

# Maliciously Secure Multi-Party PSI with Lower Bandwidth and Faster Computation

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## Abstract

Private Set Intersection (PSI) allows a set of mutually distrustful parties, each holds a private data set, to compute the intersection of all sets, such that no information is revealed except for the intersection. The state-of-the-art PSI protocol (Garimella et al., CRYPTO'21) in the multi-party setting tolerating any number of malicious corruptions requires the communication bandwidth of  $O(n\ell|\mathbb{F}|)$  bits for the central party  $P_0$  due to the star architecture, where  $n$  is the number of parties,  $\ell$  is the size of each set and  $|\mathbb{F}|$  is the size of an exponentially large field  $\mathbb{F}$ . When  $n$  and  $\ell$  are large, this forms an efficiency bottleneck (especially for networks with restricted bandwidths). In this paper, we present a new multi-party PSI protocol in dishonest-majority malicious setting, which reduces the communication bandwidth of the central party  $P_0$  from  $O(n\ell|\mathbb{F}|)$  bits to  $O(\ell|\mathbb{F}|)$  bits using a tree architecture. Furthermore, our PSI protocol reduces the expensive LPN encoding operations performed by  $P_0$  by a factor of  $n$  as well as the computational cost by  $2n\ell$  hash operations in total. Additionally, while the multi-party PSI protocol (Garimella et al., CRYPTO'21) with a single output is secure, we present a simple attack against its multi-output extension, which allows an adversary to learn more information on the sets of honest parties beyond the intersection of all sets.

## 1 Introduction

Private Set Intersection (PSI) allows a set of mutually distrustful parties, where each holds a private set, to compute the intersection of all sets, without revealing anything beyond the intersection. PSI and its variants have found a wide variety of applications, including measuring the effectiveness of online advertising [IKN<sup>+</sup>17, IKN<sup>+</sup>20], private contact discovery [KRS<sup>+</sup>19, DRRT18] and more. In the two-party setting, PSI protocols has been extensively studied and become truly practical with extremely fast implementations (see the recent work [GN19, PRTY19, PRTY20, CM20, RS21, GPR<sup>+</sup>21, RR22] and references therein). While two-party PSI is interesting for many applications, there are a lot of applications which are better suitable for the multi-party setting. For example, a) several companies intend to combine their data sets to find a target audience for an ad campaign [IKN<sup>+</sup>20]; b) a variant of multi-party PSI was recently used for cache sharing in edge computing, which allows multiple network operators to obtain a set of common data items with the highest access frequencies in the privacy-preserving way [NT21]. We refer the reader to [NTY21] for more interesting examples that are suited to the multi-party case.

The problem of multi-party PSI was first introduced in [FNP04]. The previous work [FNP04, KS05, SS07, SS08, CJS10, HV17, GN19] constructed theoretical multi-party PSI protocols, where all of these protocols are not concretely efficient (especially for large sets). The first practical PSI protocol in the multi-party setting was proposed by Kolesnikov et al. [KMP<sup>+</sup>17]. This protocol is secure against semi-honest adversaries in the dishonest-majority setting (i.e., the adversary can corrupt up to  $n - 1$  parties of the  $n$  parties but must follow the protocol specification). For semi-honest security, efficient multi-party PSI protocols were further developed, including [IOP18, KMS21] in the dishonest-majority setting based on Garbled Bloom Filter (GBF), and [CDG<sup>+</sup>21] in the honest-majority setting (i.e., the adversary can corrupt up to less than a half of parties). In this work, we focus on multi-party PSI protocols in the dishonest-majority setting in the presence of malicious adversaries (i.e., the adversary allows to run an arbitrary attack strategy in its attempt to break the protocol). This is the strongest adversary model that was considered in the previous PSI protocols.

In the dishonest-majority malicious setting, several concretely efficient PSI protocols [ZLL<sup>+</sup>19, ENOPC21, GPR<sup>+</sup>21, NTY21] have been proposed. Among these protocols, the multi-party PSI protocol by Garimella et al. [GPR<sup>+</sup>21] achieves the best efficiency in this setting. This protocol builds on the multi-party PSI protocol with augmented semi-honest security [KMP<sup>+</sup>17], and instantiates the underlying Oblivious Programmable Pseudo-Random Function (OPPRF) primitive with the Oblivious Key-Value Store (OKVS) scheme [GPR<sup>+</sup>21] and Oblivious Pseudo-Random Function (OPRF). Garimella et al. [GPR<sup>+</sup>21] use a 3-hash garbled cuckoo table to design the OKVS scheme, which is an optimized version of the PaXoS construction [PRTY20] and can achieve much better communication efficiency than GBF. The state-of-the-art protocol [RS21] to realize OPRF adopts the recent LPN-based Vector Oblivious Linear-function Evaluation (VOLE) protocol with *sublinear* communication [BCGI18, BCG<sup>+</sup>19b, BCG<sup>+</sup>19a, SGRR19, BCG<sup>+</sup>20, WYKW21, CRR21]. Garimella et al. [GPR<sup>+</sup>21] modified the augmented semi-honest PSI protocol [KMP<sup>+</sup>17] into a maliciously secure protocol by adding a random oracle to wrap the OPRF output,<sup>1</sup> and proved that the modified multi-party PSI protocol is secure against malicious adversaries. Although the state-of-the-art multi-party PSI protocol [GPR<sup>+</sup>21] is concretely efficient, we observed the following two aspects that need to be further improved and are addressed by this work.

- The multi-party PSI protocol [GPR<sup>+</sup>21] adopts the *star* architecture for communication. Specifically, the central party  $P_0$  will interact with parties  $P_1, \dots, P_n$  to compute  $n$  OPRF outputs that will include  $n$  VOLE protocol executions, and then receives  $n$  OKVS from the  $n$  parties. When  $n$  and the size  $\ell$  of each set are large, the communication bandwidth of  $P_0$  is  $O(n\ell|\mathbb{F}|)$  bits, which forms an efficiency bottleneck (especially for networks with restricted bandwidths), where  $|\mathbb{F}|$  is the size of an exponentially large field  $\mathbb{F}$ .
- The  $n$  VOLE executions for computing  $n$  OPRF outputs will require  $n$  encoding operations of Learning Parity with Noise (LPN) for the central party  $P_0$ , where the LPN encoding is computationally expensive [YWL<sup>+</sup>20] and forms a computational efficiency bottleneck of the state-of-the-art VOLE protocol with malicious security [WYKW21].

## 1.1 Our Contributions

In this paper, we propose a new multi-party PSI protocol in the dishonest-majority malicious setting, which improves the state-of-the-art multi-party PSI protocol [GPR<sup>+</sup>21] in the following two aspects.

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<sup>1</sup>A similar observation was also made by Nevo et al. [NTY21].

MP-PSI	Topology	Communication	Bandwidth	Rounds
[ENOPC21]	Star	$O(nl\kappa^2 + nl\kappa \log(\ell\kappa))$	$O(nl\kappa^2 + nl\kappa \log(\ell\kappa))$	4
[GPR+21]	Star	$O(n^2\kappa + nl(\kappa + \rho + \log \ell))$	$O(nl(\kappa + \rho + \log \ell))$	2
This work	Tree	$O(n^2\kappa + nl(\kappa + \rho + \log \ell))$	$O(\ell(\kappa + \rho + \log \ell))$	$2\lceil \log(n+1) \rceil$

Table 1: **Comparison between our protocol and recent multi-party PSI protocols tolerating all-but-one malicious corruptions.**  $n + 1$  is the number of parties,  $\ell$  is the size of each set, and  $\kappa$  and  $\rho$  are the computational and statistical security parameters respectively. The bandwidth column denotes the maximum communication sent or received by each party.

- We reduce the computation cost of LPN encoding for the central party  $P_0$  implied in the PSI protocol [GPR+21] by a factor of  $n$ . Meanwhile, we further reduce the total computation cost of their PSI protocol by  $2n\ell$  hash operations. To achieve the efficiency gain, we construct the PSI protocol by directly using VOLE in the multi-party setting (instead of calling OPRF) and integrating the repetitive operations (see Section 3.2 for a technical overview).
- Building on the above technique to improve computation, we use a *tree* architecture to reduce the communication bandwidth of the central party  $P_0$ . In particular, we reduce the bandwidth complexity of  $P_0$  from  $O(n\ell)$  field elements to  $O(\ell)$  field elements by amortizing the communication among all parties in a tree network architecture. We present two types of tree architectures: one is used to make  $P_0$  send the same message to parties  $P_1, \dots, P_n$ ; and the other is used to let  $P_1, \dots, P_n$  send the sum of  $n$  different messages to  $P_0$  in an aggregation way (see Section 3.3 for a technical overview).

Our PSI protocol requires the underlying OKVS scheme to be *linear*, which is satisfied by the recent efficient constructions [PTY20, GPR+21, RR22]. We prove security of our protocol in the Universal Composability (UC) model and random oracle model.

In addition, Garimella et al. [GPR+21] proposed a multi-output extension of their multi-party PSI protocol such that all parties can obtain the output instead of only the central party  $P_0$ . While their multi-party PSI protocol with a single output is provably secure, we present a simple attack against the multi-output extension. This attack allows an adversary to leak the information of the sets held by honest parties more than that allowing to be obtained from the intersection of the sets of all parties, even if the adversary behaves semi-honestly. In particular, if only  $P_0$  is honest, then the adversary can leak some secret items in the private set of  $P_0$ .

**Comparison of communication, bandwidth and rounds.** In Table 1, we compare our protocol with two recent multi-party PSI protocols in the dishonest-majority malicious setting (where the adversary can corrupt up to  $n$  parties of the  $n + 1$  parties). For the sake of simplicity, this table does not compare the recent PSI protocol [NTY21], as it has the same complexities as [GPR+21] in this setting. All costs are counted in the ROT/VOLE-hybrid model, where ROT represents random oblivious transfer. For the state-of-the-art ROT or VOLE protocols, the communication of these protocols is small compared to the whole communication. From this table, we can see that our protocol has the lowest bandwidth complexity while keeping the same communication complexity. In particular, our tree architecture does not increase the total communication cost, compared to the state-of-the-art multi-party PSI protocol [GPR+21]. As a trade-off of lower communication bandwidth, the round complexity of our PSI protocol is increased to  $O(\log n)$ , compared to the multi-party PSI protocol [GPR+21, NTY21] with the round complexity of  $O(1)$ .

### Functionality $\mathcal{F}_{\text{mpsi}}$

**Parameters:** The size  $\ell$  of the input set for every honest party. The maximal allowed size  $\ell' \geq \ell$  of the input set for any malicious party. Let  $\mathcal{C}$  be the set of corrupted parties.

**Execution:** This functionality runs with  $P_0, \dots, P_n$ . Upon receiving (input,  $i, X_i$ ) from every party  $P_i$  for  $i \in [0, n]$ , this functionality executes as follows:

1. If there exists some  $i \in \mathcal{C}$  such that  $|X_i| > \ell'$ , send **abort** to all parties and then abort.
2. If there exists some  $i \notin \mathcal{C}$  such that  $|X_i| > \ell$ , send **abort** to all parties and then abort.
3. Otherwise, compute  $Y := \bigcap_{i \in [0, n]} X_i$ , and then send  $Y$  to  $P_0$ .

