

# NOMADIC: Normalising MALiciously-Secure Distance with Cosine Similarity for Two-Party Biometric Authentication

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## ABSTRACT

Computing the distance between two non-normalized vectors  $\mathbf{x}$  and  $\mathbf{y}$ , represented by  $\Delta(\mathbf{x}, \mathbf{y})$  and comparing it to a predefined public threshold  $\tau$  is an essential functionality used in privacy-sensitive applications such as biometric authentication, identification, machine learning algorithms (e.g., linear regression, k-nearest neighbors, etc.), and typo-tolerant password-based authentication. Tackling a widely used distance metric, NOMADIC studies the privacy-preserving evaluation of cosine similarity in a two-party (2PC) distributed setting. We illustrate this setting in a scenario where a client uses biometrics to authenticate to a service provider, outsourcing the distance calculation to two computing servers. In this setting, we propose two novel 2PC protocols to evaluate the normalising cosine similarity between non-normalised two vectors followed by comparison to a public threshold, one in the semi-honest and one in the malicious setting. Our protocols combine additive secret sharing with function secret sharing, saving one communication round by employing a new building block to compute the composition of a function  $f$  yielding a binary result with a subsequent binary gate. Overall, our protocols outperform all prior works, requiring only two communication rounds under a strong threat model that also deals with malicious inputs via normalisation. We evaluate our protocols in the setting of biometric authentication using voice, and the obtained results reveal a notable efficiency improvement compared to existing state-of-the-art works.

## CCS CONCEPTS

• Security and privacy → Information-theoretic techniques.

## KEYWORDS

privacy-preserving protocols, malicious security, function secret sharing, cosine similarity

## ACM Reference Format:

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## 1 INTRODUCTION

Computing distance metrics of sensitive data and comparing to a predefined public threshold in a privacy-preserving way is an indispensable building block for a wide range of privacy-sensitive applications including biometric authentication and identification, machine learning (e.g., linear regression, matrix multiplication) as well as typo-tolerant (fuzzy) password based authentication. However, in the absence of trusted parties it is challenging to evaluate thresholded distance metrics over sensitive data. Although privacy-preserving computations of linear distance metrics have been widely covered in the literature, protecting non-linear computations (e.g., comparison) are not easy to compute efficiently.

In this paper, we focus on protecting the computation of *cosine similarity* between two vectors, one of the most commonly employed distance metrics, subsequently comparing the similarity score to a public threshold  $\tau$ . We present two novel privacy-preserving protocols to realize this thresholded distance functionality with high efficiency in a two-party setting (2PC) for outsourced computation, achieving passive and active security respectively.

As an ideal application of our protocols due to the sensitive nature of the data, biometric authentication consists of verifying the identity of a user by comparing a fresh compact representation of the biometric trait with a pre-existing (claimed) reference provided during an enrolment phase. The authentication is granted or denied based on whether or not the obtained similarity score is below a pre-fixed threshold. Although biometrics hold significant promise as a convenient authentication method, privacy concerns have hindered public trust and created obstacles to widespread adoption. For instance, in August 2019, over 27.8M Records exposed in BioStar 2 Data Breach, including the fingerprints of over 1 million people, as well as facial recognition information, unencrypted usernames and passwords, and personal information of employees<sup>1</sup>.

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<sup>1</sup><https://www.theguardian.com/technology/2019/aug/14/major-breach-found-in-biometrics-system-used-by-banks-uk-police-and-defence-firms/>

To showcase NOMADIC, we implement and evaluate our protocols in a biometric authentication use-case with voice biometrics. Furthermore, we provide a detailed experimental evaluation of our proposed protocol by employing benchmark speech recognition datasets (*i.e.*, VoxCeleb2). Our results show that the proposed scheme is not only efficient, requiring only two communication rounds, but also maintains the same accuracy as the plaintext systems while guaranteeing security under a strong threat model.

**Related Work.** Several techniques have been proposed in the literature to address the privacy challenges for the computation of distance metrics (such as Hamming distance [43], cosine similarity) followed by a comparison with a public threshold, based on advanced cryptographic primitives like Fully Homomorphic Encryption (FHE) [27, 28], Multi-Party Computation (MPC) [29, 56, 64] and Functional Encryption (FE) [1, 9]. FHE relies on performing an arbitrary number of arithmetic operations (*i.e.*, additions and multiplications) between ciphertexts. MPC-based techniques involve distributing data among non-colluding parties to collectively compute a desired function over the private data, covering two main security models: semi-honest or malicious depending on whether the involved parties are assumed to follow the protocol or may deviate from it. FE are public-key encryption schemes that enables authorized parties to evaluate specific linear functions (*e.g.*, inner products [3, 23, 59]) during ciphertext decryption. Efficient FE-based techniques are restricted to linear function evaluations.

**Privacy-preserving comparison:** Although linear operations such as scalar products are efficiently addressed by these techniques [31, 52, 57, 61], non-linear operations like comparisons to a public threshold are still a challenge and are often inefficient for real-time applications. Privacy-preserving comparison is possible in the FHE setting using computationally intensive polynomial approximations [18, 34]. In contrast, many MPC-based solutions and frameworks realize privacy-preserving comparison in various settings [36, 40, 53, 63]. Mixed protocols (*i.e.*, protocols that combine the use of arithmetic circuits with homomorphic encryption, garbled circuits and/or Boolean circuits) compute scalar products using arithmetic protocols and switch from arithmetic to binary/garbled circuits in order to compute comparisons [14, 16, 24, 38, 45, 49, 50]. However, the vast majority of these frameworks either requires an honest majority (3PC, 4PC) to achieve active security (against one malicious corruption) or settles with security against a semi-honest or a one-sided malicious corruption in the 2PC setting. Veugen *et al.* [62] proposed a 2PC framework that improves comparison computation in the SPDZ protocol [22] in the presence of malicious parties. However, these two solutions incur in intensive communication costs (size and/or number of rounds). Recently, Function Secret Sharing (FSS)-based protocols have been proposed to efficiently compute the comparison to a public threshold operation [10, 12, 30, 33, 55]. Nevertheless, the mentioned protocols focus mainly on the semi-honest model and barely address security against malicious adversaries.

**Privacy-preserving biometrics:** Boddeti [8] presented a privacy-preserving solution for facial recognition based on FHE. The face similarity score is computed in the encrypted domain using scalar

multiplications and additions. [32] extended FHE-based face identification to also compute the secure comparison in the FHE domain. In [46], the authors made use of an FHE scheme to compute the Hamming distance for iris authentication. The work proposed in [51] makes use of FHE to privately compute the cosine similarity for an Automatic Speaker Verification (ASV) system. However, the authors focused solely on studying the similarity computation, without carrying out the crucial step of comparing the result to the threshold. They assumed that the decision score would be received and decrypted by the client's device, which could potentially create a security vulnerability in the event of a malicious client attempting to modify the score to gain unauthorized access. Kim *et al.* [35] presented a fingerprint authentication system using FHE to compute the Square Euclidean Distance between two encrypted vectors, also performing the threshold comparison in the encrypted domain. Despite the potential shown by FHE in these works, its utilization in real-world applications is still restricted by the significant computational overheads.

Alternatively, MPC-based techniques have been successfully used to protect sensitive data like face [4, 33], iris [4, 26], and voice [48, 60]. Barni *et al.* [4] introduced a secure multi-modal biometric authentication, that combines face and iris features, based on secure two-party computation against one malicious party. The two non-colluding parties compute the Hamming distance and the Euclidean distance and later evaluate the comparison of the fused scores with a public threshold. The work in [26] proposes a secure two-party computation solution for an iris verification that is secure in the semi-honest adversary model. The Hamming distance along with the threshold comparison is computed securely.

Nautsch *et al.* [48] provided the first computationally feasible privacy-preserving ASV system with cohort score normalization based on 2PC. However, this work only provides security against a semi-honest adversary. In [60], Treiber *et al.* proposed a 2PC-based ASV system secure against a malicious client device that can change the score to get authenticated by the service provider. However, the 2PC servers are assumed to be semi-honest.

**Our Contributions.** We propose a novel two-party protocol for secure computation of cosine similarity followed by comparison to a threshold, leveraging secure 2PC and FSS primitives for both the semi-honest and the malicious setting. As shown in Table 1, our protocol outperforms prior work by requiring just two rounds of communication in the online phase. In addition, and contrary to prior work that assumes honest normalisation performed by the client, we allow the client to submit any fresh template and we incorporate a mechanism to check that the secret sharing of the fresh (as well as the reference) template was performed correctly. Our main contributions are:

- A 2PC protocol for thresholded cosine similarity computation between two vectors relying on authenticated 2PC secret sharing as well as FSS. We develop two variants: one version for the *semi-honest* setting and one for the *malicious* setting. As a distinguishing feature, the protocol (and not the input holder) carries out the normalisation of the input vectors, guaranteeing their well-formedness.
- A new building block, CondEval, to compute the composition of an input bit and a binary function  $f$  (evaluated via FSS) *i.e.*,

**Table 1: Number of communication rounds required by recent MPC protocols to compute, for  $l$ -bit inputs, a Sign, a pre-normalised thresholded inner product  $\mathcal{F}_{\text{Sign(IP}(x,y)-\tau)}$  and a full thresholded cosine similarity  $\mathcal{F}_{\text{ScaledCosAuth}}$ . Security covers one semi-honest  $\circ$  / malicious client-only  $\bullet$  / malicious  $\bullet$  (+ for fairness,  $\star$  for robustness) corruption. Primitives include Homomorphic Encryption (HE), Secret Sharing (SS), unbalanced SS (u-SS), Replicated SS (RSS), optimized SS (o-SS), (Correlated) OT extension (COTe), Garbled Circuits (GC), Binary Adder circuits (BA), Distributed Comparison Function (DCF), Interval Containment (IC), Zero-Knowledge Proofs (ZKP).**

Work	Type	Security	Protocol for IP	Protocol for Sign	$\mathcal{F}_{\text{Sign}}$	$\mathcal{F}_{\text{Sign(IP}(x,y)-\tau)}$	$\mathcal{F}_{\text{ScaledCosAuth}}$
Blaze [50]	3PC	$\bullet^+$	$\Pi_{\text{dotp}}$ , u-3SS	$\Pi_{\text{bitext}}$ , GC	$\log_2(l) + 1$	$\log_2(l) + 2$	$\log_2(l) + 4$
Falcon [63]	3PC	$\bullet$	SS	$\Pi_{\text{WA}}$ , Arith. circuit	$\log_2(l) + 3$	$\log_2(l) + 4$	$\log_2(l) + 6$
SWIFT [38]	3PC	$\bullet^\star$	$\Pi_{\text{dotp}}$ , 2SS & u-2SS	$\Pi_{\text{bitext}}$ , GC	$\log_2(l) + 1$	$\log_2(l) + 2$	$\log_2(l) + 4$
Trident [16]	4PC	$\bullet$	$\Pi_{\text{Dotp}}$ , u-4SS	[50]	$\log_2(l) + 1$	$\log_2(l) + 2$	$\log_2(l) + 4$
Flash [14]	4PC	$\bullet^\star$	$\Pi_{\text{dp}}$ , $2 \times$ u-2SS	$\Pi_{\text{msb}}$ , BA of [45]	$\log_2(l)$	$\log_2(l) + 1$	$\log_2(l) + 3$
SWIFT [38]	4PC	$\bullet^\star$	$\Pi_{\text{dotp4}}$ , 3PC & masks	$\Pi_{\text{bitext4}}$ , GC	$\log_2(l)$	$\log_2(l) + 1$	$\log_2(l) + 3$
Fantastic Four [21]	4PC	$\bullet^\star$	Mult, RSS	Share splitting, BA	$\log_2(l)$	$\log_2(l) + 1$	$\log_2(l) + 3$
Tetrad [39]	4PC	$\bullet^\star$	$\Pi_{\text{dotp}}$ , RSS & masks	$\Pi_{\text{bitext}}$ , BA of [49]	$\log_4(l) + 1$	$\log_4(l) + 2$	$\log_4(l) + 4$
ABY [24]	2PC	$\circ$	SS	GC	$\log_2(l) + 2$	$\log_2(l) + 3$	$\log_2(l) + 5$
ABY2.0 [49]	2PC	$\circ$	o-SS	BitExtraction, mi-BA	$\log_4(l) + 1$	$\log_4(l) + 2$	$\log_4(l) + 4$
Cryptflow [40]	2PC	$\circ$	SS	dReLU, Arith. circuit	$\log_2(l) + 4$	$\log_2(l) + 5$	$\log_2(l) + 7$
Cryptflow2 [53]	2PC	$\circ$	$\Pi_{\text{Umult}}$ , COTe	$\Pi_{\text{Mill}}$ , OT ext.	$\log_2(l)$	$\log_2(l) + 1$	$\log_2(l) + 3$
Muse [41]	2PC	$\bullet$	Linear layer, o-SS	Non-linear layer, GC	$\log_2(l)$	$\log_2(l) + 1$	$\log_2(l) + 3$
SIMC [15]	2PC	$\bullet$	$\Pi_{\text{Lin}}$ , HE & ZKP	$\mathcal{F}_{\text{Non-lin}}$ , GC	$\log_2(l)$	$\log_2(l) + 2$	$\log_2(l) + 6$
AriaNN [54]	2PC	$\circ$	SS	DCF gate of [12], FSS	1	2	4
Boyle et. al. [10]	2PC	$\circ$	SS	IC gate, FSS	1	2	4
Llama [30]	2PC	$\circ$	$\mathcal{G}_{\text{smult}}$ , FSS	FSS IC gate of [10]	1	2	4
Funshade [33]	2PC	$\circ$	o-SS	IC gate of [10], FSS	1	1	3
<b>Ours</b>	2PC	$\bullet$	<b>o-SS &amp; CondEval</b>	CondEval	<b>1</b>	<b>2</b>	<b>2</b>

$s \circ f(x)$  for an input value  $x$ . This building block, proven secure under both the semi-honest and malicious model, may be of independent interest for general 2PC secure computation.

- A rigorous security analysis of both protocols of NOMADIC as well as the CondEval primitive.
- An evaluation of the proposed protocols employing voice biometrics as a use case. Comparing to the existing state-of-the-art (SotA) work, our experiments demonstrate the efficiency improvement of our protocols in both the semi-honest and malicious setting thanks to the reduction of one communication round due to CondEval.

The remaining of this paper is organised as follows. Section 2 describes the problem statement, introducing the application scenario alongside the threat model, and outlining a technical overview of our solution. In Section 3, we revisit the cryptographic 2PC primitives we rely on, namely optimized additive secret sharing and function secret sharing. Section 4 outlines our technical overview, then in section 5 we introduce CondEval, a building block that allows the composition of an input bit and a binary function  $f$  evaluated via FSS, and prove its security in both the semi-honest and malicious settings. Section 6 covers our two full protocols for the privacy-preserving computation of thresholded cosine similarity in the semi-honest and the malicious settings, followed by their corresponding security analysis. Section 7 contains our experimental

evaluation, presenting and analyzing our results. We conclude the paper in Section 8.

## 2 PROBLEM STATEMENT

### 2.1 Notation

Throughout this work, we denote  $\varepsilon$  as an empty string,  $[n]$  as the set of integers  $\{1, 2, \dots, n\}$  for a positive integer  $n$ ,  $s[i]$  as the  $i^{\text{th}}$  bit of a binary string  $s$  where  $i \leq |s|$ , and  $a_i$  as the  $i^{\text{th}}$  element of a vector  $\mathbf{a}$  where  $i < |\mathbf{a}|$ . Our notation choices include:

$\text{IP}(\mathbf{x}, \mathbf{y})$  Inner product of two vectors  $\mathbf{x}, \mathbf{y}$ .

$\text{Prod}(\mathbf{x}, \mathbf{y})$  Element-wise product of two vectors  $\mathbf{x}, \mathbf{y}$ .

$\text{Shift}(x, \rho, \ell)$  Encoding in  $\mathbb{Z}_2^\ell$  of  $|\rho|$  left/right logical shifting bits over  $x$  depending on if  $\rho < 0$  or  $\rho > 0$ .

$\text{Sign}(x)$  Outputs 1 if  $x \geq 0$  and 0 otherwise.

