of Γ-signature (where m is part of a transaction defined by SIGHASH flag), which replaces the existing ECDSA signature.

Within the outputs list field of transaction, it consists of (1) **value** which is an amount of Bitcoin; and (2) **locking script** which is a cryptographic puzzle that determines the conditions required to spend the output. As for operations OP_CHECKSIG and OP_CHECKMULTISIG among locking script of P2PKH and P2SH, the ECDSA verification procedure is replaced by running $Verify(X, m, \sigma = (d, z))$ of Γ-signature.

6.3 Block

A block is a container data structure that collects transactions for inclusion in the public ledger, the blockchain. The block consists of a header, containing metadata, followed by a long list of transactions, which is shown in Table 6.

Field	Description	Size
magic-no	Value always $0xD9B4BEF9$	4 bytes
blocksize	Number of bytes following up to end of block	4 bytes
blockheader	Consists of 6 items	80 bytes
tx-counter	Counter of transactions	1-9 bytes
transactions	List of transactions	variable

Table 6. Structure of Bitcoin block

Each block is identified by a hash which is generated by running the SHA256 cryptographic hash algorithm twice on the block header. The size of block header is 80-bytes, and its structure is shown in Table 7.

Field	Description	Size
version	Block version number	4 bytes
hashPrevBlock	Hash of the previous block header	32 bytes
hashMerkleRoot	Hash of Merkle tree root in the block	32 bytes
timestamp	Current timestamp as seconds	4 bytes
bits	Current target in compact format	4 bytes
nonce	32-bit number	4 bytes

Table 7. Structure of Bitcoin blockheader

Every block in blockchain contains a summary of all the transactions using a Merkle tree. A Merkle tree, also known as a binary hash tree, is a data structure used for efficiently summarizing and verifying the integrity of large sets of data. In our modifications, we build Merkle tree with our modified txid which is the double SHA256 hash of the transaction without witness.

In the existing Bitcoin system, after validating transactions a miner will add them to the memory pool or transaction pool where transactions await until they can be included (mined) into a block. We adopt Merkle-Patricia tree (MPT) [39] to play the role of memory pool and to perform duplication check, as the elements in $\hat{T}$ and $\hat{A}$ in our aggregate Γ-signature are required to be distinct. MPT can provide a cryptographically authenticated data structure that can be used to store $(key, value)$ pairs, and enjoys a faster speed both in element searching and in outputting ordered elements. The algorithm Agg in our aggregate Γ-signature can be implemented with MPT as follows.

- Initialize two empty MPT instances $MPT_{\hat{A}}$ and $MPT_{\hat{T}}$, where $MPT_{\hat{A}}$ (resp., $MPT_{\hat{T}}$) is for the set of $\hat{A}$ (resp., $\hat{T}$).
- Traverse the received transactions and do the following. For every transaction input, extract the public key X_i and the signature $\sigma_i = (d_i, z_i)$ in the unlocking script, and m_i that is the specific part of a transaction defined by SIGHASH flag; Then, calculate A_i from σ_i, and search in MPT_A, MPT_T to check whether there already exists A_i or (X_i, m_i); Finally, verify (X_i, m_i, σ_i) with our $Verify$ algorithm.
- If there already exists A_i or (X_i, m_i), or $Verify$ algorithm outputs $REJECT$, drop the current transaction, and loop to the next transaction.
- Insert A_i and (X_i, m_i) to MPT_A and MPT_T respectively, and set $z := z + z_i \mod q$.
- When all the transactions are traversed, output the ordered list of A_i's as $\hat{A}$, the ordered list of (X_i, m_i) as $\hat{T}$, and a number $z \in Z_q$.

Now, we pay attention to P2SH unlocking script multi-signature, where N public keys are recorded in the script and at least M of them must provide signatures to unlock the funds. In order to aggregate the multi-signature, a Bitcoin node should extract each tuple (X_j, m, σ_j) from the M provided signatures on the same message m, and deals with it like a normal transaction input.