Figure 1: **Functionality for multi-party private set intersection.**

## 2 Preliminaries

### 2.1 Notation

We use  $\kappa$  and  $\lambda$  to denote the computational and statistical security parameters, respectively. For  $a, b \in \mathbb{N}$  and  $a \leq b$ , we use  $[a, b]$  to denote the set  $\{a, \dots, b\}$ . For a finite set  $S$ , we use  $x \leftarrow S$  to denote sampling  $x$  uniformly at random from  $S$ . For a vector  $\mathbf{x}$ , we denote by  $x_i$  the  $i$ -th component of  $\mathbf{x}$  with  $x_1$  the first entry. For a set  $S$ , we use  $|S|$  to denote the size of set  $S$ .

### 2.2 Security Model and Functionalities

**Security model.** We use the Universal Composability (UC) framework [Can01] to prove security in the presence of a static, malicious adversary. We say that a protocol  $\Pi$  *UC-realizes* an ideal functionality  $\mathcal{F}$  if for any Probabilistic Polynomial Time (PPT) adversary  $\mathcal{A}$ , there exists a PPT simulator  $\mathcal{S}$ , such that for any PPT environment  $\mathcal{Z}$ , the output distribution of  $\mathcal{Z}$  in the *real-world* execution where the parties interact with  $\mathcal{A}$  and execute  $\Pi$  is computationally indistinguishable from the output distribution of  $\mathcal{Z}$  in the *ideal-world* execution where the parties interact with  $\mathcal{S}$  and  $\mathcal{F}$ .

**Functionality for multi-party PSI.** In a PSI protocol, parties  $P_0, P_1, \dots, P_n$  with each having an input set  $X_i$  of size  $\ell$  compute the intersection of their input sets, i.e.,  $\bigcap_{i \in [0, n]} X_i$ . As a result of the protocol execution,  $P_0$  obtains the intersection output, and all other parties learn nothing. The multi-party PSI functionality is shown in Figure 1. Following prior work such as [RR17, PRTY20, RS21, ENOPC21], we allow the adversary who corrupts a party  $P_i$  to input a set  $X_i$  with  $\ell \leq |X_i| \leq \ell'$ .

**Functionality for VOLE.** Following the previous definition [BCG<sup>+</sup>19a, WYKW21], the functionality for Vector Oblivious Linear-function Evaluation (VOLE) is given in Figure 2. Two parties run the initialization procedure only once, and then can repeatedly call the extend procedure to obtain multiple batches of VOLE correlations. The VOLE functionality can be securely realized by the recent LPN-based protocols with *sublinear* communication [BCGI18, BCG<sup>+</sup>19b, BCG<sup>+</sup>19a, SGRR19, BCG<sup>+</sup>20, WYKW21, CRR21].

### 2.3 Oblivious Key-Value Stores

We recall the definitions of an Oblivious Key-Value Store (OKVS) proposed by Garimella et al. [GPR<sup>+</sup>21]. OKVS is a general notion, and captures the functionality and security property

**Functionality  $\mathcal{F}_{\text{vole}}$**

This functionality runs with a sender  $P_S$ , a receiver  $P_R$  and an adversary, and operates as follows:

**Initialize:** Upon receiving (init) from  $P_S$  and  $P_R$ , if  $P_S$  is honest, then sample  $\Delta \leftarrow \mathbb{F}$ , otherwise receive  $\Delta \in \mathbb{F}$  from the adversary. Store  $\Delta$  and output  $\Delta$  to  $P_S$ , and ignore all subsequent (init) commands.

**Extend:** Upon receiving (extend,  $m$ ) from  $P_S$  and  $P_R$ , do the following:

1. If  $P_S$  is honest, sample  $\mathbf{v} \leftarrow \mathbb{F}^m$ . Otherwise, receive  $\mathbf{v} \in \mathbb{F}^m$  from the adversary.
2. If  $P_R$  is honest, sample  $\mathbf{u} \leftarrow \mathbb{F}^m$  and compute  $\mathbf{w} := \mathbf{v} + \mathbf{u} \cdot \Delta$ . Otherwise, receive  $\mathbf{u}, \mathbf{w} \in \mathbb{F}^m$  from the adversary, and then recompute  $\mathbf{v} := \mathbf{w} - \mathbf{u} \cdot \Delta$ .
3. Output  $\mathbf{v}$  to  $P_S$  and  $(\mathbf{u}, \mathbf{w})$  to  $P_R$ .

Figure 2: **Functionality for vector oblivious linear-function evaluation.**

of existing constructions, including polynomials, dense matrix, garbled Bloom filter [DCW13] and PaXoS [PTY20]. Then, we roughly discuss the state-of-the-art OKVS construction as well as its communication and computation complexities.

**Definition 1** ([GPR+21]). *A key-value store is parameterized by a set  $\mathcal{K}$  of keys, a set  $\mathcal{V}$  of values, and a set  $\mathcal{H}$  of functions, and consists of the following two algorithms:*

- $\text{Encode}_{\mathcal{H}}(\{(k_i, v_i)\}_{i \in [1, \ell]})$  takes as input a set of key-value pairs  $\{(k_i, v_i)\}_{i \in [1, \ell]}$ , and outputs an object  $\mathbf{S}$  (or an error indicator  $\perp$  with statistically small probability).
- $\text{Decode}_{\mathcal{H}}(\mathbf{S}, k)$  takes as input an object  $\mathbf{S}$ , a key  $k$ , and outputs a value  $v$ .

A KVS is correct, if for all  $A \subseteq \mathcal{K} \times \mathcal{V}$  with distinct keys:

$$(k, v) \in A \text{ and } \perp \neq \mathbf{S} \leftarrow \text{Encode}_{\mathcal{H}}(A) \Rightarrow \text{Decode}_{\mathcal{H}}(\mathbf{S}, k) = v.$$

For the sake of simplicity, we choose to omit the underlying parameter  $\mathcal{H}$  in the rest of exposition, as long as the context is clear. In the known OKVS constructions, we always have that the decision whether  $\text{Encode}$  outputs  $\perp$  depends on the functions  $\mathcal{H}$  and the keys  $\{k_i\}_{i \in [\ell]}$ , and is independent of the values  $\{v_i\}_{i \in [\ell]}$ . In the following, we define the security property guaranteeing that one cannot decide whether a key  $k$  was used to generate  $\mathbf{S}$  or not.

**Definition 2** ([GPR+21]). *A KVS is an Oblivious KVS (OKVS), if for all distinct keys  $\{k_1^0, \dots, k_\ell^0\}$  and all distinct keys  $\{k_1^1, \dots, k_\ell^1\}$ , when  $\text{Encode}$  does not output  $\perp$  for both  $(k_1^0, \dots, k_\ell^0)$  and  $(k_1^1, \dots, k_\ell^1)$ , the output of  $\mathcal{R}(k_1^0, \dots, k_\ell^0)$  is computationally indistinguishable from that of  $\mathcal{R}(k_1^1, \dots, k_\ell^1)$ , where  $\mathcal{R}(k_1, \dots, k_\ell)$  is defined as:*

- For  $i \in [1, \ell]$ , sample  $v_i \leftarrow \mathcal{V}$ .
- Output  $\text{Encode}(\{(k_i, v_i)\}_{i \in [1, \ell]})$ .

From the above definition, we have that if the OKVS encodes random values, for any two sets of keys  $K^0, K^1$ , it is infeasible to distinguish the OKVS encoding of the keys of  $K^0$  from that of the keys of  $K^1$ .

Our multi-party PSI protocol requires that an OKVS has some kind of additively homomorphic property. In particular,  $\text{Decode}(\cdot, k)$  is a *linear function* for all  $k \in \mathcal{K}$ . The formal definition is recalled as follows.

**Definition 3** ([GPR+21]). An OKVS is linear over a field  $\mathbb{F}$  if  $\mathcal{V} = \mathbb{F}$  (i.e., “values” are elements of  $\mathbb{F}$ ), then the output of Encode is a vector  $\mathbf{S}$  in  $\mathbb{F}^m$ , and the Decode function is defined as:

$$\text{Decode}(\mathbf{S}, k) = \langle d(k), \mathbf{S} \rangle \stackrel{\text{def}}{=} \sum_{i=1}^m d(k)_i \cdot S_i$$

for some function  $d : \mathcal{K} \rightarrow \mathbb{F}^m$ , where  $d$  is typically defined by hash functions. Thus,  $\text{Decode}(\cdot, k)$  is a linear map from  $\mathbb{F}^m$  to  $\mathbb{F}$ .

The idea of constructing a linear OKVS is that generating a solution to the linear system of equations:

$$\begin{bmatrix} -d(k_1) - \\ -d(k_2) - \\ \vdots \\ -d(k_\ell) - \end{bmatrix} \cdot \mathbf{S}^\top = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_\ell \end{bmatrix}.$$

If Encode chooses uniformly from the set of solutions to the linear system and the values are uniform, then the output  $\mathbf{S}$  is uniformly distributed (and thus independent of the keys). That is, a linear OKVS satisfies the obliviousness property. The recent linear OKVS scheme [GPR+21] (building on the PaXoS technique [PTY20]) has the computation complexity *linear* to the number of key-value pairs, and achieves the rate of  $0.81 - o(1)$ , where the rate of an OKVS that encodes  $\ell$  items from  $\mathbb{F}$  is the ratio between  $\ell \cdot |\mathbb{F}|$  and the size of the OKVS. Very recently, Rindal and Raghuraman [RR22] significantly improved the computation efficiency of the linear OKVS scheme [GPR+21], and achieve the best concrete performance for now.

An OKVS whose parameters are chosen to encode  $N$  items may hold even more than  $N$  items, when it is generated by the adversary. In the context of PSI, this allows the adversary to encode more items than advertised. In Appendix A, we review the the definition of OKVS overfitting to bound the number of items that the adversary can “overfit” into an OKVS, which will be used in the security proof of our PSI protocol.

### 3 Technical Overview

In this section, we give an overview of our techniques to improve the communication bandwidth and computation cost of the state-of-the-art multi-party PSI protocol [GPR+21] tolerating any number of malicious corruptions. The PSI protocol by Garimella et al. [GPR+21] is constructed by transforming the augmented semi-honest PSI protocol [KMP+17] into a maliciously secure version by adding a random oracle to wrap the OPRF output, which is also observed by Nevo et al. [NTY21]. Firstly, we review the multi-party PSI protocol [GPR+21] at a high level.

#### 3.1 Overview of the Best-Known Multi-Party PSI Protocol

The state-of-the-art multi-party PSI protocol [GPR+21] tolerating any number of malicious corruptions, which builds on the augmented semi-honest protocol [KMP+17], executes as follows:

1.  $P_0, P_1, \dots, P_n$  are the  $n+1$  parties who will compute the intersection of input sets  $X_0, X_1, \dots, X_n$  with  $|X_i| = \ell$  for  $i \in [0, n]$ , where  $P_0$  will obtain the output. For any  $h$ , every party  $P_i$  can compute a zero share  $s_h^i$  such that  $\sum_{i \in [0, n]} s_h^i = 0$  by exchanging Pseudo-Random Function (PRF) keys.

