

Private Collaborative Data Cleaning via Non-Equi PSI

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Abstract

We introduce and investigate the privacy-preserving version of collaborative data cleaning. With collaborative data cleaning, two parties want to reconcile their data sets to filter out badly classified, misclassified data items. In the privacy-preserving (private) version of data cleaning, the additional security goal is that parties should only learn their misclassified data items, but nothing else about the other party’s data set. The problem of private data cleaning is essentially a variation of private set intersection (PSI), and one could employ recent circuit-PSI techniques to compute misclassifications with privacy. However, we design, analyze, and implement three new protocols tailored to the specifics of private data cleaning that significantly outperform a circuit-PSI-based approach. With the first protocol, we exploit the idea that a small additional leakage (the size of the intersection of data items) allows for runtime and communication improvements of more than one order of magnitude over circuit-PSI. The other two protocols convert the problem of finding a mismatch in data classifications into finding a match, and then follow the standard technique of using oblivious pseudo-random functions (OPRF) for computing PSI. Depending on the number of data classes, this leads to either total runtime or communication improvements of up to two orders of magnitude over circuit-PSI.

1 Introduction

Data cleaning [16] is the most time-consuming task in data science. Current estimates range from 45% to 80% of the total time in data science is spent on data cleaning [28, 40]. Data cleaning is necessary to prepare high-quality data sets that result in high-quality machine learning models. The more data sources can be used to clean data, improve data quality, and reduce errors, the higher the accuracy of the final model.

A standard collaborative data cleaning scenario consists of two data sources, each comprising a set of pairs of data elements and their corresponding classifications (labels). One wants to find those data elements that have been misclassified, i.e., classified differently in each set. More formally, given two sets $\mathcal{S}_A = \{(x_1, u_1), \dots, (x_n, u_n)\}$ and $\mathcal{S}_B = \{(y_1, v_1), \dots, (y_n, v_n)\}$ where x_i, y_j are data elements and u_i, v_j their corresponding labels, compute the set of classification errors $\mathcal{S}_E = \{(y_j, v_j) \in B \mid \exists (x_i, u_i) \in A : x_i = y_j \wedge u_i \neq v_j\}$.

Examples for detecting such classification errors (“misclassifications”) are plentiful: malware classifiers, where two Security Operation Centers (SOCs) detect attacks by the same malware but have assigned it to different classes of malware, medical image classifiers, where experts (doctors) have classified public medical images but have come to different conclusions, and many more. Finding misclassified data elements is crucial, as these cannot be used in their corresponding application, but have to be cleaned (removed, re-classified) instead.

However, often different data sets come from different, untrusted parties, so data cannot be exchanged in clear text. Ideally, different parties would jointly compute misclassified data from their sets, while at the same time not learning anything else about the other party’s set. Private set intersection (PSI) [9] is a common tool to link two data sources without revealing anything but the intersection. Yet, for data cleaning, the intersection may be too revealing. While private data cleaning is a restricted form of PSI,

PARAMETERS: Input length n

1. Wait for input $\langle (x_1, u_1), \dots, (x_n, u_n) \rangle$ from sender S and $\langle (y_1, v_1), \dots, (y_n, v_n) \rangle$ from receiver R .
2. Output $\{i|\exists j : (x_j = y_i) \wedge (u_j \neq v_i)\}$ to R .

Figure 1: Ideal misclassification functionality $\mathcal{F}^{\text{MCLASS}}$

reducing collaborative private data cleaning to private set intersection at scale is non-trivial. As we will see, one might use a general form of circuit-PSI [3, 19, 31] and construct a strawman protocol (Section 1.3) for detecting mismatching labels $u_i \neq v_j$. Yet, the resulting performance is disappointing (Section 5) since it resorts to generic two-party computation. More clever reductions are not straightforward, since turning a label mismatch into a match (as found by PSI) would require comparing a label to the entire set of possible labels.

In this paper, we present two new ideas how to improve the performance of Private Data Cleaning and scale it to big data sizes of 2^{20} inputs. These two ideas use different insights into the problem of data cleaning which makes them interesting to study in parallel. The first (Section 3) results in protocol PDC_1 that has optimal round complexity ($O(1)$, 1.5 rounds) as well as optimal communication, and optimal computation complexity ($O(n)$). On the downside, PDC_1 brings a small additional leakage (the size of the intersection is revealed to one party), and it uses $O(n)$ public key operations. Hence, the protocol’s concrete runtime is sub-optimal.

Consequently, we present a second protocol PDC_2 which does not suffer from any leakage and only uses only a small, constant number of public key operations, and then reverts to symmetric key techniques for the bulk of the work. PDC_2 also features optimal round complexity ($O(1)$, 1 or 2 rounds depending on the oblivious pseudo-random function (OPRF) used), but has $O(n \log |\mathcal{L}|)$ communication and computation complexity, where \mathcal{L} is the set of all possible labels. Yet, as we show by our extensive benchmarks (Section 5), PDC_2 ’s concrete runtime is much better than the one of PDC_1 , rendering it practical for many real-world scenarios.

In summary, we make four contributions:

- We formalize the problem of private collaborative data cleaning and show its relation to private set intersection (Section 1.1)
- We present a new protocol with optimal round, communication and computation complexity, but small additional leakage and non-optimal use of public-key cryptography (Section 3).
- We present a new protocol with optimal round complexity, leakage, and use of public-key cryptography, but with higher communication and computation complexity (Section 4.4). However, this protocol has the best concrete runtime in our experiments.
- We evaluate our open-source implementation of these protocols and the baseline and report runtimes and communication cost (Section 5). Overall, we show an improvement of up to two orders of magnitude over circuit-PSI.

The remainder of the paper is structured as follows. In the remainder of this section, we formally define the problem, introduce our notation and some preliminaries and provide the baseline strawman protocol using circuit-PSI. In Section 2, we describe our two high-level ideas of our protocols to improve over circuit-PSI. In the subsequent sections 3 and 4, we provide the details of our protocols including all optimizations. We present our experimental results based on our open-source implementation in Section 5. Finally, we review related work in Section 6 and summarize our conclusions in Section 7.

