Low-Communication Updatable PSI from Asymmetric PSI and PSU

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Abstract. Private Set Intersection (PSI) allows two mutually untrusted parties to compute the intersection of their private sets without revealing additional information. In general, PSI operates in a static setting, where the computation is performed only once on the input sets of both parties. Badrinarayanan et al. (*PoPETs* 2022) initiated the study of Updatable PSI (UPSI), which extends this capability to dynamically updating sets, enabling both parties to securely compute the intersection as their sets are modified while incurring significantly less overhead than re-executing a conventional PSI. However, existing UPSI protocols either do not support arbitrary deletion of elements or incur high computational and communication overhead. In this work, we combine asymmetric PSI with Private Set Union (PSU) to present a novel UPSI protocol. Our UPSI protocol supports arbitrary additions and deletions of elements, offering a flexible approach to update sets. Furthermore, our protocol enjoys extremely low communication overhead, scaling linearly with the size of the update set while remaining independent of the total set size. We implement our protocol and compare it against state-of-the-art conventional PSI and UPSI protocols. Experimental results demonstrate that our UPSI protocol incurs 587 to 755 times less communication overhead than the recently proposed UPSI protocol (*AsiaCrypt* 2024) that supports arbitrary additions and deletions. Moreover, our UPSI protocol has a significant advantage in low-bandwidth environments due to the exceptionally low communication overhead. Specifically, with an input size of 2^{22} and the size of the addition/deletion set being 2^{10}, the existing UPSI protocol requires approximately 1650.45, 1789.5, and 3458.1 seconds at bandwidths of 200 Mbps, 50 Mbps, and 5 Mbps, respectively, whereas our UPSI protocol only requires around 13.01, 13.75, and 22.53 seconds under the same conditions. Our open-source implementation is available at: https://github.com/ShallMate/upsi.

1 Introduction

Private Set Intersection (PSI) enables two parties, each holding a private set, to compute the intersection of their sets while revealing nothing beyond the intersection itself (except for the size of their inputs). It has found broad application in a variety of scenarios, such as data mining on private data [1], measuring ad conversion rates [2], and private contact discovery [3]. Over the past decade, PSI has made remarkable progress, with numerous efficient PSI protocols having been proposed [4,5,6,7,8,9,10,11,12]. The most efficient PSI protocol [10] can compute the intersection in approximately one second for input sizes on the order of one million, requiring only tens of megabytes of communication overhead.

Despite the significant performance breakthroughs achieved by these efficient PSI protocols, they are restricted to a static setting. Specifically, if the set of either party changes, even by a single element, a complete PSI execution is required to obtain the updated intersection. However, in practical applications, the inputs to the PSI protocol are often subject to continuous changes. For example, a typical application of a PSI protocol is sample alignment prior to vertical federated learning [13]. Moreover, the data on both parties may need to be updated continuously or periodically. If each update to the sets requires re-executing the PSI protocol, it would result in a significant waste of resources. A recent work by Badrinarayanan et al. [14] initiates the study of Updatable PSI (UPSI), which enables two parties to compute the intersection of updated sets without re-executing a conventional PSI. However, this UPSI protocol only supports the addition of elements, while deletions are implemented through a periodic refresh mechanism, referred to as "weak deletion". Very recently, Badrinarayanan et al. [15] proposed a revised version of this protocol, which supports the addition and deletion of elements in UPSI. However, from the experimental results presented in Table 4 of [15], we observe that this protocol introduces new challenges, including high computational and communication overhead. Specifically, it only outperforms a re-execution of the most efficient PSI protocol [10] under particular conditions, namely when the total input is very large (i.e., 2^{22}), the update set is minimal (i.e., 2^4), and the available bandwidth is significantly constrained (i.e., 5 Mbps). Therefore, this raises a natural question:

Could we construct a UPSI protocol that supports arbitrary additions and deletions of elements and ensures faster performance than re-executing a conventional PSI in most cases rather than being limited to highly specific conditions?

1.1 Our Results

This work constructs a UPSI protocol that allows both parties to arbitrarily add or delete elements from their input sets while incurring significantly lower computational and communication overhead than the deletion-supporting UPSI protocol in [15]. Our UPSI is inspired by the UPSI framework based on structured encryption [16,17] proposed by Agarwal et al. [18]. However, our UPSI protocol does not rely on structured encryption but instead borrows the idea of asymmetric PSI and requires an efficient Private Set Union (PSU) protocol. Additionally, the UPSI protocol proposed by Agarwal et al. [18] provides only a complexity analysis, with neither experimental results nor code implementation available, leaving its concrete performance unclear. In Table 1, we compare our UPSI protocol with previous UPSI protocols [14,15,18] in terms of support for addition and deletion of elements, computational and communication complexity, availability of code implementations and experimental results, and practical communication.

Table 1: Summary of our results in comparison to previous work, including support for addition and deletion of elements, computational and communication complexity, whether code implementations and experiments, and practical communication ratio are provided. N denotes the size of the entire sets, and N_u denotes the size of the updated sets. t denotes the number of updates when parties refresh their sets with weak deletion. k denotes a constant. Since BMX22 [14] and ACG$^+$24 [18] did not provide code, we cannot compare practical communication with them.

Protocol	Addition/Deletion	Comp. Complexity	Comm. Complexity	Code/Experiment	Practical Communication Ratio
BMX22 (addition-only) [14]	Addition	$\mathcal{O}(N_u)$	$\mathcal{O}(N_u)$	Experiment	-
BMX22 [14]	Weak Deletion	$\mathcal{O}(N_u \cdot t)$	$\mathcal{O}(N_u \cdot t)$	Experiment	-
BMS$^+$24 (addition-only) [15]	Addition	$\mathcal{O}(N_u \cdot \log N)$	$\mathcal{O}(N_u \cdot \log N)$	Code & Experiment	5 - 6
BMS$^+$24 [15]	Addition & Deletion	$\mathcal{O}(N_u \cdot \log^2 N)$	$\mathcal{O}(N_u \cdot \log^2 N)$	Code & Experiment	587 - 755
ACG$^+$24 [18]	Addition & Deletion	$\mathcal{O}((\log N)^k)$	$\mathcal{O}(N_u)$	-	-
Ours	Addition & Deletion	$\mathcal{O}(N_u \cdot \log N)$	$\mathcal{O}(N_u)$	Code & Experiment	1

In summary, our work is the first UPSI protocol to provide comprehensive experimental results, an open-source implementation, and support for adding and deleting elements while guaranteeing performance. Furthermore, we present our results in terms of experiments, computation, and communication.

• **Experiments**. We implement our UPSI protocol and provide a comprehensive report on its performance under various input sizes, update set sizes, and bandwidth conditions to strengthen reader confidence in our work. Moreover, we also compare our protocol with the state-of-the-art UPSI and conventional PSI protocols, demonstrating the performance advantages of our UPSI protocol. Finally, we evaluate the update size threshold at which re-executing the conventional PSI becomes more efficient than our UPSI protocol.

• **Computation**. Our UPSI protocol outperforms existing UPSI protocols in terms of computational overhead. For example, when the input set size is 2^{20} and the update set size is 2^{10}, our UPSI protocol achieves over 1300 times faster runtime compared to the addition/deletion-supporting version in [15], and over most 20 times faster compared to the version that supports only addition. Furthermore, our protocol can be up to 18$\times$ faster than the state-of-the-art conventional PSI protocol [10] with a bandwidth of 5 Mbps.

• **Communication**. Our protocol reduces communication overhead by a factor of 587 to 755 compared to the state-of-the-art UPSI protocol [15] that supports both addition and deletion. Additionally, it achieves a 5 to 6 times reduction compared to the version that supports only addition. Additionally, our protocol outperforms the conventional PSI protocol [10] by 10 to 2066 times in all settings.