$M_b \leftarrow \mathcal{F}_{\text{OT}}(b, (M_0, M_1))$  Given a pair of messages  $(M_0, M_1)$  from a sender and a choice bit  $b \in \{0, 1\}$  from a receiver, it returns  $M_b$  to the receiver.

$\mathbf{r} \leftarrow \mathcal{F}_{\text{rand}}(1^\lambda, n, \mathbb{U}_N)$  Given a security parameter  $\lambda$ , a positive integer  $n$  and a pre-defined input domain  $\mathbb{U}_N$ , it outputs a random vector  $\mathbf{r} \in \mathbb{U}_N^n$ .

$\mathcal{F}_{\text{Permu}}(\pi, \{a_0, a_1\})$  Given a list  $\{a_0, a_1\}$  and a permutation bit  $\pi \in \{0, 1\}$ , it outputs  $\{a_0, a_1\}$  if  $\pi = 0$  and  $\{a_1, a_0\}$  otherwise.

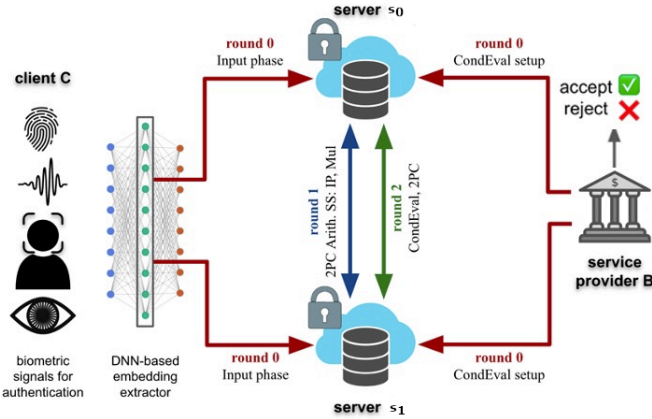
$[x]^B$  Boolean secret sharing of  $x \in \mathbb{Z}_2$ .

- $[x]^A$  Arithmetic secret sharing of  $x \in U_N$  where  $U_N$  indicates a pre-defined input domain.
- $\langle x \rangle^A$  Optimized arithmetic secret sharing of  $x \in U_N$  where  $U_N$  indicates a pre-defined input domain.
- $[x]_b^A, [x]_b^B, \langle x \rangle_b^A$  Share of  $x$  held by party  $b$ .

## 2.2 Application Scenario

Let us examine a scenario in which a client C requests access to a service, such as an online bank, that uses a biometric-based authentication system. During the authentication process, the client's fresh biometric template is compared to the stored reference biometric template using the cosine similarity distance, and the authentication decision is taken based on a pre-determined threshold. This application scenario is depicted in Fig. 1.

For privacy reasons, the client does not want to disclose his biometric data in clear to any party, considering their sensitive nature and the high risk of a data breach in a centralised database. We therefore consider two outsourced non-colluding servers,  $S_0$  and  $S_1$ , in charge of collecting and privately storing the secret shares of the clients' reference templates upon their registration during the enrollment phase, performing the biometric verification process (*i.e.*, matching of the templates) and disclosing the decision output to the service provider B (*i.e.*, the bank).



**Figure 1: Privacy-preserving biometric authentication system based on cosine similarity in the 2PC setting.**

## 2.3 Threat Model and System Design

In this paper, we assume that the client has already registered and submitted her biometric data (*e.g.*, voice reference template) in a privacy-preserving manner to the two non-colluding servers. Then, in the authentication process the comparison between the fresh biometric template and the registered reference template is performed by the two servers. Note that neither any server nor the service provider have access to any biometric templates in

cleartext. We also assume that the protocol is running through secure channels providing security against any external adversary that can compromise the transmission.

In our considered setting of Fig. 1, we have four players playing three different roles. We assume a **semi-honest dealer** (*e.g.*, the central bank B) distributing reliable correlated randomness to two computing servers **in the setup phase**. We consider that the client may act maliciously *i.e.*, may attempt to impersonate a legitimate user and deduce information for the corresponding templates. Thus, **in the input phase**, we require the client to submit a non-normalised reference template and we incorporate a mechanism to check that the secret sharing of the fresh (as well as the reference) template have been secret shared correctly.

Then, we consider an outsourced online secure evaluation among two non-colluding servers, where we assume both **semi-honest servers in a semi-honest setting** as well as a **malicious setting** that there is at most one active adversary deviating from the correct execution of the protocol (*i.e.*, perform wrong computation in the matching process between the fresh and stored template and/or try to infer information about the fresh and/or stored biometric templates). Finally, the secret sharing of the evaluation result is returned to the bank for reconstruction in the semi-honest setting, or validation and reconstruction in the malicious setting.

## 2.4 Cosine Similarity for Authentication

Let  $\mathbf{x}$  and  $\mathbf{y}$  of dimension  $n$  denote the fresh template received from the user requesting to authenticate and the reference template submitted upon enrolment, respectively. The cosine similarity metric between them would then be computed as:

$$\cos(\mathbf{x}, \mathbf{y}) = \frac{\text{IP}(\mathbf{x}, \mathbf{y})}{\|\mathbf{x}\| \cdot \|\mathbf{y}\|} = \frac{\prod_{i=1}^n x_i \cdot y_i}{\prod_{i=1}^n x_i^2 \cdot \prod_{i=1}^n y_i^2} \quad (1)$$

To authenticate the owner of  $\mathbf{x}$  against the reference  $\mathbf{y}$ , the resulting score of  $\cos(\mathbf{x}, \mathbf{y})$  would then be compared with a public threshold  $\tau \in [-1, 1]$ . If the similarity score is greater than or equal to  $\tau$ , then the user is successfully authenticated, otherwise, the authentication fails. Overall, we can define the biometric authentication functionality as:

$$\mathcal{F}_{\text{CosAuth}}(\mathbf{x}, \mathbf{y}, \tau) = \text{Sign}(\cos(\mathbf{x}, \mathbf{y}) - \tau) \quad (2)$$

In designing a privacy-preserving protocol for this functionality, we tweak  $\mathcal{F}_{\text{CosAuth}}$  to avoid expensive non-linear operations (division, square-root), obtaining an equivalent MPC-friendly circuit  $c = C(\mathbf{x}, \mathbf{y}, \tau)$  as follows:

$$\begin{aligned} c_1 &= \text{Sign}([\text{IP}(\mathbf{x}, \mathbf{y})]^A) \\ c_2 &= \text{Sign}([1/\tau^2 \cdot \text{IP}(\mathbf{x}, \mathbf{y})^2 - \text{IP}(\mathbf{x}, \mathbf{x}) \cdot \text{IP}(\mathbf{y}, \mathbf{y})]^A) \\ c &= \begin{cases} c_1 \wedge c_2 & \text{if } \tau > 0 \\ c_1 & \text{if } \tau = 0 \\ c_1 \vee \neg c_2 & \text{if } \tau < 0 \end{cases} \end{aligned} \quad (3)$$

One can easily verify the equivalence  $C \equiv \mathcal{F}_{\text{CosAuth}}$ . For the remainder of this paper, *w.l.o.g.*, we assume  $\tau > 0$ , and define the following functionality accordingly:

$$\begin{aligned} \mathcal{F}_{\text{ScaledCosAuth}}(\mathbf{x}, \mathbf{y}, \tau) &= \text{Sign}(\text{IP}(\mathbf{x}, \mathbf{y})) \wedge \\ &\quad \text{Sign}(1/\tau^2 \cdot \text{IP}(\mathbf{x}, \mathbf{y})^2 - \text{IP}(\mathbf{x}, \mathbf{x}) \cdot \text{IP}(\mathbf{y}, \mathbf{y})) \end{aligned} \quad (4)$$

### 3 PRELIMINARIES

#### 3.1 Two-Party Secure Computation

Secure two-party computation allows two non-colluding parties  $P_0$  and  $P_1$  to compute a function  $f$  on their private inputs (e.g.,  $x$  and  $y$ ) without revealing any information beyond the final value of  $f(x, y)$ . 2PC protocols are based on either: (i) Secret Sharing (SS) techniques i.e., arithmetic SS, like additive SS [5] and replicated SS [2], or Boolean SS like GMW [44]; or (ii) Garbled Circuits (CG) like Yao's GCs [64], BMR [6].

**Optimized Additive SS.** We make use of an optimized additive SS with function dependent pre-processing as our building block [7]. In the setup phase, correlated random offset shares are generated for input wires and output wires of each gate of the arithmetic circuit. These shares are input-independent and can be executed at any time before the online/data-dependent phase where the actual function  $f$  is evaluated. In the evaluation phase, efficient online secure computation is performed. We denote by  $\langle x \rangle^\Delta$  the optimized additive SS of a secret  $x$  in a two-party setting of  $S_0, S_1$ , where  $\forall b \in \{0, 1\}$ ,  $S_b$  holds partial shares

$$\langle x \rangle_b^\Delta : (\delta_x, [r_x]_b^\Delta)$$

in which  $\delta_x = x + r_x$ ,  $[r_x]_b^\Delta$  is the associated random offset of the secret value  $x$ . One share alone does not disclose any information about  $x$  but when summed together, they reconstruct it ( $x = \delta_x - [r_x]_0^\Delta - [r_x]_1^\Delta$ ). The parties interact with each other to compute any desired function from the input secret shares. We denote by  $\mathcal{F}_{\text{Reveal}}(\langle x \rangle^\Delta)$  the functionality that inputs an arithmetic secret sharing of  $x$  from two servers and outputs  $x$ . Given  $\langle x \rangle^\Delta$  and  $\langle y \rangle^\Delta$ , we demonstrate how addition and multiplication gates are performed in the following.

**Addition.** It requires no communication between the two parties. The two parties simply locally add the shares they hold. More precisely, each party  $b \in \{0, 1\}$  computes:

$$\langle x + y \rangle_b^\Delta = (\delta_x + \delta_y, [r_x]_b^\Delta + [r_y]_b^\Delta)$$

**Multiplication.** It requires interaction between the two parties and relies on additional input-independent but function-dependent random pre-computed secret shares named Beaver's triples [6]. To compute  $[z]^\Delta$  where  $z = xy$ , with the corresponding Beaver's triples  $[r_x]^\Delta, [r_y]^\Delta, [r_x r_y]^\Delta$  generated in the setup phase, the parties are able to locally compute the additive secret shares of  $z$ . i.e.,  $\forall b \in \{0, 1\}$ ,  $S_b$  holds  $\{\langle x \rangle_b^\Delta = (\delta_x, [r_x]_b^\Delta), \langle y \rangle_b^\Delta = (\delta_y, [r_y]_b^\Delta), [r_x r_y]_b^\Delta\}$  in which  $\delta_x = x + r_x, \delta_y = y + r_y$ . Then,  $S_b$  computes:

$$[z]_b^\Delta = b \cdot \delta_x \delta_y - \delta_x [r_x]_b^\Delta - \delta_y [r_y]_b^\Delta + [r_x r_y]_b^\Delta. \quad (5)$$

However, to conform the multiplication output to an optimized secret sharing format, the parties locally compute  $[z + r_z]^\Delta$  and reveal  $z + r_z$  in one round, where  $r_z$  is a pre-generated random output wire offset in the setup phase. Thus, obtaining  $\langle z \rangle^\Delta = (z + r_z, [r_z]^\Delta)$ .

Throughout this paper, we denote  $[x]^\Delta \leftarrow \text{SS.Share}(x, Z_{2\ell})$  as the algorithm dividing a secret into secret sharing in the domain  $Z_{2\ell}$ , where  $x$  can be a single value or an  $n$ -sized vector;  $\text{SS.MUL}(\langle x \rangle^\Delta, \langle y \rangle^\Delta, [r_x r_y]^\Delta)$  as the multiplication process shown

in Eq. 5,  $\text{SS.IP}(\langle x \rangle^\Delta, \langle y \rangle^\Delta, [u]^\Delta)$  as the inner product of two vectors  $x, y$  where  $u$  is the corresponding correlated randomness.

**SPDZ2k protocol.** The SPDZ2k protocol [20] is a MPC protocol that works in a dishonest majority setting. We utilize this protocol in a two-party computation (2PC) context in this work. It ensures secure operations by using authenticated secret sharing in a ring structure and detects any cheating attempts through a secret message authentication key that is shared between the two servers, denoted as  $\Delta$ . Throughout this paper, in the context of 2PC, we define an authenticated arithmetic secret sharing of a secret  $x$  as

$$\text{Jxk}^\Delta : ([x]^\Delta, [\Delta \cdot x]^\Delta)$$

and  $\langle\langle x \rangle\rangle^\Delta$  as the authenticated optimized arithmetic secret sharing of  $x$ . Naturally, we use notations  $\text{Jxk}_b^\Delta$  and  $\langle\langle x \rangle\rangle_b^\Delta$  to denote the share held by  $S_b$ . After the online secure computation, two servers work together to verify the integrity of every revealed value in the secure computation by using a final MAC verification algorithm, as shown in Fig.6 of [20]. In the case when the verification fails, malicious behaviors are detected and the parties abort the protocol.

When working with authenticated optimized secret sharing in the malicious setting, we use same notation  $\text{SS.MUL}(\langle\langle x \rangle\rangle^\Delta, \langle\langle y \rangle\rangle^\Delta, \text{Jr}_x \text{r}_y \text{k}^\Delta)$  and  $\text{SS.IP}(\langle\langle x \rangle\rangle^\Delta, \langle\langle y \rangle\rangle^\Delta, \text{Juk}^\Delta)$  to denote secure multiplication and secure inner product, respectively.

#### 3.2 Secure Truncation

Given an arithmetical secret sharing  $[x]^\Delta$  where  $x \in Z_{2\ell}$  and a truncation parameter  $\rho \in Z^+$ , a secret sharing based truncation operation aims to compute the truncated secret sharing  $[\text{Shift}(x, \rho, \ell)]^\Delta$ . In the proposed protocols of section 6, we applied a secure truncation operation within our final implementation in the pre-processing model. We employ the truncation pair  $(r, \text{Shift}(r, \rho, \ell))$  introduced in [45]. More precisely, the life cycle of a truncation pair  $(r, \text{Shift}(r, \rho, \ell))$  are as follows:

- (1) In the setup phase, a random offset  $r \in Z_{2\ell}$  is generated, both  $[r]^\Delta$  and  $[\text{Shift}(r, \rho, \ell)]^\Delta$  are distributed among  $S_0$  and  $S_1$ ;
- (2) In the evaluation phase, to truncate a secret sharing  $[x]^\Delta$  where  $x \in Z_{2\ell}$ , the servers run  $\mathcal{F}_{\text{reveal}}([x+r]^\Delta)$  to obtain  $x+r$ ,  $\forall b \in \{0, 1\}$  then  $S_b$  computes  $\text{Shift}(x+r, \rho, \ell) - [\text{Shift}(r, \rho, \ell)]_b^\Delta$  as  $[\text{Shift}(x, \rho, \ell)]_b^\Delta$ .

Remarkably, using such a truncation pair results in an approximate result, meanwhile it requires a significant gap between the modulus and the maximum size of the underlying secret  $x$ . Moreover, as noted in [42], this type of probabilistic truncation protocol introduces a security vulnerability. This issues arises because the same randomness is employed both to protect the privacy of the secret value to be truncated, and sample the 1-bit rounding error for the truncated value probabilistically, which makes the protocol not simulatable in terms of security. Despite these concerns, when evaluating the final efficiency of our full protocols, in order to use function secret sharing with a relatively short input domain, universally we apply this building block into our protocols as well as other existing state-of-the-arts.

#### 3.3 Function Secret Sharing

Function Secret Sharing (FSS) was first introduced by Boyle et al. [11] as a cryptographic primitive that secretly shares a function

$f$ . Informally, in a two party setting with a function  $f : D \rightarrow \mathbb{R}$ , FSS comprises a key generation algorithm  $\text{Gen}^f(1^\lambda)$  producing a key pair  $(k_0, k_1)$ , and an evaluation algorithm  $\text{Eval}^f$ , which takes in one shared key and a value  $x$ , ensuring that  $\forall x \in D$  that the sum of  $\text{Eval}^f(k_0, 0, x)$  and  $\text{Eval}^f(k_1, 1, x)$  is equal to  $f(x)$ .