1.1 Problem Definition

We consider two parties, a sender S and a Receiver R . Sender S has a sequence of pairs $\mathcal{S}_S = \langle (x_1, u_1), \dots, (x_n, u_n) \rangle$, and R has a sequence of pairs $\mathcal{S}_R = \langle (y_1, v_1), \dots, (y_n, v_n) \rangle$. We call the $x_i, y_j \in \mathcal{D}$ data elements, e.g., x-ray images or cryptographic hashes of malware. Here, \mathcal{D} denotes the domain of all possible

data elements. Similar to PSI, we assume that all x_i are unique in \mathcal{S}_S , and all y_j are unique in \mathcal{S}_R . We call the $u_i, v_j \in \mathcal{L}$ labels, e.g., classes in a classification task that come from domain \mathcal{L} . Labels do not need to be unique within either \mathcal{S}_S or \mathcal{S}_R . Our goal is to allow receiver R to compute all elements (y_j, v_j) representing a classification error with respect to sequence \mathcal{S}_S of sender S . That is, R should be able compute those (y_j, v_i) pairs where y_j matches an x_i , but the corresponding labels u_i and v_i differ: $\mathcal{S}_E = \{(y_j, v_i) \in \mathcal{S}_R \mid \exists (x_i, v_i) \in \mathcal{S}_S : x_i = y_j \wedge u_i \neq v_i\}$. We require that this computation is secure in the sense that it reveals nothing else to either S or R . In particular, elements (x_i, u_i) which are also in \mathcal{S}_S (and would be revealed by running PSI on $\{x_i\}$ and $\{y_j\}$) are not revealed.

We formalize our intuition of an ideal misclassification functionality in Figure 1. Note that data elements x_i, y_j are typically bit strings of some length L , $x_i, y_j \in \{0, 1\}^L$ with $L = \log |\mathcal{D}|$, and similarly $u_i, v_j \in \{0, 1\}^\ell$ with $\ell = \log |\mathcal{L}|$. To work with smaller data types and also save communication, a standard trick is to hash arbitrary long inputs into bit strings of length $\log n + \sigma$, where σ is a statistical security parameter. Similar to related work [3] against which we compare, we implicitly use this technique (if not specified otherwise), too.

1.2 Preliminaries

Throughout the paper, λ denotes a computational security parameter, and σ denotes a statistical security parameter.

We write $i \in [n]$ as a shorthand for $i \in \{1, \dots, n\}$ and $\langle x_i \rangle_{i \in [n]}$ for sequence $\langle x_1, \dots, x_n \rangle$. Throughout the paper, we will often use sequences instead of sets to represent data, as sequences allow indexing over individual elements.

For a length ℓ bit string $B = b_1 \dots b_\ell$, $\text{Prefix}_i(B) = b_1 \dots b_i$ denotes the length i prefix of B . We write $\text{Prefix}_i(B) \oplus 1$ as a shorthand for the length i prefix of B where the last bit of the prefix is flipped, i.e., $\text{Prefix}_i(B) \oplus 1 = b_1 \dots (b_i \oplus 1)$.

We write $B[i]$ to denote the i^{th} bit of B .

For a keyed pseud-random function (PRF) $\text{PRF} : \{0, 1\}^\lambda \times \{0, 1\}^\lambda \rightarrow \{0, 1\}^\lambda$, a cryptographic hash function $H : \{0, 1\}^* \rightarrow \{0, 1\}^\lambda$ (modeled as a random oracle), and s variable-length bit strings $x_i \in \{0, 1\}^{\ell_i}$, we will write $\text{PRF}_K(x_1 || \dots || x_s)$ as a shorthand for $\text{PRF}_K(H(x_1 || \dots || x_s))$. Here, “||” denotes an unambiguous pairing of inputs, e.g., concatenation with padding.

Similarly, for Elgamal encryption $\text{Enc} : \mathcal{K} \times \mathcal{M} \rightarrow \mathbb{G}$ over key space \mathcal{K} , plaintext space \mathcal{M} , and message space \mathbb{G} , we write $\text{Enc}_K(x_1 || \dots || x_s)$ as a shorthand for $\text{Enc}_K(H^{\mathbb{G}}(x_1 || \dots || x_s))$, where $H^{\mathbb{G}} : \{0, 1\}^* \rightarrow \mathbb{G}$ is a cryptographic hash function.

If obvious from the context, we will also omit writing key K for a keyed PRF $\text{PRF}_K(\cdot)$ on input x and simply write $\text{PRF}(x)$.

For two families of random variables (probability ensembles) $\{X\}_{\lambda \in \mathbb{N}}$ and $\{Y\}_{\lambda \in \mathbb{N}}$ we write $X \stackrel{c}{\equiv} Y$ if they are computationally indistinguishable.

1.3 A Strawman Solution using Circuit-PSI

There already exist variations of PSI that one can use in combination with generic two-party computation (2PC) to perform private data cleaning. On input of sequence $\mathcal{S}_S = \langle x_1, \dots, x_n \rangle$ by sender S and sequence $\mathcal{S}_R = \langle y_1, \dots, y_n \rangle$ by receiver R , a circuit-PSI protocol [3, 19, 31] offers the following functionality. For each $x_i \in \mathcal{S}_S$, S receives a value α_i , and R receives a value β_i such that $\alpha_i = \beta_i$, iff $x_i \in \mathcal{S}_S \cap \mathcal{S}_R$. After executing a circuit-PSI protocol, parties can then use 2PC to compute any functionality on top of the intersection, using the α_i, β_i as input to the 2PC.

Previous works use Oblivious Programmable Pseudo-Random Functions (OPPRFs) to realize circuit-PSI which, interestingly, also allows to include *associate payloads* in the computation of the intersection functionality with 2PC. Here, the associate payload of an input x_i is u_i as in our definition of data cleaning, i.e., parties’ inputs are pairs (x_i, u_i) and (y_i, v_i) . As we use a circuit-PSI-based strawman construction for data cleaning as our benchmark and to compare our techniques against, we give a brief overview about how such a construction could work. For more details on circuit-PSI with associated payload, refer to Section 6 of Pinkas et al. [31].