1.2 Technical Overview

From a high-level perspective, our UPSI protocol requires executing an asymmetric PSI protocol twice, with computational and communication overhead depending solely on the party with the smaller set. Unlike previous asymmetric PSI protocols [19,20,21,22], our protocol outputs the result to the party with the larger set. Furthermore, this process does not require fully homomorphic encryption [23,24] to protect data privacy and hashing for element alignment between both parties. Meanwhile, our protocol requires a PSU protocol as a foundational component, for which several efficient solutions exist, such as those presented in [26,27,28,29,30]. Now, let us introduce our UPSI protocol step by step from a high-level perspective. We would like to emphasize that certain specific computational steps are omitted here to maintain clarity for the reader.

Initialization. Let $\mathcal{P}_X$ and $\mathcal{P}_Y$ denote the parties holding the original sets X and Y, respectively. $\mathcal{P}_X$ and $\mathcal{P}_Y$ can execute an existing two-party PSI protocol, such as those in [6,8,10], to obtain the intersection $I = X \cap Y$.

Deletion. Let $\mathcal{P}_X$ and $\mathcal{P}_Y$ intend to delete the sets X^- and Y^-, respectively.

Addition. Let $\mathcal{P}_X$ and $\mathcal{P}_Y$ intend to add the sets X^+ and Y^+, respectively.

Compute the intermediate intersection. Let the updated sets of $\mathcal{P}_X$ and $\mathcal{P}_Y$ be denoted as X_1 and Y_1, respectively. $\mathcal{P}_X$ and $\mathcal{P}_Y$ execute our asymmetric PSI protocol using X_1 and Y^+, with $\mathcal{P}_X$ obtaining $T = Y^+ \cap X_1$. Similarly, $\mathcal{P}_X$ and $\mathcal{P}_Y$ execute the asymmetric PSI protocol again using X^+ and Y_1, with $\mathcal{P}_Y$ obtaining $V = X^+ \cap Y_1$. Note that the computational and communication overhead of this step is linear in $|X^+|$ and $|Y^+|$, and independent of the size of the entire sets held by both parties.

Compute the intermediate union. $\mathcal{P}_X$ and $\mathcal{P}_Y$ use T and V as inputs, respectively, and invoke an existing PSU protocol, such as those in [26,27,28,29,30], with both parties obtaining $U = T \cup V$. Subsequently, $\mathcal{P}_X$ and $\mathcal{P}_Y$ locally compute $X^- \cap I$ and $Y^- \cap I$, respectively. Both parties invoke the PSU protocol again, and each receives $U' = (X^- \cap I) \cup (Y^- \cap I)$.

Compute the updated intersection. Finally, each party can locally compute $I_1 = (I \backslash U') \cup U$ as the result of $X_1 \cap Y_1$.

It can be observed that our UPSI protocol requires executing a conventional PSI protocol as a base PSI during the first intersection between the two parties. After both parties update their sets, the UPSI protocol can be executed repeatedly by following the same steps (except for initialization). Our protocol is formally described in Figure 4 and is proven secure in the semi-honest model. It achieves worst-case communication and computation complexity that grows linearly with the size of the updates and poly-logarithmically with the size of the entire sets.

2 Related Work

We provide a brief review of PSI, asymmetric PSI, PSU, and UPSI.

PSI. Early PSI protocols were primarily constructed based on the DH key agreement [31,32]. Due to continuous optimizations in scalar multiplication on elliptic curves, primarily through two efficient curves, Curve25519 [33] and Four$\mathbb{Q}$ [34], the efficiency of DH-based PSI protocols has been significantly improved. The advantages of DH-based PSI protocols are their ease of implementation and relatively low communication overhead. Consequently, some modern DH-based PSI protocols [35,36,37] have been proposed in recent years. The state-of-the-art DH-based PSI protocol, proposed by Rosulek et al. [35], is known to be one of the fastest and most communication-efficient protocols for small input sizes. Pinkas et al. [4] constructed an efficient PSI protocol based on Oblivious Transfer (OT) extension [38], which can be considered the origin of highly efficient OT-based PSI protocols. Since then, many efficient PSI protocols [5,6,7,8] have been developed, with KKRT16 [6] and CM20 [8] being among the most efficient. However, compared to DH-based PSI protocols, these protocols sacrifice communication efficiency in exchange for higher computational efficiency. Fortunately, this changed with the advent of OKVS [7,39], which provides a convenient way to represent private sets and facilitates subsequent intersection calculations. Initially, OKVS was introduced to address the challenge of achieving maliciously secure PSI protocols, as cuckoo hashing [40] was unsuitable for this purpose. For further details, please refer to [39]. Consequently, the communication overhead of the first OKVS-based PSI protocols was high. Garimella et al. [41] addressed this issue, and Rindal et al. [9] subsequently combined OKVS with a VOLE protocol [42] to create an efficient PSI protocol with very low communication. Shortly afterward, Raghuraman et al. [10] improved the OKVS in [9] and combined it with a more efficient VOLE protocol [43], resulting in a state-of-the-art PSI protocol with extremely low computational and communication overhead.

Asymmetric PSI. Asymmetric PSI, also known as unbalanced PSI, is a special case of PSI where the set held by one party is significantly smaller than the set held by the other. In general, the intersection in an unbalanced PSI protocol is obtained by the party with the smaller input set. The current asymmetric PSI protocols are primarily constructed based on fully homomorphic encryption [23,24]. Chen et al. [19] introduced optimizations to reduce the multiplicative depth of the function evaluated homomorphically, thereby enhancing efficiency. Chen et al. [20] and Cong et al. [21] employ a combination of OPRF and fully homomorphic encryption, building upon [19]. This not only enhances performance but also extends security to the malicious model and enables the protocol to handle elements of arbitrary bit length. Recently, Mahdavi et al. [22] further optimized unbalanced PSI through a combination of constant-weight encoding and hashing techniques.

PSU. Currently, known PSU protocols are generally constructed using two approaches: additively homomorphic encryption [44] and OT extension [38,43]. We primarily focus on OT-based PSU protocols due to their higher efficiency. Kolesnikov et al. [26] proposed the first efficient PSU protocol, which features good practical performance and is several orders of magnitude faster than previous PSU protocols. Subsequently, Garimella et al. [27] proposed a new PSU protocol based on oblivious switching [45]. Jia et al. [28] also proposed two shuffle-based PSU protocols built on the oblivious switching, which they referred to as the Permute + Share subprotocol. Consequently, the performance of their protocols is similar to that of [27]. Recently, Zhang et al. [29] proposed two general constructions for PSU protocols with linear computational and communication complexity.

UPSI. In contrast to the primitives mentioned above, research on UPSI has only begun in the past two years, initially proposed and defined by Badrinarayanan et al. [14]. The UPSI protocol in [14] essentially only supports the addition of elements, while deletion is achieved through a periodic refreshing method, which they refer to as weak deletion. Recently, Badrinarayanan et al. [15] appear to have addressed this issue by proposing new UPSI protocols that support both the addition and deletion of elements. However, we observe that their protocols only demonstrate an advantage under very restrictive conditions, namely, very low bandwidth, tiny update set sizes, and large input sets. Around the same time, Agarwal et al. [18] also propose a UPSI protocol based on structured encryption [16,17]. However, they do not provide experimental results and code, only theoretical analysis. As a result, the concrete performance of this protocol remains unclear.

3 Preliminaries

3.1 Notation

Let κ and λ denote the computational and statistical security parameters, respectively. Let p be a large prime. Let $\mathbb{G}$ be a group of prime order p with generator g. Let $\mathsf{H} : \{0,1\}^* \to \mathbb{G}$ be a hash function. Let $\mathcal{P}_X$ and $\mathcal{P}_Y$ denote the parties holding the original sets X and Y, respectively. Each party intends to add sets X^+ and Y^+, and delete sets X^- and Y^-. The updated sets are denoted as X_1 and Y_1. Let the input size and update size for both parties be N and N_u, respectively.