Shortly after, a secure computation scheme with pre-processing via FSS was proposed by Boyle *et al.* [12]. In this scheme, nonlinear gates (e.g., for equality tests, integer comparison, etc.) can be computed with a relatively small amount of communication in one round. We give an example how in this model a secure equality check is conducted between  $S_0$  and  $S_1$ .

Let us consider the case of a point function  $f_a : \{0, 1\}^\ell \rightarrow \{0, 1\}$  corresponding to a special point  $a \in \{0, 1\}^\ell$ ,  $\forall x \in \{0, 1\}^\ell$  that outputs 1 if  $x = a$  and outputs 0 otherwise. In the pre-processing phase, a trusted dealer distributes  $S_0, S_1$  additive secret shares  $r_0, r_1$  of a random mask value  $r \leftarrow \{0, 1\}^\ell$  and FSS key shares  $k_0, k_1$  correspond to the random point function  $f_r$ . In the online phase,  $\forall i \in \{0, 1\}$ ,  $P_i$  holds  $x_i, r_i$  and  $k_i$ , and  $P_i$  exchanges  $x_i + r_i$  with  $P_{1-i}$ . After revealing the masked value  $x + r$ ,  $\forall i \in \{0, 1\}$ ,  $P_i$  computes  $f_{r,i}(x + r - a)$  and thus, obtains secret shares of  $f_a(x)$ . Boyle *et al.* also proposed FSS schemes for comparison functions in [11, 13].

In constructing all our protocols, we make use the interval containment function secret sharing (IC-FSS) ([10], Section 4.1) as our secure comparison primitive, as the same as in Funshade [33].

**Definition 1.** *Interval containment function secret sharing.* There is a key generation algorithm  $\text{Gen}_\ell^{[p,q]}(\cdot)$ , and an evaluation algorithm  $\text{Eval}_\ell^{[p,q]}(\cdot)$ , given an interval containment  $[p, q]$  where  $p, q \in \mathbb{Z}_{2^\ell}$  and  $p < q$ ,  $\forall \alpha \in \mathbb{Z}_{2^\ell}$ ,  $\forall \beta_1, \beta_2 \in \mathbb{U}_N$ ,

$$(k_0, k_1) \leftarrow \text{Gen}_\ell^{[p,q]}(1^\lambda, \alpha, \beta_1, \beta_2, \mathbb{U}_N)$$

for  $\forall x \in \mathbb{Z}_{2^\ell}$  denote  $\delta_x = x + \alpha$ , it holds that:

$$\bigcirc_{b=0} \text{Eval}_\ell^{[p,q]}(k_b, b, \delta_x) = \begin{cases} \beta_1, & \text{if } x \in [p, q] \\ \beta_2, & \text{if } x \text{ not in } [p, q] \end{cases}$$

## 4 TECHNICAL OVERVIEW

To realize  $\mathcal{F}_{\text{ScaledCosAuth}}$  with passive security, we study its required components and immediately reach two preliminary protocols,  $\Pi_{\text{NaiveSH}}$  and  $\Pi_{\text{OptimSH}}$ . Further enhancements in this paper will later lead to our novel NOMADIC protocols. The online evaluation steps for all the protocols described in this section are outlined in Table 2, with associated online costs detailed in Table 3.

As a first approach [10, 30, 54],  $\Pi_{\text{NaiveSH}}$  employs additive SS and FSS. This protocol requires four communication rounds.

Next, we follow the guidelines of Funshade [33] to get  $\Pi_{\text{OptimSH}}$ , introducing optimized secret sharing paired with FSS comparison gates. By its online phase,  $\Pi_{\text{OptimSH}}$  necessitates pre-prepared, circuit-dependent correlated randomness, generated offline by a dealer and distributed to two computing servers. As highlighted in Funshade [33], this approach improves communication efficiency over  $\Pi_{\text{NaiveSH}}$  in terms of rounds and data volume. For instance, computing  $[c_1]^B$  of  $\mathcal{F}_{\text{ScaledCosAuth}}$  in  $\Pi_{\text{OptimSH}}$  requires only one round and a single group element exchange, compared to  $\Pi_{\text{NaiveSH}}$  (two rounds and  $2n$  group elements). Consequently,  $\Pi_{\text{OptimSH}}$  reduces to three communication rounds: the first for  $c_1$ , the second for  $c_2$ , and the third for the final output  $c = c_1 \wedge c_2$ .

Further refining  $\Pi_{\text{OptimSH}}$ , we decrease communication rounds to two using a novel component, CondEval. This new primitive introduces a conditional evaluation gate:  $(c, x, f, \wedge) \rightarrow c \wedge f(x)$ , traditionally executed in two rounds but here accomplished in one. It is designed for computing the composition of a function  $f(x)$  with a logical AND ( $\wedge$ ) or OR ( $\vee$ ) operation, integrating optimized secret sharing and novel FSS key generation steps. This innovation enables efficient computation of  $c = c_1 \wedge c_2$  in the second round, thus saving a communication round.

We specify and prove the security of the CondEval primitive in the semi-honest setting, incorporating it to the full protocol in  $\Pi_{\text{NomadicSH}}$ . As a result,  $\Pi_{\text{NomadicSH}}$  encompasses the following four distinct phases:

- (1) **Setup:** Generating correlated randomness for  $\mathcal{F}_{\text{ScaledCosAuth}}$ .
- (2) **Input:** Both the client and entity B independently input their secret vectors,  $\mathbf{x}$  and  $\mathbf{y}$ , respectively.
- (3) **Evaluation:** Conducted by two servers, this phase includes:
  - *First round:* Computation of a Boolean sharing  $[c_1]^B$ .
  - *Second round:* Computation of the final Boolean sharing  $[c]^B = [c_1]^B \wedge f(x)$ . In a malicious setting, this includes generating  $[c]^B$  and a proof  $[\sigma]^A$  for verification.
- (4) **Reveal:** The service provider B constructs the value of  $c$  in clear-text. In a malicious context, B also validates  $[c]^A$  and assesses the proof  $[\sigma]^A$ .

Additionally, we extend the construction and security proof of CondEval to the malicious setting. To provide active security in the FSS gates we resort to the parallel execution of multiple instances for the conditional evaluation gate with independent FSS keys. A subset of these instances, chosen at FSS key generation and unknown to the computing servers, act as "trap" instances, allowing the detection of malicious behavior in a single computing server. For a fully maliciously-secure protocol  $\Pi_{\text{NomadicM}}$ , we employ homomorphic MACs when using additive secret shares (as in the SPDZ2k protocol [20] but we use a simplified variant). The full maliciously-secure protocol  $\Pi_{\text{NomadicM}}$  follows a similar online evaluation pipeline to what in  $\Pi_{\text{NomadicSH}}$ , thus, we do not repeat it in Tab. 2.

## 5 REALIZING CONDEVAL IN 2PC

We introduce CondEval as key element to optimize our thresholded cosine similarity protocols. Following circuit  $C$  (Eq. 3), the evaluation of a biometric matching protocol splits into (i) computing shares of boolean values  $c_1$  and  $c_2$  (see Equation 3) and (ii) computing  $c_1 \wedge c_2$  or  $c_1 \vee \neg c_2$  depending on the value of  $\tau$ . FSS can be used to efficiently compute the Sign for  $c_1$  and  $c_2$ . Computing the last boolean gate ( $\wedge$  or  $\vee$ ) in secret shares would require at least one communication round. CondEval integrates this boolean gate into the FSS evaluation of  $c_2$ , saving up one communication round in the pre-processing model. We formalize this functionality as:

$$[s \circ f(x)]^B \leftarrow \text{CondEval}(o, \mathcal{K}_o, [s]^B, [x]^A)$$

where we compute a two-input boolean gate with a bit  $s$  and a function  $f(x)$  (instantiated with FSS). We would then apply CondEval to  $\mathcal{F}_{\text{ScaledCosAuth}}$  by setting  $s = c_1$  and  $f(x) = c_2 = \text{Sign}([1/\tau^2 \cdot \text{IP}(\mathbf{x}, \mathbf{y})^2 - \text{IP}(\mathbf{x}, \mathbf{x}) \cdot \text{IP}(\mathbf{y}, \mathbf{y})]^A)$ .

**Table 2: Online steps of protocols for  $\mathcal{F}_{\text{ScaledCosAuth}}$ .**

	Round-1	Round-2	Round-3	Round-4
$\Pi_{\text{NaiveSH}}$	$[\text{IP}(\mathbf{x}, \mathbf{y})]^A$ $[\text{IP}(\mathbf{x}, \mathbf{x})]^A$ $[\text{IP}(\mathbf{y}, \mathbf{y})]^A$	$[c_1]^B \leftarrow \text{Sign}([\text{IP}(\mathbf{x}, \mathbf{y})]^A)$ $[u_1]^A \leftarrow [\text{IP}(\mathbf{x}, \mathbf{x}) \cdot \text{IP}(\mathbf{y}, \mathbf{y})]^A$ $[u_2]^A \leftarrow [\text{IP}(\mathbf{x}, \mathbf{y})^2]^A$ $[u]^A \leftarrow [u_1 - 1/\tau^2 \cdot u_2]^A$	$[c_2]^B \leftarrow \text{Sign}([u]^A)$	$[c]^B \leftarrow [c_1 \wedge c_2]^B$
$\Pi_{\text{OptimSH}}$	$[\text{IP}(\mathbf{x}, \mathbf{y})]^A$ $[\text{IP}(\mathbf{x}, \mathbf{x})]^A$ $[\text{IP}(\mathbf{y}, \mathbf{y})]^A$ $[c_1]^B \leftarrow \text{Sign}([\text{IP}(\mathbf{x}, \mathbf{y})]^A)$	$[u_1]^A \leftarrow [\text{IP}(\mathbf{x}, \mathbf{x}) \cdot \text{IP}(\mathbf{y}, \mathbf{y})]^A$ $[u_2]^A \leftarrow [\text{IP}(\mathbf{x}, \mathbf{y})^2]^A$ $[u]^A \leftarrow [u_1 - 1/\tau^2 \cdot u_2]^A$ $[c_2]^B \leftarrow \text{Sign}([u]^A)$	$[c]^B \leftarrow [c_1 \wedge c_2]^B$	-
$\Pi_{\text{NomadicSH}}$	$[\text{IP}(\mathbf{x}, \mathbf{y})]^A$ $[\text{IP}(\mathbf{x}, \mathbf{x})]^A$ $[\text{IP}(\mathbf{y}, \mathbf{y})]^A$ $[c_1]^B \leftarrow \text{Sign}([\text{IP}(\mathbf{x}, \mathbf{y})]^A)$	$[u_1]^A \leftarrow [\text{IP}(\mathbf{x}, \mathbf{x}) \cdot \text{IP}(\mathbf{y}, \mathbf{y})]^A$ $[u_2]^A \leftarrow [\text{IP}(\mathbf{x}, \mathbf{y})^2]^A$ $[u]^A \leftarrow [u_1 - 1/\tau^2 \cdot u_2]^A$ $[c]^B \leftarrow \text{CondEval}([c_1]^B, \text{Sign}([u]^A))$	-	-

**Table 3: Comparative overview of online costs for  $\Pi_{\text{NaiveSH}}$ ,  $\Pi_{\text{OptimSH}}$ ,  $\Pi_{\text{Mal1}}$ ,  $\Pi_{\text{NomadicSH}}$  and  $\Pi_{\text{NomadicM}}$ . Here "Comm." refers to the total communication volume among two servers,  $n$  is the length of the input vector,  $\ell$  the length of the operational ring,  $L$  the communication volume for a single evaluation of CondEval, and  $\kappa$  the amount of parallel CondEval executions in  $\Pi_{\text{NomadicM}}$ .**

	Model	Primitive	Comm.	#Rounds
$\Pi_{\text{NaiveSH}}$	SH	SS + FSS	$6n\ell + 4n\ell + 2\ell + 2$	4
$\Pi_{\text{OptimSH}}$	SH	Optimized SS + FSS	$6\ell + 2$	3
$\Pi_{\text{NomadicSH}}$	SH	Optimized SS + CondEval	$7\ell + L$	2
$\Pi_{\text{NaiveM}}$	M	Authenticated (Optimized SS + FSS)	$6\ell + 2$	3
$\Pi_{\text{NomadicM}}$	M	Authenticated (Optimized SS + CondEval)	$6\ell + \kappa(\ell + L)$	2

In the reminder of this section, *w.l.o.g.*, we consider the  $\wedge$  operation and we describe how CondEval computes  $s \wedge f(x)$  in a symmetric 2PC setting in both the semi-honest and malicious models, we ignore on the  $\vee$  operation since  $s \wedge f(x) = \neg(\neg s \wedge \neg f(x))$ . It is worth noting that CondEval can be used as an independent building block to compute the boolean composition of a bit  $s$  and a function  $f(x)$  evaluated with FSS for 2PC protocols.

## 5.1 High-level Overview

CondEval seeks to evaluate  $s \wedge f(x)$  using FSS with two computing servers  $S_0$  and  $S_1$ . The inputs to this protocol are mainly secret shares of a bit  $s$  and an input  $x$  know to both parties. To avoid the additional communication round that the  $\wedge$  gate would require, we aim to merge it with the FSS evaluation of  $x$ . Indeed, from Table 4 (which corresponds to the truth table of  $s \wedge f(x)$ ), we observe that if  $s = 0$ , then the output is 0 and if  $s = 1$  then the output is the actual output of  $f(x)$ . In cases where the output range is in  $\mathbb{Z}_2$ ,  $f(x)$  is obtained when each server  $S_b$  ( $b \in \{0, 1\}$ ) runs  $\text{Eval}^f(k_b, x)$  and the output would reconstruct with an XOR of these. If each server runs  $\text{Eval}^f$  over the same keys, the reconstructed output would become 0, which corresponds to the case when  $s = 0$ . Hence, the overall idea is to make sure that when running Eval, the two servers use the same FSS key if  $s = 0$  and different FSS keys if  $s = 1$ . This is illustrated in the 2PC setting (note that  $s$  is secretly shared among the two servers as well) in Table 4. Each server must retrieve the correct FSS key according to  $s$ , without leaking any information neither about  $s$ , nor the other FSS key.

**Table 4: Truth table of  $[s \wedge f(x)]^B$ .**

$s$	$s \wedge f(x)$	$s_0$	$s_1$	$[y]_0^B$	$[y]_1^B$	$[y]_0^B \oplus [y]_1^B$
0	0	0	0	$\text{Eval}^f(k_0, 0, \delta_x)$	$\text{Eval}^f(k_0, 0, \delta_x)$	0
		1	1	$\text{Eval}^f(k_1, 1, \delta_x)$	$\text{Eval}^f(k_1, 1, \delta_x)$	
1	$f(x)$	1	0	$\text{Eval}^f(k_0, 0, \delta_x)$	$\text{Eval}^f(k_1, 1, \delta_x)$	$f(x)$
		0	1	$\text{Eval}^f(k_1, 1, \delta_x)$	$\text{Eval}^f(k_0, 0, \delta_x)$	

Such a protocol can be designed if the two FSS keys  $k_0$  and  $k_1$  generated by the bank<sup>2</sup> were randomly mapped to the values of  $s$  ( $k_{1-s}$  if  $s = 0$ ,  $k_s$  if  $s = 1$ ). Then, each server would run a private information retrieval protocol with the bank to retrieve the actual key corresponding to its share without revealing the actual share. In order not to involve an additional party on the computation and save in the number of communication rounds, we propose to store these two keys at both servers in an encrypted manner. More precisely, server  $S_0$  will store the two FSS keys, encrypted beforehand and will receive one decryption key from server  $S_1$  based on its share of  $s$  and vice versa. Hence, the key required to decrypt the FSS key (stored on one server) is stored on the other server. This implies that each server  $S_b$  will store both encrypted FSS keys but will only be able to decrypt one of them according to the value of  $s$ . The goal is to receive and decrypt the correct FSS key, without disclosing  $s$  nor the FSS key not used by  $S_b$ .

Now, we need to prevent any leakage about  $[s]_b^B$  from  $S_{1-b}$ , and the other FSS key not used by  $S_{1-b}$  (where  $b \in \{0, 1\}$ ):

<sup>2</sup>The entity in charge of generating pre-processing material (Fig. 1).