Table 1: n : number of data elements/labels $\langle x_i, u_i \rangle_{i \in [n]}, \langle y_i, v_j \rangle_{i \in [n]}$, $|\text{DH}|$: size of element of DDH-group, $\ell = |u_i| = |v_j| \leq \log n + \sigma$ for statistical security parameter σ , λ : computational security parameter, m : size of stash-free cuckoo hash table with three hash functions ($m = 1.27n$), SK: symmetric key operations, PK: public-key operations

Scheme	Communication Complexity	Computational Complexity	Leakage
Circuit-PSI + 2PC	$O(n\ell)$	$O(\lambda)\text{PK} + O(n\ell)\text{SK}$	None
PDC ₁	$13n \cdot \text{DH} $	$O(n)\text{PK} + O(n)\text{SK}$	$ X \cap Y $
Ours DH-PDC ₂	$n\ell \cdot (2 \cdot \text{DH} + H)$	$O(n\ell)\text{PK} + O(n\ell)\text{SK}$	None
KKRT-PDC ₂	$(5m + 3n)\ell\lambda$	$O(\lambda)\text{PK} + O(n\ell)\text{SK}$	None

Parties S and R run a first circuit-PSI instance using $\mathcal{S}_S = \langle x_1, \dots, x_n \rangle$ and $\mathcal{S}_R = \langle y_1, \dots, y_n \rangle$ as their input. Internally, S chooses n random values t_i and functions $f_i(y)$ such that $f_i(y) = t_i$ if $\exists j : y = x_j$, else $f_i(y)$ is random. Sender S and receiver R obviously evaluate the f_i such that R receives $r_i = f(y_i)$ for y_i .

Parties run a second circuit-PSI instances using $\mathcal{S}_S = \langle (x_1, u_1), \dots, (x_n, u_n) \rangle$ and $\mathcal{S}_R = \langle (y_1, v_1), \dots, (y_n, v_n) \rangle$ as their input. Internally, S chooses n random values t'_i and functions $f'_i(y)$ such that $f'_i(y) = t'_i \oplus u_j$, if $\exists j : y = x_j$, else $f'_i(y)$ is random. Parties obviously evaluate the f'_i such that R receives $r'_i = f'(y_i)$ for each y_i .

Finally, parties use 2PC and run for each (y_i, v_i) a circuit outputting

$$F(r_i, r'_i, v_i, t_i, t'_i) = \begin{cases} 1, & \text{if } r_i = t_i \wedge v_i \neq r'_i \oplus t'_i \\ 0, & \text{otherwise} \end{cases}$$

to R . So, R learns whether (y_i, v_i) is a misclassification if it has received 1 as an output.

In conclusion, one can realize the private data cleaning functionality by essentially running two instances of circuit-PSI and subsequently evaluating a 2PC circuit. While conceptually simple, it turns out that this strawman construction is expensive in practice, as the 2PC computation has high bandwidth requirements, see our evaluation in Section 5. In this paper, we significantly improve both computation and communication overhead for realistic values of $\ell = \log \mathcal{L}$, i.e., the domain of all possible labels.

2 Technical overview of our constructions

We present, analyze, and implement two new ideas to construct more efficient protocols that do not need to resort to generic, expensive 2PC.

2.1 PDC₁

Our first insight is that it suffices if receiver R cannot distinguish between two cases in a PSI protocol: the case $y_j \notin \mathcal{S}_S$, when their element is not in set \mathcal{S}_S of sender S , and the case $x_i = y_j \wedge u_i = v_j$, when their element is in set \mathcal{S}_S of sender S , but has the same label u_i as the corresponding element x_i from S .

Hence, with our first protocol PDC₁ we construct a variant of the basic Diffie-Hellman (DH) key exchange based PSI that evaluates an OPRF [1, 14, 25].

In PDC₁, S and R run OPRFs such that S receives PRF outputs for all data elements from \mathcal{S}_S and \mathcal{S}_R using receiver R 's key. At the same time, S also receives Elgamal ciphertexts for all labels in \mathcal{S}_S and \mathcal{S}_R , encrypted under R 's key. In case there is a PRF output from a data element \mathcal{S}_R matching a PRF output for a data element in \mathcal{S}_S , we have $x_i = y_j$. In that case, S sends this PRF output together with the two encrypted labels back to R . Receiver R decrypts these labels, and if they do not match, R has found a misclassification. In case there is no matching PRF output for a data element from \mathcal{S}_R in \mathcal{S}_S , S also sends the PRF output to R , but now together with: 1) the corresponding Elgamal encrypted label, and 2) an Elgamal re-encryption of this encrypted label. Again R decrypts the two encrypted labels and finds matching labels.

Note the security properties that PDC₁ provides. Receiver R obtains, for each y_j , two Elgamal ciphertexts. In case the ciphertexts encrypt different plaintexts, R has found a misclassification. In case they encrypt

the same plaintexts, *either* there is a matching x_i , but the labels of x_i and y_i are the same, *or* there is no matching x_i for y_i . Receiver R cannot distinguish the two cases and consequently does not learn any information besides what is specified by ideal functionality $\mathcal{F}^{\text{MCLASS}}$. However, we stress that S learns the intersection cardinality of the data elements, i.e., $|\{x_i\} \cap \{y_j\}|$ which is more than specified by $\mathcal{F}^{\text{MCLASS}}$. As we will see, PDC_1 is still interesting, as it outperforms all other protocols considered in this paper when the length $\ell = |u_i| = |v_i|$ of classification strings increases. Moreover, its additional leakage can be protected by differential privacy and hence be acceptable depending on the scenario.

For the sake of giving an overview, we here omit several additionally required security techniques for PDC_1 like *blinding*, such that, e.g., R cannot compute the actual label. We refer to Section 3 for all technical details.

2.2 PDC_2

Our second insight is that, in order to compare labels for misclassification, we only need to compare their bit representation. In particular, for each mismatching pair of labels there exists unique prefixes of the two label bit strings that differ only in the last bit.