3.2 Private Set Intersection

The ideal functionality of the PSI protocol is presented in Figure 1. In this protocol, both parties, $\mathcal{P}_X$ and $\mathcal{P}_Y$, hold private sets, denoted as X and Y, respectively. They aim to compute the intersection $X \cap Y$ without revealing any additional information. Specifically, each party should only learn $|X|$, $|Y|$, and $X \cap Y$, while remaining unaware of $Y \backslash X$ and $X \backslash Y$. The concept of PSI originated with Meadows et al. [31], initially inspired by the DH key agreement. In recent years, efficient PSI protocols [6,7,8,9,10] have primarily relied on the oblivious pseudorandom function protocol.

3.3 Private Set Union

We use the PSU protocol as one of the core components of our UPSI protocol. In contrast to the rapid development of PSI over the past decade, PSU has only

$\mathcal{F}_{\mathsf{PSI}}$

There are two parties, $\mathcal{P}_X$ and $\mathcal{P}_Y$, who hold private sets X and Y, respectively. Both parties receive the intersection $I = X \cap Y$ as the output.

Fig. 1: Ideal functionality $\mathcal{F}_{\mathsf{PSI}}$.

recently started to attract attention. Fortunately, several highly efficient OT-based PSU protocols have already been proposed [26,27,28,29,30]. As shown in Figure 2, in the PSU protocol, $\mathcal{P}_X$ holds X and $\mathcal{P}_Y$ holds Y, and they securely compute $X \cup Y$ for both parties without revealing $X \cap Y$. Similar to the PSI protocol, the PSU protocol also reveals the input sizes of both parties, i.e., $|X|$ and $|Y|$.

$\mathcal{F}_{\mathsf{PSU}}$

There are two parties, $\mathcal{P}_X$ and $\mathcal{P}_Y$, who hold private sets X and Y, respectively. Both parties receive the union $U = X \cup Y$ as the output.

Fig. 2: Ideal functionality $\mathcal{F}_{\mathsf{PSU}}$.

3.4 Updatable Private Set Intersection

UPSI is a variant of PSI that allows both parties to compute the intersection on dynamically updating sets. The concept of UPSI was recently introduced by Badrinarayanan et al. [14], who also provided an improved version [15]. In this work, we define UPSI protocols that support both addition and deletion operations in Figure 3. In our definition, we require an ideal PSI to perform the initial intersection between both parties. In other words, our UPSI protocol essentially operates on the updated input sets and the intersection obtained from the most recent intersection computation.

3.5 Secure Model

The semi-honest model [46] is used in this paper, where a semi-honest adversary may corrupt parties before executing the protocol. In other words, the parties in the protocol will honestly execute the protocol as agreed. However, a party corrupted by the adversary will attempt to extract additional information from its view. Consider a two-party protocol for computing the function $\mathcal{F}_{\Pi}(X, Y)$, where $\mathcal{P}_X$ has private input X and $\mathcal{P}_Y$ has private input Y. For $\mathcal{P}_X$, let $\mathsf{VIEW}^{\Pi}_{\mathcal{P}_X}(1^\kappa, X, Y)$ denote the view of party $\mathcal{P}_X$ during an honest execution

$\mathcal{F}_{\mathsf{UPSI}}$

There are two parties, $\mathcal{P}_X$ and $\mathcal{P}_Y$, who hold private sets X and Y, respectively. Both parties have obtained $I = X \cap Y$ using either an ideal $\mathcal{F}^{\mathsf{PSI}}$ or an ideal $\mathcal{F}^{\mathsf{UPSI}}$. $\mathcal{F}^{\mathsf{PSI}}$ should be used when computing the intersection for the first time. $\mathcal{P}_X$ updates its set to X_1 by adding X^+ and deleting X^-. Similarly, $\mathcal{P}_Y$ updates its set to Y_1 by adding Y^+ and deleting Y^-. Both parties receive the updated intersection $I_1 = X_1 \cap Y_1$ as the output.

Fig. 3: Ideal functionality $\mathcal{F}_{\mathsf{UPSI}}$.

of Π on input X. This view consists of the input, the random tape, and all messages exchanged by $\mathcal{P}_X$ as part of the Π protocol. Similarly, $\mathsf{VIEW}^{\Pi}_{\mathcal{P}_Y}(1^\kappa, X, Y)$ represents the view of $\mathcal{P}_Y$. I_X and I_Y denote the outputs of $\mathcal{P}_X$ and $\mathcal{P}_Y$ in $\mathcal{F}_{\Pi}(X, Y)$, respectively.

Definition 1. *(Semi-Honest Model)* [46]. *Π securely realizes $\mathcal{F}_{\Pi}$ in the presence of semi-honest adversaries if there exists two simulators $\mathsf{SIM}^{\Pi}_{\mathcal{P}_X}$ and $\mathsf{SIM}^{\Pi}_{\mathcal{P}_Y}$ such that*

$$\mathsf{SIM}^{\Pi}_{\mathcal{P}_X}(1^\kappa, X, I_X) \approx \mathsf{VIEW}^{\Pi}_{\mathcal{P}_X}(1^\kappa, X, Y),$$
$$\mathsf{SIM}^{\Pi}_{\mathcal{P}_Y}(1^\kappa, Y, I_Y) \approx \mathsf{VIEW}^{\Pi}_{\mathcal{P}_Y}(1^\kappa, X, Y),$$

where $\approx$ denotes computational indistinguishability with respect to the security parameter κ.

4 Updatable PSI from Asymmetric PSI and PSU

In this section, we formally describe our updatable PSI protocol, which supports arbitrary additions and deletions based on our asymmetric PSI and any PSU protocol. Our UPSI protocol can be completed in a constant number of rounds and is secure in the semi-honest model.

4.1 Component Overview

We begin by introducing the individual components and then combine them to form our UPSI protocol.

Base PSI. We require an efficient conventional two-party PSI protocol to perform the initial intersection between the two parties. For efficiency in the overall protocol, we recommend using the state-of-the-art RR22 [10] to accomplish this task. In practice, other efficient PSI protocols can also be used as the Base PSI here, such as those in [6,7,8,9].

Asymmetric PSI. In our asymmetric PSI protocol, the party with the larger input set obtains the intersection. For example, when $|X| \gg |Y|$, $\mathcal{P}_X$ and $\mathcal{P}_Y$

execute the asymmetric PSI protocol, with $\mathcal{P}_X$ receiving $X \cap Y$ while $\mathcal{P}_Y$ learns nothing except $|X|$.

We achieve this asymmetric PSI protocol based on the DH agreement. Specifically, $\mathcal{P}_X$ and $\mathcal{P}_Y$ can locally compute $H_X = \{\mathsf{H}(x)^{k_x}\}$ and $H_Y = \mathsf{H}(y)^{k_y}$, respectively, for all $x \in X$ and $y \in Y$. Then, $\mathcal{P}_Y$ can send H_Y to $\mathcal{P}_X$, allowing $\mathcal{P}_X$ to compute $E_Y = \{h_y^{k_x}\}$ for all $h_y \in H_Y$ and send it back to $\mathcal{P}_Y$. Subsequently, $\mathcal{P}_Y$ locally computes $H'_Y = \{e_y^{k_y^{-1}}\}$ for all $e_y \in E_Y$ and sends it to $\mathcal{P}_X$. Finally, $\mathcal{P}_X$ obtains the intersection $X \cap Y$ by computing $H_X \cap H'_Y$. It is evident that the communication overhead of the above process is solely linear with respect to $|Y|$. Moreover, since H_X can be precomputed independently by $\mathcal{P}_X$, the computational overhead in the above process is also linearly dependent on the input size of the party with the smaller set.

