Differential Cryptanalysis of a Lightweight Block Cipher LELBC

Manjeet Kaur, Tarun Yadav, Manoj Kumar Dhananjoy Dey

Abstract—In this study, we investigate the newly developed low energy lightweight block cipher (LELBC), specifically designed for resource-constrained Internet of Things (IoT) devices in smart agriculture. The designers conducted a preliminary differential cryptanalysis of LELBC through mixed-integer linear programming (MILP). This paper further delves into LELBC's differential characteristics in both single and related-key frameworks using MILP, identifying a nine-round differential characteristic with a probability of 2^{-60} in a single-key framework and a 12round differential characteristic with a probability of 2^{-60} in a related-key framework.

Index Terms—Differential Cryptanalysis, LELBC, Lightweight Block Cipher, MILP.

I. INTRODUCTION

Nowadays, the Internet of Things (IoT) plays a crucial role in various sectors, including agriculture, healthcare, industry, and transportation. However, IoT devices face limitations in resources, such as computing power and energy, making traditional cryptographic techniques inadequate for ensuring data security. To address this problem, lightweight cryptography has emerged, leading to the development of numerous lightweight block ciphers like PRESENT [1], SKINNY [2], GIFT [3], WARP [4], IVLBC [5], and HDLBC [6] etc. Recently, a low energy lightweight block cipher (LELBC) has been proposed by Song et al. [7] for IoT devices with limited resources, particularly for smart agriculture. This cipher is based on a novel structure called the 'permutation substitution permutation', incorporating two symmetric diffusion layers and a sublayer. The diffusion process involves left circular shifts and XOR operations, while the Sublayer is made up of 16 involutory 4-bit S-boxes. To ensure encryption and decryption are consistent, a permutation is applied after executing all 16 rounds. The designers conducted the security evaluation of LELBC, measuring its resilience against a range of attacks including dependency and avalanche effect, differential and linear attacks [8], algebraic attack, as well as meet-in-themiddle and slide attacks. In their differential cryptanalysis, the designers determined the minimal number of active Sboxes through a bit-oriented mixed integer linear programming

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Dhananjoy Dey is an Associate Professor in the Department of Mathematics, Indian Institute of Information Technology, Lucknow, India–226 002. Email: ddey@iiitl.ac.in (MILP) program for up to 12 rounds, asserting that the fullround LELBC is resistant to differential attack. However, their evaluation did not take into account the probabilities associated with the S-box difference distribution table (DDT), an essential factor in determining the cipher's strength more accurately. Additionally, they did not analyze the cipher's resistance to related-key differential attack, in which attackers exploit differences in keys to slightly propagate state differences. These gaps highlight the basis of our investigation, which aims to conduct an in-depth security analysis of the cipher, specifically focusing on its resilience to differential attack in both single and related-key frameworks. Currently, differential cryptanalysis is studied through methods such as Boolean Satisfiability and Satisfiability Modulo Theory (SAT/SMT), MILP, and Constraint Programming (CP) [9]. In this study, we apply the MILP method to assess the security of LELBC against differential attack. This involves transforming the problem of finding the minimum number of active S-boxes and optimal differential characteristics into a MILP problem, which is then addressed through the Gurobi¹ solver.

Contribution

In this paper, we study the LELBC against differential cryptanalysis in single and related-key frameworks through the MILP method including the probabilities of DDT of LELBC's S-box. We determine the exact lower bounds on the number of active S-boxes and probabilities up to 13 rounds, providing a nine-round differential characteristic in a single-key framework. Further, we identify valid differential characteristics up to 12 rounds and present a 12-round differential characteristic in a related-key framework.

Organization

The rest of the paper is organized as follows: Section II describes the related work to our study. The comprehensive details of the LELBC lightweight block cipher are elaborated in Section III. Section IV discusses the MILP modeling of LELBC for identifying differential characteristics of LELBC in both single and related-key frameworks. Subsequent findings and analyses of these characteristics are presented in Section V. Finally, the paper is concluded in Section VI.

II. RELATED WORK

This section reviews existing studies related to the cryptanalysis of lightweight block ciphers.