- since  $\text{Eval}^f$  also takes as input the index of the FSS key (0 or 1), this index should not leak any information about  $s$ . Hence, the mapping between the FSS key index and  $[s]_b^B$  needs to be protected as well, this is realized by set the initial FSS key pair as  $(k_t, k_{1-t})$  where  $t \leftarrow_s \{0, 1\}$ .
  - to protect  $[s]_b^B$ , a random permutation is applied to  $(k_t, k_{1-t})$ ;
  - to protect the unused FSS key, the permuted keys are one-time-pad encrypted before their storage and only one decryption key will be sent from  $S_b$  to  $S_{1-b}$  in the key decryption step.
- Overall,  $\text{CondEval}$  is thus specified with four algorithms:
- During the **setup phase**, a trusted dealer generates the keying material using  $\text{CondEval.KeyGen}$  and protects the FSS key pair with  $\text{CondEval.KeyEnc}$ . These encrypted FSS keys and the keying material are distributed to the servers;
  - During the **online phase**, each server who has received the share of  $s$  and the randomized input  $\delta_x = x + r$ , will send the correct decryption key  $sk$  according to its share of  $s$  to the other server. Once the correct FSS key is decrypted using  $\text{CondEval.KeyDec}$ , the server finishes the protocol by running  $\text{CondEval.Eval}$  to obtain the share of the output  $s \wedge f(x)$ .

Note that, while we only demonstrate the construction of  $\text{CondEval}$  for  $s \wedge f(x)$ , the protocol for computing  $s \vee f(x)$  is defined similarly and can be performed either by using

$$s \vee f(x) = \neg(\neg s \wedge \neg f(x)).$$

## 5.2 CondEval in the Semi-Honest setting

We propose the building block  $\text{CondEval}$ , designed to securely evaluate  $s \wedge f(x)$  in a semi-honest setting. It works in the pre-processing model, and is composed of two functionalities:

$$\begin{aligned} \mathcal{K}_\lambda &\leftarrow \text{CondEval.Setup}(\wedge, 1^\lambda, \ell) \text{ and} \\ [y]^B &\leftarrow \text{CondEval.Eval}(\wedge, \mathcal{K}_\lambda, [s]^B, [x]^A) \end{aligned}$$

that are run respectively in the setup and online phase. The construction of  $\text{CondEval.Setup}$  is shown in Table 5.  $\text{CondEval.Setup}$ , inputs a security parameter  $1^\lambda$  and outputs  $\mathcal{K}_\lambda$ . After  $\text{KeyGen}$  and  $\text{KeyEnc}$ , it outputs correlated random keys which are then distributed to the two servers.  $\text{CondEval.Eval}(\wedge, \mathcal{K}_\lambda, [s]^B, [x]^A)$  inputs  $\mathcal{K}_\lambda$  generated by  $\text{CondEval.Setup}$  as well as  $[s]^B, [x]^A$ . Then, within one round of communication,  $\delta_x$  is revealed and one clear-text FSS key is obtained from both servers with  $\text{KeyDec}$  being performed, and finally it outputs  $[y]^B$ .

**Theorem 1. Correctness.** If two servers follow  $\text{CondEval.Eval}(\wedge, \mathcal{K}_\lambda, [s]^B, [x]^A)$  honestly, then it outputs  $[y]^B = [s \wedge f(x)]^B$ .

**PROOF.** If  $s = 0$ , then  $([s]_0^B, [s]_1^B)$  is equal to either  $(0, 0)$  or  $(1, 1)$ . Thus, in the online phase after the decryption of FSS keys, the servers obtain either  $\{k_t, k_t\}$  or  $\{k_{1-t}, k_{1-t}\}$ , which implies:

$$y = \begin{cases} \text{Eval}^f(k_0, 0, \delta_x) \oplus \text{Eval}^f(k_0, 0, \delta_x) \text{ or} \\ \text{Eval}^f(k_1, 1, \delta_x) \oplus \text{Eval}^f(k_1, 1, \delta_x). \end{cases}$$

In either case,  $y = 0$ , as desired. On the other hand, if  $s = 1$ , then  $([s]_0^B, [s]_1^B)$  equals either  $(0, 1)$  or  $\{1, 0\}$ , which implies that  $y = \oplus_{t=0}^1 \text{Eval}^f(k_t, t, \delta_x)$ . In either case, we obtain  $y = f(x)$ , as desired. Thus,  $y = s \wedge f(x)$ .

**Table 5: The construction of  $\mathcal{K}_\lambda \leftarrow \text{CondEval.Setup}(\wedge, 1^\lambda, \ell)$  in the semi-honest setting, where  $\lambda$  is the security parameter used in generating FSS keys,  $\ell$  defines the domain of the secret sharing.**

KeyGen	$r \leftarrow_s Z_{2\ell}, [r]^A \leftarrow \text{SS.Share}(r, Z_{2\ell})$ $(k_0, k_1) \leftarrow \text{Gen}^f(1^\lambda, r, 1, 0, Z_2) ( k_0  =  k_1  = K);$ $t \leftarrow_s Z_2, \pi_0 \leftarrow_s Z_2, \pi_1 \leftarrow_s Z_2; L := K + 1;$ $\text{sk}_0^{(0)} \parallel \text{sk}_1^{(0)} \parallel \text{sk}_0^{(1)} \parallel \text{sk}_1^{(1)} \leftarrow_s \{0, 1\}^{4L},$ $ \text{sk}_0^{(0)}  =  \text{sk}_1^{(0)}  =  \text{sk}_0^{(1)}  =  \text{sk}_1^{(1)}  = L.$
KeyEnc	$m_0 = \{\text{sk}_0^{(0)} \oplus (k_t \parallel t), \text{sk}_1^{(0)} \oplus (k_{1-t} \parallel (1-t))\}$ $C_0 = \mathcal{F}_{\text{Permu}}(\pi_0, m_0); \text{SK}_0 = \{(\text{sk}_0^{(1)}, \text{sk}_1^{(1)}), \pi_1\}$ $m_1 = \{\text{sk}_0^{(1)} \oplus (k_t \parallel t), \text{sk}_1^{(1)} \oplus (k_{1-t} \parallel (1-t))\}$ $C_1 = \mathcal{F}_{\text{Permu}}(\pi_1, m_1); \text{SK}_1 = \{(\text{sk}_0^{(0)}, \text{sk}_1^{(0)}), \pi_0\}$
Outputs $\mathcal{K}_\lambda = \{C_i, \text{SK}_i, [r]_i^A\}_{i \in \{0,1\}}$	

**Functionality**  $[s \wedge f(x)]^B \leftarrow \text{CondEval.Eval}(\wedge, \mathcal{K}_\lambda, [s]^B, [x]^A)$

**Players:**  $S_0, S_1$ .

**Functionality:**  $[s \wedge f(x)]^B \leftarrow \text{CondEval.Eval}(\wedge, \mathcal{K}_\lambda, [s]^B, [x]^A)$ .

**Input:**  $[s]^B, [x]^A$  from two servers, and  $\mathcal{K}_\lambda$  prepared in the setup phase as shown in Table 5 where  $\mathcal{K}_\lambda = \{C_b, \text{SK}_b, r_b\}_{b \in \{0,1\}}$ , more specifically each  $S_b$  ( $b \in \{0, 1\}$ ) inputs  $\{C_b, \text{SK}_b, r_b\}$ . (Note that  $(r_0, r_1)$  constitutes  $[r]^A$ ).

**Output:**  $[s \wedge f(x)]^B$ .

- 1:  $\delta_x \leftarrow \mathcal{F}_{\text{Reveal}}([x]^A + [r]^A)$
- 2: **for**  $b = 0$  to 1 **do**
- 3:  $S_b: p^{(1-b)} \leftarrow [s]_b^B \oplus \pi_{1-b}, \text{sk}^{(1-b)} \leftarrow \text{sk}_{[s]_b^B}^{(1-b)}$ .
- 4:  $S_b: \text{Sends } p^{(1-b)} \parallel \text{sk}^{(1-b)} \text{ to } S_{1-b}. \quad \triangleright \text{Key decryption}$
- 5: **for**  $b = 0$  to 1 **do**
- 6:  $S_b: k^{(b)} \parallel \text{id}^{(b)} \leftarrow C_b[p^{(b)}] \oplus \text{sk}^{(b)}$
- 7:  $S_b: [y]_b^B \leftarrow \text{Eval}^f(k^{(b)}, \text{id}^{(b)}, \delta_x)$
- 8: Outputs  $[y]^B$ .

**Theorem 2. Security.** In the presence of a passive PPT adversary  $\mathcal{A}$  corrupting one of the two servers in  $\text{CondEval.Eval}(\wedge, \mathcal{K}_\lambda, [s]^B, [x]^A)$ , we assert that  $\mathcal{A}$  learns nothing about the inputs  $x, s$ , nor about the output  $s \wedge f(x)$ .

**PROOF.** After online evaluation, for each  $b \in \{0, 1\}$  we denote  $S_b$ 's transcript view as:

$$\text{View}_b := \{[r]_b^A, \delta_x, k_{t \oplus [s]_{1-b}^B} \parallel (t \oplus [s]_{1-b}^B), [s]_{1-b}^B \oplus \pi_b\}$$

From this transcript,  $\{k_{t \oplus [s]_{1-b}^B}, [r]_b^A, \delta_x\}$  correspond to the FSS evaluation in the pre-processing model, and we resort to the security proof in [12] (Definition 2) of Boyle *et al.* to argue the computational indistinguishability of the ideal and real-world executions; Regarding the remaining items of the view transcripts, since both  $\pi_b$  and  $t$  are uniformly selected, both  $[s]_{1-b}^B \oplus \pi_b$  and  $t \oplus [s]_{1-b}^B$  provide information-theoretic secrecy for  $[s]_{1-b}^B$ . Therefore, from  $\text{View}_b$ ,  $\mathcal{A}$  learns nothing about the input  $x, s$ , nor the output  $s \wedge f(x)$ .



### 5.3 CondEval in the Malicious Setting

We extend the setting to an active adversary. Since the two servers are symmetric, for the sake of clarity, we consider that  $S_0$  is malicious. Assume we run  $\text{CondEval.Eval}(\wedge, \mathcal{K}_\wedge, [s]^\text{B}, [x]^\text{A})$  in the malicious setting where  $S_0$  is malicious and  $\mathcal{K}_\wedge$  is the output of  $\text{CondEval.Setup}(\wedge, 1^\text{A}, Z_{2^r})$ , then we report three disruptions from  $S_0$  that might flip the final authentication bit:

- $S_0$  may dishonestly report  $[x + r]_0^\text{A}$  when revealing  $x + r$  to introduce errors and thus disrupt the computation;
- During the online evaluation phase,  $S_0$  may flip  $[s]_0^\text{B}$  during the key decryption step, thus potentially flipping  $[y]^\text{B}$  to be equal to  $[\neg(s \wedge f(x))]^\text{B}$ .
- After the online evaluation,  $S_0$  may submit  $\neg[y]_0^\text{B}$  to  $B$ , potentially resulting in a flipped authentication bit  $y = \neg(s \wedge f(x))$ .

To counter the first attack, we incorporate a homomorphic MAC scheme from the `spd2k` framework [20]. More specifically, during the setup phase, a trustworthy dealer generates random offset shares and the corresponding authenticated shares. This enables an additional verification step to be performed over all partially disclosed intermediate values at the end of the online evaluation phase. The final output is only deemed valid if the verification is successful, thereby deterring any fraudulent disclosure of secret sharing. In order to safeguard against the remaining attacks, we require a separate scheme that protects Boolean secret sharings. As a solution, we propose the use of *authenticated Boolean secret sharing* defined as below.

**Definition 2.** An *authenticated Boolean secret sharing*  $(v)^m$  of bit  $v \in \{0, 1\}$  is a list of Boolean secret shares defined with a secret authentication key  $\psi \in \{0, 1\}^m$ ,  $m \in \mathbb{Z}^+$ , denoted as:

$$(v)^m : \{[v_1]^\text{B}, \dots, [v_m]^\text{B}\}$$

where  $\forall i \in [m]$  that  $v_i = v \cdot (1 - \psi[i])$ .

By extending the semi-honest  $\text{CondEval}$  constructions with the above two authentication schemes, we realize  $\text{CondEval}$  in a malicious setting which contains three functionalities:

$$\begin{aligned} \mathcal{K}_\wedge^* &\leftarrow \text{CondEval.Setup}^*(\wedge, 1^\text{A}, \ell_0, \ell_1, m), \\ (y)^m, [\sigma]^\text{A} &\leftarrow \text{CondEval.Eval}^*(\wedge, \mathcal{K}_\wedge^*, (s)^m, \text{JxK}^\text{A}) \text{ and} \\ z &\leftarrow \text{CondEval.Verify}((y)^m, [\sigma]^\text{A}, \psi) \end{aligned}$$

$\text{CondEval.Setup}^*$  is shown in Table 6, where a trustful dealer prepares  $m$  instances of FSS key pairs based on a uniform random string  $\psi \leftarrow_s \{0, 1\}^m$ . More precisely, for all  $i \in [m]$ , let us denote  $K_i$  as the  $i^{\text{th}}$  FSS key pair prepared. Then, if  $\psi[i] = 0$ , the dealer outputs  $K_i$  as a *normal* FSS key pair, which means that the evaluation result is equal to  $s \wedge f(x)$ ; Otherwise, if  $\psi[i] = 1$ , the dealer outputs  $K_i$  as a *trap* FSS key pair which guarantees that the corresponding evaluation result will be equal to 0 for whatever value of  $x$ . Furthermore  $\text{CondEval}^*.\text{Setup}$  generates additional proof string differently due to the FSS key type (*normal* or *trap*) which are appended at each FSS key. With these designs, any manipulation from  $\mathcal{A}$  in the `KeyDec` step of  $\text{CondEval.Eval}^*$  will be captured with high probability (assuming that  $\mathcal{A}$  does not know  $\psi$ ). As a result, the probability that  $\mathcal{A}$  flips  $(s \wedge f(x))^m$  to  $(\neg(s \wedge f(x)))^m$  without being detected is in negligible probability. In conclusion, compared

with the construction of  $\text{CondEval.Eval}$ , this extended construction  $\text{CondEval.Eval}^*$  comes at a cost of  $m$  times the computation time and communication volume when dealing with FSS gates; Nevertheless this malicious construction retains the advantage of the optimized one round communication.

The concrete construction of  $\text{CondEval.Eval}^*$  is shown in the following pseudo code, where it takes in the desired operation  $\wedge$ , as well as  $\mathcal{K}_\wedge^*, (s)^m, \text{JxK}^\text{A}$  from the two servers; During the online evaluation, for each  $i \in [m]$ , within one round of communication  $\delta_{x,i}$  is revealed, and simultaneously one clear-text FSS key is obtained by each of the two servers by decrypting one FSS key from other party's choice; By line 16 of  $\text{CondEval.Eval}^*$ , we run  $\mathcal{F}_{\text{MacVryGen}}$  in Fig. 4 to generate a proof to verify that each mask value  $\delta_{x(i)}$  for  $i \in [m]$  is revealed honestly. Finally,  $\text{CondEval.Eval}^*$  outputs  $(y)^m$  and an associated proof  $[\sigma]^\text{A}$  which can only be validated if they could pass the verification procedure in functionality  $\text{CondEval.Verify}$  as shown below.

---

**Functionality**  $(y)^m, [\sigma]^\text{A} \leftarrow \text{CondEval.Eval}^*(\wedge, \mathcal{K}_\wedge^*, (s)^m, \text{JxK}^\text{A})$

---

**Players:**  $S_0, S_1$ .

**Functionality:**  $(y)^m, [\sigma]^\text{A} \leftarrow \text{CondEval.Eval}^*(\wedge, \mathcal{K}_\wedge^*, (s)^m, \text{JxK}^\text{A})$ .

**Input:** For each  $b \in \{0, 1\}$  that  $\mathcal{K}_\wedge^*[b] = \{\{C_{i,b}, \text{SK}_{i,b}, \text{Jr}_i^{\text{K}^\text{A}}\}_{i \in [m]}, [\psi]^\text{B}, [\Delta]^\text{A}\}$  are obtained by  $S_b$  in the setup phase, and authenticated secret sharing  $(s)^m$  whose secret authentication key is equal to  $\psi$  in  $\mathcal{K}_\wedge^*$ .