We would like to emphasize that if an alternative asymmetric PSI protocol is available, where the party with the larger set receives the intersection, and both computational and communication overheads scale linearly with the size of the smaller set, it can directly replace the asymmetric PSI described here. Additionally, we note that the asymmetric PSI protocol proposed by Angelou et al. [47] is similar to the one presented here. However, in [47], the party with the smaller set receives the intersection, which does not meet the requirements for constructing UPSI in our context. Furthermore, due to the integration of the Bloom filter in [47], the protocol suffers from false positives and makes updating the set (either adding or deleting elements) challenging.

PSU. To instantiate our UPSI protocol, we recommend employing the PSU protocol by Zhang et al. [29] due to its linear computational complexity. Furthermore, the use of the OKVS proposed by Bienstock et al. [11] can further reduce communication costs. However, this may come at the expense of increased computational overhead. In fact, using these PSU protocols [26,27,28,30] to instantiate our UPSI protocol is feasible, as they have linear communication costs and do not result in significant performance loss.

4.2 Protocol Construction

We combine all the components mentioned earlier to construct our UPSI protocol, as illustrated in Figure 4. Our UPSI protocol reveals only the size of the addition and deletion sets while enabling intersection computation in a constant number of rounds.

Correctness. In fact, proving the correctness of our protocol essentially involves demonstrating that the intersection $I_1 = (I \setminus U') \cup U$ holds for X_1 and Y_1, where $U = T \cup V$, $T = Y^+ \cap X_1$, $V = X^+ \cap Y_1$, and $U' = (X^- \cap I) \cup (Y^- \cap I)$.

Theorem 1. *Let X and Y be sets with intersection $I = X \cap Y$. We let $X_1 = (X \setminus X^-) \cup X^+, Y_1 = (Y \setminus Y^-) \cup Y^+, T = Y^+ \cap X_1, V = X^+ \cap Y_1, U = T \cup V$, and $U' = (X^- \cap I) \cup (Y^- \cap I)$. Then, the updated intersection $I_1 = X_1 \cap Y_1$ satisfies $I_1 = (I \setminus U') \cup U$.*

Π^{UPSI}

Initialization:
1. $\mathcal{P}_X$ and $\mathcal{P}_Y$ randomly sample k_x and k_y from $\mathbb{Z}_p^*$, respectively.
2. $\mathcal{P}_X$ and $\mathcal{P}_Y$ locally compute $H_X = \{\mathsf{H}(x)^{k_x}\}$ and $H_Y = \{\mathsf{H}(y)^{k_y}\}$ for all $x \in X$ and $y \in Y$, respectively.
3. $\mathcal{P}_X$ and $\mathcal{P}_Y$ invoke an ideal $\mathcal{F}^{\mathsf{PSI}}$, and both parties receive the intersection $I = X \cap Y$.

Deletion:
4. If $X^- \neq \emptyset$, then $\mathcal{P}_X$ computes $D_X = \{\mathsf{H}(x^-)^{k_x}\}$ for all $x^- \in X^-$, and then updates $H_X = H_X \backslash D_X$.
5. If $Y^- \neq \emptyset$, then $\mathcal{P}_Y$ computes $D_Y = \{\mathsf{H}(y^-)^{k_y}\}$ for all $y^- \in Y^-$, and then updates $H_Y = H_Y \backslash D_Y$.

Addition:
6. If $X^+ \neq \emptyset$, then $\mathcal{P}_X$ computes $A_X = \{\mathsf{H}(x^+)^{k_x}\}$ for all $x^+ \in X^+$, and then updates $H_X = H_X \cup A_X$.
7. If $Y^+ \neq \emptyset$, then $\mathcal{P}_Y$ computes $A_Y = \{\mathsf{H}(y^+)^{k_y}\}$ for all $y^+ \in Y^+$, and then updates $H_Y = H_Y \cap A_Y$.

Compute the intermediate intersection:
8. $\mathcal{P}_Y$ randomly samples k'_y from $\mathbb{Z}_p^*$. Subsequently, $\mathcal{P}_Y$ computes $A'_Y = \{\mathsf{H}(y^+)^{k'_y}\}$ for all $y^+ \in Y^+$ and sends A'_Y to $\mathcal{P}_X$. Then, $\mathcal{P}_X$ computes $E_Y = \{{a'_y}^{k_x}\}$ for all $a'_y \in A'_Y$ and returns it to $\mathcal{P}_Y$. After that, $\mathcal{P}_Y$ computes $H_Y^+ = e_y^{{k'_y}^{-1}}$ for all $e_y \in E_Y$. Finally, $\mathcal{P}_Y$ sends H_Y^+ to $\mathcal{P}_X$ in a random order. $\mathcal{P}_X$ then computes $H_Y^+ \cap H_X$ to obtain $T = Y^+ \cap X_1$.
9. Similarly, $\mathcal{P}_Y$ can obtain $V = X^+ \cap Y_1$.

Compute the intermediate union:
10. $\mathcal{P}_X$ and $\mathcal{P}_Y$ invoke an ideal $\mathcal{F}^{\mathsf{PSU}}$, and both parties then receive $U = T \cup V$.
11. $\mathcal{P}_X$ and $\mathcal{P}_Y$ locally compute $X^- \cap I$ and $Y^- \cap I$, respectively.
12. $\mathcal{P}_X$ and $\mathcal{P}_Y$ invoke an ideal $\mathcal{F}^{\mathsf{PSU}}$, and both parties then receive $U' = (X^- \cap I) \cup (Y^- \cap I)$.

Compute the updated intersection:
13. $\mathcal{P}_X$ and $\mathcal{P}_Y$ can locally compute $I_1 = (I \backslash U') \cup U$.

The next UPSI operation:
14. $\mathcal{P}_X$ and $\mathcal{P}_Y$ prepare the new X^+, X^- and Y^+, Y^-, respectively, and redefine the current X_1 as X and the current Y_1 as Y. They then re-execute all the phases above **except for the initialization**.

Fig. 4: Our complete UPSI protocol.

Proof. We will demonstrate that $I_1 = (I \setminus U') \cup U$ by proving both inclusions:

$$I_1 \subseteq (I \setminus U') \cup U,$$
$$(I \setminus U') \cup U \subseteq I_1.$$

• $I_1 \subseteq (I \setminus U') \cup U$: Let $z \in I_1$. Then $z \in X_1$ and $z \in Y_1$.

Case 1: If $z \in I$, then $z \in X$ and $z \in Y$. Since $z \in X_1$, it must either not be deleted from X (i.e., $z \notin X^-$) or be re-added (i.e., $z \in X^+$). Similarly, $z \in Y_1$ implies $z \notin Y^-$ or $z \in Y^+$. Since $z \in I$ and $z \in I_1$, $z \in I \setminus U'$ holds if $z \notin U'$ (i.e., $z \notin X^-$ and $z \notin Y^-$).

Case 2: If $z \notin I$, then for $z \in X_1 \cap Y_1$ while $z \notin I$, it must be the case that z was added to at least one of the sets: If $z \in Y^+$ and $z \in X_1$, then $z \in T$. If $z \in X^+$ and $z \in Y_1$, then $z \in V$. Therefore, $z \in U = T \cup V$.

Combining both cases, we have $z \in (I \setminus U') \cup U$.

• $(I \setminus U') \cup U \subseteq I_1$: Let $z \in (I \setminus U') \cup U$.

Case 1: If $z \in I \setminus U'$, then $z \in I$ and $z \notin U'$ hold. Therefore, $z \notin X^-$ and $z \notin Y^-$ due to $z \notin U'$, implying $z \in X_1$ and $z \in Y_1$. Hence, $z \in I_1$.

Case 2: If $z \in U$, then $z \in T$ and $z \in V$ due to $U = T \cup V$. If $z \in T = Y^+ \cap X_1$, then $z \in Y^+$ implies $z \in Y_1$, and $z \in X_1$. If $z \in V = X^+ \cap Y_1$, then $z \in X^+$ implies $z \in X_1$, and $z \in Y_1$. Combining both subcases, we have $z \in I_1$.