In PDC_2 , for each of their labels u_i and v_j , S and R create all possible prefixes. In addition, S flips the last bits of their prefixes. As a result, for a *mismatching* combination $u_i \neq v_j$, there exist exactly one matching combination of prefix from R and modified prefix from S . For a *matching* pair of labels $x_i = v_j$, there exists no matching combination of prefix from R and modified prefix from S .

For each (modified) prefix, PDC_2 then hashes the (modified) prefix together with the corresponding data element x_i or y_i . For length- ℓ classifications $u_i, v_j, |u_i| = |v_j| = \ell$, S computes ℓ hash values $H(x_i || \text{ModifiedPrefix}_1(u_i)), \dots, H(x_i || \text{ModifiedPrefix}_\ell(u_i))$, and R computes ℓ hash values $H(y_j || \text{Prefix}_1(v_j)), \dots, H(y_j || \text{Prefix}_\ell(v_j))$.

DH- PDC_2 . The idea is now to simply compute private set intersection cardinality (PSI-CA) over two sets of hash values. The resulting cardinality (either 0 or 1) indicates a misclassification.

Again, we omit additional techniques, e.g., how to avoid computing PSI-CA for $O(n^2)$ pairs of sets of size ℓ , but instead compute essentially one PSI-CA for a set of size $n \cdot \ell$. We refer to Section 4 for all details.

Even though we only compute one PSI-CA, a crucial technical challenge for PDC_2 remains its efficiency. While computing PSI-CA with a DH-based OPRF approach, dubbed DH- PDC_2 (Section 4.2), results in low concrete communication complexity, its concrete computational complexity is high, requiring $3\ell \cdot n$ public key operations.

KKRT- PDC_2 . Thus, our main construction, dubbed KKRT- PDC_2 (Section 4.4), computes PSI-CA with the highly efficient KKRT OPRF [20] for our specific setting.

Applying the KKRT OPRF turns out to be a major technical problem. First, while the DH OPRF allows sender S to re-use the same key for multiple inputs from receiver R , the KKRT OPRF outputs a separate key to S for each input from R . That is, the order of inputs from R to the OPRF now matter. Second, the DH OPRF allows S to *shuffle* inputs from R before returning (shuffled) PRF outputs back to R . After S also sends PRFs outputs of their inputs, R then can elegantly compute PSI-CA by counting the number of matching PRF outputs, while not learning which of their inputs match. While ordering of inputs can be realized by a standard application of Cuckoo hashing [3, 10, 20, 21, 29–32], the lack of a receiver input shuffling functionality in the KKRT OPRF turns out to be the main challenge.

In KKRT- PDC_2 , our idea is that both S and R use stash-less Cuckoo hashing with three hash functions to hash their data elements into separate Cuckoo hash tables. With Cuckoo hashing and three hash functions, data elements $x_i = y_j$ might end up in three different buckets in their hash tables. For example, let b_1, b_2, b_3 be the three possible bucket indexes that x_i or $y_j = x_i$ can be hashed to. Sender S maps x_i to, e.g., b_2 , but R maps y_j to b_3 . Our idea is now that R place three “replicas” of $H(y_j || \text{Prefix}_k(v_j))$ into bucket b_3 , each blinded with the PRF output of y_i using a PRF key specific to bucket b_1, b_2 , and b_3 . R sends the resulting blinded table to S .

At the same time, S places $H(x_i || \text{Prefix}_k(u_j))$ into b_2 . Then S runs an OPRF with R for each of S ’s Cuckoo table buckets, using the bucket contents as input and receiving the PRF output with R ’s bucket specific key. For each data element x_i , S takes the three buckets b_1, b_2, b_3 from R ’s blinded table, and “unblinds” the replica that corresponds to the bucket R has the PRF output from. To avoid that parties learn at which bit position u_i and v_j differ, there is an additional blinding step (from S) and unblinding