### Functionality $\mathcal{F}_{\text{mvole}}$

Let  $\mathcal{C}$  be the set of corrupted parties. This functionality runs with parties  $P_0, P_1, \dots, P_n$  and an adversary, and operates as follows:

**Initialize:** For each  $i \in [1, n]$ , upon receiving  $(\text{init}, i)$  from  $P_i$ , if  $P_i$  is honest, then sample  $\Delta_i \leftarrow \mathbb{F}$ , else receive  $\Delta_i \in \mathbb{F}$  from the adversary. Store  $(\Delta_1, \dots, \Delta_n)$  and output  $\Delta_i$  to  $P_i$  for each  $i \in [1, n]$ , and ignore all subsequent  $(\text{init})$  commands.

**Extend:** Upon receiving  $(\text{extend}, m)$  from all parties, execute as follows:

1. For  $i \in [1, n]$ , if  $P_i$  is honest, then sample  $\mathbf{v}_i \leftarrow \mathbb{F}^m$ , else receive  $\mathbf{v}_i \in \mathbb{F}^m$  from the adversary.
2. If  $P_0$  is honest, sample  $\mathbf{u} \leftarrow \mathbb{F}^m$  and compute  $\mathbf{w}_i := \mathbf{v}_i + \mathbf{u} \cdot \Delta_i$  for  $i \in [1, n]$ .
3. If  $P_0$  is corrupted, receive  $\mathbf{u}_i \in \mathbb{F}^m$  for all  $i \in [1, n]$  and  $\mathbf{w}_i \in \mathbb{F}^m$  for all  $i \notin \mathcal{C}$  from the adversary, and then recompute  $\mathbf{v}_i := \mathbf{w}_i - \mathbf{u}_i \cdot \Delta_i$  for each  $i \notin \mathcal{C}$  and compute  $\mathbf{w}_i := \mathbf{v}_i + \mathbf{u}_i \cdot \Delta_i$  for each  $i \in \mathcal{C}$ .
4. Output  $\{(\mathbf{u}_i, \mathbf{w}_i)\}_{i \in [1, n]}$  to  $P_0$  and  $\mathbf{v}_i$  to  $P_i$  for  $i \in [1, n]$ .

Figure 3: **Functionality for multi-party VOLE.**

2. For each  $i \in [1, n]$ ,  $P_0$  and  $P_i$  call an oblivious PRF functionality  $\mathcal{F}_{\text{opr}}f$  with malicious security, and then  $P_i$  obtains the key denoted by  $\text{PRF}_i$  and  $P_0$  gets the set  $\{\text{PRF}_i(h) \mid h \in X_0\}$ .
3. Let  $(\text{Encode}, \text{Decode})$  be an OKVS scheme which maps  $\ell$  items to  $m$  slots, and  $\text{H} : \{0, 1\}^* \rightarrow \mathbb{F}$  be a random oracle. For each  $i \in [1, n]$ ,  $P_i$  computes an OKVS  $\mathbf{Q}_i$  as

$$\mathbf{Q}_i = \text{Encode} \left( \left\{ (h, \text{H}(\text{PRF}_i(h), h) + s_h^i) \mid h \in X_i \right\} \right),$$

and then sends  $\mathbf{Q}_i$  to  $P_0$ .

4.  $P_0$  computes an OKVS  $\mathbf{Q}_0$  as

$$\mathbf{Q}_0 = \text{Encode} \left( \left\{ \left( h, - \sum_{i=1}^n \text{H}(\text{PRF}_i(h), h) + s_h^0 \right) \mid h \in X_0 \right\} \right).$$

5. After receiving the OKVS from other  $n$  parties,  $P_0$  computes the output as

$$\left\{ h \in X_0 \mid \sum_{i=0}^n \text{Decode}(\mathbf{Q}_i, h) = 0 \right\}.$$

## 3.2 Our Approach to Improve Computation Efficiency

The state-of-the-art OPRF protocol [RS21] is constructed by combining VOLE with an OKVS scheme called PaXoS [PRTY20], where the efficiency of OKVS can be further improved using the recent constructions [GPR<sup>+</sup>21, RR22]. At a high level, this protocol running between  $P_i$  and  $P_0$  works as follows:

1.  $P_i$  and  $P_0$  call functionality  $\mathcal{F}_{\text{vole}}$  (shown in Figure 2), which returns  $\mathbf{v}_i$  to  $P_i$  and  $(\mathbf{u}, \mathbf{w}_i)$  to  $P_0$ , where  $\mathbf{w}_i = \mathbf{v}_i + \mathbf{u} \cdot \Delta_i$ .
2. For an input set  $X_0$ ,  $P_0$  computes an OKVS as:

$$\mathbf{S} = \text{Encode} \left( \left\{ (h, \text{H}(h)) \mid h \in X_0 \right\} \right).$$

3.  $P_0$  computes  $\mathbf{d} := \mathbf{S} - \mathbf{u} \in \mathbb{F}^m$  and sends it to  $P_i$  who computes  $\mathbf{V}_i := \mathbf{v}_i - \mathbf{d} \cdot \Delta_i$ . Let  $\mathbf{W}_i = \mathbf{w}_i$ , and thus  $\mathbf{W}_i = \mathbf{V}_i + \mathbf{S} \cdot \Delta_i$ .
4.  $P_0$  computes  $\text{PRF}_i(h) = \text{H}(\text{Decode}(\mathbf{W}_i, h), h)$  for  $h \in X_0$ , and  $P_i$  can compute  $\text{PRF}_i(x) = \text{H}(\text{Decode}(\mathbf{V}_i, x) + \text{H}(x) \cdot \Delta_i, x)$  for any  $x$  in the domain.

If integrating the above OPRF protocol into the state-of-the-art multi-party PSI protocol with malicious security shown in the previous section,  $P_0$  needs to call  $\mathcal{F}_{\text{vole}}$  with  $n$  different parties. Note that the LPN encoding is the computational efficiency bottleneck for the state-of-the-art VOLE protocol [WYKW21] in the malicious setting. When instantiating functionality  $\mathcal{F}_{\text{vole}}$ , the multi-party PSI protocol requires  $P_0$  to perform  $n$  operations of LPN encoding, which is computationally expensive.

**Our solution.** Our approach reduces the cost of LPN encoding associated with vector  $\mathbf{u}$  by a factor of  $n$ . We make an important observation that a single random vector  $\mathbf{u}$  is sufficient to mask the OKVS  $\mathbf{S}$ , and thus it is unnecessary to compute  $n$  random vectors  $\mathbf{u}_1, \dots, \mathbf{u}_n$ . Thus,  $P_0$  needs to generate  $n$  VOLE correlations with the same vector  $\mathbf{u}$  by running a protocol with  $n$  different parties. We model this as a multi-party VOLE functionality  $\mathcal{F}_{\text{mvole}}$  shown in Figure 3. Note that when  $P_0$  is corrupted, the adversary allows to choose different vectors  $\mathbf{u}$ . That is, we do not require the consistency check of vector  $\mathbf{u}$ . Furthermore, we do *not* require  $P_0$  to broadcast the vector  $\mathbf{d} = \mathbf{S} - \mathbf{u}$ . These are sufficient to design multi-party PSI protocols in the malicious setting (see the security proof of our protocol given in Section 4.3). Such a functionality  $\mathcal{F}_{\text{mvole}}$  can be UC-realized by calling the standard VOLE functionality (shown in Figure 2)  $n$  times and programming the input/output of  $P_0$  to keep the consistency of  $\mathbf{u}$  in the honest case (see [BCG<sup>+</sup>19b] for more details).

Furthermore, we further reduce the computation cost of the multi-party PSI protocol described as above by  $2n\ell$  H operations. In particular, we construct the PSI protocol by directly using  $\mathcal{F}_{\text{mvole}}$  as the underlying primitive instead of the OPRF primitive. We adopt  $\mathcal{F}_{\text{mvole}}$  and an OKVS to obtain an OPRF, but simplify the computation of  $\text{H}(\text{PRF}_i(h), h) = \text{H}(\text{H}(\text{Decode}(\mathbf{V}_i, h) + \text{H}(h) \cdot \Delta_i, h), h)$  performed by  $P_i$  for  $i \in [1, n]$ ,  $h \in X_i$  in the previous construction as

$$\text{H}(\text{Decode}(\mathbf{V}_i, h) + \text{H}(h) \cdot \Delta_i, h).$$

As such, the computation of  $\sum_{i=1}^n \text{H}(\text{PRF}_i(h), h) = \sum_{i=1}^n \text{H}(\text{H}(\text{Decode}(\mathbf{W}_i, h), h), h)$  performed by  $P_0$  for  $h \in X_0$  can be simplified as

$$\sum_{i=1}^n \text{H}(\text{Decode}(\mathbf{W}_i, h), h).$$

In other words, we combine two hash operations into one, which removes the redundant hash operations and significantly improves the computation efficiency.

### 3.3 Our Approach to Reduce Communication Bandwidth

The state-of-the-art multi-party PSI protocol [GPR<sup>+</sup>21] adopts the *star* network architecture. In particular, the central party  $P_0$  will have to send the messages  $\{\mathbf{d}_i = \mathbf{S} - \mathbf{u}_i \in \mathbb{F}^m\}_{i \in [1, n]}$  to  $n$  parties for computing OPRF values and receive the OKVSs  $\mathbf{Q}_1, \dots, \mathbf{Q}_n \in \mathbb{F}^m$  from  $n$  parties. These make the communication bandwidth of  $P_0$  be  $nm|\mathbb{F}|$  bits per protocol execution, where  $m = \ell/(0.81 - o(1))$  for the recent OKVS scheme [GPR<sup>+</sup>21] and  $\ell$  is the size of a set. For a large number  $n$  of parties and a large size  $\ell$  of input sets, this forms an efficiency bottleneck of the PSI protocol (especially

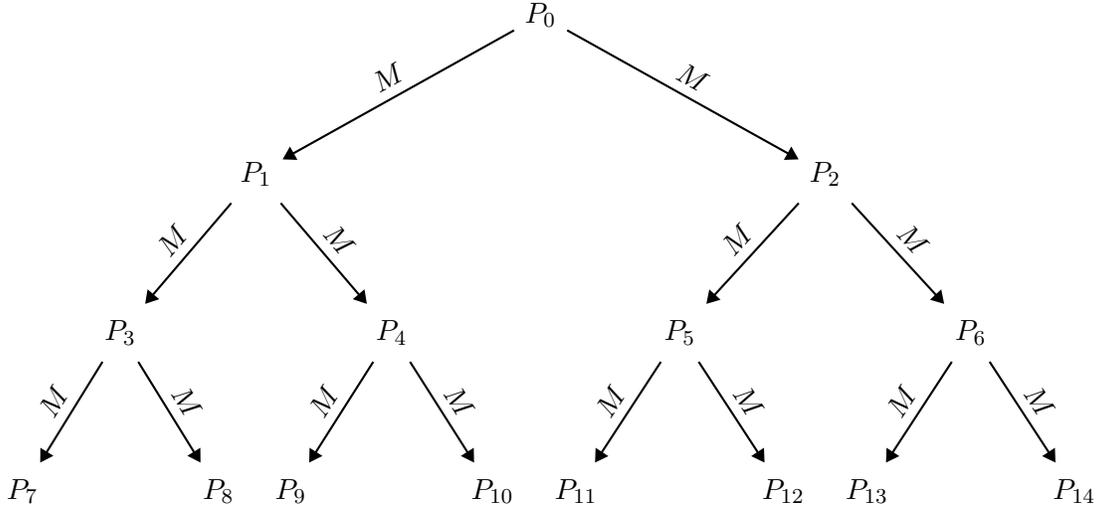


Figure 4: **Transmission of a message  $M$  using a binary tree when  $n = 14$ .**

for networks with restricted bandwidths). For example, when  $\ell = 2^{24}$ ,  $n = 20$  and  $|\mathbb{F}| = 128$ ,  $P_0$  needs about 53 gigabits of communication bandwidth. To reduce the communication bandwidth, one may consider using the Cuckoo hashing approach [KMP<sup>+</sup>17] to divide a large set into multiple bins. However, this approach is *not* secure against malicious adversaries [NTY21].