Since both inclusions hold, we conclude that: $I_1 = (I \setminus U') \cup U$.

4.3 Complexity Analysis

We comprehensively analyze the computational and communication complexities of Protocol 4 here. We conduct a detailed analysis of the complexities at each phase of the UPSI protocol to determine its overall complexity. We would like to emphasize that this analysis does not include the initialization phase, as it comprises a base PSI and preparatory steps that can be implemented using an efficient conventional PSI protocol, such as those in [6,7,8,9,10]. The repeated execution of the remaining phases alone constitutes our UPSI protocol. For the sake of discussion, we assume $|X| = |Y| = N$ and $|X^+| = |Y^+| = |X^-| = |Y^-| = N_u$. Let h denote the cost of hashing, and e denote the cost of exponentiation.

Deletion. Both parties perform $\mathcal{O}(N_u \cdot (h + e) + N_u \log N)$ computations. This phase incurs no communication overhead.

Addition. The computational complexity of this phase is the same as that of the deletion phase, with no communication overhead.

Compute the intermediate intersection. In step 8, $\mathcal{P}_Y$ and $\mathcal{P}_X$ performs $\mathcal{O}(N_u \cdot (h + 2e))$ and $\mathcal{O}(N_u \cdot e)$ computations, respectively. The communication overhead is $\mathcal{O}(3 \cdot |\mathbb{G}| \cdot N_u)$. Step 9 is identical to Step 8, except that $\mathcal{P}_X$ and $\mathcal{P}_Y$ interchange roles. Therefore, both $\mathcal{P}_X$ and $\mathcal{P}_Y$ perform $\mathcal{O}(N_u \cdot (h + 3e))$ computations in this phase, requiring a total communication cost of $\mathcal{O}(6 \cdot |\mathbb{G}| \cdot N_u)$.

Compute the intermediate union. This phase mainly involves two invocations of the PSU protocol. We can see that the input size for these two PSU protocol invocations is at most N_u in the worst case. Therefore, according to the existing efficient PSU protocols [26,27,28,29,30], the computational complexity and communication complexity of this phase are $\mathcal{O}(N_u)$ (or possibly $\mathcal{O}(N_u \cdot \log N_u)$) and $\mathcal{O}(N_u)$, respectively.

Compute the updated intersection. The computational cost for both parties in this step is $\mathcal{O}(N_u)$, with no communication cost required.

In summary, the computational complexity and communication complexity of our UPSI protocol can be summarized as $\mathcal{O}(N_u \cdot \log N)$ and $\mathcal{O}(N_u)$, respectively.

4.4 Updatable PSI Security Proof

It is evident that the number of elements each party adds and deletes, i.e., $|X^+|$, $|Y^+|$, $|Y^-|$, and $|X^-|$, is revealed. This might be an unavoidable form of leakage, as it has been a persistent issue in previous UPSI protocols [14,15,18]. Our UPSI protocol relies on the DDH assumption.

Decisional Diffie-Hellman (DDH) Assumption. Let $\mathbb{G}$ be a cyclic group of prime order p with generator g. Let a, b, c be sampled uniformly at random from $\mathbb{Z}_p$. The DDH assumption states that

$$(g^a, g^b, g^{ab}) \approx (g^a, g^b, g^c).$$

Theorem 2. *Let H be a random oracle. The protocol Π^{UPSI} realizes $\mathcal{F}_{\mathsf{UPSI}}$ against a semi-honest adversary under the DDH assumption, assuming the existence of ideal $\mathcal{F}_{\mathsf{PSI}}$ and $\mathcal{F}_{\mathsf{PSU}}$ in the semi-honest model.*

Proof. Since $\mathcal{P}_X$ and $\mathcal{P}_Y$ are symmetric, we only need to prove that no additional information is revealed when either party is corrupted. Without loss of generality, we assume that $\mathcal{P}_X$ is the corrupted party, while $\mathcal{P}_Y$ remains honest. Before Step 8 of the protocol, that is, prior to the **Compute the intermediate intersection** phase, all operations are local computations except for the invocation of the ideal $\mathcal{F}_{\mathsf{PSI}}$. Thus, simulating this part of the process is trivial. $\mathcal{P}_X$ will receive $A'_Y = \{\mathsf{H}(y^+)^{k'_y}\}$ for all $y^+ \in Y^+$ from $\mathcal{P}_Y$. Using a sequence of hybrid arguments, we show that the corrupted $\mathcal{P}_X$ cannot distinguish the elements in A'_Y from random values in $\mathbb{G}$.

$\mathcal{H}_0$: This is the view of $\mathcal{P}_X$ in the real execution of Π^{UPSI} when it receives A'_Y.

$\mathcal{H}_{1,i}$: For $i \in \{1, \cdots, N_u\}$, the same as $\mathcal{H}_0$ except that we replace $\mathsf{H}(y^+)^{k'_y}$ in A'_Y with random $g_i \in \mathbb{G}$.

$\mathcal{H}_2$: The view of $\mathcal{P}_X$ as output by the simulator when it finishes receiving A'_Y.

We argue that $\mathcal{H}_{1,i-1}$ and $\mathcal{H}_{1,i}$ are indistinguishable to $\mathcal{P}_X$. If any PPT adversary $\mathcal{A}$ can distinguish the two hybrids, we devise a challenger $\mathcal{C}$ who can break the DDH assumption. $\mathcal{C}$ is given (g, g^a, g^b, g^c) and needs to decide whether c is random or $c = ab$. $\mathcal{C}$ can program $\mathsf{H}(\cdot)$ to return g^b on input y^+, and we let $g^a = g^{k'_y}$. $\mathcal{C}$ receives the challenge mask ϵ. Note that $\mathcal{C}$ does not know that ϵ belongs to $\mathcal{H}_{1,i-1}$ or $\mathcal{H}_{1,i}$. $\mathcal{C}$ sends ϵ to $\mathcal{A}$, and then $\mathcal{A}$ determines whether ϵ belongs to $\mathcal{H}_{1,i-1}$ or $\mathcal{H}_{1,i}$. If $c = ab$, the mask $\epsilon = g^c$, otherwise $\epsilon = g_i$ (since g_i is random). If $\mathcal{A}$ judges that ϵ belongs to $\mathcal{H}_{1,i-1}$, then $\mathcal{C}$ outputs that $c = ab$; otherwise outputs that c is random. Therefore, we can see that if $\mathcal{A}$ can distinguish the mask part of two hybrids, then $\mathcal{C}$ can break the DDH assumption with the same probability.

$\mathcal{P}_X$ also receives $E_X = \{{a'_x}^{k_y}\}$ for all $a'_x \in A'_X$, where $A'_X = \{\mathsf{H}(x^+)^{k'_x}\}$. Note that this process in Step 9 is omitted in the description of Protocol 4, as it is almost identical to Step 8. Additionally, $\mathcal{P}_X$ will receive $H_Y^+ = \{e_y^{{k'_y}^{-1}}\}$ for all $e_y \in E_Y$ from $\mathcal{P}_Y$. The methods used to prove that $\mathcal{P}_X$ cannot distinguish the elements in E_X and H_Y^+ from random values in $\mathbb{G}$ are similar to those for A'_Y, and thus will not be repeated here.

The remaining parts of the protocol consist entirely of local computations, except for the two invocations of the ideal $\mathcal{F}_{\mathsf{PSU}}$. Therefore, the simulation of the remaining parts of the protocol is trivial.

In summary, our UPSI protocol is secure against semi-honest adversaries.

5 Evaluation

In this section, we provide a comprehensive evaluation of our work, including the performance of our UPSI protocol under different network environments and various parameters, the conditions under which a new conventional PSI protocol should be reused when the updated set size exceeds a certain threshold, and a comparison with state-of-the-art protocols. Our implementation is available on GitHub: https://github.com/ShallMate/upsi.