Cao and Zhang [10] introduced related-key differential characteristics for the two reduced versions of block cipher GIFT (GIFT-64 and GIFT-128) through MILP model, utilizing these characteristics to execute key recovery attacks on a reduced version of GIFT-64. Weng et al. [11] analyzed LBlock's resistance to related-key differential attack, providing tighter security bounds. Their SMT-based analysis determined the minimum number of active S-boxes and the upper bounds of probabilities for up to 19 rounds of LBlock. Additionally, they demonstrated key-recovery attacks on both 22- and 23round of LBlock. Chan et al. [12] assessed the differential attack resilience of SLIM and LCB, two lightweight block ciphers. For SLIM, they provided differential characteristics for all rounds and demonstrated a key recovery attack on 14 rounds. Moreover, they identified LCB's linearity due to its S-box design, subsequently improving LCB by substituting its S-box with PRESENT. Teh et al. [13] presented new differential cryptanalysis results for the lightweight block cipher BORON, including high-probability differentials and key recovery attacks against BORON-80/128. For WARP, Teh and Biryukov [14] presented differential distinguishers and mounted key-recovery differential attacks in single and related-key frameworks. Further, they identified a boomerang distinguisher for rectangle attack on WARP. Mahzoun et al. [15] presented differential attack on a variant of K-Cipher with a complexity of $2^{29.7}$ for specific parameters. Zhang et al. [16] discussed an impossible differential attack on FBC-128, a block cipher. The authors constructed 9-round truncated impossible differentials, leading to a 13-round key recovery attack. Cui et al. [17] identified differential characteristics in MANTIS, Midori-128, and QARMA-64 using the SAT method, incorporating Matsui's bounding conditions and the technique of dichotomy. Furthermore, they provided a keyrecovery differential attack for Midori-128 and QARMA-64. Nobuyuki et al. [18] assessed RBFK's vulnerability to differential, linear, and meet-in-the-middle attacks, concluding it lacks security against such attacks. They demonstrated keyrecovery attacks on full-round RBFK, highlighting weaknesses in key scheduling and round functions, and suggested improvements to withstand such attacks. Li et al. [19] constructed fullround differential characteristics with MILP for both Shadow-32 and Shadow-64, versions of the lightweight block cipher Shadow. Additionally, they demonstrated a full-round keyrecovery attack on both versions. ILTER and SELCUK [20] proposed two alternative MILP models for multiple XOR operations. They found differential and linear characteristics for Klein and Prince in single-key frameworks.

III. LELBC

The key generation and encryption algorithms of LELBC are discussed in this section.

A. Key Generation

Initially, the key generation process is employed on the key (K) to derive the first round subkey (SK_1) . Subsequently,

Algorithm 1 Key Generation

Require: $K = k_0 k_1 k_3 \dots k_{126} k_{127}$
Ensure: $SK_i, \forall i \in \{1, 2,, 16\}$
for $i \leftarrow 1$ to 16 do
$K_0 K_1 K_2 K_3 \leftarrow K$
$k_{124} k_{125} k_{126} k_{127} \leftarrow S(k_{124} k_{125} k_{126} k_{127})$
$k_{121} k_{122} k_{123} k_{124} \leftarrow (k_{121} k_{122} k_{123} k_{124}) \oplus (i-1)$
$k_0 k_1 k_3 \dots k_{126} k_{127} \leftarrow k_{64} k_{65} k_{66} \dots k_{62} k_{63} $
$k_0 k_1 k_3 \dots k_{62} k_{63} \leftarrow k_9 k_{10} k_{11} \dots k_7 k_8$
$k_{64} k_{65} k_{66} \dots k_{126} k_{127} \leftarrow k_{79} k_{80} k_{81} \dots k_{77} k_{78} k_{77} k_{78} k_{78} k_{77} k_{78} k_{78} k_{77} k_{78} k_{78} k_{77} k_{78} k_{$
$SK_i \leftarrow K$
end for

TABLE I S-box								
x	0	1	2	3	4	5	6	7
S(x)	C	E	6	A	4	F	2	7
x	8	9	A	B	C	D	E	F
S(x)	9	8	3	B	0	D	1	5

for each round, the key generation process utilizes the subkey from the preceding round to obtain the subsequent round's subkey. This process comprises three operations: an S-box for substitution, an addition of a round constant, and left circular shift, as elaborated in Algorithm 1. In i^{th} round, the high 64bit $(K_0||K_1)$ of the subkey (SK_i) is used, where K_0 and K_1 undergo XOR operations with the left and right 32-bit of X, respectively.