<p>INPUT OF S: n pairs $\langle (x_1, u_1), \dots, (x_n, u_n) \rangle, x_i \in \{0, 1\}^*, u_i \in \{0, 1\}^\ell$, keys $\alpha_1, \alpha_2 \in \mathbb{Z}_p$, public keys g^{β_1}, g^{β_2}</p> <p>INPUT OF R: n pairs $\langle (y_1, v_1), \dots, (y_n, v_n) \rangle, y_i \in \{0, 1\}^*, v_i \in \{0, 1\}^\ell$, keys $\beta_1, \beta_2 \in \mathbb{Z}_p$, public keys $g^{\alpha_1}, g^{\alpha_2}$</p> <p>PARAMETERS: Security parameter λ, DDH group \mathbb{G} of prime order p, $p = \lambda$, generator g, Elgamal encryption $\text{Enc}, \text{Dec}, \text{UnMask}, \text{UnPeel}, \text{AddEnc}, \text{ReEnc}$, DH-based pseudorandom function family PRF, hash function $H : \{0, 1\}^* \rightarrow \mathbb{G}$</p> <p>PROTOCOL:</p> <ol style="list-style-type: none"> For all $i \in [n]$, S hashes inputs (x_i, u_i) to $X_i = H(x_i)$ and $\Upsilon_i = H(x_i u_i)$. R hashes inputs (y_i, v_i) to $Y_i = H(y_i)$ and $\Phi_i = H(y_i v_i)$. S computes the sequence of tuples $\langle (\text{PRF}_{\alpha_1}(X_i), \text{Enc}_{\alpha_2}(\Upsilon_i)) \rangle_{i \in [n]}$ and sends it to R. R computes the two sequences $\mathcal{R}_1 = \langle (\text{PRF}_{\beta_1 \alpha_1}(X_i) = \text{PRF}_{\beta_1}(\text{PRF}_{\alpha_1}(X_i)), \text{Enc}_{\alpha_2 \beta_2}(\Upsilon_i) = \text{AddEnc}_{\beta_2}(\text{Enc}_{\alpha_2}(\Upsilon_i))) \rangle_{i \in [n]}$ $\mathcal{R}_2 = \langle (\text{PRF}_{\beta_1}(Y_i), \text{Enc}_{\beta_2}(\Phi_i)) \rangle_{i \in [n]}.$ <p>R shuffles \mathcal{R}_1 and shuffles \mathcal{R}_2 and sends both to S.</p> S computes $\mathcal{S} = \langle (\text{PRF}_{\beta_1}(X_i) = \text{UnMask}_{\alpha_1}(\text{PRF}_{\beta_1 \alpha_1}(X_i)), \text{Enc}_{\beta_2}(\Upsilon_i) = \text{UnPeel}_{\alpha_2}(\text{Enc}_{\alpha_2 \beta_2}(\Upsilon_i))) \rangle_{i \in [n]}.$ <p>S creates an empty sequence Σ. For each pair $(\text{PRF}_{\beta_1}(Y_i), \text{Enc}_{\beta_2}(\Phi_i)) \in \mathcal{R}_2$,</p> <ul style="list-style-type: none"> If $\exists (\text{PRF}_{\beta_1}(X_j), \text{Enc}_{\beta_2}(\Upsilon_j)) \in \mathcal{S} : \text{PRF}_{\beta_1}(X_j) = \text{PRF}_{\beta_1}(Y_i)$, then S computes tuple $(a_i = \text{PRF}_{\beta_1}(Y_i), b_i = \text{PRF}_{\alpha_1}(\text{Enc}_{\beta_2}(\Phi_i)), c_i = \text{PRF}_{\alpha_1}(\text{Enc}_{\beta_2}(\Upsilon_j)))$ and appends it to Σ. If $\nexists (\text{PRF}_{\beta_1}(X_j), \text{Enc}_{\beta_2}(\Upsilon_j)) \in \mathcal{S} : \text{PRF}_{\beta_1}(X_j) = \text{PRF}_{\beta_1}(Y_i)$, then S computes tuple $(a_i = \text{PRF}_{\beta_1}(Y_i), b_i = \text{PRF}_{\alpha_1}(\text{Enc}_{\beta_2}(\Phi_i)), c_i = \text{PRF}_{\alpha_1}(\text{ReEnc}_{g^{\beta_2}}(b_i)))$ and appends it to Σ. <p>S sends Σ to R.</p> R computes $\langle (a_i, b'_i = \text{Dec}_{\beta_2}(b_i), c'_i = \text{Dec}_{\beta_2}(c_i)) \rangle_{i \in [n]}$. For each Y_i from R's input, where $\text{PRF}_{\beta_1}(Y_i) = a_i$, but $b'_i \neq c'_i$, R outputs i.
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Figure 2: Linear misclassification protocol PDC_1

(from R) required. For more details, we refer to Section 4.4.

We summarize the communication and computational complexities in Table 1.

3 PDC_1

With protocol PDC_1 , we improve over the strawman solution by avoiding secure 2PC and only rely on a (modified) PSI protocol. Our key insight is that the protocol view of receiver R is secure if they cannot distinguish between the two cases when 1) their element y_j is not in the input set of sender S and 2) when y_j is in input set of S ($\exists x_i : x_i = y_j$), but the two corresponding labels are equal, i.e., $u_i = v_j$. The idea behind PDC_1 is based on the standard DH-based PSI protocol for computing the size of the intersection (PSI capacity, PSI-CA [6]) and computes a permuted and blinded set intersection of the data elements. We then augment this set intersection protocol with (Elgamal) encrypted labels, such that S can select R 's label if data element x_i is *not* in the intersection, and the label from S if it is.

3.1 Tools: DH-based PRF and Elgamal Encryption

Let \mathbb{G} be a DDH group of prime order p , $|p| = \lambda$, and let key $k \xleftarrow{\$} \mathbb{Z}_p$ be chosen randomly such that $k^{-1} \bmod (p-1)$ exists (k co-prime to $p-1$). For inputs $\chi \in \mathbb{G}$, the output of function

$$\text{PRF}_k(\chi) = \chi^k$$

is indistinguishable from randomly chosen elements of \mathbb{G} . This is a simple *DH-based PRF*, essentially masking χ by k [1, 14, 25]. As a side note, one can convert such a DH-based PRF into a regular PRF by a standard application of the leftover hash lemma [12].

There exists an interesting commutativity feature for DH-based PRFs which we will exploit later. For keys $\alpha_1, \beta_1 \in \mathbb{Z}_p$, we have

$$(\text{PRF}_{\alpha_1}(x))^{\beta_1} = (\text{PRF}_{\beta_1}(x))^{\alpha_1} = \text{PRF}_{\alpha_1\beta_1}(x).$$

The masking by raising to a key can also be undone: given $\text{PRF}_{\alpha_1\beta_1}(x)$ and α_1 ,

$$\text{UnMask}_{\alpha_1}(\text{PRF}_{\alpha_1\beta_1}(x)) = Y^{\alpha_1^{-1}} = \text{PRF}_{\beta_1}(x)$$

and similarly for β_1 .

Let $\alpha_2, \beta_2 \xleftarrow{\$} \mathbb{Z}_p$ again be chosen randomly. They serve as secret keys for Elgamal encryptions Enc_{α_2} and Enc_{β_2} . The corresponding public keys g^{α_2} and g^{β_2} are distributed in advance to S and R . For key $k \in \{\alpha_2, \beta_2\}$ and any input $\chi \in \mathbb{G}$, we define Elgamal encryption as

$$\text{Enc}_k(\chi) = \langle g^r, \chi g^{rk} \rangle,$$

where $r \xleftarrow{\$} \mathbb{Z}_p$. Note that $\text{Enc}_k(\chi)$ is a probability ensemble indexed by the length p , $|p| = \text{poly}(\lambda)$. Consequently, there are many ciphertexts $\text{Enc}_k(\chi)$ for a plaintext χ which we regard as an ensemble. We only need to store one representative of the ensemble, since all other elements in the ensemble can be generated from it.