5.1 Experimental Setup

We used workstations with Intel(R) Xeon(R) Gold 6230R CPU @ 2.10 GHz, with 52 cores, and having 128 GB RAM. The experiment runs on the CentOS system. Our UPSI protocols is implemented using the YACL library[1], which is a C++ library that contains common cryptography, network and I/O modules. We set the computational security parameter $\kappa = 128$ and the statistical security parameter $\lambda = 40$. We evaluate the performance of our protocol under both LAN and WAN settings. We simulate the WAN connection using the Linux "tc" command. We simulate the LAN connection with 0.2 ms RTT network latency and 1 Gbps network bandwidth. For the bandwidth in the WAN setting, we follow the configuration of Badrinarayanan et al. [15] and present our results at 200 Mbps, 50 Mbps, and 5 Mbps with an RTT latency of 80 ms. We use the Four$\mathbb{Q}$ curve [34] to instantiate $\mathbb{G}$ and SHA512 for the hash function H. We use the PSU protocol proposed by Kolesnikov et al. [26] to instantiate our UPSI instead of the linear-computation-cost PSU proposed by Zhang et al. [29], as their provided source code is written in Java[2]. We observe that our protocol remains efficient using [26]. We would like to emphasize that if the PSU protocol by Zhang et al. [29] were used, the performance of our protocol might be even better than what is demonstrated in this paper. For the base PSI used to compute the initial intersection between both parties, we choose the state-of-the-art two-party PSI proposed by Raghuraman et al. [10]. For the sake of presenting the results, we assume $|X| = |Y| = N$ and $|X^+| = |Y^+| = |X^-| = |Y^-| = N_u$.

5.2 The Performance of Our UPSI

We present the specific performance of our UPSI protocol under different values of N and N_u, as well as various network environments. We set $N \in \{2^{19}, 2^{20}, 2^{21}, 2^{22}, 2^{23}, 2^{24}\}$ and $N_u \in \{2^7, 2^8, 2^9, 2^{10}, 2^{11}, 2^{12}, 2^{13}, 2^{14}, 2^{15}, 2^{16}\}$. In previous works [14,15], the performance was shown only up to $N = 2^{22}$ and $N_u = 2^{12}$. We provide as many parameters as possible to give readers a better understanding of the performance of our protocol.

[1] https://github.com/secretflow/yacl
[2] https://github.com/alibaba-edu/mpc4j

Table 2: Communication cost (in MB) and running time (in seconds) of the initialization phase under different values of N.

N	2^{19}	2^{20}	2^{21}	2^{22}	2^{23}	2^{24}
Running Time (s)	3.46	7.67	16.7	36.9	82.7	176.7
Comm. (MB)	26.1	51.9	103.4	206.6	412.9	825.6

Table 3: Communication cost (in MB) and running time (in seconds) of our protocol under different values of N and N_u, and various network environments.

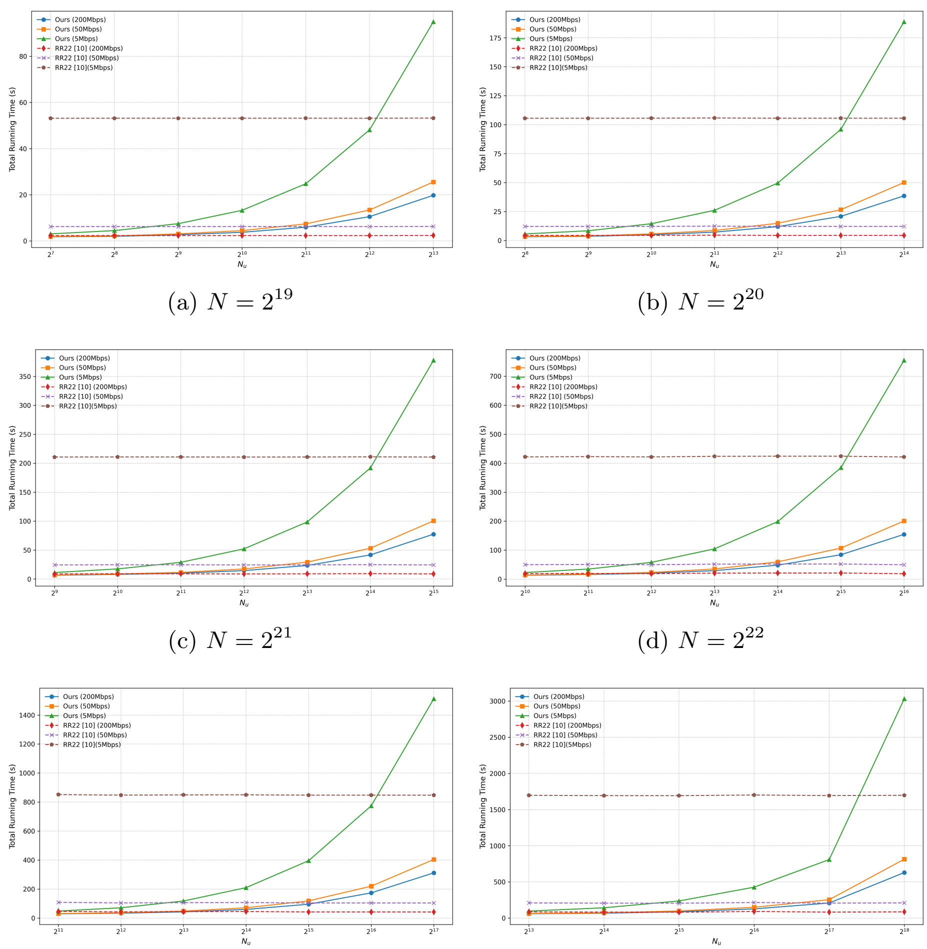
N	N_u	Comm. (MB)	Total Running Time (s)			
			LAN	200Mbps	50Mbps	5Mbps
2^{19}	2^{7}	0.65	1.37	1.81	1.91	3.08
	2^{8}	1.27	1.55	2.01	2.2	4.5
	2^{9}	2.45	2.18	2.7	3.07	7.49
	2^{10}	4.88	3.12	3.76	4.49	13.27
	2^{11}	9.65	5.08	5.96	7.41	24.79
2^{20}	2^{8}	1.27	2.75	3.21	3.4	5.7
	2^{9}	2.45	3.11	3.63	4.0	8.41
	2^{10}	4.88	4.33	4.98	5.71	14.49
	2^{11}	9.65	6.35	7.24	8.68	26.06
	2^{12}	19.3	10.6	11.97	14.87	49.57
2^{21}	2^{9}	2.45	5.94	6.47	6.84	11.25
	2^{10}	4.88	7.38	8.03	8.76	17.54
	2^{11}	9.65	9.19	10.07	11.52	28.9
	2^{12}	19.3	13.2	14.58	17.47	52.18
	2^{13}	38.5	21.1	23.47	29.25	98.61
2^{22}	2^{10}	4.88	12.5	13.18	13.92	22.7
	2^{11}	9.65	15.0	15.89	17.34	34.72
	2^{12}	19.3	18.8	20.24	23.13	57.83
	2^{13}	38.5	26.9	29.3	35.08	104.44
	2^{14}	77.1	43.8	48.09	59.65	198.33
2^{23}	2^{11}	9.65	28.4	29.27	30.72	48.09
	2^{12}	19.3	31.9	33.3	36.19	70.9
	2^{13}	38.5	40.0	42.37	48.16	117.52
	2^{14}	77.1	55.1	59.39	70.95	209.63
	2^{15}	153.9	87.2	95.33	118.42	395.6
2^{24}	2^{12}	19.3	58.6	60.0	62.89	97.59
	2^{13}	38.5	65.6	67.97	73.75	143.11
	2^{14}	77.1	83.2	87.42	98.98	237.66
	2^{15}	153.9	119.5	127.62	150.72	427.89
	2^{16}	307.8	193.1	208.93	255.11	809.28

First, we present the costs of our UPSI protocol during the initialization phase. Essentially, this is not part of the pre-computation for our UPSI protocol and only needs to be executed once. The cost of this phase mainly depends on N and is linear with respect to it. This phase primarily requires a single execution of the base PSI protocol, as well as pre-computation by both parties for the masks used in the subsequent asymmetric PSI. We present the communication cost (in MB) and running time (in seconds) of the initialization phase of our UPSI under different values of N in Table 2. We can see that when $N = 2^{24}$, both parties require 176.7 seconds and 825.6 MB of communication to complete the initialization phase. Fortunately, this phase is essentially a setup phase and only needs to be executed once. Therefore, we do not include the costs of the initialization phase in the subsequent UPSI evaluations. Note that the communication cost for the base PSI here is slightly higher than executing a one-way PSI protocol by Raghuraman et al. [10] alone, as the party receiving the intersection needs to send it to the other party.