**Our solution.** In the following, we present an efficient approach to reduce the communication bandwidth of  $P_0$  to  $2m|\mathbb{F}|$  bits by amortizing the communication among all parties in a *binary-tree* architecture. Specifically, we first apply the optimized approach described in the previous section to the multi-party PSI protocol. In this case,  $P_0$  needs to send the same message  $\mathbf{d} = \mathbf{S} - \mathbf{u}$  to all other parties. Therefore, we can use a binary-tree structure to send  $\mathbf{d}$ . The parties  $P_0, P_1, \dots, P_n$  constitute a binary tree with the root  $P_0$ . Party  $P_0$  can only send  $\mathbf{d}$  to its two child nodes  $P_1$  and  $P_2$ . Then, for each level of the tree, every parent node  $P_{j-1}$  sends  $\mathbf{d}$  to its children  $P_{2j-1}$  and  $P_{2j}$ . See Figure 4 for an example of  $n = 14$  where the message  $M = \mathbf{d}$ . Note that here we do *not* require to broadcast vector  $\mathbf{d}$ .

To reduce the bandwidth that communicates the OKVSs  $\mathbf{Q}_1, \dots, \mathbf{Q}_n$ , we require that the OKVS scheme is *linear*, which is satisfied by the recent highly-efficient constructions [RS21, GPR<sup>+</sup>21, RR22]. In this case, we can aggregate these OKVSs into one OKVS through a binary tree. For each level of the tree (from bottom to top), two child nodes  $P_{2j-1}$  and  $P_{2j}$  respectively send  $\mathbf{Q}_{2j-1}$  and  $\mathbf{Q}_{2j}$  to their parent node  $P_{j-1}$  who updates its OKVS as  $\mathbf{Q}_{j-1} := \mathbf{Q}_{j-1} + \mathbf{Q}_{2j-1} + \mathbf{Q}_{2j}$ . Finally, the root node  $P_0$  obtains the OKVS  $\mathbf{Q}_0 := \sum_{i=0}^n \mathbf{Q}_i$ . See Figure 5 for an example of  $n = 14$ , where  $P_i$  holds an OKVS  $M_i = \mathbf{Q}_i \in \mathbb{F}^m$  for  $i \in [0, n]$ . According to the linearity of the OKVS scheme, we have  $\sum_{i=0}^n \text{Decode}(\mathbf{Q}_i, h) = \text{Decode}(\sum_{i=0}^n \mathbf{Q}_i, h)$ . Therefore, the OKVS  $\sum_{i=0}^n \mathbf{Q}_i$  that has been aggregated allows  $P_0$  to obtain the correct output. The message-aggregation approach can also be used for the recent multi-party PSI protocols based on garbled Bloom filters [ENOPC21, KMS21], where the approach can reduce the communication bandwidth of the central party  $P_0$  in the protocol [ENOPC21] by a factor of  $O(n)$  and the rounds in the protocol [KMS21] from  $O(n)$  to  $O(\log n)$ . In addition, our tree-architecture approach is able to be applied in the multi-party PSI protocol [NTY21] for any corruption threshold  $t \leq n$ .

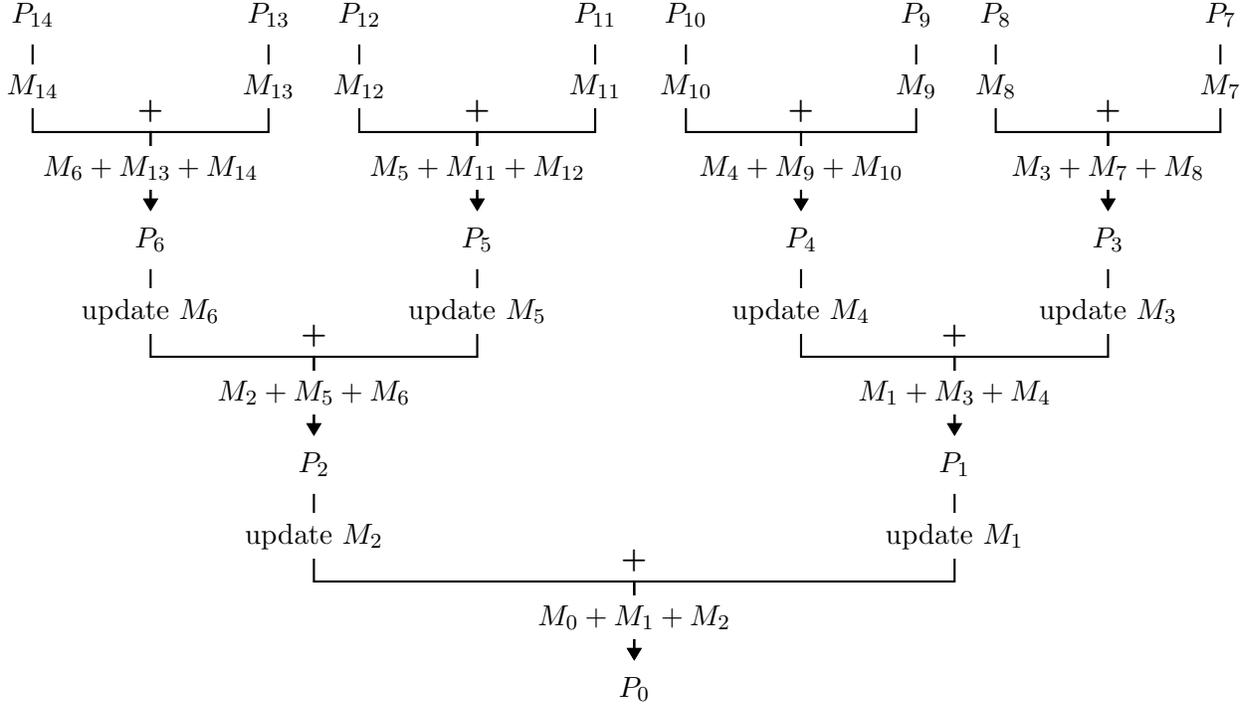


Figure 5: Message addition aggregation using a binary tree when  $n = 14$

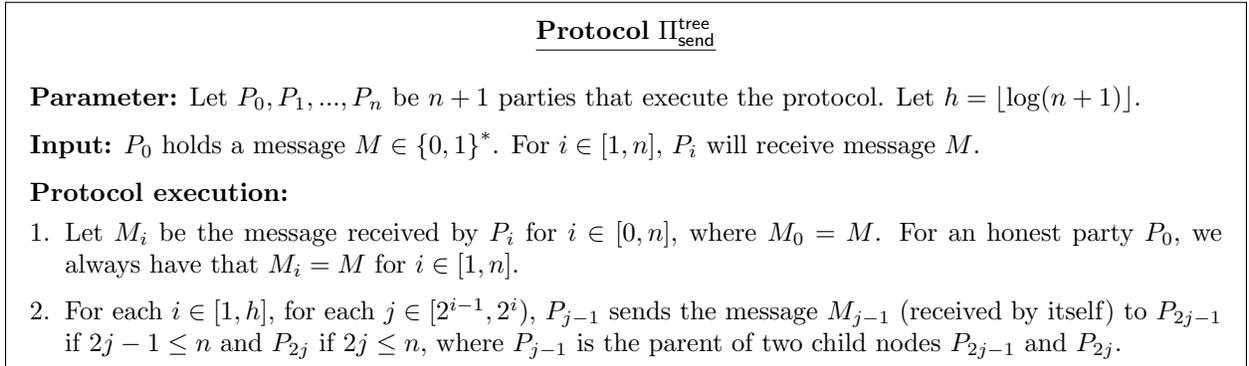


Figure 6: Protocol for sending messages with constant bandwidth using a binary tree.

## 4 Maliciously Secure Multi-Party PSI Protocol

We first describe two sub-protocols which are used to send and aggregate messages respectively in a binary-tree architecture. Then, we present the detailed construction of our multi-party PSI protocol with malicious security. Finally, we give a formal proof of security for our PSI protocol.

### 4.1 Protocols for Sending and Aggregating Messages Using Tree Architecture

We first describe the sub-protocol  $\Pi_{\text{send}}^{\text{tree}}$  shown in Figure 6, which allows the central party  $P_0$  to send a message  $M$  to all other parties  $P_1, \dots, P_n$ . The message is transmitted in a binary-tree architecture, which enables us to obtain  $O(1)$  communication bandwidth instead of  $O(n)$ . As a trade-off, the round complexity is increased from  $O(1)$  to  $O(\log n)$ . A malicious party  $P_i$  for

**Protocol  $\Pi_{\text{aggregate}}^{\text{tree}}$**

**Parameter:** Let  $P_0, P_1, \dots, P_n$  be  $n + 1$  parties that run the protocol. Let  $h = \lfloor \log(n + 1) \rfloor$ .

**Inputs:** For  $i \in [1, n]$ ,  $P_i$  holds a message  $M_i \in \mathbb{F}^m$ . Party  $P_0$  will obtain the output  $\sum_{i \in [1, n]} M_i$ .

**Protocol execution:**

1.  $P_0$  sets  $M_0 := 0$ .
2. From  $i = h$  to  $i = 1$ , for each  $j \in [2^{i-1}, 2^i)$ ,  $P_{2j-1}$  sends  $M_{2j-1}$  to  $P_{j-1}$  (if  $2j - 1 \leq n$ ) and  $P_{2j}$  sends  $M_{2j}$  to  $P_{j-1}$  (if  $2j \leq n$ ), and then  $P_{j-1}$  computes  $M'_{j-1} := M_{j-1} + M_{2j-1} + M_{2j}$  and updates  $M_{j-1} := M'_{j-1}$ .
3.  $P_0$  outputs the final message  $M_0$ .

Figure 7: **Protocol for aggregating messages with constant bandwidth using a binary tree.**

$i \in [1, n]$  may send  $M' \neq M$  to its left child and  $M'' \neq M$  to its right child. In the next subsection, we will show that such malicious behavior is harmless for the security of our PSI protocol.