**Output:**  $((s \wedge f(x))^m, [\sigma]^\text{A})$ .

1:  $(\{[s_1]^\text{B}, \dots, [s_m]^\text{B}\}) \leftarrow (s)^m$

2:  $\mathbf{V} \leftarrow [\mathbf{0}]^m$

3: **for**  $b = 0$  to  $1$  **do**

4:  $[\sigma]_b^\text{A} \leftarrow \mathbf{0}$

5:  $\{C_b, \text{SK}_b, \text{Jr}_i^{\text{K}^\text{A}}, [\psi]^\text{B}\} \leftarrow \mathcal{K}_\wedge^*[b]$

6: **for**  $i = 1$  to  $m$  **do**

7:  $\text{J}\delta_{x(i)}^{\text{K}^\text{A}} \leftarrow \text{JxK}^\text{A} + \text{Jr}_i^{\text{K}^\text{A}}$

8:  $\delta_{x(i)} \leftarrow \mathcal{F}_{\text{Reveal}}([\delta_{x(i)}^{\text{K}^\text{A}}])$

9:  $\mathbf{V}[i] \leftarrow (\delta_{x(i)}, [\Delta \cdot \delta_{x(i)}]^\text{A})$

10: **for**  $b = 0$  to  $1$  **do**

11:  $S_b$ : Sends  $\{[s_i]_b^\text{B} \oplus \pi_{1-b}, \text{sk}_{[s_i]_b}^{(1-b)}\}$  to  $S_{1-b}$  as  $\{p^{(1-b)}, \text{sk}^{(1-b)}\}$

► Key decryption

12: **for**  $b = 0$  to  $1$  **do**

13:  $S_b$ :  $k^{(b)} \parallel \text{id}^{(b)} \parallel \xi^{(b)} \leftarrow C_{i,b}[p^{(b)}] \oplus \text{sk}^{(b)}$

14:  $[\sigma]_b^\text{A} \leftarrow [\sigma]_b^\text{A} + \xi^{(b)}$

15:  $[y_i]_b^\text{B} \leftarrow \text{Eval}^f(k^{(b)}, \text{id}^{(b)}, \delta_{x(i)})$

16:  $[\zeta]^\text{A} \leftarrow \mathcal{F}_{\text{MacVryGen}}([\Delta]^\text{A}, \mathbf{V})$

17:  $[\sigma]^\text{A} \leftarrow [\zeta]^\text{A} + [\sigma]^\text{A}$

18:  $(y)^m \leftarrow \{[y_1]^\text{B}, \dots, [y_m]^\text{B}\}$

19: Output  $(y)^m, [\sigma]^\text{A}$

---

We have the theorem of correctness, security and soundness for  $\text{CondEval}$  that work in the malicious setting in the following, however, due to page limit, we have attached their associated proofs in Appendix A. It is important to note that there may be a potential integrity attack on our scheme from  $S_0$ , which could go undetected by our  $\text{CondEval.Verify}$  procedure. This type of attack might occur if  $S_0$  maliciously modifies  $z < m$  partial decryption keys before transferring them to  $S_1$ . Specifically, if these altered keys cause all corresponding decrypted Function Secret Sharing

**Table 6: The construction of  $\mathcal{K}_\lambda^* \leftarrow \text{CondEval.Setup}^*(\wedge, 1^\lambda, \ell_0, \ell_1, m)$  in the malicious setting, it inputs a security parameter  $\lambda$ ,  $\ell_0$  and  $\ell_1$ , where  $\ell_0$  is both used in generating FSS keys and it also defines the domain of the input secrets,  $\ell_1$  defines the domain of authenticated secret key  $\Delta$ ,  $m \in \mathbb{Z}^+$  defines the secret key for authenticated Boolean secret sharing.**

Let $\psi \leftarrow_s \{0, 1\}^m$ , $[\psi]_0^B \leftarrow_s \{0, 1\}^m$ , $[\psi]_1^B \leftarrow \psi \oplus [\psi]_0^B$ ; $\Delta \leftarrow_s \mathbb{Z}_{2^{\ell_1}}$ , $[\Delta]^\wedge \leftarrow \text{SS.share}(\Delta, \mathbb{Z}_{2^{\ell_0+\ell_1}})$ then it does <i>KeyGen</i> and <i>KeyEnc</i> for each $i \in [m]$ as follows:		
KeyGen	If $\psi[i] = 0$ ( <b>Normal</b> )	Otherwise if $\psi[i] = 1$ ( <b>Trap</b> )
	$\beta_1 \leftarrow 1, \beta_2 \leftarrow 0$	$\beta_1 \leftarrow 0, \beta_2 \leftarrow 0$
	$r_i \leftarrow_s \mathbb{Z}_{2^{\ell_0}}$ , $[r_i]^\wedge \leftarrow \text{SS.share}(r_i, \mathbb{Z}_{2^{\ell_0+\ell_1}})$ , $Jr_i^{\mathcal{K}^\wedge} \leftarrow \{[r_i]^\wedge, \text{SS.share}(r_i \cdot \Delta, \mathbb{Z}_{2^{\ell_0+\ell_1}})\}$ ; $(k_0, k_1) \leftarrow \text{Gen}(1^\lambda, r_i, \beta_1, \beta_2, \mathbb{Z}_2)( k_0  =  k_1  = K)$ $t \leftarrow_s \mathbb{Z}_2$ , $\pi_0 \leftarrow_s \mathbb{Z}_2$ , $\pi_1 \leftarrow_s \mathbb{Z}_2$ ; $\omega \leftarrow_s \mathbb{Z}_2$ ; $\xi_1 \leftarrow_s \mathbb{Z}_{2^{\ell_0+\ell_1}}$ , $\xi_2 \leftarrow_s \mathbb{Z}_{2^{\ell_0+\ell_1}}$ ; $L := K + \ell_0 + \ell_1 + 1$ ; $\text{sk}_0^{(0)} \parallel \text{sk}_1^{(0)} \parallel \text{sk}_0^{(1)} \parallel \text{sk}_1^{(1)} \leftarrow_s \{0, 1\}^{4L}$ , $ \text{sk}_0^{(0)}  =  \text{sk}_1^{(0)}  =  \text{sk}_0^{(1)}  =  \text{sk}_1^{(1)}  = L$	
KeyEnc	$M_{i,0} = \{k_t \parallel t \parallel \xi_1, k_{1-t} \parallel (1-t) \parallel \xi_1\}$	$M_{i,0} = \{k_t \parallel t \parallel \xi_1, k_{1-t} \parallel (1-t) \parallel \xi_2\}$
	$M_{i,1} = \{k_t \parallel t \parallel (-\xi_1), k_{1-t} \parallel (1-t) \parallel (-\xi_1)\}$	$M_{i,1} = \{k_t \parallel t \parallel (-\xi_1), k_{1-t} \parallel (1-t) \parallel (-\xi_2)\}$
	$C_{i,0} = \mathcal{F}_{\text{Permu}}(\pi_0, \{M_{i,0}[0] \oplus \text{sk}_0^{(0)}, M_{i,0}[1] \oplus \text{sk}_1^{(0)}\})$ , $\text{SK}_{i,0} = \{(\text{sk}_0^{(1)}, \text{sk}_1^{(1)}), \pi_1\}$ $C_{i,1} = \mathcal{F}_{\text{Permu}}(\pi_1, \{M_{i,1}[0] \oplus \text{sk}_0^{(1)}, M_{i,1}[1] \oplus \text{sk}_1^{(1)}\})$ , $\text{SK}_{i,1} = \{(\text{sk}_0^{(0)}, \text{sk}_1^{(0)}), \pi_0\}$	
Outputs $\psi, \mathcal{K}_\lambda^* = \{\{C_{i,j}, \text{SK}_{i,j}, Jr_i^{\mathcal{K}^\wedge}\}_{i \in [m], j \in \{0,1\}}, [\Delta]^\wedge\}$		

**Functionality**  $z \leftarrow \text{CondEval.Verify}((y)^m, [\sigma]^\wedge, \psi)$

**Players:**  $S_0, S_1, B$ .

**Functionality:**  $s \leftarrow \text{CondEval.Verify}(\{(y)_b^m, [\sigma]_b^\wedge\}_{b \in \{0,1\}}, \psi)$ .

**Input:**  $((y)_b^m, [\sigma]_b^\wedge)$  from  $S_b$  for each  $b \in \{0, 1\}$ , and  $\psi$  from  $B$ .

**Output:**  $z \in \{-1, 0, 1, \perp\}$ .

```

1:  $z \leftarrow -1$ 
2: if  $\psi < \{1\}^m$  then
3:   if  $[\sigma]_0^\wedge + [\sigma]_1^\wedge = 0$  then
4:     for  $i = 1$  to  $m$  do
5:        $y_i \leftarrow [y_i]_0^B \oplus [y_i]_1^B$ 
6:       if  $\psi[i] = 0$  then
7:         if  $z = -1$  then
8:            $z \leftarrow y_i$ 
9:         else
10:        if  $y_i < s$  then
11:           $z \leftarrow \perp$ , abort
12:        else
13:        if  $y_i < 0$  then
14:           $z \leftarrow \perp$ , abort
15:        else
16:         $z \leftarrow \perp$ , abort
17: Outputs  $z$ .
```

(FSS) keys—despite having partially flipped bits—to produce the same evaluation result as the original, unaltered FSS keys, the attack could succeed without detection. Notably, the probability of such an integrity attack succeeding decreases as the number  $z$  of altered keys increases. However, this potential vulnerability does not affect the overall correctness or soundness of our scheme, and therefore, we have not addressed it further in this discussion.

**Theorem 3. Correctness.** Assuming all servers indeed honestly follow  $\text{CondEval.Eval}^*(\wedge, \mathcal{K}_\lambda^*, (s)^m, Jx^{\mathcal{K}^\wedge})$ , i.e., none of the servers deviate from the protocol description, then they obtain  $((y)^m, [\sigma]^\wedge)$ . Denote  $p$  the probability that  $\text{CondEval.Verify}((y)^m, [\sigma]^\wedge, \psi)$  is equal to  $s \wedge f(x)$ , we claim  $p = 1 - 1/2^m$ .

**Theorem 4. Security.** In the presence of an active PPT adversary  $\mathcal{A}$  among two servers in  $\text{CondEval.Eval}^*(\wedge, \mathcal{K}_\lambda^*, (s)^m, Jx^{\mathcal{K}^\wedge})$ , where

$\mathcal{K}_\lambda^*$  comes from the output of functionality  $\text{CondEval.Setup}^*$ , we assert that  $\mathcal{A}$  learns no information about  $x, s, s \wedge f(x)$  or  $\psi$ .

**Theorem 5. Soundness.** Assuming the existence of an active PPT adversary  $\mathcal{A}$  ( $S_0$  or  $S_1$ ) when performing  $\text{CondEval.Eval}^*(\wedge, \mathcal{K}_\lambda^*, (s)^m, Jx^{\mathcal{K}^\wedge})$  that outputs  $((y)^m, [\sigma]^\wedge)$ . We denote by  $p$  the probability that  $\text{CondEval.Verify}((y)^m, [\sigma]^\wedge, \psi) = \neg(s \wedge f(x))$ . We claim

$$p < 2^{-\ell_1 + \log(\ell_1 + 1)} + 2^{1-m} + 2^{-(\ell_0 + \ell_1)}$$

where  $\ell_1$  denotes the length of the message authentication key.

## 6 SECURE COSINE SIMILARITY COMPUTATION AND VERIFICATION

In this section, we provide comprehensive constructions for cosine similarity back-end verification (matching fresh and reference templates) in both semi-honest and malicious settings, utilizing building blocks from Section 4. For example, in the semi-honest setting within a pre-processing model, our proposed protocol initially calculates a Boolean secret sharing  $[c_1]^B$  and an arithmetic secret sharing  $[z]^\wedge$  using optimized secret sharing and FSS. Then, it employs  $\text{CondEval.Eval}(\wedge, [c_1]^B, [x]^\wedge)$  to compute  $[c_1 \wedge \text{Sign}(z)]^B$  as the final authentication bit.

The solutions we propose for evaluating  $\mathcal{F}_{\text{ScaledCosAuth}}$  in a 2PC model has the dual objectives of minimizing the number of communication rounds and communication volume. Our method leverages optimized secret sharing and  $\text{CondEval}$  to perform secure addition, multiplication, and comparison operations within  $\mathcal{F}_{\text{ScaledCosAuth}}$ . The online evaluation process requires two communication rounds, with a communication cost of four ring elements (corresponds to the four times of  $\mathcal{F}_{\text{reveal}}$  invocations) and two decryption keys transmitted for decrypting the FSS keys.

When using floating-point numbers as input for a secure computation framework, these secret floating-point numbers need to be converted to fixed-point representation by multiplying them by a precision parameter  $2^\rho$ . This process occurs before entering the secure computation framework, where  $\rho$  defines the preserved precision in the fraction part. In a standard way, a secure truncation

operation is required to obtain the same precision when performing one multiplication over two fixed point values. This is meaningful in the sense of performing an operation within a smaller ring and also outputs an arithmetical result with the same precision. However, in our actual evaluation of  $\mathcal{F}_{\text{ScaledCosAuth}}$ , where the input vectors are small float point numbers, as already mentioned in 3.2 in order to reduce the cost of the underlying FSS module, we perform truncation once over  $1/\tau^2 \cdot \text{IP}(\mathbf{x}, \mathbf{y})^2 - \text{IP}(\mathbf{x}, \mathbf{x}) \cdot \text{IP}(\mathbf{y}, \mathbf{y})$  in  $\mathcal{F}_{\text{ScaledCosAuth}}$ , such that it results in a smaller input domain of FSS.

**Full protocol in the semi-honest setting.** In the semi-honest setting, our final full protocol  $\Pi_{\text{NomadicSH}}$  comprises of three sub-protocols: Setup,  $\mathcal{F}_{\text{Input}}$  and Eval, executed in the Setup, Input, and Evaluation phases, respectively. Sub-protocol 1 details the setup phase, integrating CondEval.Setup and generating correlated randomness depends on  $\mathcal{F}_{\text{ScaledCosAuth}}$ . Next, the client and B input their secret vectors  $\mathbf{x}, \mathbf{y}$  by running  $\mathcal{F}_{\text{Input}}$  in Fig. 5. Lastly, with correlated randomness and input prepared, Eval produces the Boolean secret sharing of the desired authentication bit  $[y]^B$ , where  $y = \text{Sign}(\cos(\mathbf{x}, \mathbf{y}) - \tau)$ . Please refer to the complete full protocol construction and its corresponding correctness/security analysis in the semi-honest setting in Appendix B.1.

**Full protocol in the malicious setting.** Due to page limitations, please refer to Appendix B.2 for the complete protocol  $\Pi_{\text{NomadicM}}$  which implements  $\mathcal{F}_{\text{ScaledCosAuth}}$  in a malicious setting.

## 7 EXPERIMENT

In this section, we aim to test the improved efficiency of  $\Pi_{\text{NomadicSH}}$  over the naive protocol  $\Pi_{\text{OptimSH}}$  in the semi-honest setting, as well as  $\Pi_{\text{NomadicM}}$  over the naive protocol  $\Pi_{\text{NaiveM}}$  in the malicious setting. To achieve that, we implement all four protocols in Python 3.10 [17] and have performed assessments of them through a use case of voice biometric authentication. In the following, we first describe the dataset being used, then the experimental setting, and finally present the performance comparison of all protocols.

### 7.1 Data

We employ a pre-trained model<sup>3</sup> from ECAPA-TDNN [25] to extract speaker embeddings. The model was trained using the development part of the VoxCeleb2 dataset [19] with 5994 speakers. The datasets RIR [37] and MUSAN [58] were also used for data augmentation. The model is composed of 3 SE-Res2Block modules. The channel size and the dimension of the bottleneck in the SEBlock are set to 1024 and 256, respectively. The entire utterance is fed into the ECAPA-system to finally obtain a vector of 192-dimensional speaker embedding. We get our experiments' input from the test set of VoxCeleb1-E [47], which consists of 37720 pairs of enrolment and verification ECAPA-TDNN embeddings, in which we have approximately half target and half non-target speakers. It is important to note that the resulting threshold  $\tau$  in  $\mathcal{F}_{\text{ScaledCosAuth}}$  was established at a point where the False Acceptance Rate (FAR) is 1.15%, the False Rejection Rate (FRR) is 1.14%, and the Equal Error Rate (EER) is 1.15%.