Next, in Table 3, we present the communication cost (in MB) and running time (in seconds) of our protocol under different values of N and N_u and various network environments. We can see that the communication cost of the proposed UPSI protocol is independent of N and primarily grows linearly with N_u. Since our UPSI protocol enjoys extremely low communication costs, it can quickly obtain the updated intersection even under very low bandwidth conditions. For example, with a bandwidth of 5 Mbps, both parties can complete the UPSI protocol in 5.7 seconds when $N = 2^{20}$ and $N_u = 2^8$. When dealing with large-scale data, our protocol can quickly complete the computation of the updated intersection. For example, when $N = 2^{24}$ and $N_u = 2^{12}$, our UPSI protocol completes the intersection computation in 58.6, 60.0, 62.89, and 97.59 seconds under LAN, 200 Mbps, 50 Mbps, and 5 Mbps WAN settings, respectively. It can be observed that our protocol is efficient even with large-scale data and under low bandwidth conditions.

5.3 The Threshold of N_u

In most cases, N_u should be much smaller than N. However, determining the threshold at which N_u grows large enough that running UPSI becomes less efficient than executing a conventional PSI with updated inputs is crucial, and previous works [14,15,18] seem to have overlooked this aspect. We experiment with different values of N and N_u to evaluate the threshold of N_u at which our UPSI protocol becomes less efficient than directly re-executing the state-of-the-art conventional two-party PSI protocol [10]. We consider this a crucial experiment, as there may be a threshold value of N_u beyond which executing the UPSI protocol could be less efficient than re-running a new conventional two-party PSI. As shown in Figure 5, we present a comparison of our UPSI protocol with [10] under different bandwidths and updated set sizes when $N \in \{2^{19}, 2^{20}, 2^{21}, 2^{22}, 2^{23}, 2^{24}\}$. We summarize the threshold N_u^t results for different values of N and various bandwidths in Table 4 for the convenience of readers. For example, when $N = 2^{24}$, as long as $|N^+| \leq 2^{17}$ and $|N^-| \leq 2^{17}$, executing our UPSI protocol with a bandwidth of 5 Mbps will definitely be faster than executing [10]. Although 2^{17} still has a significant gap compared to 2^{24}, this is an impressive result compared to previous works. For example, the current state-of-the-art UPSI [15] that supports addition and deletion has an actual N_u^t of only 2^4 under the same parameters. In other words, we have expanded this threshold by at least 2^{13} (8192) times in this setting. Therefore, it is clear that our protocol indeed addresses the problem we posed in the introduction. We have successfully constructed a UPSI that supports arbitrary additions and deletions of elements while ensuring faster performance than re-executing a conventional PSI in most cases, rather than being limited to highly specific conditions.



(a) $N = 2^{19}$

(b) $N = 2^{20}$

(c) $N = 2^{21}$

(d) $N = 2^{22}$

(e) $N = 2^{23}$

(f) $N = 2^{24}$

Fig. 5: Comparison with RR22 [10] under different bandwidths and updated set sizes when $N \in \{2^{19}, 2^{20}, 2^{21}, 2^{22}, 2^{23}, 2^{24}\}$.

Table 4: Let N_u^t be a threshold for N_t, beyond which it becomes more efficient to re-run RR22 [10] rather than executing our UPSI protocol.

N	2^{19}	2^{20}	2^{21}	2^{22}	2^{23}	2^{24}
N_u^t (200 Mbps)	2^{8}	2^{9}	2^{10}	2^{11}	2^{13}	2^{14}
N_t^t (50 Mbps)	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{16}
N_t^t (5 Mbps)	2^{12}	2^{13}	2^{14}	2^{15}	2^{16}	2^{17}

Table 5: Communication cost (in MB) and running time (in seconds) of our protocol in comparison with prior work. If the existing work is optimal, we highlight it in green; if our work is optimal, we highlight it in red. We did not record experimental results with a running time exceeding two hours.

N	N_u	Protocol	Comm. (MB)	Total Running Time (s)			
				LAN	200Mbps	50Mbps	5Mbps
2^{20}	-	RR22 [10]	51.88	1.41	4.4	12.19	105.57
	2^{4}	BMS$^+$24 [15] (addition-only)	0.50	1.52	1.95	2.02	2.52
	2^{6}		1.95	5.49	5.99	6.28	9.79
	2^{8}		7.57	22.8	23.58	24.71	38.34
	2^{10}		29.61	87.6	89.48	93.92	147.22
	2^{4}	BMS$^+$24 [15] (addition & deletion)	58.7	101.2	104.54	113.34	219.0
	2^{6}		231	363.3	375.25	409.9	825.7
	2^{8}		922	1398.4	1445.25	1584.6	3256.8
	2^{10}		3687	5543.5	5727.75	6280.8	-
	2^{4}	Ours	0.10	2.18	2.58	2.6	2.78
	2^{6}		0.34	2.34	2.76	2.81	3.42
	2^{8}		1.28	2.70	3.16	3.36	5.66
	2^{10}		4.88	4.18	4.82	5.56	14.34
2^{22}	-	RR22 [10]	206.65	8.17	18.87	49.77	420.57
	2^{4}	BMS$^+$24 [15] (addition-only)	0.53	1.62	2.05	2.13	3.08
	2^{6}		2.06	5.45	5.95	6.26	9.97
	2^{8}		8.03	23.6	24.4	25.61	40.06
	2^{10}		31.5	92.4	94.38	99.1	155.8
	2^{4}	BMS$^+$24 [15] (addition & deletion)	61.6	108.3	111.78	121.02	231.9
	2^{6}		243	402	414.55	451.0	888.4
	2^{8}		927	1603.7	1650.45	1789.5	3458.1
	2^{10}		-	-	-	-	-
	2^{4}	Ours	0.10	10.13	10.54	10.55	10.73
	2^{6}		0.34	10.48	10.9	10.95	11.56
	2^{8}		1.28	10.87	11.33	11.53	13.83
	2^{10}		4.88	12.37	13.01	13.75	22.53

5.4 Comparison with State-of-The-Art Protocols

We compare our UPSI protocol with the existing optimal conventional PSI protocol [10] and UPSI protocol [15]. In [15], there is also a UPSI protocol that only supports addition operations, which we have included in the comparison to demonstrate the efficiency of our protocol. We present the communication cost (in MB) and running time (in seconds) of our protocol in comparison with [10,15] in Table 5.

We summarize our experimental results in terms of communication improvement, computation improvement, and overall running time.