In Figure 7, we describe the sub-protocol  $\Pi_{\text{aggregate}}^{\text{tree}}$ , which allows  $n$  parties to send the sum of their messages to the central party  $P_0$ . These messages are aggregated by addition operations in the binary-tree network architecture, which reduces the communication bandwidth from  $O(n)$  to  $O(1)$ . Similarly, the round complexity is increased from  $O(1)$  to  $O(\log n)$ .

In both of two sub-protocols, the message is always transmitted between a parent node and two child nodes. While the parent node sends the same message to two child nodes in the protocol  $\Pi_{\text{send}}^{\text{tree}}$ , two child nodes send two different messages to the parent node who sums the two messages and its message as an aggregation message in the protocol  $\Pi_{\text{aggregate}}^{\text{tree}}$ .

**Extending to  $k$ -ary trees.** Although we describe our protocols in the binary-tree architecture, they are easy to be extended to work in the  $k$ -ary tree architecture with any  $2 \leq k \leq n$ . For a  $k$ -ary tree, while the communication bandwidth will be increased by a factor of  $k/2$ , the round complexity will be reduced from  $O(\log n)$  to  $O(\log_k n)$ . In particular, the star architecture used in prior work can be considered as the special case of  $n$ -ary trees.

## 4.2 Our PSI protocol with Efficient Bandwidth and Computation

In Figure 8, we describe the details of our multi-party PSI protocol in the dishonest-majority malicious setting. The communication bandwidth of the central party  $P_0$  is  $O(\ell|\mathbb{F}|)$  bits, where the state-of-the-art protocol [GPR<sup>+</sup>21] that builds on the technique [KMP<sup>+</sup>17] requires the bandwidth of  $O(n\ell|\mathbb{F}|)$  bits for  $P_0$ . This protocol works in the  $\mathcal{F}_{\text{mvole}}$ -hybrid model, and is executed by  $n + 1$  parties  $P_0, P_1, \dots, P_n$ . We separate the protocol into two phases: preprocessing phase where the input sets are unknown, and online phase in which the sets are known. In this protocol, only the central party  $P_0$  obtains the output.

**Correctness.** To see that the PSI protocol as described in Figure 8 is correct in the honest case, we first note that for any  $h$ ,

$$\sum_{i=0}^n s_h^i = \sum_{i=0}^n \left( \sum_{j<i} \text{PRF}(k_{i,j}, h) - \sum_{j>i} \text{PRF}(k_{j,i}, h) \right) = 0.$$

**Protocol  $\Pi_{\text{mpsi}}$**

**Parameters:**

- A large finite field  $\mathbb{F}$  with  $|\mathbb{F}| \geq 2^\kappa$ .
- Linear OKVS scheme (Encode, Decode) that maps  $\ell$  items to  $m$  slots.
- A cryptographic hash function  $H : \{0, 1\}^* \rightarrow \mathbb{F}$  modeled as a random oracle.
- Let  $\text{PRF} : \{0, 1\}^\kappa \times \{0, 1\}^\kappa \rightarrow \mathbb{F}$  be a Pseudo-Random Function (PRF).

**Inputs:** For  $n + 1$  parties  $P_0, P_1, \dots, P_n$ , every party  $P_i$  holds a set  $X_i$  such that  $X_i \in \mathbb{F}$  and  $|X_i| = \ell$ . Let  $P_0$  be the designated party to receive the output  $\bigcap_{i \in [0, n]} X_i$ .

**Preprocessing:** This procedure can be executed when the sets are *unknown*.

1. Every party  $P_i$  samples  $k_{i,j} \leftarrow \{0, 1\}^\kappa$  as a random PRF key, and then sends it to  $P_j$  over a private channel for each  $j < i$ . (The zero-sharing setup phase is run only once.)
2. For each  $i \in [1, n]$ ,  $P_i$  sends  $(\text{init}, i)$  to the multi-party VOLE functionality  $\mathcal{F}_{\text{mvole}}$  (shown in Figure 3), which returns  $\Delta_i$  to  $P_i$ .
3. The parties  $P_0, P_1, \dots, P_n$  send  $(\text{extend}, m)$  to functionality  $\mathcal{F}_{\text{mvole}}$ , which returns  $\mathbf{u} \in \mathbb{F}^m$  and  $\{\mathbf{w}_i\}_{i \in [1, n]}$  to  $P_0$ , and outputs  $\mathbf{v}_i \in \mathbb{F}^m$  and  $\Delta_i \in \mathbb{F}$  to  $P_i$  for each  $i \in [1, n]$ , such that  $\mathbf{w}_i = \mathbf{v}_i + \mathbf{u} \cdot \Delta_i$  for  $i \in [1, n]$ .

**Online:** This procedure is run when the sets are *known*.

4.  $P_0$  computes OKVS via

$$\mathbf{S} = \text{Encode}(\{(h, H(h)) \mid h \in X_0\}),$$

and calculates  $\mathbf{d} = \mathbf{S} - \mathbf{u} \in \mathbb{F}^m$ . Then  $P_0$  sends  $\mathbf{d}$  to  $P_i$  for each  $i \in [1, n]$  by running the sub-protocol  $\Pi_{\text{send}}^{\text{tree}}$  (shown in Figure 6) in a bandwidth-efficient way.

5. For each  $i \in [1, n]$ ,  $P_i$  computes  $\mathbf{V}_i := \mathbf{v}_i - \mathbf{d} \cdot \Delta_i$ .  $P_0$  sets  $\mathbf{W}_i := \mathbf{w}_i$  for  $i \in [1, n]$  where  $\mathbf{W}_i = \mathbf{V}_i + \mathbf{S} \cdot \Delta_i$ .
6. For each  $i \in [0, n]$ , for each  $h \in X_i$ , every party  $P_i$  computes the following:

$$s_h^i := \sum_{j < i} \text{PRF}(k_{i,j}, h) - \sum_{j > i} \text{PRF}(k_{j,i}, h).$$

7. For each  $i \in [1, n]$ ,  $P_i$  computes  $z_{i,h} = H(\text{Decode}(\mathbf{V}_i, h) + H(h) \cdot \Delta_i, h)$  for each  $h \in X_i$ , and then computes OKVS  $\mathbf{Q}_i$  via

$$\mathbf{Q}_i = \text{Encode}(\{(h, z_{i,h} + s_h^i) \mid h \in X_i\}).$$

8. Parties  $P_1, \dots, P_n$  run the sub-protocol  $\Pi_{\text{aggregate}}^{\text{tree}}$  (shown in Figure 7) to send  $\mathbf{Q} = \sum_{i=1}^n \mathbf{Q}_i \in \mathbb{F}^m$  to  $P_0$  in a bandwidth-efficient way.
9.  $P_0$  computes  $z'_{i,h} = H(\text{Decode}(\mathbf{W}_i, h), h)$  for each  $h \in X_0$ , and then outputs

$$\left\{ h \in X_0 \mid \text{Decode}(\mathbf{Q}, h) + s_h^0 - \sum_{i=1}^n z'_{i,h} = 0 \right\}.$$

Figure 8: **Maliciously secure multi-party PSI protocol in the  $\mathcal{F}_{\text{mvole}}$ -hybrid model.**

Then, we observe that for each  $h \in \bigcap_{i \in [0, n]} X_i, i \in [1, n]$ , the following holds:

$$\begin{aligned} z_{i,h} - z'_{i,h} &= H(\text{Decode}(\mathbf{V}_i, h) + H(h) \cdot \Delta_i, h) - H(\text{Decode}(\mathbf{W}_i, h), h) \\ &= H(\text{Decode}(\mathbf{V}_i, h) + H(h) \cdot \Delta_i, h) - H(\text{Decode}(\mathbf{V}_i + \mathbf{S} \cdot \Delta_i, h), h) \\ &= H(\text{Decode}(\mathbf{V}_i, h) + H(h) \cdot \Delta_i, h) - H(\text{Decode}(\mathbf{V}_i, h) + \text{Decode}(\mathbf{S}, h) \cdot \Delta_i, h) \\ &= H(\text{Decode}(\mathbf{V}_i, h) + H(h) \cdot \Delta_i, h) - H(\text{Decode}(\mathbf{V}_i, h) + H(h) \cdot \Delta_i, h) = 0. \end{aligned}$$

Therefore, for each  $h \in \bigcap_{i \in [0, n]} X_i$ , we have

$$\begin{aligned}
\text{Decode}(\mathbf{Q}, h) + s_h^0 - \sum_{i=1}^n z'_{i,h} &= \text{Decode}\left(\sum_{i=1}^n \mathbf{Q}_i, h\right) + s_h^0 - \sum_{i=1}^n z'_{i,h} \\
&= s_h^0 + \sum_{i=1}^n \text{Decode}(\mathbf{Q}_i, h) - \sum_{i=1}^n z'_{i,h} \\
&= s_h^0 + \sum_{i=1}^n (z_{i,h} + s_h^i) - \sum_{i=1}^n z'_{i,h} \\
&= \sum_{i=0}^n s_h^i + \sum_{i=1}^n (z_{i,h} - z'_{i,h}) = 0.
\end{aligned}$$

### 4.3 Proof of Security

In the following, we prove the security of our PSI protocol in the multi-party malicious setting. Our proof of security will use the following lemma, which has been proven in [GPR<sup>+</sup>21].

**Lemma 1** ([GPR<sup>+</sup>21]). *Given a set of parties that run the zero-sharing setup (i.e., the step 1 in Figure 8) such that a pair of parties  $P_i, P_j$  are honest and the adversary's view is independent of the  $P_i$ 's share  $s_h^i$ , then the  $P_j$ 's share  $s_h^j$  is computationally indistinguishable from a uniform value.*

Based on the above lemma, we prove the following theorem.

**Theorem 1.** *Let  $H$  be a random oracle and  $(\text{Encode}, \text{Decode})$  be a linear OKVS scheme. Let PRF be a pseudo-random function. Then protocol  $\Pi_{\text{mpsi}}$  shown in Figure 8 UC-realizes functionality  $\mathcal{F}_{\text{mpsi}}$  in the presence of a malicious adversary corrupting up to  $n$  of the  $n+1$  parties in the  $\mathcal{F}_{\text{mvole}}$ -hybrid model.*

*Proof.* We first consider the case of a malicious central party  $P_0$ , and then consider the case of an honest  $P_0$ . In each case, we construct a PPT simulator  $\mathcal{S}$  given access to functionality  $\mathcal{F}_{\text{mpsi}}$ , running a PPT adversary  $\mathcal{A}$  as a subroutine when emulating  $\mathcal{F}_{\text{mvole}}$ . For each case,  $\mathcal{S}$  simulates the random oracle  $H$  by responding the queries made by  $\mathcal{A}$  with random values when keeping the consistency of responses. We always implicitly consider that whenever  $\mathcal{A}$  aborts,  $\mathcal{S}$  sends **abort** to functionality  $\mathcal{F}_{\text{mpsi}}$  and then aborts. For each case, we show that no PPT environment  $\mathcal{Z}$  can distinguish the real-world execution from the ideal-world execution. Let  $\mathcal{C} \subset [0, n]$  denote the set of corrupted parties.