### 7.2 Setting

We have performed all experiments on a relatively low-spec machine with 12th Gen Intel(R) Core(TM) i7-12700K processor, and 2 x 16 GB dual-channel DDR5 4400 MHz RAM. The hosting machine runs Ubuntu 22.04 LTS. We separately compared the performance of all protocols in the offline and online phase. Specifically, for the evaluation of the online phase, we created three simulated network environments using traffic control tools to emulate real-world network conditions.

To adapt the raw data from the ECAPA-TDNN embeddings [47] with float-point numbers to our protocols that deal with integers, we multiply the floating-point numbers with  $2^8$  ( $\rho = 8$ ) and keep only the integer digits of the multiplication result to maintain precision in the fractional part of the raw data. We remark that employing a conversion parameter of  $\rho = 8$  yields a non-significant deviation (0.003%) in terms of false acceptance rate (FAR) and false rejection rate (FRR), when compared to directly using floating-point numbers.

Regarding the FSS module underlying our protocols, which includes the interval containment FSS that implements Sign and our CondEval, we have universally set the security parameter  $\kappa = 128$ . The input domain is configured as  $\{0, 1\}^{32}$ , which sufficiently covers our use case, and we do implementation of the interval containment FSS gate as per in Fig.3 of [10].

Specifically, we perform all the secure computation operation in a ring of 64 bits for protocols  $\Pi_{\text{OptimSH}}$  and  $\Pi_{\text{NomadicSH}}$  in the semi-honest setting; in the malicious setting we perform all operations in a ring of  $64 + 32$  bits, where 64 bits covers the circuit computation and 32 bits is the size of the secret authentication parameter  $\alpha$  we selected. Additionally, in the implementation of  $\Pi_{\text{NomadicM}}$  we set  $m = 30$  which is marginally smaller than the length of the secret authentication parameter  $\alpha$ , according to Theorem 5 it gives us a soundness  $p < 2^{-26}$ .

### 7.3 Results

We measured the offline computation time and communication volume as illustrated in Fig. 2, required by executing each protocol once. These results are quantified per server; specifically, the communication volume represents the data transmitted from the dealer to each server, while the computation time pertains to the duration required to generate correlated randomness data for each server. It is important to note that, across all four protocols, the dealer distributes correlated randomness in accordance with  $\mathcal{F}_{\text{ScaledCosAuth}}$ . This distribution is independent of the actual inputs used later in the online phase. Therefore, it is typically acceptable to incur higher costs during the offline phase. As depicted in Fig. 2, when comparing our protocol  $\Pi_{\text{NomadicSH}}$  with protocol  $\Pi_{\text{OptimSH}}$ ,  $\Pi_{\text{NomadicSH}}$  incurs a marginally higher cost in both computation time and communication volume. In contrast, for our protocol  $\Pi_{\text{NomadicM}}$  compared to  $\Pi_{\text{OptimSH}}$ , there is a noticeable increase in both computation time and communication volume. This escalation is attributed to the utilization of multiple FSS keys in computing  $c_1$  and  $c_2$  of  $\mathcal{F}_{\text{ScaledCosAuth}}$ , and is considered an acceptable trade-off for the offline phase.

Fig. 3 presents the online execution time to run  $\mathcal{F}_{\text{ScaledCosAuth}}$  with one pair of enrolment (reference) and verification (fresh) vectors as input in different network settings for all protocols. These

<sup>3</sup><https://github.com/TaoRuijie/ECAPA-TDNN>

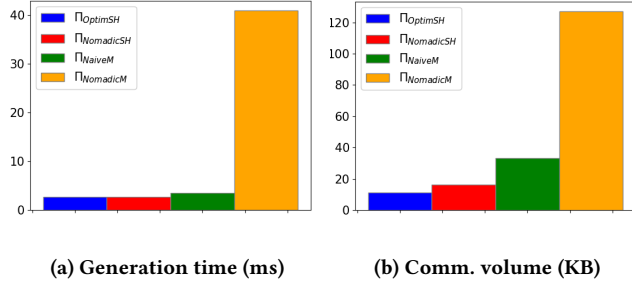


Figure 2: Offline costs measured per server

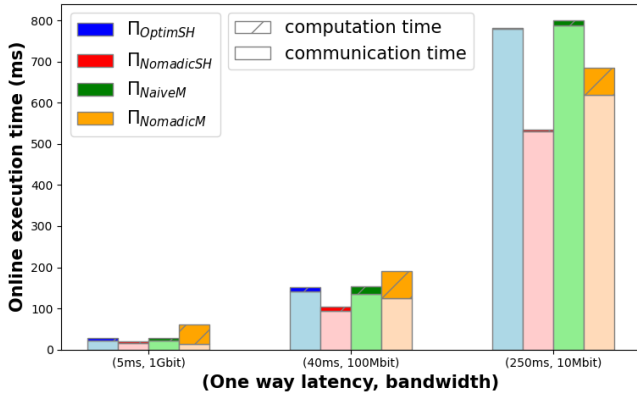


Figure 3: Online performance comparison of NOMADIC protocols VS existing protocols for local/LAN/WAN networks.

measurements started from the moment all input secret sharing was in place and continued until the final secret sharing of the output bit  $c$  of  $\mathcal{F}_{ScaledCosAuth}$  was obtained. Here, the execution time for each protocol is further divided into two parts, the local computation time and communication time. We have set up three common network settings with different network latency and bandwidth, which are denoted as LAN (10ms, 1Gbps), WAN1 (80ms, 100Mbps) and WAN2 (500ms, 10Mbps). The cost is measured for 20 runs on average. The online communication volume measured from our experiments are 0.09 KB, 0.69 KB, 0.21 KB, and 20.77 KB respectively for protocol  $\Pi_{OptimSH}$ ,  $\Pi_{NomadicSH}$ ,  $\Pi_{NaiveM}$  and  $\Pi_{NomadicM}$ .

In the semi-honest setting,  $\Pi_{NomadicSH}$  demonstrates a significant improvement in efficiency over the naive protocol  $\Pi_{OptimSH}$ . Specifically,  $\Pi_{NomadicSH}$  achieves a 31% reduction in online execution time compared to  $\Pi_{OptimSH}$ , even though it costs more communication volume (0.69 KB) than what in  $\Pi_{OptimSH}$  (0.09 KB). This efficiency is attributed to the reduced number of rounds required;  $\Pi_{NomadicSH}$  operates in two rounds, as opposed to  $\Pi_{OptimSH}$  that requires three rounds.

In the malicious setting,  $\Pi_{NomadicM}$  requires less communication time compared to the naive protocol  $\Pi_{NaiveM}$  in each of the network settings, however, it does not exhibit efficiency gains over the naive

protocol  $\Pi_{NaiveM}$  in total except for the WAN2 setting. This performance characteristic primarily results from the use of multiple FSS keys, specifically  $m = 30$  in our experiments. Our tests revealed a total computational cost of approximately 45ms for completing evaluations of  $c_1$  and  $c$  in our protocol  $\Pi_{NomadicM}$ . This computational burden, however, is not intractable. One approach, as demonstrated in Funshade [33], involves using an efficiency-focused programming language. Their C implementation, for instance, requires a local computation time of just 0.55 ms, including the evaluation of one interval containment gate and a local inner product. Based on this, the local computation time for the multiple FSS components in our protocol  $\Pi_{NomadicM}$  is estimated to be no more than 16.5ms if implemented in C. Alternatively, using more powerful hardware can certainly reduce the computation overhead.

Considering that reducing wide area network latency will likely remain challenging in the near future, our  $\Pi_{NomadicM}$  protocol becomes a preferable choice. This is due to one round reduction compared to  $\Pi_{NaiveM}$ , making it more effective despite potentially higher computational demands. In summary, our experimental results demonstrate an online evaluation efficiency improvement for our proposed protocols over the SotA when computing the cosine similarity functionality in the squared domain, especially in the WAN setting, where the network latency is a bottleneck for efficient secure computation, making it suitable for real-world applications.

## 8 CONCLUSION

Privacy-preserving cosine similarity computation and comparison to a predefined threshold is an important building block that has multiple applications (e.g., biometric authentication and identification, privacy-preserving machine learning). In this paper, we introduce two novel protocols to compute the cosine similarity and compare to a threshold in a privacy-preserving way, i.e.,  $\Pi_{NomadicSH}$  for the semi-honest setting, and  $\Pi_{NomadicM}$  the malicious setting. Both  $\Pi_{NomadicSH}$  and  $\Pi_{NomadicM}$  rely on the recent advances of FSS [10, 12]. And  $\Pi_{NomadicM}$  additionally relies on 2PC authenticated secret sharing and our proposed use of multiple instances of random FSS keys (either *normal* or *trap*). All our protocols are based on a new primitive  $CondEval$  that allows computing the composition of an input bit  $s$  and a binary function  $f$  (evaluated via FSS) on an input  $x$  i.e.,  $s \circ f(x)$ .  $CondEval$  is provably secure under both the semi-honest and malicious setting, is general and of independent interest; Thus, could be used in general 2PC computations.

Furthermore, we provide a detailed security analysis of the proposed protocols, introduced building blocks and evaluate the proposed protocols for the use of biometric authentication. Our results show that the proposed protocols are not only round-efficient, necessitating merely two communication rounds, but also proved to exhibit enhanced efficiency though our bench-marking in comparison to SotA.

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## A SUPPLEMENT FOR CONDEVAL IN THE MALICIOUS SETTING

**A proof of theorem 3.** Assume all servers follow  $\text{CondEval.Eval}^*(\wedge, \mathcal{K}_\lambda^*, (s)^m, \mathcal{J}_x^{\mathcal{K}^\wedge})$  honestly.  $\forall i \in [m]$ , if  $\psi[i] = 0$ . From Theorem 1, we know  $[y_i]^B$  is equal to  $[s \wedge f(x)]^B$  as expected. Otherwise if  $\psi[i] = 1$ , since  $[s]_0^B, [s]_1^B$  is either  $\{0, 0\}$  or  $\{1, 1\}$ , which implies  $y_i = \text{Eval}^f(k_t, t, \delta_x) \oplus \text{Eval}^f(k_t, t, \delta_x)$  or  $y_i = \text{Eval}^f(k_{1-t}, 1-t, \delta_x) \oplus \text{Eval}^f(k_{1-t}, 1-t, \delta_x)$ , then  $[y_i]^B$  is equal to  $[0]^B$ . Thus, the functionality outputs a valid authenticated Boolean secret sharing. However, in the case of  $\psi = \{0, 1\}^m$  which has a probability of  $1/2^m$  the protocol  $\text{CondEval.Verify}((y)^m, [\sigma]^A, \psi)$  outputs  $-1$ . Thus,  $p$  is equal to  $1 - 1/2^m$ .

**A proof of theorem 4.** After online evaluation, for each  $b \in \{0, 1\}$ ,  $S_b$ 's transcript view is composed of transcript views of

$\text{CondEval.Eval}(\mathcal{K}_\lambda^*[i], [s_i]^B, [x]^A)$  for each  $i \in [m]$ . From Theorem 2, we know that  $\text{CondEval.Eval}^*(\wedge, \mathcal{K}_\lambda^*, (s)^m, \mathcal{J}_x^{\mathcal{K}^\wedge})$  does not leak any information about  $x$ ,  $s$ , or  $s \wedge f(x)$ . Furthermore, owing to the computational indistinguishability of FSS keys from entirely random keys, based on the security of the pseudorandom generator [10], a probabilistic polynomial-time (PPT) the probability that  $\mathcal{A}$  distinguishes between a trap key instance, a completely random key, or a normal key instance is negligible. Consequently,  $\mathcal{A}$  gains no information about  $\psi$ .

**A proof of theorem 5.** If  $\text{CondEval.Verify}((y)^m, [\sigma]^A, \psi) = \neg(s \wedge f(x))$ , w.l.o.g., we assume  $S_0$  is the malicious server in the functionality  $\text{CondEval}^*$ . The actual computation can be performed in the following two ways, only:

- Case 1:  $S_0$  honestly follows  $\text{CondEval.Eval}^*(\wedge, \mathcal{K}_\lambda^*, (s)^m, \mathcal{J}_x^{\mathcal{K}^\wedge})$ .
- Case 2:  $S_0$  arbitrarily deviates from  $\text{CondEval.Eval}^*(\wedge, \mathcal{K}_\lambda^*, (s)^m, \mathcal{J}_x^{\mathcal{K}^\wedge})$ ,
  - either by flipping  $[s_i]_0^B (i \in [m])$  when performing the *KeyDec* step OT,
  - or, by dishonestly flipping partial shares when reporting  $(s)^m$  to the receiver.

We denote by  $p_1$  and  $p_2$  the probability that  $\text{CondEval.Verify}((y)^m, [\sigma]^A, \psi) = \neg(s \wedge f(x))$  under cases 1 and 2, respectively.

In the first case, from Theorem 3 we know that the protocol outputs  $-1$  with probability  $1/2^m$ , and  $s \wedge f(x)$  with probability of  $1 - 1/2^m$  it outputs. Thus, the protocol never outputs  $\neg(s \wedge f(x))$  and hence,  $p_1 = 0$ .

In the second case, we claim there are three events A, B, C in which  $S_0$  that could manipulate the protocol and thus, which may result in outputting a flipped authentication bit  $(\neg(s \wedge f(x)))^m$ :

- (1) A:  $S_0$  reports one or more incorrect arithmetic shares, submits a manipulated proof  $\sigma_0^*$  such that, still,  $\sigma_0^* + \sigma_1 = 0$ , and resulting to a valid flipped authenticated bit  $(y^*)^m$  where  $y^* = \neg(s \wedge f(x))$ .
- (2) B:  $S_0$  manages to flip the computation result  $(y)^m$  to  $(-y)^m$  by flipping  $[s_i]_0^B$  for all  $\psi[i] = 0$ ,  $i \in [m]$  when performing the *KeyDec* step, and also presents a correct proof that passes the final verification.
- (3) C:  $\forall i \in [m]$ ,  $S_0$  manages to flip each  $[y_i]_0^B$  for  $\psi[i] = 0$ , and keeps  $[y_i]_0^B$  unchanged for  $\psi[i] = 1$ . In the end,  $S_0$  submits his manipulated sharing of  $(y)^m$  to B.

Each event is considered independently. Concerning event A, according to [20], it is known that the probability that event A occurs is no greater than  $2^{-\ell_1 + \log(\ell_1 + 1)}$ .