For this Elgamal encryption, we can add another layer of encryption as follows. With $\langle A, B \rangle \leftarrow \text{Enc}_{\alpha_2}(\chi)$, we define a double-layer Elgamal ciphertext $\text{Enc}_{\beta_2\alpha_2}(\chi)$ as

$$\begin{aligned} \text{Enc}_{\beta_2\alpha_2}(\chi) &= \text{AddEnc}_{\beta_2}(\langle A, B \rangle) \\ &= \langle Ag^{r'}, BA^{\beta_2} \cdot (g^{\alpha_2})^{r'} \cdot (g^{\beta_2})^{r'} \rangle, \end{aligned}$$

with $r' \in \mathbb{Z}_p$ chosen randomly. Note that knowledge of β_2 is required. Adding another layer of encryption is commutative. Let $\langle A, B \rangle \leftarrow \text{Enc}_{\alpha_2}(\chi)$ and $\langle A', B' \rangle \leftarrow \text{Enc}_{\beta_2}(\chi)$. We have

$$\text{AddEnc}_{\beta_2}(\langle A, B \rangle) \stackrel{c}{\equiv} \text{AddEnc}_{\alpha_2}(\langle A', B' \rangle).$$

To re-encrypt (re-randomize) a given ciphertext $\langle A, B \rangle \leftarrow \text{Enc}_{\alpha_2}(\chi)$ using public key g^{β_2} , we use

$$\text{ReEnc}_{g^{\beta_2}}(\langle A, B \rangle) = \langle Ag^r, B \cdot (g^{\beta_2})^r \rangle,$$

with $r \in \mathbb{Z}_p$ chosen randomly.

Furthermore, we combine our DH-based PRF with Elgamal encryption. For key $k \in \{\alpha_2, \beta_2\}$ and a ciphertext $\langle A, B \rangle$, we define our DH-based PRF variant for Elgamal ciphertexts as

$$\text{PRF}_k(\langle A, B \rangle) = \langle A^k, B^k \rangle.$$

Again, Elgamal encryption and our DH-based PRF offer commutativity. Let $\langle A, B \rangle \leftarrow \text{Enc}_{\alpha_2}(\chi)$ and $\chi' = \text{PRF}_{\beta_2}(\chi)$. Then,

$$\text{PRF}_{\beta_2}(\langle A, B \rangle) \stackrel{c}{\equiv} \text{Enc}_{\alpha_2}(\chi').$$

For some double-layer ciphertext $\langle A, B \rangle \leftarrow \text{Enc}_{\alpha_2\beta_2}$, we can peel-off one layer of encryption from key $k \in \{\alpha_2, \beta_2\}$ by

$$\text{UnPeel}_k(\langle A, B \rangle) = \langle A, BA^{-k} \rangle.$$

Finally, we also decrypt any Elgamal ciphertext $\langle A, B \rangle$ using key $k \in \{\alpha_2, \beta_2\}$ as

$$\text{Dec}_k(\langle A, B \rangle) = BA^{-k}.$$

Observe that $\text{PRF}_{\beta_2}(\chi) = \text{Dec}_{\alpha_2}(\text{PRF}_{\beta_2}(\text{Enc}_{\alpha_2}(\chi)))$.

3.2 Main Protocol

Our protocol proceeds in the following five main steps.

In Step 1, both parties hash their data elements and labels into elements from an appropriate DDH group. This enables parties to use the DH-based PRF and Elgamal constructions from the previous section.

In Step 2, sender S computes the first part of the DH-based OPRF and sends their input data elements, masked by a PRF, to R . For each data element, S also sends an Elgamal encryption of the corresponding label to R .

In Step 3, R completes the DH-based OPRF. That is, R applies their PRF to the masked data elements and encrypted labels from S , shuffles the result, and sends it back to S . Also, R sends PRF outputs of their data elements together with encryptions of corresponding labels to S .

In Step 4, S un.masks the PRF output from R for data elements of S . Then, S correlates the two received sets of PRF outputs. S learns which PRF outputs are in both sets. However, S cannot determine which of their inputs correspond to each PRF output. For each PRF output that is in both sets, S sends to R : the PRF output, a masked version of the corresponding Elgamal encryption of the label as received from R , and a masked version of the Elgamal encryption of the corresponding label from S . For each PRF output not in both sets, S sends to R : the PRF output, a masked version of the corresponding Elgamal encryption of the label as received from R , and a masked version of the re-encryption of the corresponding Elgamal encryption of the label a received from R .

In Step 5, R decrypts Elgamal ciphertexts and verifies, for each PRF output, whether the two corresponding Elgamal ciphertexts match (no misclassification) or not (misclassification found).

We formalize our protocol in Figure 2. Observe that, in Step 4, sender S creates the two indistinguishable cases of *either* $x_i \neq y_j$ *or* $x_i = y_j$, but $u_i = v_j$.

3.3 Security in IND-CDP-2PC

He et al. [13] define a security model IND-CDP-2PC for two-party computations in the computational differential privacy setting [26]. In this model the leakage of the set intersection cardinality to the sender S can be avoided. We modify the protocol as follows. In addition to $\langle x_i, u_i \rangle$ S inputs $d = 2\lceil -\log \delta/\epsilon \rceil$ elements $\langle \chi_1, v_1, \dots, \chi_d, v_d \rangle$. Correspondingly, R chooses a random number d' from a shifted and bounded Laplace distribution $\min(d, \max(0, \text{Lap}(-\log \delta/\epsilon, 1/\epsilon)))$. R 's additional input is $\langle \chi_1, v_1, \dots, \chi_{d'}, v_{d'} \rangle$ and $\langle \psi_1, v_{d'+1}, \dots, \psi_{d-d'}, v_d \rangle$ where $\psi_i \neq \chi_j$. They run the protocol as in Figure 2. R 's output is unmodified, since none of the additional inputs has a matching element with mismatching labels. Let $|\mathcal{S}_I|$ be the set intersection cardinality of $\langle x_i \rangle_{i \in [n]}$ and $\langle y_i \rangle_{i \in [n]}$. S learns, i.e., its view includes, $|\mathcal{S}_I| + d'$. This satisfies IND-CDP-2PC, since the set intersection cardinality of two neighbouring input databases \mathcal{S}_R and \mathcal{S}'_R may differ by at most 1. The shifted and bounded Laplace mechanism is (ϵ, δ) -differentially private with respect to a change (sensitivity) of 1 (see [38]) and hence the view of S in the protocol is differentially private with respect to \mathcal{S}_R .