**Malicious central party  $P_0$ .** When emulating  $\mathcal{F}_{\text{mvole}}$ ,  $\mathcal{S}$  interacts with adversary  $\mathcal{A}$  as follows:

1. In the zero-sharing setup, on behalf of every honest party  $P_i$ ,  $\mathcal{S}$  samples  $k_{i,j} \leftarrow \{0, 1\}^\kappa$  and sends it to  $\mathcal{A}$  for each  $j < i, j \in \mathcal{C}$ , as well as receives  $k_{j,i} \in \{0, 1\}^\kappa$  from  $\mathcal{A}$  for each  $j \in \mathcal{C}, j > i$ . Thus, for any  $h$ ,  $\mathcal{S}$  can compute the share  $s_h^i$  for  $i \notin \mathcal{C}$ , when it also samples the PRF keys between any two honest parties.
2.  $\mathcal{S}$  emulates the (init) command of  $\mathcal{F}_{\text{mvole}}$  and receives  $\Delta_i$  for  $i \in \mathcal{C}$  from  $\mathcal{A}$ . Then,  $\mathcal{S}$  emulates the (extend) command of  $\mathcal{F}_{\text{mvole}}$  by receiving  $\mathbf{u}_i, \mathbf{w}_i \in \mathbb{F}^m$  for  $i \in [1, n]$  from  $\mathcal{A}$ . For  $i \in [1, n]$ ,  $\mathcal{S}$  sets  $\mathbf{W}_i = \mathbf{w}_i$ .

3. During the protocol execution of  $\Pi_{\text{send}}^{\text{tree}}$ ,  $\mathcal{S}$  receives  $\mathbf{d}_i \in \mathbb{F}^m$  for  $i \notin \mathcal{C}$  from  $\mathcal{A}$ , and then computes  $\mathbf{S}_i := \mathbf{d}_i + \mathbf{u}_i$  for  $i \notin \mathcal{C}$ . Then, for  $i \notin \mathcal{C}$ ,  $\mathcal{S}$  computes the following set

$$X_0^i := \{h \mid \mathcal{A} \text{ queried H at } h \text{ and } \text{Decode}(\mathbf{S}_i, h) = \text{H}(h)\}.$$

4.  $\mathcal{S}$  sends  $\tilde{X}_0 := \bigcap_{i \notin \mathcal{C}} X_0^i$  to functionality  $\mathcal{F}_{\text{mpsi}}$ , as the input of all corrupted parties, and then receives  $Y = \tilde{X}_0 \cap (\bigcap_{i \notin \mathcal{C}} X_i)$  as the output.
5. For each  $i \notin \mathcal{C}$ ,  $\mathcal{S}$  chooses an arbitrary different  $h \notin Y$  from the domain of  $X_i$  for each unknown value in set  $X_i \setminus Y$ , and then computes the OKVS  $\mathbf{Q}_i$  of honest party  $P_i$  as follows:

$$\mathbf{Q}_i = \begin{cases} \text{Encode}(h, \text{H}(\text{Decode}(\mathbf{W}_i, h), h) + s_h^i) & \text{if } h \in Y \\ \text{Encode}(h, r_{i,h}) \text{ with } r_{i,h} \leftarrow \mathbb{F} & \text{if } h \notin Y \end{cases} \quad (1)$$

On behalf of every honest party  $P_i$ ,  $\mathcal{S}$  sends  $\mathbf{Q}_i$  to  $\mathcal{A}$  when simulating the protocol execution of  $\Pi_{\text{aggregate}}^{\text{tree}}$ .

**Hybrid argument.** In the following, we use a series of hybrid games to prove that the real-world execution is computationally indistinguishable from the ideal-world execution.

**Hybrid<sub>0</sub>:** This is the real-world execution, where every honest party  $P_i$  computes the OKVS  $\mathbf{Q}_i$  as follows:

$$\mathbf{Q}_i = \text{Encode}\left(\left\{ (h, \text{H}(\text{Decode}(\mathbf{V}_i, h) + \text{H}(h) \cdot \Delta_i, h) + s_h^i) \mid h \in X_i \right\}\right).$$

**Hybrid<sub>1</sub>:** In this hybrid, the simulator  $\mathcal{S}$  simulates the zero-sharing setup honestly, and emulates functionality  $\mathcal{F}_{\text{mvole}}$  by recording the values sent by  $\mathcal{A}$  as described above.

Note that for each  $i \in \mathcal{C} \cap [1, n]$ , letting  $\mathcal{A}$  choose the output vector  $\mathbf{w}_i$  for corrupted party  $P_0$  is equivalent to making  $\mathcal{A}$  choose  $\mathbf{v}_i$  for corrupted party  $P_i$ , as  $\mathbf{w}_i = \mathbf{v}_i + \mathbf{u}_i \cdot \Delta_i$  holds. Thus, **Hybrid<sub>1</sub>** has the identical distribution as **Hybrid<sub>0</sub>**.

**Hybrid<sub>2</sub>:** In this hybrid,  $\mathcal{S}$  extracts the sets  $X_0^i$  for all  $i \notin \mathcal{C}$  as described above. Then, for each  $i \notin \mathcal{C}$ ,  $\mathcal{S}$  computes  $T_i := X_i \cap X_0^i$ , where the simulator knows the input sets of honest parties in the hybrid. On behalf of every honest party  $P_i$ ,  $\mathcal{S}$  computes the OKVS  $\mathbf{Q}_i$  as follows:

$$\mathbf{Q}_i = \begin{cases} \text{Encode}(h, \text{H}(\text{Decode}(\mathbf{W}_i, h), h) + s_h^i) & \text{if } h \in T_i \\ \text{Encode}(h, r_{i,h}) \text{ with } r_{i,h} \leftarrow \mathbb{F} & \text{if } h \in X_i \setminus T_i \end{cases}$$

Then, for  $i \notin \mathcal{C}$ ,  $\mathcal{S}$  sends  $\mathbf{Q}_i$  to  $\mathcal{A}$  in the tree-aggregation way during the execution of  $\Pi_{\text{aggregate}}^{\text{tree}}$ .

From the simulation, we can see that  $\mathbf{W}_i = \mathbf{V}_i + \mathbf{S}_i \cdot \Delta_i$  for  $i \notin \mathcal{C}$ . From the linearity of the Decode algorithm, for  $h \in X_0^i$ , we have the following:

$$\begin{aligned} \text{Decode}(\mathbf{W}_i, h) &= \text{Decode}(\mathbf{V}_i, h) + \text{Decode}(\mathbf{S}_i, h) \cdot \Delta_i \\ &= \text{Decode}(\mathbf{V}_i, h) + \text{H}(h) \cdot \Delta_i. \end{aligned}$$

Therefore, for  $i \notin \mathcal{C}$ , for  $h \in T_i = X_i \cap X_0^i$ , the corresponding element of  $\mathbf{Q}_i$  in **Hybrid<sub>2</sub>** has the identical distribution as that in **Hybrid<sub>1</sub>**. Below, we analyze the case of  $h \in X_i \setminus T_i$ . While  $\mathbf{Q}_i = \text{Encode}(h, \text{H}(\text{Decode}(\mathbf{V}_i, h) + \text{H}(h) \cdot \Delta_i, h) + s_h^i)$  in **Hybrid<sub>1</sub>**,  $\mathbf{Q}_i = \text{Encode}(h, r_{i,h})$  in **Hybrid<sub>2</sub>**. From the above equation, we know that  $\text{Decode}(\mathbf{V}_i, h) + \text{H}(h) \cdot \Delta_i = \text{Decode}(\mathbf{W}_i, h) + (\text{H}(h) - \text{Decode}(\mathbf{S}_i, h)) \cdot \Delta_i$  for  $h \notin X_0^i$ . Since  $\text{Decode}(\mathbf{S}_i, h) \neq \text{H}(h)$  and  $\Delta_i \in \mathbb{F}$  is uniformly random from

the adversary's view, the probability that  $\mathcal{A}$  makes a query  $(\text{Decode}(\mathbf{W}_i, h) + (\mathbf{H}(h) - \text{Decode}(\mathbf{S}_i, h)) \cdot \Delta_i, h)$  to random oracle  $\mathbf{H}$  is at most  $q/|\mathbb{F}|$  that is negligible in security parameter  $\kappa$ , where  $q$  is the number of  $\mathbf{H}$  queries. Therefore, for  $h \notin X_0^i$ ,  $\mathbf{H}(\text{Decode}(\mathbf{V}_i, h) + \mathbf{H}(h) \cdot \Delta_i, h) = \mathbf{H}(\text{Decode}(\mathbf{W}_i, h) + (\mathbf{H}(h) - \text{Decode}(\mathbf{S}_i, h)) \cdot \Delta_i, h)$  is uniform from the adversary's view in the random oracle model. Further, we have that the distribution of the elements of  $\mathbf{Q}_i$  associated with  $h \in X_i \setminus T_i$  in **Hybrid**<sub>1</sub> is computationally indistinguishable from that in **Hybrid**<sub>2</sub>.

**Hybrid**<sub>3</sub>: In this hybrid,  $\mathcal{S}$  changes the way that creates the OKVS  $\mathbf{Q}_i$  of every honest party  $P_i$  as follows:

$$\mathbf{Q}_i = \begin{cases} \text{Encode}(h, \mathbf{H}(\text{Decode}(\mathbf{W}_i, h), h) + s_h^i) & \text{if } h \in \bigcap_{i \notin \mathcal{C}} T_i \\ \text{Encode}(h, r_{i,h}) \text{ with } r_{i,h} \leftarrow \mathbb{F} & \text{if } h \in X_i \setminus \bigcap_{i \notin \mathcal{C}} T_i \end{cases} \quad (2)$$

Clearly, if there is only one honest party, then the distribution of **Hybrid**<sub>3</sub> is identical to that of **Hybrid**<sub>2</sub>. In the following, we assume that there exists at least two honest parties. It is easy to see that if  $h \in \bigcap_{i \notin \mathcal{C}} T_i$ , then **Hybrid**<sub>3</sub> is identical to **Hybrid**<sub>2</sub>. For each  $i \notin \mathcal{C}$ , the computation of  $\mathbf{Q}_i$  is different in two hybrids for a value  $h \in T_i$ , where  $h \notin T_j$  for some different  $j \notin \mathcal{C}$ . When  $h \notin T_j$ , we note that in **Hybrid**<sub>2</sub>, the computation of  $\mathbf{Q}_j$  does not use the value  $s_h^j$  anywhere. From Lemma 1, we have that  $s_h^i$  is computationally indistinguishable from a uniformly random value. Thus, for  $h \in T_i$  and  $h \notin T_j$ , the view of  $\text{Encode}(h, \mathbf{H}(\text{Decode}(\mathbf{W}_i, h), h) + s_h^i)$  is indistinguishable from  $\text{Encode}(h, r_{i,h})$  for a uniform value  $r_{i,h} \in \mathbb{F}$ . Therefore, the distribution of **Hybrid**<sub>3</sub> is computationally indistinguishable from that of **Hybrid**<sub>2</sub>.