Regarding event B, the final outcome is altered when  $f(x) = 1$  and only when all the *normal* indexes ( $\psi[i] = 0$ ) are flipped during the *KeyDec* step. We analyse the probability that  $S_0$  successfully flips all *normal* indexes without being detected as follows: First, we distinguish two complementary events  $B_1$  and  $B_2$  that are covered by B. We denote by  $B_1$  the event that  $S_0$  manages to flip the corresponding bit  $[s_i]_0^B$  for each  $i \in [m]$  where  $\psi[i] = 0$ , and keeps the corresponding bit  $[s_i]_0^B$  unchanged for each  $i \in [m]$  where  $\psi[i] = 1$ . On the other hand,  $B_2$  corresponds to the event when  $S_0$  manages to flip the corresponding bit  $[s_i]_0^B$  for each  $i \in [m]$  where  $\psi[i] = 0$ , and at least one of the corresponding bit  $[s_i]_0^B$  is wrongly flipped

**FUNCTIONALITY**  $[\zeta]^A \leftarrow \mathcal{F}_{\text{MacVryGen}}([\Delta]^A, L)$   
**Players:**  $S_0, S_1$ .  
**Input:** An arithmetic secret sharing of the MAC authentication key  $[\Delta]^A$ , along with a list  $L = (x_i, [\Delta \cdot x_i]^A)_{i \in [n]}$ , in which  $\forall i \in [n]$  that  $x_i, r_i$  is public and  $[\Delta \cdot x_i]^A$  is the arithmetic secret sharing over  $\Delta \cdot x_i$ .  
**Output:**  $[\zeta]^A$   
 1: **for**  $b = 0$  to **do**  
 2:      $[\zeta]_b^A \leftarrow 0$   
 3:     **for**  $i = 1$  to **do**  
 4:          $[\zeta]_b^A \leftarrow [\zeta]_b^A + [\Delta]_b^A \cdot x_i - [\Delta \cdot x_i]_b^A$   
 5:  $S_0$  and  $S_1$  re-randomize  $[\zeta]^A$  locally.  
 6: **Outputs**  $[\zeta]^A$

**Figure 4: Our Homomorphic MAC Verification proof generation in the malicious setting.**

for  $\psi[i] = 1$ , i.e.,  $S_0$  does not perfectly correctly guesses all  $\psi[i]$  where  $\psi[i] = 1$ , instead  $S_0$  wrongly determines  $\psi[i] = 0$  by at least one index  $i$  where in fact  $\psi[i]$  is equal to 1. The reason why we distinguish  $B_1$  from  $B_2$  is because for event  $B_1$  the adversary  $S_0$  does not need to do anything to pass the verification. However, for event  $B_2$  the adversary  $S_0$  needs to select a random offset value  $e$  added to  $[\sigma]_0^A$  to pass the verification (it happens with probability  $2^{-(\ell_0 + \ell_1)}$ ), the reason is because in event  $B_2$  that  $S_0$  wrongly assumed  $\psi[i] = 0$  by at least one index  $i$  for  $\psi[i] = 1$ , under this case servers obtain an associated secret share of a non-zero random value.

Because we know  $\psi$  is uniformly generated, say  $S_0$  flips each bit  $[s_i]_0^B$  independently for each  $i \in [m]$  with probability  $q$ . Then, we have the probability of event  $B_1$

$$P(B_1) = \frac{q}{2} + \frac{1-q}{2} m = 2^{-m},$$

and the probability of event  $B_2$

$$P(B_2) = \frac{q}{2} + \frac{1}{2} m - P(B_1) \cdot 2^{-(\ell_0 + \ell_1)} < 2^{-(\ell_0 + \ell_1)}.$$

Let  $p^* \in [0, 1]$  be the probability when the input  $f(x)$  is equal to 1, then we have

$$P(B) = P^* \cdot (P(B_1) + P(B_2)) < P^* \cdot (2^{-m} + 2^{-(\ell_0 + \ell_1)})$$

Denote  $P(B^*)$  the probability that an occurrence of event  $B$  results in an flipped result  $y = \neg(s \wedge f(x))$ , which we know it happens only when  $\psi < \{1\}^m$ , so we have  $P(B^*) < P(B)$ .

For event  $C$ ,  $P(C) = 1/2^m$ . Thus,

$$\begin{aligned}
 p_2 &< P(A) + P(B^*) + P(C) \\
 &< P(A) + P(B) + P(C) \\
 &< 2^{-\ell_1 + \log(\ell_1 + 1)} + \frac{p^* + 1}{2^m} + 2^{-(\ell_0 + \ell_1)} \\
 &< 2^{-\ell_1 + \log(\ell_1 + 1)} + 2^{1-m} + 2^{-(\ell_0 + \ell_1)}.
 \end{aligned}$$

In conclusion,

$$p = p_1 + p_2 < 2^{-\ell_1 + \log(\ell_1 + 1)} + 2^{1-m} + 2^{-(\ell_0 + \ell_1)}.$$

**FUNCTIONALITY**  $\langle x \rangle^A$  or  $\perp \leftarrow \mathcal{F}_{\text{Input}}(x, [r_{\text{in}}]^A)$ :  
**Players:** A client  $C$ ,  $S_0$  and  $S_1$ .  
**Input:** A secret  $x$  from  $C$ , and  $[r_{\text{in}}]^A$  from  $S_0, S_1$ .  
**Output:**  $\langle x \rangle^A$  or  $\perp$ .  
 1:  $\forall b \in \{0, 1\}$ ,  $S_b$  sends  $[r_{\text{in}}]_b^A$  to  $C$ .  
 2:  $\forall b \in \{0, 1\}$ ,  $C$  returns  $\delta_x^b = x + [r_{\text{in}}]_0^A + [r_{\text{in}}]_1^A$  to  $S_b$ .  
 3:  $\forall b \in \{0, 1\}$ ,  $S_b$  exchanges  $\delta_x^b$  with  $S_{1-b}$  to check if  $\delta_x^b$  is equal to  $\delta_x^{1-b}$ . If both the verification pass, denote  $\delta_x = \delta_x^0 = \delta_x^1$ , then servers output  $\langle x \rangle^A = (\delta_x, [r_{\text{in}}]_b^A)$ , otherwise output  $\perp$ .

**Figure 5: Input validation protocol in the semi-honest server setting.**

## B FULL PROTOCOL CONSTRUCTIONS FOR BIOMETRIC AUTHENTICATION AND THEIR ANALYSIS

In the following, we present the full protocol construction and corresponding analysis for biometric authentication respectively in the semi-honest setting and malicious setting.

### B.1 Full protocol in the semi-honest setting

**B.1.1  $\mathcal{F}_{\text{Input}}$  for  $\mathcal{F}_{\text{ScaledCosAuth}}$  in the semi-honest setting.** Fig. 5 details the client input validation protocol in a semi-honest server setting. The protocol ensures the validity of the client's input by having the client broadcast a masked value  $\delta_x$  to both servers. The servers, which gain no knowledge of the actual input  $x$ , verify the consistency of  $\delta_x$  with each other to detect any potential malicious client.

**B.1.2 Correctness.** From Theorem 1 we know that Protocol 2 outputs  $[c]^B$  where  $c = c_1 \wedge c_2$ ,  $c_1 = \text{Sign}(\text{IP}(x, y))$  and  $c_2 = \text{Sign}(1/t^2 \cdot \text{IP}(x, y)^2 - \text{IP}(x, x) \cdot \text{IP}(y, y))$ , thus computes  $\text{Sign}(\cos(x, y) - \tau)$  as shown in Eq. 3.

**B.1.3 Security.** Our goal is to prove that sub-protocols 1 and 2 provide a secure implementation of  $\mathcal{F}_{\text{ScaledCosAuth}}$  when faced with a semi-honest PPT adversary  $\mathcal{A}$  in a 2PC setting. We assert that, with the correlated randomness provided in sub-protocol 1, and following the online evaluation of sub-protocol 2,  $\mathcal{A}$  learns nothing about the input  $x, y$ , or  $\text{Sign}(\cos(x, y) - \tau)$ .

**PROOF.** By applying Theorem 2, we ensure that there is no information leakage from the internal view tapes of  $\text{CondEval.Eval}_{\text{trunc}}(\wedge, \mathcal{K}_\wedge, [c_1]^B, [z]^A)$ . However, we need to consider the other view transcripts  $\text{View}_b$  for each  $b \in \{0, 1\}$ , where:

$$\text{View}_b := \{k_b^{(1)}, \delta_u, \delta_v, \delta_w\}$$

As  $\delta_v$  and  $\delta_w$  hide the correlated intermediate values of  $x$  and  $y$ , we argue that the view  $k_b^{(1)}$  is pseudo-random (computationally indistinguishable from the real random key) as proved in Fig. 1 from [10], thereby concealing the information of  $\alpha^{(1)}$  that is contained in  $(k_0^{(1)}, k_1^{(1)})$  from  $\mathcal{A}$ .

---

**Protocol 1**  $([\mathcal{R}]^A, \mathcal{K}_\lambda, \mathcal{K}_0) \leftarrow \text{Setup}(1^\lambda, \ell, \rho)$ 


---

**Players:** The central bank B.**Functionality:**  $([\mathcal{R}]^A, \mathcal{K}) \leftarrow \text{Setup}(1^\lambda, \ell, \rho)$ .**Input:** A security parameter  $\lambda$ , element encoding length  $\ell$ , and a truncation parameter  $\rho \in \mathbb{Z}^+$  where  $\rho < \ell$ .**Output:**  $([\mathcal{R}]^A, \mathcal{K})$ .

- 1:  $(\mathbf{x}_{in}, \mathbf{y}_{in}) \leftarrow \mathbb{S} Z_{2\ell}^{2n}, \mathbf{u}_{in} \leftarrow \text{Prod}(\mathbf{x}_{in}, \mathbf{y}_{in})$
  - 2:  $\mathbf{v}_{in} \leftarrow \text{Prod}(\mathbf{x}_{in}, \mathbf{x}_{in}), \mathbf{w}_{in} \leftarrow \text{Prod}(\mathbf{y}_{in}, \mathbf{y}_{in})$
  - 3:  $\mathcal{K}_\lambda \leftarrow \text{CondEval.Setup}(\lambda, 1^\lambda, \ell)$
  - 4:  $\{C_i, \text{SK}_i, [r]_i^A\}_{i \in \{0,1\}} \leftarrow \mathcal{K}_\lambda$
  - 5:  $\alpha^{(2)} \leftarrow [r]_0^A + [r]_1^A$
  - 6:  $v \leftarrow \mathbb{S} Z_{2\rho}, [\eta]^A \leftarrow \text{SS.Share}(\text{Shift}(\alpha^{(2)}, -\rho, \ell) + v, Z_{2\ell+\rho})$
  - 7:  $\tilde{\mathcal{K}}_\lambda \leftarrow \{C_i, \text{SK}_i, [\eta]_i^A\}_{i \in \{0,1\}}$
  - 8:  $\alpha^{(1)} \leftarrow \mathbb{S} Z_{2\ell}$
  - 9:  $(k_0, k_1) \leftarrow \text{Gen}_\ell^{[0,2^{\ell-1}]}(1^\lambda, \alpha^{(1)}, 1, 0, Z_2)$
  - 10:  $\mathcal{R} \leftarrow \{\mathbf{x}_{in}, \mathbf{y}_{in}, \mathbf{u}_{in}, \mathbf{v}_{in}, \mathbf{w}_{in}, (r_1, r_2, r_1 r_2), \alpha^{(1)}, (\alpha^{(1)})^2\}$
  - 11:  $\mathcal{K}_0 \leftarrow \{k_0, k_1\}$
  - 12: Outputs  $([\mathcal{R}]^A, \tilde{\mathcal{K}}_\lambda, \mathcal{K}_0)$
- 

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**Protocol 2**  $[c]^B \leftarrow \text{Eval}([\mathcal{R}]^A, \tilde{\mathcal{K}}_\lambda, \mathcal{K}_0, \langle \mathbf{x} \rangle^A, \langle \mathbf{y} \rangle^A, \tau, \rho, \ell)$ 


---

**Players:**  $S_0, S_1$ .**Functionality:**  $[c]^B \leftarrow \text{Eval}([\mathcal{R}]^A, \tilde{\mathcal{K}}_\lambda, \mathcal{K}_0, \langle \mathbf{x} \rangle^A, \langle \mathbf{y} \rangle^A, \tau, \rho, \ell)$ .**Input:**  $[\mathcal{R}]^A, \mathcal{K}_\lambda, \mathcal{K}_0, \langle \mathbf{x} \rangle^A, \langle \mathbf{y} \rangle^A$  from  $S_0$  and  $S_1$  respectively. A public threshold  $\tau \in (0, 1]$ , element encoding length  $\ell \in \mathbb{Z}^+$ , and a truncation parameter  $\rho \in \mathbb{Z}^+$  where  $\rho < \ell$ .**Output:**  $[c]^B$ 

- 1:  $\{k_0, k_1\} \leftarrow \mathcal{K}_0$
  - 2:  $\{\mathbf{x}_{in}, \mathbf{y}_{in}, \mathbf{u}_{in}, \mathbf{v}_{in}, \mathbf{w}_{in}, (r_1, r_2, r_1 r_2), \alpha^{(1)}, (\alpha^{(1)})^2\} \leftarrow \mathcal{R}$
  - 3:  $[u]^A \leftarrow \text{SS.IP}(\langle \mathbf{x} \rangle^A, \langle \mathbf{y} \rangle^A, [\mathbf{u}_{in}]^A)$
  - 4:  $[v]^A \leftarrow \text{SS.IP}(\langle \mathbf{x} \rangle^A, \langle \mathbf{x} \rangle^A, [\mathbf{v}_{in}]^A)$
  - 5:  $[w]^A \leftarrow \text{SS.IP}(\langle \mathbf{y} \rangle^A, \langle \mathbf{y} \rangle^A, [\mathbf{w}_{in}]^A)$
  - 6:  $\delta_u \leftarrow \mathcal{F}_{\text{Reveal}}([u + \alpha^{(1)}]^A)$  ▷ First round
  - 7:  $\langle u \rangle^A \leftarrow (\delta_u, [\alpha^{(1)}]^A)$
  - 8:  $\langle v \rangle^A \leftarrow (\mathcal{F}_{\text{Reveal}}([v + r_1]^A), [r_1]^A)$  ▷ First round
  - 9:  $\langle w \rangle^A \leftarrow (\mathcal{F}_{\text{Reveal}}([w + r_2]^A), [r_2]^A)$  ▷ First round
  - 10: **for**  $b = 0$  to  $1$  **do**
  - 11:  $[c_1]_b^B \leftarrow \text{Eval}_\ell^{[0,2^{\ell-1}]}(k_b, b, \delta_u)$
  - 12:  $T \leftarrow \text{Shift}(\frac{1}{2}, -\rho, \ell)$  ▷ after shifting,  $T \in Z_{2\ell}$
  - 13:  $[z]^A \leftarrow T \cdot \text{SS.MUL}(\langle u \rangle^A, \langle u \rangle^A, [(\alpha^{(1)})^2]^A)$
  - 14:  $[z]^A \leftarrow [z]^A - 2\rho \cdot \text{SS.MUL}(\langle v \rangle^A, \langle w \rangle^A, [r_1 r_2]^A)$
  - 15:  $[c]^B \leftarrow \text{CondEval.Eval}_{\text{trunc}}(\lambda, \tilde{\mathcal{K}}_\lambda, [c_1]^B, [z]^A)$  ▷ Second round
  - 16: Outputs  $[c]^B$
- 

## B.2 Full protocol for $\mathcal{F}_{\text{ScaledCosAuth}}$ in the malicious setting

We present sub-protocols 3 and 4 which evaluate  $\mathcal{F}_{\text{ScaledCosAuth}}$  in a stronger threat model, with one malicious server who might deviate from the protocol description in sub-protocol 2. More specifically:

- In sub-protocol 3, B first runs  $\text{CondEval}^*.Setup(1^\lambda)$ , then from those results it obtains  $\Delta$  and  $\psi$ , which are secret keys for authenticating an arithmetical revelation and a Boolean revelation in the later online evaluation. Afterwards, B additionally generates an authenticated random secret sharing  $\mathcal{I}_x, \mathcal{I}_y$  to be used in input phase. It also generates a function dependent authenticated correlated random secret sharing

$\mathcal{J}\mathcal{R}K^A$  from  $\Delta$ , and random FSS keys  $\mathcal{K}_0$  from  $\psi$  for computing  $c_1$ ;

- After the setup phase, whenever the input is ready, (*i.e.*, the client provides her fresh template  $\mathbf{x}$ ), the client and the two servers coordinately run:

$$\langle\langle \mathbf{x}_i \rangle\rangle^A \leftarrow \mathcal{F}_{\text{Input}^*}(\mathbf{x}_i, [\Delta]^A, \mathcal{J}\mathbf{r}_{in}^{(x)}[i]K^A, \mathcal{J}\mathbf{x}_{in}[i]K^A)$$

where  $\mathcal{F}_{\text{Input}^*}$  is shown in Fig. 6;

- Then in the evaluation phase, the two servers evaluate the on-line sub-protocol 4  $\text{Eval}^*$  in which we incorporate a variant version of  $\text{CondEval.Eval}^*$  denoted as  $\text{CondEval.Eval}_{\text{trunc}}^*$ , whose sole difference to the former is to perform a local truncation after each  $\delta_x$  is revealed (similarly to the process applied in the semi-honest setting);
- After completing the evaluation phase in Protocol 4, the servers obtain a final authenticated boolean secret sharing  $(y)^m$  and an associated proof  $[\sigma]^A$ .
- Upon receiving  $(y)^m$  and  $[\sigma]^A$  from the servers, with value  $\psi$  already known in the setup phase, B runs  $\text{CondEval.Verify}((y)^m, [\sigma]^A, \psi)$  and outputs a symbol  $z \in \{-1, 0, 1, \perp\}$ . B accepts the authentication bit as  $z$  if  $z$  is not in  $\{-1, \perp\}$ .