3.4 Security Analysis

Semi-Honest Security Model. We operate in the semi-honest, i.e., passive, security model. Security against malicious adversaries is an open problem for circuit-PSI protocols. All current circuit-PSI protocols follow the same construction [3, 19, 31] of a reactive 2PC after computing secret shares of the intersection. Since this construction is composed of reactive functionalities, it is also only secure in the semi-honest model by default. Efficient constructions secure against malicious adversaries that do not resort to 2PC for the entire protocol are still an open problem. Hence, we believe it is justified to also consider the semi-honest model for our protocols.

Let \mathcal{S}_I be the intersection of elements in $\langle x_i \rangle_{i \in [n]}$ and $\langle y_i \rangle_{i \in [n]}$. For protocol PDC_1 , we assume the leakage of the size of this intersection \mathcal{S}_I given to the sender S . We prove security against a semi-honest adversaries by constructing two simulators $\text{Sim}_R(\mathcal{S}_R, \mathcal{S}_E)$ and $\text{Sim}_S(\mathcal{S}_S, |\mathcal{S}_I|)$ for the receiver and sender, respectively. The output of each simulator is computationally indistinguishable from the respective party's

view VIEW_R or VIEW_S , i.e., its messages received, during the execution of the real protocol:

$$\begin{aligned} \text{Sim}_R(\mathcal{S}_R, \mathcal{S}_E) &\stackrel{c}{\equiv} \text{VIEW}_R^{\text{PDC}_1} \\ \text{Sim}_S(\mathcal{S}_S, |\mathcal{S}_I|) &\stackrel{c}{\equiv} \text{VIEW}_S^{\text{PDC}_1} \end{aligned}$$

Security Proof.

Theorem 1. *Protocol PDC_1 securely implements¹ functionality $\mathcal{F}^{\text{MCLASS}}$ in the semi-honest security model.*

Proof. We prove the existence of the two simulators by construction.

$\text{Sim}_R(\mathcal{S}_R, \mathcal{S}_E)$: R receives n PRF outputs and Elgamal ciphertexts under S 's keys α_1 and α_2 . By the definition of the primitives (pseudo-random functions and semantically secure encryption) these can be simulated using $3n$ uniformly random numbers from the DDH group \mathbb{G} . In the second round, R receives again n PRF outputs and $2n$ Elgamal ciphertexts under R 's keys β_1 and β_2 . The simulator computes the n PRF outputs from \mathcal{S}_R and R 's key β_1 . The $2n$ Elgamal ciphertexts decrypt to (and their plaintext can be simulated by) PRF outputs of the element concatenated with the label under S ' key α_1 . For each element $y_i \in \mathcal{S}_E$, the simulator outputs the same uniformly chosen group element (in \mathbb{G}) twice. For each element $y_i \in \mathcal{S}_R \setminus \mathcal{S}_E$, the simulator outputs two independently, uniformly chosen group elements. The message received by R are Elgamal ciphertexts under R 's key β_2 of these group elements.

$\text{Sim}_S(\mathcal{S}_S, |\mathcal{S}_I|)$: S receives $2n$ pseudo-random functions and Elgamal ciphertexts under R 's keys β_1 and β_2 . The Elgamal ciphertexts can be simulated using $4n$ group elements from \mathbb{G} . However, the joint distribution of the PRF outputs reveals the set intersection cardinality, while each PRF output remains indistinguishable from a random group element. The simulator uniformly chooses $|\mathcal{S}_I|$ group elements from \mathbb{G} and puts them into set \mathcal{S}_1 . It then chooses $2(n - |\mathcal{S}_I|)$ group elements and puts them into set \mathcal{S}_2 . The simulator divides set \mathcal{S}_2 into two equal-sized sets \mathcal{S}_3 and \mathcal{S}_4 . Then it forms two new sets $\mathcal{S}_5 = \mathcal{S}_1 \cup \mathcal{S}_3$ and $\mathcal{S}_6 = \mathcal{S}_1 \cup \mathcal{S}_4$. It randomly shuffles \mathcal{S}_5 and \mathcal{S}_6 . These sequences follow the same joint distribution as the sequences received during the real protocol execution. \square

4 PDC₂

The high efficiency of OPRF-based PSI [3, 20, 34] stems from the fact that the actual intersection is computed “in the clear”. Receiver R receives (O)PRF outputs for their input, and sender S sends PRF values for their input to R . The actual intersection of elements is then computed by R “in the clear” on the two sets of PRF outputs. This avoids reverting to expensive cryptographic techniques such as 2PC or FHE during computation of the intersection and yields high efficiency. The only expensive operation is computing the OPRFs for the parties' inputs. We will stick to this “OPRF-and-comparing-in-the-clear” principle also for PDC_2 .

4.1 Overview

Our main idea is based on the following observation. If two bit strings u from S and v from R , $|u| = |v| = \ell$, differ in the bit at position i , then $u[i] \oplus 1 = v[i]$. Using an OPRF, R could receive set

$$\mathcal{R} = \{\text{PRF}_K(v[1]||1), \dots, \text{PRF}_K(v[\ell]||\ell)\}$$

as output, and S could send set

$$\mathcal{S} = \{\text{PRF}_K(u[1] \oplus 1||1), \dots, \text{PRF}_K(u[\ell] \oplus 1||\ell)\}$$

in shuffled order to R . Receiver R would now check in the clear whether there is an element in \mathcal{R} that is also in \mathcal{S} . If there is such a match, then $u \neq v$. However, with this approach, R would learn additional information, namely how many bits are different between u and v .

¹With leakage of the set intersection cardinality $|\mathcal{S}_I|$ to S

To remedy, consider the following two sets

$$\begin{aligned}\mathcal{R} &= \{(\text{Prefix}_1(v)||1), \dots, (\text{Prefix}_\ell(v)||\ell)\} \text{ and} \\ \mathcal{S} &= \{(\text{Prefix}_1(u) \oplus 1||1), \dots, (\text{Prefix}_\ell(u) \oplus 1||\ell)\}.\end{aligned}$$

These are the sets of all prefixes of u and v , where each time the last bit of the prefixes of u is flipped. It is crucial to observe that for these two sets we have $|\mathcal{S} \cap \mathcal{R}| = 1$ if and only if $u \neq v$, otherwise $|\mathcal{S} \cap \mathcal{R}| = 0$. That is, either there exists a single match between an element in \mathcal{R} which is also in \mathcal{S} ($u \neq v$), or there is no such match ($u = v$).