B. Encryption

LELBC, a block cipher, operates on a 128-bit key and a 64-bit plaintext to produce a 64-bit ciphertext over 16 rounds. Each round consists of XOR, add round key, sublayer (see Table I), and left circular shift operations. The process starts by splitting the plaintext into two branches: left (X_0) and right (X_1). Initially, the left branch, X_0 , is combined with the key K_0 using an XOR operation. Subsequently, the right branch, X_1 , is XORed with the result of applying a 5-bit left circular shift to the updated X_0 . Following the application of the sublayer to both branches, then X_0 is XORed with the result of a 5-bit left circular shift applied to X_1 , and X_1 is XORed with K_1 . After the final round, X_0 and X_1 are swapped to generate the ciphertext. The encryption process is illustrated in Figure 1 and Algorithm 2.

IV. MILP MODEL OF LELBC

The differential attack in block ciphers is currently addressed using the mixed-integer linear programming (MILP) method. This method converts the cipher's operations, including XOR, S-boxes, and left circular shift operation, into linear inequalities, as follows:

A. XOR Operation in Terms of Linear Inequalities

For the XOR operation with input bit differences (x, y)and output difference z, this operation is described by the

Algorithm 2 Encryption of LELBC

 Require: X, SK_i ; $\forall i \in \{1, 2, ..., 16\}$

 Ensure: Y

 for $i \leftarrow 1$ to 16 do

 $K_0 ||K_1||K_2||K_3 \leftarrow SK_i$
 $X_0 \leftarrow K_0 \oplus X_0$
 $X_1 \leftarrow X_1 \oplus (X_0 <<<5)$
 $X_0 ||X_1 \leftarrow S(X_0)||S(X_1)$
 $X_0 \leftarrow K_0 \oplus (X_1 <<<5)$
 $X_1 \leftarrow K_1 \oplus X_1$

 end for

 $Y \leftarrow X_1 ||X_0$

 return Y



Fig. 1. Encryption Process

following linear inequalities:

$$-x + y + z \ge 0$$

$$x - y + z \ge 0$$

$$x + y - z \ge 0$$

$$x + y + z \le 2.$$
(1)

B. S-box in Terms of Linear Inequalities

Given (u_0, u_1, u_2, u_3) and (y_0, y_1, y_2, y_3) as input and output differences for a 4-bit S-box, we consider the input-output differential as a point $(u_0, u_1, u_2, u_3, y_0, y_1, y_2, y_3) \in \mathbb{F}_2^8$. Then compute the H-representation of the convex hull of all possible input-output differentials of the S-box using the double description method. This process yields numerous linear inequalities, some of which may be redundant and hinder model efficiency. To minimize these inequalities, we employ a reduction algorithm leveraging the MILP technique, as suggested by Sasaki and Todo [21]. There are 21 linear

inequalities to represent the S-box of LELBC as follows:



To find the exact probabilities of the differential characteristics of a cipher, we consider the differential behavior of the S-box with probabilities. There are three entries 2, 4, and 16 in the DDT of the S-box. Therefore, we need to take two more variables (p_0, p_1) (see Equation 3) such that the inputoutput differential $(u_0, u_1, u_2, u_3, y_0, y_1, y_2, y_3, p_0, p_1) \in \mathbb{F}_2^{10}$. Similarly, there are 21 linear inequalities through the MILPbased reduction algorithm as shown in Equation 4.

$$(p_0, p_1) = \begin{cases} (0,0) \text{ if } Pr = 1\\ (0,1) \text{ if } Pr = 2^{-3}\\ (1,0) \text{ if } Pr = 2^{-2} \end{cases}$$
(3)