We note here that via sub-protocol 4, we are able to capture a client who may act maliciously *i.e.*, may attempt to impersonate a legitimate user and deduce information for the corresponding templates. Thus, we incorporate a mechanism to check that the secret sharing of the fresh (as well as the reference) template have been secret shared correctly.

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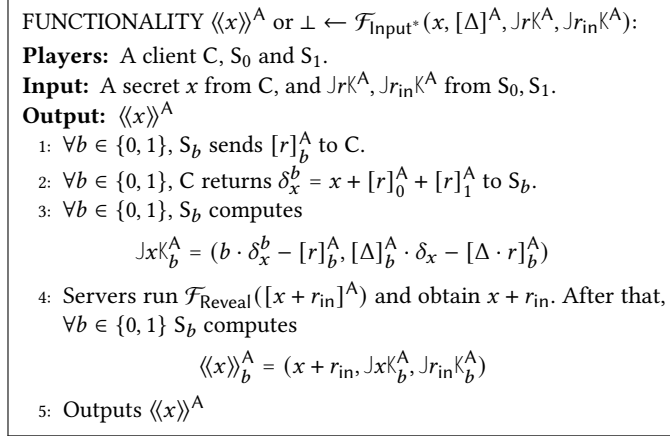
**Protocol 3**  $(\mathcal{I}_x, \mathcal{I}_y, \mathcal{J}\mathcal{R}K^A, \tilde{\mathcal{K}}_\lambda^*, \mathcal{K}_0) \leftarrow \text{Setup}^*(1^\lambda, m, \ell_0, \ell_1, \rho)$ 


---

**Players:** B.**Functionality:**  $(\mathcal{I}_x, \mathcal{I}_y, \mathcal{J}\mathcal{R}K^A, \tilde{\mathcal{K}}_\lambda^*, \mathcal{K}_0) \leftarrow \text{Setup}^*(1^\lambda, m, \ell_0, \ell_1, \rho)$ .**Input:** A security parameter  $\lambda$ ,  $\ell, \ell_1, \rho \in \mathbb{Z}^+$ .**Output:**  $(\mathcal{I}_x, \mathcal{I}_y, \mathcal{J}\mathcal{R}K^A, \tilde{\mathcal{K}}_\lambda^*, \mathcal{K}_0)$ .

- 1:  $\mathcal{K}_\lambda^* \leftarrow \text{CondEval}^*.Setup(\lambda, 1^\lambda, \ell_0, \ell_1, m)$
  - 2:  $\{\{C_{i,j}, \text{SK}_{i,j}, \mathcal{J}\mathbf{r}_{in}^{(x)}\}_{i \in [m], j \in \{0,1\}}, [\psi]^B, [\Delta]^A\} \leftarrow \mathcal{K}_\lambda^*$
  - 3:  $\Delta \leftarrow [\Delta]_0^A + [\Delta]_1^A, \psi \leftarrow [\psi]_0^A \oplus [\psi]_1^A$
  - 4:  $\mathbf{r}_{in}^{(x)} \leftarrow \mathbb{S} Z_{2\ell}^n, \mathcal{J}\mathbf{r}_{in}^{(x)K^A} \leftarrow \text{SS.Share}(\{\mathbf{r}_{in}^{(x)}, \Delta \cdot \mathbf{r}_{in}^{(x)}\}, Z_{2\ell_0+\ell_1})$
  - 5:  $\mathbf{r}_{in}^{(y)} \leftarrow \mathbb{S} Z_{2\ell}^n, \mathcal{J}\mathbf{r}_{in}^{(y)K^A} \leftarrow \text{SS.Share}(\{\mathbf{r}_{in}^{(y)}, \Delta \cdot \mathbf{r}_{in}^{(y)}\}, Z_{2\ell_0+\ell_1})$
  - 6:  $(\mathbf{x}_{in}, \mathbf{y}_{in}) \leftarrow \mathbb{S} Z_{2\ell}^{2n}, \mathbf{u}_{in} \leftarrow \text{Prod}(\mathbf{x}_{in}, \mathbf{y}_{in})$
  - 7:  $\mathbf{v}_{in} \leftarrow \text{Prod}(\mathbf{x}_{in}, \mathbf{x}_{in}), \mathbf{w}_{in} \leftarrow \text{Prod}(\mathbf{y}_{in}, \mathbf{y}_{in})$
  - 8:  $(r_1, r_2) \leftarrow \mathbb{S} Z_{2\ell}^2$
  - 9: **for**  $i = 1$  to  $m$  **do**
  - 10:  $\alpha^{(1,i)} \leftarrow \mathbb{S} Z_{2\ell}$
  - 11:  $r \leftarrow \mathbb{S} Z_{2\rho}$
  - 12:  $\alpha^{(2,i)} \leftarrow [r]_0^A + [r]_1^A$
  - 13:  $\eta_i \leftarrow \text{Shift}(\alpha^{(2,i)}, -\rho, \ell) + r$
  - 14:  $(k_0^{(1,i)}, k_1^{(1,i)}) \leftarrow \text{Gen}_\ell^{[0,2^{\ell-1}]}(1^\lambda, \alpha^{(1,i)}, 1 - \psi[i], 0, Z_2)$
  - 15:  $\mathcal{J}\eta_i K^A \leftarrow (\text{SS.Share}(\eta_i, Z_{2\ell_0+\ell_1}), \text{SS.Share}(\Delta \cdot \eta_i, Z_{2\ell_0+\ell_1}))$
  - 16:  $\tilde{\mathcal{K}}_\lambda^* \leftarrow \{\{C_{i,j}, \text{SK}_{i,j}, \mathcal{J}\eta_i K^A\}_{i \in [m], j \in \{0,1\}}, [\psi]^B, [\Delta]^A\}$
  - 17:  $\mathcal{R} \leftarrow \{\mathbf{x}_{in}, \mathbf{y}_{in}, \mathbf{u}_{in}, \mathbf{v}_{in}, \mathbf{w}_{in}, (r_1, r_2, r_1 r_2), (\alpha^{(1,1)})^2\}$
  - 18:  $\mathcal{J}\mathcal{R}K^A \leftarrow (\text{SS.Share}(\mathcal{R}, Z_{2\ell_0+\ell_1}), \text{SS.Share}(\mathcal{R} \cdot \Delta, Z_{2\ell_0+\ell_1}))$
  - 19:  $\mathcal{K}_0 \leftarrow \{k_0^{(1,i)}, k_1^{(1,i)}\}_{i \in [m]}$
  - 20:  $\mathcal{I}_x \leftarrow (\mathcal{J}\mathbf{r}_{in}^{(x)K^A}, \mathcal{J}\mathbf{x}_{in}K^A), \mathcal{I}_y \leftarrow (\mathcal{J}\mathbf{r}_{in}^{(y)K^A}, \mathcal{J}\mathbf{y}_{in}K^A)$
  - 21: Outputs  $(\mathcal{I}_x, \mathcal{I}_y, \mathcal{J}\mathcal{R}K^A, \tilde{\mathcal{K}}_\lambda^*, \mathcal{K}_0)$
-





**Figure 6: Input validation protocol in the malicious setting.**

---

**Protocol 4**  $((y)^m, [\sigma]^A) \leftarrow \text{Eval}^*(J\mathcal{R}K^A, \langle\langle \mathbf{x} \rangle\rangle^A, \langle\langle \mathbf{y} \rangle\rangle^A, \mathcal{K}_\lambda^*, \mathcal{K}_0, \tau, \rho)$

---

**Players:**  $S_0, S_1$ .

**Functionality:**  $((y)^m, [\sigma]^A) \leftarrow \text{Eval}^*(J\mathcal{R}K^A, \langle\langle \mathbf{x} \rangle\rangle^A, \langle\langle \mathbf{y} \rangle\rangle^A, \mathcal{K}_\lambda^*, \mathcal{K}_0, \tau, \rho)$ .

**Input:** Secret sharing  $[\Delta]^A, JrK^A, \langle\langle \mathbf{x} \rangle\rangle^A, \langle\langle \mathbf{y} \rangle\rangle^A$  and  $\mathcal{K}_1$  from  $S_0, S_1, \mathcal{K}_2$  from B, a public threshold  $\tau \in (0, 1]$  and a truncation parameter  $\rho \in \mathbb{Z}^+$ .

**Output:**  $((y)^m, [\sigma]^A)$ .

- 1:  $\mathbf{p} \leftarrow [\emptyset]^{m+2}$
- 2:  $JuK^A \leftarrow \text{SS.IP}(\langle\langle \mathbf{x} \rangle\rangle^A, \langle\langle \mathbf{y} \rangle\rangle^A, J\mathbf{u}_{\text{in}}K^A)$
- 3:  $JvK^A \leftarrow \text{SS.IP}(\langle\langle \mathbf{x} \rangle\rangle^A, \langle\langle \mathbf{x} \rangle\rangle^A, J\mathbf{v}_{\text{in}}K^A)$
- 4:  $JwK^A \leftarrow \text{SS.IP}(\langle\langle \mathbf{y} \rangle\rangle^A, \langle\langle \mathbf{y} \rangle\rangle^A, J\mathbf{w}_{\text{in}}K^A)$
- 5:  $\delta_v \leftarrow \mathcal{F}_{\text{Reveal}}([v + r_1]^A)$  ▷ First round
- 6:  $\delta_w \leftarrow \mathcal{F}_{\text{Reveal}}([w + r_2]^A)$  ▷ First round
- 7:  $\mathbf{p}[1] \leftarrow (\delta_v, [\Delta \cdot \delta_v]^A), \mathbf{p}[2] \leftarrow (\delta_w, [\Delta \cdot \delta_w]^A)$
- 8:  $\langle\langle v \rangle\rangle^A \leftarrow (\delta_v, JvK^A, Jr_1K^A)$
- 9:  $\langle\langle w \rangle\rangle^A \leftarrow (\delta_w, JwK^A, Jr_2K^A)$
- 10:  $\{C_{i,j}, SK_{i,j}, Jr_iK^A\}_{i \in [m], j \in \{0,1\}}, [\psi]^B, [\Delta]^A \leftarrow \mathcal{K}_\lambda^*$
- 11:  $\{k_0^{(1,i)}, k_1^{(1,i)}\}_{i \in \{0,1\}} \leftarrow \mathcal{K}_0$
- 12: **for**  $i = 1$  **to**  $m$  **do**
- 13:  $J\delta_{u(i)}K^A \leftarrow JuK^A + J\alpha^{(1,i)}K^A$
- 14:  $\delta_{u(i)} \leftarrow \mathcal{F}_{\text{Reveal}}([\delta_{u(i)}]^A)$  ▷ First round
- 15: **for**  $b = 0$  **to**  $1$  **do**
- 16:  $[s_i]_b^B \leftarrow \text{Eval}_t^{[0, 2^{\ell-1}]}(k_b^{(1,i)}, b, \delta_{u(i)})$
- 17:  $\mathbf{p}[2+i] \leftarrow (\delta_{u(i)}, [\Delta \cdot \delta_{u(i)}]^A)$
- 18:  $(s)^m \leftarrow \{[s_1]^B, \dots, [s_m]^B\}$
- 19:  $T \leftarrow \text{Shift}(\frac{1}{2}, -\rho, \ell)$
- 20:  $JzK^A \leftarrow T \cdot \text{SS.MUL}(\langle\langle u^{(1)} \rangle\rangle^A, \langle\langle u^{(1)} \rangle\rangle^A, J(\alpha^{(1,1)})^2K^A)$
- 21:  $JzK^A \leftarrow JzK^A - 2\rho \cdot \text{SS.MUL}(\langle\langle v \rangle\rangle^A, \langle\langle w \rangle\rangle^A, Jr_1r_2K^A)$
- 22:  $((y)^m, [\sigma]^A) \leftarrow \text{CondEval.Eval}_{\text{trunc}}^*(\wedge, \mathcal{K}_\lambda^*, (s)^m, JzK^A)$  ▷ Second round
- 23:  $[\zeta]^A \leftarrow \mathcal{F}_{\text{MacVryGen}}([\Delta]^A, \mathbf{p})$
- 24:  $[\sigma]^A \leftarrow [\sigma]^A + [\zeta]^A$
- 25: Outputs  $((y)^m, [\sigma]^A)$ .

---

**B.2.1 Soundness.** In sub-protocol 4, assuming an active adversary  $\mathcal{A}$ , the soundness of functionality  $\text{CondEval.Eval}_{\text{trunc}}^*$  has

been proven in Theorem 5. Furthermore, thanks to the protection provided by the MAC scheme of other operations using authenticated arithmetic sharing, let  $p$  represent the probability that  $\text{CondEval.Verify}((y)^m, [\sigma]^A, [\psi]^B) = 1 - \text{Sign}(\cos(\mathbf{x}, \mathbf{y}) - \tau)$ . Consequently, we can deduce

$$p < 2^{-\ell_1 + \log(\ell_1 + 1)} + 2^{1-m} + 2^{-(\ell_0 + \ell_1)}.$$

**B.2.2 Security.** We assert that, sub-protocols 3 and 4 provide a secure implementation of  $\mathcal{F}_{\text{ScaledCosAuth}}$  when faced with an active PPT  $\mathcal{A}$  in the 2PC setting. Specifically, with the correlated randomness provided in Protocol 3, following the online evaluation of sub-protocol 4,  $\mathcal{A}$  learns nothing about the inputs  $\mathbf{x}, \mathbf{y}$ , or  $\text{Sign}(\cos(\mathbf{x}, \mathbf{y}) - \tau)$ .

**PROOF.** From Theorem 4, we first exclude information leakage within  $\text{CondEval.Eval}_{\text{trunc}}^*(\wedge, \mathcal{K}_\lambda^*, (s)^m, JzK^A)$ . Still, we need to prove that the view transcript

$$\text{View}_b := \{\{k_b^{(1,i)}, \delta_{u(i)}\}_{i \in [m]}, \delta_v, \delta_w\}$$

of  $S_b$  for each  $b \in \{0, 1\}$  does not leak any information. This is true, as  $\delta_v, \delta_w \sim \cup_{\mathcal{N}}$  information theoretically hide the associated intermediate computation results; Nevertheless, for  $\{(k_b^{(1,i)}, \delta_{u(i)})\}_{i \in [m]}$  which are correlated, we argue that  $\mathcal{A}$  has only the view  $k_b^{(1,i)}$  which is pseudo-random (computationally indistinguishable from a real random key) as proved in Fig. 1 in [10]; Thus, hiding the information of  $\alpha^{(1,i)}$  contained in  $(k_0^{(1,i)}, k_1^{(1,i)})$  from  $\mathcal{A}$ .

**B.2.3 Truncating  $\delta_x$ .** Note that while executing Eval, a variant version of  $\text{CondEval.Eval}$  which we denote as  $\text{CondEval.Eval}_{\text{trunc}}$  is incorporated whereby  $\delta_x$  is truncated additionally. Namely, it performs

$$\delta_x \leftarrow \text{Shift}(\delta_x, 4\rho, \ell)$$

in  $\text{CondEval.Eval}_{\text{trunc}}$  while the remaining steps in  $\text{CondEval.Eval}$  remain unchanged. Previously, when dealing with the truncation of  $[x]^A, [\eta]^A$  was generated and corresponds to the secret sharing result of the sum of the shifting of  $r$  to the left by  $\rho$  and a random value  $v \in \mathbb{Z}_{2\rho}$ ; In the above equation, the fractional part of  $\delta_x$  is truncated by  $4\rho$  bits due to the accumulation of the three multiplications over secret sharing and one multiplication over a public scalar value ( $T, 2^\rho$  resp. in line 12,14). Such a truncation does not imply any additional communication round and is performed during the second communication round. Furthermore, as it will be shown in section 7.3, such a truncation has almost no impact on the actual accuracy of the protocol.