**Hybrid**<sub>4</sub>: In this hybrid,  $\mathcal{S}$  computes  $\tilde{X}_0 := \bigcap_{i \notin \mathcal{C}} X_0^i$ , and then receives the output  $Y = \tilde{X}_0 \cap (\bigcap_{i \notin \mathcal{C}} X_i)$  from functionality  $\mathcal{F}_{\text{mpsi}}$ . For each  $i \notin \mathcal{C}$ ,  $\mathcal{S}$  chooses an arbitrary different  $h \notin Y$  as each unknown value in set  $X_i \setminus Y$ , and then computes the OKVS  $\mathbf{Q}_i$  of honest party  $P_i$  as follows:

$$\mathbf{Q}_i = \begin{cases} \text{Encode}(h, \mathbf{H}(\text{Decode}(\mathbf{W}_i, h), h) + s_h^i) & \text{if } h \in Y \\ \text{Encode}(h, r_{i,h}) \text{ with } r_{i,h} \leftarrow \mathbb{F} & \text{if } h \notin Y \end{cases}$$

On behalf of every honest party  $P_i$ ,  $\mathcal{S}$  sends  $\mathbf{Q}_i$  to  $\mathcal{A}$  during the protocol execution of  $\Pi_{\text{aggregate}}^{\text{tree}}$ .

It is easy to see that  $Y = \tilde{X}_0 \cap (\bigcap_{i \notin \mathcal{C}} X_i) = \bigcap_{i \notin \mathcal{C}} (\tilde{X}_0 \cap X_i) = \bigcap_{i \notin \mathcal{C}} T_i$ . Thus, for each  $h \in Y$ , the distribution of  $\mathbf{Q}_i$  in **Hybrid**<sub>3</sub> is identical to that in **Hybrid**<sub>4</sub>. For the case of  $h \notin Y$ , the only difference between **Hybrid**<sub>3</sub> and **Hybrid**<sub>4</sub> is that  $h \in X_i \setminus Y$  in **Hybrid**<sub>3</sub>, while  $h$  is chosen arbitrarily from the domain in **Hybrid**<sub>4</sub>. From the security of the OKVS scheme ( $\text{Encode}$ ,  $\text{Decode}$ ), we have that the only difference is negligible in  $\kappa$ .

**Honest central party**  $P_0$ . Given access to functionality  $\mathcal{F}_{\text{mpsi}}$ ,  $\mathcal{S}$  interacts with  $\mathcal{A}$  as follows:

1.  $\mathcal{S}$  simulates the zero-sharing setup just as in the case of malicious  $P_0$ , and thus can compute the share  $s_h^i$  for any  $h$  and each  $i \notin \mathcal{C}$ .
2. Similarly,  $\mathcal{S}$  emulates the (init) command of  $\mathcal{F}_{\text{mvole}}$  and receives  $\Delta_i$  for  $i \in \mathcal{C}$  from  $\mathcal{A}$ . Then,  $\mathcal{S}$  emulates the (extend) command of  $\mathcal{F}_{\text{mvole}}$  by receiving  $\mathbf{v}_i \in \mathbb{F}^m$  for  $i \in \mathcal{C}$  from  $\mathcal{A}$ .
3. On behalf of honest party  $P_0$ ,  $\mathcal{S}$  samples a uniform vector  $\mathbf{d} \leftarrow \mathbb{F}^m$ , and then sends it to  $\mathcal{A}$  during the protocol execution of  $\Pi_{\text{send}}^{\text{tree}}$ .
4. For  $i \in \mathcal{C}$ ,  $\mathcal{S}$  computes  $\mathbf{V}_i = \mathbf{v}_i - \mathbf{d} \cdot \Delta_i \in \mathbb{F}^m$  following the protocol description.

5. If there are at least two honest parties, for each  $i \notin \mathcal{C}, i \neq 0$ ,  $\mathcal{S}$  chooses an arbitrary input set  $X'_i$ , and samples  $r_{i,h} \leftarrow \mathbb{F}$  for  $h \in X'_i$ . Then, for  $i \notin \mathcal{C}, i \neq 0$ ,  $\mathcal{S}$  computes the OKVS  $\mathbf{Q}_i$  as

$$\mathbf{Q}_i = \text{Encode}(\{(h, r_{i,h}) \mid h \in X'_i\}).$$

On behalf of every honest party  $P_i$  ( $i \neq 0$ ),  $\mathcal{S}$  sends  $\mathbf{Q}_i$  during the protocol execution of  $\Pi_{\text{aggregate}}^{\text{tree}}$ . Note that in the binary-tree architecture, the OKVS  $\mathbf{Q}_i$  from an honest party  $P_i$  may be sent to adversary  $\mathcal{A}$  who corrupts the parent node  $P_j$ .

6. From the protocol execution of  $\Pi_{\text{aggregate}}^{\text{tree}}$ ,  $\mathcal{S}$  computes the sum of OKVSs  $\sum_{i \in \mathcal{C}} \mathbf{Q}_i$ .
7. According to the queries of random oracle  $\mathsf{H}$ ,  $\mathcal{S}$  defines the set

$$O = \left\{ h \mid \mathcal{A} \text{ made a query of the form } \mathsf{H}(\text{Decode}(\mathbf{V}_i, h) + \mathsf{H}(h) \cdot \Delta_i, h) \text{ for } i \in \mathcal{C} \right\},$$

and then computes

$$\tilde{X} := \left\{ h \in O \mid \text{Decode}\left(\sum_{i \in \mathcal{C}} \mathbf{Q}_i, h\right) = \sum_{i \in \mathcal{C}} \mathsf{H}(\text{Decode}(\mathbf{V}_i, h) + \mathsf{H}(h) \cdot \Delta_i, h) - \sum_{i \notin \mathcal{C}} s_h^i \right\}.$$

8.  $\mathcal{S}$  sends  $\tilde{X}$  to functionality  $\mathcal{F}_{\text{mpsi}}$  as the input for every corrupted party. Then,  $\mathcal{F}_{\text{mpsi}}$  returns the output  $\tilde{X} \cap (\bigcap_{i \notin \mathcal{C}} X_i)$  to honest party  $P_0$ .

**Hybrid argument.** In the following, we use a series of hybrid games to prove that the real-world execution is computationally indistinguishable from the ideal-world execution.

**Hybrid<sub>0</sub>:** This is the real-world execution, where honest party  $P_0$  computes the output as:

$$\left\{ h \in X_0 \mid \sum_{i \in \mathcal{C}} \text{Decode}(\mathbf{Q}_i, h) + \sum_{i \notin \mathcal{C}} \text{Decode}(\mathbf{Q}_i, h) + s_h^0 - \sum_{i=1}^n \mathsf{H}(\text{Decode}(\mathbf{W}_i, h), h) = 0 \right\}.$$

**Hybrid<sub>1</sub>:** In this hybrid, how  $P_0$  computes the output is modified as:

$$\left\{ h \in \bigcap_{i \notin \mathcal{C}} X_i \mid \sum_{i \in \mathcal{C}} \text{Decode}(\mathbf{Q}_i, h) + \sum_{i \notin \mathcal{C}} \text{Decode}(\mathbf{Q}_i, h) + s_h^0 - \sum_{i=1}^n \mathsf{H}(\text{Decode}(\mathbf{V}_i, h) + \mathsf{H}(h) \cdot \Delta_i, h) = 0 \right\}.$$

From the linearity of  $\text{Decode}$ , we have that for  $i \in [1, n]$ , for  $h \in X_0 \subseteq \bigcap_{i \notin \mathcal{C}} X_i$ ,

$$\begin{aligned} \text{Decode}(\mathbf{W}_i, h) &= \text{Decode}(\mathbf{V}_i + \mathbf{S} \cdot \Delta_i, h) \\ &= \text{Decode}(\mathbf{V}_i, h) + \text{Decode}(\mathbf{S}, h) \cdot \Delta_i \\ &= \text{Decode}(\mathbf{V}_i, h) + \mathsf{H}(h) \cdot \Delta_i. \end{aligned}$$

Therefore, the computation of the output only differs for values  $h \in X_0$  where  $h \notin X_i$  for some  $i \notin \mathcal{C}$ . Such an  $h$  will never satisfy the equation in **Hybrid<sub>1</sub>** as  $h \notin X_i$ . For such an  $h$ , since  $h \notin X_i$ ,  $\mathbf{Q}_i$  is generated independently of  $s_h^i$ . Nevertheless, the output computed by  $P_0$  at  $h$  contains a term  $s_h^0$ , which renders the whole equation uniformly random in the computational sense (by Lemma 1). Thus, the probability that  $h$  satisfies the equality in **Hybrid<sub>0</sub>** is at most  $1/|\mathbb{F}|$  which is negligible in  $\kappa$ .

Note that if  $h \in \bigcap_{i \notin \mathcal{C}} X_i$ , then  $\text{Decode}(\mathbf{Q}_i, h) = \mathsf{H}(\text{Decode}(\mathbf{V}_i, h) + \mathsf{H}(h) \cdot \Delta_i, h) + s_h^i$  for  $i \notin \mathcal{C}$ . Therefore, we can rewrite the equation as follows:

$$\left\{ h \in \bigcap_{i \notin \mathcal{C}} X_i \mid \sum_{i \in \mathcal{C}} \text{Decode}(\mathbf{Q}_i, h) = \sum_{i \in \mathcal{C}} \mathsf{H}(\text{Decode}(\mathbf{V}_i, h) + \mathsf{H}(h) \cdot \Delta_i, h) - \sum_{i \notin \mathcal{C}} s_h^i \right\}.$$

**Hybrid<sub>2</sub>**: In this hybrid, how  $P_0$  computes the output is modified again. Specifically, the simulator computes the following set

$$O = \left\{ h \mid \mathcal{A} \text{ made a query of the form } \mathbf{H}(\text{Decode}(\mathbf{V}_i, h) + \mathbf{H}(h) \cdot \Delta_i, h) \text{ for } i \in \mathcal{C} \right\}.$$

Then, on behalf of  $P_0$ , the simulator computes the output with a modified computation as follows:

$$\left\{ h \in O \cap \left( \bigcap_{i \notin \mathcal{C}} X_i \right) \mid \sum_{i \in \mathcal{C}} \text{Decode}(\mathbf{Q}_i, h) = \sum_{i \in \mathcal{C}} \mathbf{H}(\text{Decode}(\mathbf{V}_i, h) + \mathbf{H}(h) \cdot \Delta_i, h) - \sum_{i \notin \mathcal{C}} s_h^i \right\}.$$

It suffices to show that if  $h \notin O$ , then the probability that  $h$  would have satisfied the equation in **Hybrid<sub>1</sub>** is negligible in  $\kappa$ . Indeed, if  $h \notin O$ , then when the adversary sends  $\mathbf{Q}_i$  for  $i \in \mathcal{C}$ , the value  $\mathbf{H}(\text{Decode}(\mathbf{V}_i, h) + \mathbf{H}(h) \cdot \Delta_i, h)$  is distributed independently of the adversary's view. Since the condition in **Hybrid<sub>1</sub>** is a linear equality including the term, the probability that it is satisfied is at most  $1/|\mathbb{F}|$  which is negligible in  $\kappa$ . We can rewrite the above equation as

$$\left\{ h \in O \mid \sum_{i \in \mathcal{C}} \text{Decode}(\mathbf{Q}_i, h) = \sum_{i \in \mathcal{C}} \mathbf{H}(\text{Decode}(\mathbf{V}_i, h) + \mathbf{H}(h) \cdot \Delta_i, h) - \sum_{i \notin \mathcal{C}} s_h^i \right\} \cap \left( \bigcap_{i \notin \mathcal{C}} X_i \right) = \tilde{X} \cap \left( \bigcap_{i \notin \mathcal{C}} X_i \right).$$

**Hybrid<sub>3</sub>**: In this hybrid, the simulator  $\mathcal{S}$  simulates the zero-sharing setup honestly, and emulates functionality  $\mathcal{F}_{\text{mvole}}$  as described above, as well as sends a uniform vector  $\mathbf{d} \in \mathbb{F}^m$  to adversary  $\mathcal{A}$  via executing protocol  $\Pi_{\text{send}}^{\text{tree}}$  with  $\mathcal{A}$ .