 $1u_0 - 1u_1 + 0u_2 - 1u_3 - 1y_0 + 0y_1 + 0y_2 + 0y_3 + 3p_0 + 2p_1 \ge 0$ $1u_0 - 1u_1 + 0u_2 - 1u_3 - 1y_0 + 1y_1 - 1y_2 + 1y_3 + 4p_0 + 2p_1 \ge 0$ $1u_0 + 1u_1 + 0u_2 + 2u_3 + 1y_0 - 3y_1 - 1y_2 + 2y_3 + 0p_0 + 4p_1 \ge 0$ 0 $1u_0 - 1u_1 - 2u_2 - 1u_3 + 5y_0 + 2y_1 + 4y_2 + 2y_3 - 3p_0 + 0p_1 \ge 0$ 0 $6u_0 + 1u_1 + 2u_2 + 1u_3 + 1y_0 - 2y_1 - 3y_2 - 2y_3 + 0p_0 + 3p_1 \ge 0$ 0 $3u_0 + 3u_1 + 0u_2 - 1u_3 - 1y_0 - 2y_1 - 2y_2 - 1y_3 + 3p_0 + 5p_1 \ge 0$ 0 $2u_0 + 5u_1 + 2u_2 + 5u_3 - 2y_0 - 1y_1 + 0y_2 - 1y_3 + 0p_0 - 1p_1 \ge 0$ 0 $-1u_0 + 3u_1 + 0u_2 + 2u_3 - 4y_0 + 2y_1 + 1y_2 - 1y_3 + 4p_0 + 2p_1 \ge$ 0 $3u_0 + 1u_1 + 4u_2 + 3u_3 - 1y_0 + 3y_1 - 2y_2 - 4y_3 - 1p_0 + 4p_1 \ge$ 0 $3u_0 - 1u_1 + 0u_2 + 3u_3 - 1y_0 - 1y_1 - 2y_2 - 2y_3 + 3p_0 + 5p_1 \ge 0$ 0 $-2u_0 + 2u_1 - 2u_2 + 2u_3 - 1y_0 + 1y_1 + 0y_2 + 1y_3 + 5p_0 + 1p_1 \ge 0$ 0 $-5u_0 + 2u_1 + 1u_2 - 2u_3 - 1y_0 + 4y_1 + 0y_2 + 3y_3 + 5p_0 + 3p_1 \ge 0$ 0 $-1u_0 - 1u_1 - 2u_2 - 2u_3 + 3y_0 - 1y_1 + 0y_2 + 3y_3 + 3p_0 + 5p_1 \ge 0$ $-5u_0 - 2u_1 + 1u_2 + 2u_3 - 1y_0 + 3y_1 + 0y_2 + 4y_3 + 5p_0 + 3p_1 \ge 0$ 0 $-1u_0 - 2u_1 - 2u_2 - 1u_3 + 3y_0 + 3y_1 + 0y_2 - 1y_3 + 3p_0 + 5p_1 \ge$ 0 $1u_0+2u_1-1u_2-3u_3+1y_0+2y_1+0y_2+1y_3+0p_0+4p_1\geq$ 0 $1u_0-1u_1-1u_2+2u_3-2y_0-3y_1+3y_2+3y_3+5p_0+4p_1\geq$ 0 $-1u_0 - 1u_1 + 0u_2 + 1u_3 - 2y_0 - 1y_1 + 2y_2 - 3y_3 + 5p_0 + 8p_1 \ge 0$ 0 $-1u_0 - 1u_1 + 0u_2 - 1u_3 + 1y_0 - 2y_1 + 0y_2 - 2y_3 + 6p_0 + 5p_1 \ge 0$ 0 $-2u_0 - 1u_1 + 1u_2 - 3u_3 + 0y_0 - 1y_1 + 1y_2 + 1y_3 + 4p_0 + 7p_1 \ge 0$ 0 $0u_0 + 0u_1 + 0u_2 + 0u_3 + 0y_0 + 0y_1 + 0y_2 + 0y_3 - 1p_0 - 1p_1 \ge -1$ (4)

C. Left Circular Shift in Terms of Linear Inequalities

Consider the sequences $(x_0, x_1, \ldots, x_{31})$ and $(y_0, y_1, \ldots, y_{31})$ represents the input and output bit differences of a 5-bit left circular shift, respectively. The relationship

 $y_i = x_{(i+5)\%32}$ for $0 \le i \le 31$ provides a set of linear equations characterizing this operation.