Communication Improvement. Our protocol outperforms RR22 [10] by 10 - 2066 $\times$ in all settings. When $N = 2^{22}$ and $N_u = 2^4$, the communication cost of RR22 is 206.65 MB, whereas our protocol requires only 0.1 MB. Additionally, our protocol achieves a 5 - 6$\times$ reduction compared to the version that supports only addition in [15]. When $N = 2^{22}$ and $N_u = 2^{10}$, it requires 31.5 MB of communication, whereas our UPSI needs only 4.88 MB. Our protocol reduces communication overhead by 587 - 755$\times$ compared to the state-of-the-art UPSI protocol [15] that supports both addition and deletion. When $N = 2^{20}$ and $N_u = 2^{10}$, it requires 3687 MB of communication, whereas our UPSI still needs only 4.88 MB. It can be observe that our protocol enjoys extremely low communication cost compared to existing works. Therefore, our UPSI protocol has a significant advantage in terms of communication overhead.

Computation Improvement. Our protocol achieves up to a 21$\times$ reduction in computation overhead compared to the version that supports only addition in [15]. Specifically, when $N = 2^{20}$ and $N_u = 2^{10}$, it takes 87.6 seconds to complete the protocol, whereas our UPSI protocol requires only 4.18 seconds in the LAN setting. Furthermore, our protocol reduces computation overhead by 46 - 1326$\times$ compared to the state-of-the-art UPSI protocol [15] that supports both addition and deletion. For example, when $N = 2^{20}$ and $N_u = 2^{10}$, it takes 5543.5 seconds to complete the protocol, whereas our UPSI protocol still requires only 4.18 seconds. It is evident that our UPSI protocol also has a significant advantage in terms of computation overhead compared to [15].

Overall Running Time. The end-to-end running time of our protocol begins to outperform the version that supports only addition in [15] when $N_u \geq 2^6$ for $N = 2^{20}$ and $N_u \geq 2^8$ for $N = 2^{22}$. In these settings, our protocol can be over 20$\times$ faster than the latter. For example, when $N = 2^{20}$ and $N_u = 2^{10}$, the latter requires 93.92 seconds to complete the protocol, whereas our protocol only needs 5.56 seconds with a bandwidth of 50 Mbps. In comparison with the version that supports both addition and deletion in [15], our UPSI demonstrates an advantage in overall running time across all settings. Our protocol can be up to 1188$\times$ faster than the state-of-the-art UPSI protocol [15] that supports both addition and deletion. Specifically, when $N = 2^{20}$ and $N_u = 2^{10}$, it takes 5727.75 seconds to complete the intersection computation, whereas our UPSI requires only 4.82 seconds with a bandwidth of 200 Mbps. Furthermore, our UPSI also has an advantage in overall running time compared to state-of-the-art conventional PSI [10] across all settings. Our protocol can be up to 18$\times$ faster than [10]. Specifically, when $N = 2^{22}$ and $N_u = 2^{10}$, [10] takes 420.57 seconds to complete the intersection computation, whereas our UPSI requires only 22.53 seconds with a bandwidth of 5 Mbps. Therefore, we can see that our protocol also has an advantage in end-to-end running time due to its extremely low communication.

In summary, our protocol has significant advantages over the current optimal protocols in terms of communication improvement, computation improvement, and overall running time.

6 Limitations and Discussion

This section primarily discusses some limitations of our UPSI protocol and outlines our future work.

First, our UPSI protocol is secure in the semi-honest model but does not provide security against malicious adversaries. To the best of our knowledge, no maliciously secure UPSI protocol has been proposed so far. Therefore, we leave the construction of a maliciously secure UPSI protocol as an open problem and a direction for our future work.

Secondly, our UPSI protocol is a two-way UPSI, meaning that both parties obtain the intersection. Therefore, our work may not be suitable for PSI scenarios where only one party needs to obtain the intersection. However, this does not imply that our work is without value. For example, in vertical federated learning [13], the first step is usually to use a PSI protocol for sample alignment, where all parties typically need to obtain the intersection. In the next section, we also provide some application scenarios for the UPSI protocol proposed in this paper to illustrate its strong practical significance. However, we still want to improve our UPSI protocol to allow only one party to receive the intersection, which is currently included in our future work.

We plan to address the above two limitations in future work and welcome further research aimed at improving our UPSI protocol.

7 Applications

In this section, we present three application scenarios for our UPSI protocol to illustrate its practical role.

Vertical Federated Learning. Vertical federated learning [13] is a highly favored approach for joint model training in the industry. It refers to multiple companies possessing different feature spaces for the same set of samples, aiming to improve model accuracy through feature expansion. The prerequisite for vertical federated learning is achieving privacy-preserving entity alignment, which involves identifying the common sample IDs across all companies. In fact, this task is typically accomplished using a PSI protocol. However, in practice, data is not stable and actually requires continuous updates. Here we cite a passage from Meta's paper [48]: "*But a typical scenario is for one party's dataset of records to be large and stable for some time, while the other party's dataset arrives in a* *streaming fashion and in small batches. For example, parameters of a machine learning model can be continuously updated as new batches of records arrive.*" This means that if we use a conventional PSI protocol, we would need to execute it multiple times to continually perform privacy-preserving entity alignment. Although they also use a PSI variant called streaming PSI to attempt to address this issue, this construction is not efficient due to the expensive homomorphic encryption involved (taking up to about two hours to complete the intersection computation for input sizes less than 2^{24} on a c5.18xlarge AWS instance). As shown in Table 3, with our protocol, the entire computation can be completed in less than 20 seconds for an update size of 2^{12} (4096), even taking only around 97 seconds under a 5 Mbps bandwidth. Furthermore, in the vertical federated learning scenario, all participating parties need to obtain the intersection (otherwise, they cannot know which samples are involved in model training). Thus, our UPSI protocol is well-suited for vertical federated learning.

Medical Data Sharing. PSI is also frequently used for privacy-preserving medical data sharing [49,50]. At the same time, we know that medical data, such as epidemic monitoring and case tracking, requires frequent data updates. For example, during the COVID-19 pandemic, the number of new hospital cases and test results can change rapidly. The updatable feature of the UPSI protocol enables hospitals to quickly add the latest data to the intersection, assisting relevant departments and hospitals in obtaining real-time analyses. Furthermore, the capability for dynamic updates also facilitates long-term collaborative case analysis. Suppose a hospital identifies a new case or updates test data. In that case, it can swiftly integrate the new information into the intersection via the UPSI protocol, ensuring that all collaborating parties receive the most timely information. If multiple hospitals identify cases of the same patient with a specific disease, the parties can conduct in-depth analyses based on the intersection data to explore information such as causes and treatment options. This cross-hospital collaboration facilitates knowledge sharing and enhances overall diagnostic and treatment outcomes. Due to the considerable performance advantages of our UPSI compared to previous works, it can assist hospitals in quickly performing these frequently updated intersection computations.

Social Network Analysis. PSI has been widely used in social networks [51,52,53]. Social platforms can identify overlapping users and update their social graphs without disclosing user privacy, thereby providing users with a richer social experience and cross-platform services. When building user social graphs across multiple social platforms, each platform has a large user base, making it relatively expensive to perform a PSI protocol. However, the friend lists and interests of these users may change frequently. Therefore, using a conventional PSI protocol would result in significant resource waste. Each platform can use UPSI to address this issue. Moreover, since each platform needs to update its own social graph, they all require to obtain the intersection. As a result, our UPSI protocol is a viable solution worth considering for this scenario, as it demonstrates highly efficient performance compared to previous works in a setting that requires updating sets.

In fact, the application scenarios for UPSI are not limited to the three we have listed. Since data inherently requires constant updates, the usefulness of our UPSI protocol becomes increasingly evident.

8 Conclusion

In this work, we construct a UPSI protocol that supports arbitrary additions and deletions of elements, offering faster performance compared to re-executing a conventional PSI in most scenarios without being restricted to specific conditions. Experimental results demonstrate that our UPSI protocol is substantially more efficient than the existing state-of-the-art UPSI protocol, achieving two to three orders of magnitude improvements in both computational and communication costs. Finally, we also provide some applications of the proposed UPSI protocol to demonstrate how our protocol can play a significant role in practical scenarios.

9 Acknowledge

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