After collecting enough transactions, the miner constructs a candidate block, with the only witness of aggregate signature $(\hat{A}, z)$ being placed at the end of block as specified by segregated witness (SegWit).[3] This way, our result inherits all the advantages of SegWitness, besides enjoying a more compact witness.

When a miner finds a solution nonce (that is inserted into the block header) such that the block header hash is less than the target, the miner transmits the candidate block to all its peers immediately. By the consensus mechanism of Bitcoin, every node independently validates the new block before propagating it to its peers, which ensures that only valid blocks are propagated on the network. Instead of individually validating all the transactions within the block, with our modifications, each node only needs to verify one aggregate signature with algorithm $AggVerify(\hat{T}, \hat{A}, z)$, as follows.

- Note that both $\hat{T}$ and $\hat{A}$ are ordered. In order to ensure the elements within are distinct, just traverse the lists $\hat{T}$ and $\hat{A}$ to confirm that every two adjacent elements are different and are monotonically incremented.
- If the elements are not distinct in the above step, abort and output $REJECT$. Otherwise, continue with the next procedure.
- Execute the aggregate signature validation, and output $ACCEPT$ if the verification is successful. Otherwise, output $REJECT$.

With our modifications and improvements, the process of signature verification becomes much simpler, and the signature verification time can be even reduced by about 80% compared with verifying signatures individually.

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A A Variant of Aggregate Γ-Signature

A variant of the aggregate Γ-signature scheme is described in Table 8.

Here, the algorithms $(KeyGen, Sign, Verify)$ constitute a variant of the Γ-signature scheme presented in Section 2.3, where the value A (rather than $d = H(A)$ in the original Γ-signature) is output as part of the signature. This change does not affect the provable security of Γ-signature, but the result signature, i.e.,

KeyGen(1^l)
$x \leftarrow Z_q^*$
$X := xP$
return (x, X)

Sign(X, x, m)
$r \leftarrow Z_q$
$A := rP$
$d := H(A)$
$e := H(X, m)$
$z := -(rd + ex) \mod q$
return $\sigma = (A, z)$

Verify$(X, m, \sigma = (A, z))$
$d := H(A)$
$e := H(X, m)$
if $(zP + dA + eX) \neq \infty$
return $REJECT$
else
return $ACCEPT$

Agg$(\{(X_1, m_1, \sigma_1), ..., (X_n, m_n, \sigma_n)\})$
$\hat{T} := \emptyset, \hat{A} := \emptyset, z := 0$
for $i = 1$ *to* n
if $Verify(X_i, m_i, \sigma_i) = ACCEPT \wedge (X_i, m_i) \notin \hat{T} \wedge A_i \notin \hat{A}$
$\hat{T} := \hat{T} \cup \{(X_i, m_i)\}$
$\hat{A} := \hat{A} \cup \{A_i\}$
$z := z + z_i \mod q$
return $(\hat{T}, \hat{A}, z)$

AggVerify$(\hat{T}, \hat{A}, z)$
if *elements in* $\hat{T}$ *are not distinct*
return $REJECT$
if *elements in* $\hat{A}$ *are not distinct*
return $REJECT$
if $|\hat{T}| \neq |\hat{A}|$
return $REJECT$
$n' := |\hat{T}| = |\hat{A}|$
for $j = 1$ *to* n'
$d_j := H(A_j), e_j := H(X_j, m_j)$
if$(zP + \sum_{j=1}^{n'}(d_j A_j + e_j X_j)) \neq \infty$
return $REJECT$
return $ACCEPT$

Table 8. Variant of Aggregate Γ-signature

(A, z), is one bit longer than (d, z). Specifically, the bit length of d is $l = \log q = 256$, while that of A is $l + 1 = 257$ over the secp256k1 curve. This variant has some advantages, on the following grounds: (1) the verification of individual signatures and that of aggregate signature are more compatible; (2) it can be more efficient for signature aggregation.