D. Add Round Key in Terms of Linear Inequalities

In a related-key differential attack, adversaries exploit plaintext differences along with key differences. Therefore, we include the linear constraints for the add round key in MILP models to search related-key differential characteristics. For this operation, consider an input bit x, a key bit k, and the output bit z such that $z = k \oplus x$. The following are the linear inequalities corresponding to this operation:

$$-k + x + z \ge 0$$

$$k - x + z \ge 0$$

$$k + x - z \ge 0$$

$$k + x + z \le 2.$$
(5)

E. Objective Functions

To determine the minimum number of active S-boxes and the maximum probability for differential characteristics, objective functions are formulated as

$$\min \sum_{i} A_i$$
 and $\min \sum_{i} (2p_{i,0} + 3p_{i,1})$.

V. RESULT

Utilizing the Gurobi solver for MILP models, this study determines precise lower bounds on active S-boxes and probabilities of differential characteristics for up to 13 rounds in a single-key framework, as detailed in Table II. We determine valid single-key differential characteristics of LELBC up to nine rounds, including a nine-round differential characteristic with a probability of 2^{-60} shown in Table III. Additionally, our analysis identifies valid related-key differential characteristics up to 12 rounds, with a 12-round differential characteristic of probability 2^{-60} as depicted in Table IV.

 TABLE II

 Comparison between number of active S-boxes (#AS) and

 probabilities of differential characteristics up to 13 rounds

Dound	This l	Paper (Single-key)	Designers' Claim #AS	
Kouliu	#AS	Probability		
1	1	2^{-2}	1	
2	3	2^{-6}	3	
3	6	2^{-15}	6	
4	9	2^{-23}	9	
5	12	2^{-34}	13	
6	14	2^{-38}	14	
7	16	2^{-43}	17	
8	19	2^{-51}	20	
9	22	2^{-60}	26	
10	25	2^{-69}	27	
11	27	2^{-73}	30	
12	30	2^{-85}	34	
13	33	2^{-93}	-	

VI. CONCLUSION

This study conducted a thorough analysis of the LELBC lightweight block cipher's resilience to differential attacks under both single and related-key frameworks, utilizing MILP to

TABLE III 9-ROUND DIFFERENTIAL CHARACTERISTIC

Round	Input Difference	Probability
1	4000 0000 0020 a008	2^{-0}
2	0004 0000 0080 2000	2^{-7}
3	0000 0000 0000 4000	2^{-6}
4	0002 0000 0000 1000	2^{-3}
5	1004 0000 0080 1000	2^{-9}
6	5000 0100 0000 2008	2^{-11}
7	4000 0000 0000 0008	2^{-8}
8	4000 0000 0000 0000	2^{-3}
9	1000 0040 0000 0002	2^{-5}
10	4000 4010 0000 0200	2^{-8}

 TABLE IV

 12-round differential characteristic

Round	Input Difference	Key Difference	Probability
1	0004 0000 0080 0000	0000 0000 0100 0000	2^{-0}
2	0001 0000 0000 0000		2^{-3}
3	0000 0000 0000 0000		2^{-0}
4	0000 0000 0000 0000	0000 0000 0000 0000	2^{-0}
5	0000 0000 0000 0100	0000 0000 0000 0100	2^{-0}
6	0000 4000 0000 0200		2^{-3}
7	0020 200d 0001 0180	0000 0000 0000 0000 0000 0000 0000 0000 0000	2^{-14}
8	0140 0000 0808 0080	0000 0000 0000 0000 0000	2^{-16}
9	0000 0800 0001 0040	0100 0000 0000 0000 0000	2^{-9}
10	0000 0000 0000 0010	0000 0000 0000 0000 0000 0000 0000 0000 0000	2^{-5}
11	0000 0800 0001 0040	0000 0000 0000 0080	2^{-3}
12	0000 0100 0000 0020	0000 0000 0000 0000 0000	2^{-5}
13	0000 1400 0000 00a0	0000 0100 0000 0000 0000 0000 0000 0000	2^{-2}

discover differential characteristics. Our MILP models incorporated DDT probabilities to find out characteristics for up to nine and 12 rounds in single-key and related-key frameworks, respectively. This investigation presented nine-round (single-key) and 12-round (related-key) differential characteristics, each with a probability of 2^{-60} .

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