Clearly, the simulation of the zero-sharing setup and  $\mathcal{F}_{\text{mvole}}$  is perfect. In **Hybrid<sub>2</sub>**, the vector  $\mathbf{d} = \mathbf{S} - \mathbf{u}$  is uniformly random, as  $\mathbf{u}$  is uniform in the presence of the adversary's view. Therefore, **Hybrid<sub>3</sub>** has the identical distribution as **Hybrid<sub>2</sub>**.

**Hybrid<sub>4</sub>**: In this hybrid, if there exists at least two honest parties, then for each  $i \notin \mathcal{C}, i \neq 0$ ,  $\mathcal{S}$  computes and sends the OKVS  $\mathbf{Q}_i$  to  $\mathcal{A}$  via running protocol  $\Pi_{\text{aggregate}}^{\text{tree}}$  with  $\mathcal{A}$ , in the way that an arbitrary input set  $X'_i$  and random values  $r_{i,h} \in \mathbb{F}$  for  $h \in X'_i$  are used in the computation of OKVS  $\mathbf{Q}_i$  as described above.

If there are at least two honest parties, the simulator needs to simulate the OKVS of honest parties before seeing the OKVS of corrupted parties, as the OKVS is transmitted in a binary-tree architecture. For each  $i \notin \mathcal{C}, i \neq 0$ , for each  $h \in X_i$ , the value  $z_{i,h} + s_h^i$  is uniformly random in **Hybrid<sub>3</sub>**, since  $s_h^i$  is uniform for the case of honest  $P_i$  and  $P_0$  from the adversary's view. From the security of the OKVS scheme ( $\text{Encode}, \text{Decode}$ ),  $\mathcal{A}$  cannot distinguish between the real input set  $X_i$  and the dummy input set  $X'_i$  after seeing the OKVS  $\mathbf{Q}_i$  for each  $i \notin \mathcal{C}, i \neq 0$ . Overall, **Hybrid<sub>4</sub>** is computationally indistinguishable from **Hybrid<sub>3</sub>**.

**Hybrid<sub>5</sub>**: In this hybrid,  $\mathcal{S}$  extracts the set  $\tilde{X}$  following the way as described above, and then sends  $\tilde{X}$  to functionality  $\mathcal{F}_{\text{mpsi}}$  as the input for every corrupted party.

According to the definition of the output of  $P_0$  in the previous hybrid, it is easy to see that **Hybrid<sub>5</sub>** has the identical distribution as **Hybrid<sub>4</sub>**, where  $P_0$  will obtain the same output in two hybrids.

In conclusion, no PPT environment  $\mathcal{Z}$  can distinguish between the real-world execution and the ideal-world execution, which completes the proof.  $\square$

## 5 An Attack against Multi-Output Extension of A PSI Protocol

In the malicious setting, it is a non-trivial task to extend a multi-party PSI protocol to support multiple outputs where every party (instead of only party  $P_0$ ) will obtain the output. This is because the parties cannot be trusted to deliver the intersection output faithfully. In the multi-party malicious setting, Garimella et al. [GPR<sup>+</sup>21] extended their PSI protocol with a single output to a protocol supporting multiple outputs that achieves the best efficiency. In this section, we present a simple and practical attack for the multi-output extension [GPR<sup>+</sup>21], which allows the attacker to reveal more information of the sets of honest parties than that obtained from the intersection of the sets of all parties. Specifically, we first review the multi-output extension by Garimella et al. [GPR<sup>+</sup>21]. This extension modifies the multi-party PSI protocol shown in Section 3.1 to realize that all parties obtain the output in the following procedure:

- All parties  $P_0, P_1, \dots, P_n$  publicly commit to their OKVSs  $\{\mathbf{Q}_i\}_{i \in [0, n]}$ . That is, every party  $P_i$  broadcasts a commitment  $com_i = \text{Commit}(\mathbf{Q}_i; r_i)$  to all other parties where  $r_i$  is a randomness.
- After all the commitments that have been made, the parties open these commitments. That is, for  $i \in [0, n]$ , every party  $P_i$  sends  $(\mathbf{Q}_i, r_i)$  to all other parties, and then verifies the correctness of all the openings (i.e., checking that  $com_j = \text{Commit}(\mathbf{Q}_j; r_j)$  for all  $j \neq i$ ).
- Every party  $P_i$  with  $i \in [0, n]$  computes the output as

$$\left\{ h \in X_i \mid \sum_{j=0}^n \text{Decode}(\mathbf{Q}_j, h) = 0 \right\}.$$

**An attack to leak the information of the sets held by honest parties.** Below, we show a simple but practical attack against the above multi-output extension to reveal more information of the sets held by honest parties beyond the intersection of all sets. Suppose that  $P_0$  is honest. Let  $\mathcal{C} \subseteq [1, n]$  denote the set of corrupted parties. In particular, an adversary  $\mathcal{A}$  (who even behaves semi-honestly) is able to perform the following attack:

1.  $\mathcal{A}$  receives the OKVS  $\mathbf{Q}_i$  for all  $i \notin \mathcal{C}$  that contain  $\mathbf{Q}_0$ . Recall that the OKVS  $\mathbf{Q}_i$  for  $i \notin \mathcal{C}, i \neq 0$  is defined as follows:

$$\mathbf{Q}_i = \text{Encode}\left(\left\{ (h, \text{H}(\text{PRF}_i(h), h) + s_h^i) \mid h \in X_i \right\}\right).$$

The OKVS  $\mathbf{Q}_0$  is computed as

$$\mathbf{Q}_0 = \text{Encode}\left(\left\{ (h, s_h^0 - \sum_{i=1}^n \text{H}(\text{PRF}_i(h), h) \mid h \in X_0) \right\}\right).$$

2.  $\mathcal{A}$  computes  $\mathbf{Q} := \sum_{i \notin \mathcal{C}} \mathbf{Q}_i$ . According to the definition of zero sharings, we know that  $\sum_{i \in [0, n]} s_h^i = 0$  for any  $h$ . Therefore, for each  $h \in \bigcap_{i \notin \mathcal{C}} X_i$ , we obtain the following:

$$\sum_{i \notin \mathcal{C}} \text{Decode}(\mathbf{Q}_i, h) = - \sum_{i \in \mathcal{C}} s_h^i - \sum_{i \in \mathcal{C}} \text{H}(\text{PRF}_i(h), h). \quad (3)$$

3.  $\mathcal{A}$  who corrupts  $P_i$  for  $i \in \mathcal{C}$  can compute the shares  $s_h^i$  and values  $\text{PRF}_i(h)$  for all  $i \in \mathcal{C}$  when any  $h$  is known. Then, for any  $h \notin \bigcap_{i \in [0, n]} X_i$ ,  $\mathcal{A}$  is able to decide whether  $h \in \bigcap_{i \notin \mathcal{C}} X_i$  by checking if the equation (3) holds. For the items in the set  $\bigcap_{i \notin \mathcal{C}} X_i$  that have a low entropy,  $\mathcal{A}$  can directly reveal these items by enumerating the items and then checking their correctness, and thus obtain the secret data included in  $\bigcap_{i \notin \mathcal{C}} X_i$ .

In the special case that only  $P_0$  is honest and all other parties are corrupted (i.e.,  $|\mathcal{C}| = n$ ), the adversary can leak some secret items in the set  $X_0$  by performing the above attack. The reason behind the successful attack is that the OKVS  $\mathbf{Q}_0$  is revealed by the honest central party  $P_0$ . This does not occur in the original multi-party PSI protocol [GPR<sup>+</sup>21] that only  $P_0$  obtains the output, where  $P_0$  only uses the OKVS  $\mathbf{Q}_0$  *locally* in the original protocol. To prevent the attack as describe above, one may need to change the way computing the OKVS  $\mathbf{Q}_0$  and introduce some high-entropy secrets into  $\mathbf{Q}_0$ . We leave it as an interesting future work to design a concretely efficient multi-party PSI protocol supporting multiple outputs in the malicious setting.

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## A OKVS Overfitting

For the security proof of a maliciously secure PSI protocol, the simulator obtains an OKVS from a corrupted party, and needs to extract the keys that are encoded in the OKVS. In general, this is done by defining  $v_i = H(k_i)$  for  $i \in [1, \ell]$  where  $H$  is a random oracle. Then, the simulator can observe the queries to  $H$  made by the adversary, and then check which of the keys  $k$  satisfy  $\text{Decode}(\mathbf{S}, k) = H(k)$ . An OKVS whose parameters are chosen to encode  $\ell$  keys may often hold even more than  $\ell$  keys, when it is generated by the adversary. In the context of PSI, this allows the adversary to encode more keys than advertised. Therefore, we need to bound the number of keys that the adversary can “overfit” into an OKVS. Following the previous work [GPR<sup>+</sup>21], we model the property in the following definition.

**Definition 4** ([GPR<sup>+</sup>21]). *The  $(\ell, \ell')$ -OKVS overfitting game is defined as follows.*

- Let  $(\text{Encode}, \text{Decode})$  be an OKVS with parameters chosen to support  $\ell$  items, and  $\mathcal{A}$  be any PPT adversary. Let  $H : \mathcal{K} \rightarrow \mathcal{V}$  be a random oracle.
- Run  $\mathbf{S} \leftarrow \mathcal{A}^{H(\cdot)}(1^\kappa)$ .
- Define the following set:

$$X = \{k \mid \mathcal{A} \text{ made a query } (k) \text{ to } H \text{ and } \text{Decode}(\mathbf{S}, k) = H(k)\}.$$

- If  $|X| > \ell'$ , then the adversary  $\mathcal{A}$  wins.

We say that the  $(\ell, \ell')$ -OKVS overfitting problem is hard for an OKVS, if no PPT adversary wins this game except with negligible probability.

For  $\kappa = 128$  and  $\lambda = 40$ , according to the analysis [PTY20], when  $H : \mathcal{K} \rightarrow \mathbb{F}$  is used to define the values, a linear OKVS with a field size  $|\mathbb{F}| = 128$  can guarantee that the successful probability of the adversary in the above overfitting game is less than  $1/2^{40}$ , even though the adversary is allowed to make  $2^{80}$  queries to  $H$ .