Consequently, for a single pair of length ℓ bit strings u from S and v from R , parties run ℓ instances of the OPRF. Here, R 's inputs are $\text{Prefix}_1(v)$ to $\text{Prefix}_\ell(v)$ such that R receives set

$$\mathcal{R} = \{\text{PRF}_K(\text{Prefix}_1(v)), \dots, \text{PRF}_K(\text{Prefix}_\ell(v))\}$$

as output. After the OPRF evaluations, S sends

$$\mathcal{S} = \{\text{PRF}_K(\text{Prefix}_1(u) \oplus 1), \dots, \text{PRF}_K(\text{Prefix}_\ell(u) \oplus 1)\}$$

to R . Then, R computes $|\mathcal{S} \cap \mathcal{R}|$ in the clear to determine whether $u = v$ and without learning anything else about u .

Integrating data elements. Our approach above essentially converts the problem of finding a *mismatch* between labels u and v into that of finding a *match*. We expand this technique to also determine equality of the corresponding x and y data elements at the same time. In our situation, a match is a pair of tuples (x, u) and (y, v) with $x = y$ and simultaneously $u \neq v$, so we change the above computation of OPRFs as follows. For R 's input element (y, v) , S and R run ℓ instances of an OPRF where R 's input is $\{(y||\text{Prefix}_1(v)), \dots, (y||\text{Prefix}_\ell(v))\}$, and R receives

$$\mathcal{R} = \{\text{PRF}_K(y||\text{Prefix}_1(v)), \dots, \text{PRF}_K(y||\text{Prefix}_\ell(v))\}$$

as output. Sender S then sends

$$\mathcal{S} = \{\text{PRF}_K(x||\text{Prefix}_1(u) \oplus 1), \dots, \text{PRF}_K(x||\text{Prefix}_\ell(u) \oplus 1)\}.$$

Finally, R again computes $|\mathcal{S} \cap \mathcal{R}|$ in the clear, which is either 0 or 1. Observe that now $|\mathcal{S} \cap \mathcal{R}| = 1$ iff $x = y \wedge u \neq v$.

To support n inputs $\{(x_1, u_1), \dots, (x_n, u_n)\}$ from S and $\{(y_1, v_1), \dots, (y_n, v_n)\}$ from R , parties run $n\ell$ instances of the OPRF, ℓ for each of R 's inputs (y_i, v_i) . Similarly, S sends ℓ PRF outputs for each input (x_i, u_i) .

Reducing complexity. Conceptually, a PRF and an OPRF for this PRF support arbitrary long inputs using the standard trick of applying a cryptographic hash function to the input before using it as input to the PRF or OPRF. Hashing reduces arbitrary length inputs down to λ bits. As a result, for standard OPRF-based PSI protocols, the length of inputs does not matter for the complexity of the protocol. However for our OPRF-based PSI misclassification protocols, the situation is different. For an input (x, u) (and also (y, v)), the lengths of data elements x and y do not matter, but label length $\ell = |u| = |v|$ does. Now, communication and communication complexity increase by a factor of ℓ . While this is acceptable for smaller values of ℓ , performance will suffer for larger values of $\ell \leq \lambda$. To reduce both asymptotic and concrete complexity, we use a probabilistic variation of the above idea which does not require $\ell = \lambda$ OPRFs for arbitrary long labels, but only $\min(\ell, \log n + \sigma)$, where σ is a statistical security parameter. As the technical features of this variation are not crucial for understanding our OPRF-based constructions in detail, assume for now that for any label length of u or v , we will perform ℓ OPRFs.

Caveats. We stress that, in the above high-level description of our OPRF-based constructions, we omit important security aspects. For example, R must receive (O)PRF outputs in shuffled order. Also, the KKRT OPRF requires a certain ordering of inputs and also shuffling of (some) output. We will come back to these during the detailed description of the constructions (Section 4.2 and Section 4.4).

<p>INPUT OF S: n pairs $\langle (x_1, u_1), \dots, (x_n, u_n) \rangle, x_i \in \{0, 1\}^*, u_i \in \{0, 1\}^\ell$ INPUT OF R: n pairs $\langle (y_1, v_1), \dots, (y_n, v_n) \rangle, y_i \in \{0, 1\}^*, v_i \in \{0, 1\}^\ell$ PARAMETERS: Security parameter λ, group \mathbb{G} of prime order p where the DDH assumption is believed to be hard, $p = \lambda$, g is a generator of \mathbb{G}, hash function $H : \{0, 1\}^* \rightarrow \mathbb{G}$ PROTOCOL:</p> <ol style="list-style-type: none"> 1. S selects $K \xleftarrow{\\$} \mathbb{Z}_p$. R selects $r \xleftarrow{\\$} \mathbb{Z}_p$. 2. For each $(y_i, v_i)_{i \in [n]}$, R computes $\langle h_{i,j} = H(y_i \text{Prefix}_j(v_i))^{r-1} \rangle_{j \in [\ell]}$. R sends all ℓn values $h_{i,j}$ to S. 3. For $i \in [n]$: <ol style="list-style-type: none"> (a) S selects random permutation $\pi_i : [\ell] \rightarrow [\ell]$. (b) For $j \in [\ell]$: <ol style="list-style-type: none"> i. S sends $h'_{i,j} = (h_{i,\pi_i(j)})^K$ to R, and R computes $\gamma_{i,j} = (h'_{i,j})^{r^{-1}}$. ii. S computes $\gamma'_{i,j} = (H(x_i \text{Prefix}_j(u_i) \oplus 1)^K$. 4. S sends all ℓn values $\gamma'_{i,j}$ in randomly shuffled order to R. 5. R outputs $\{i \exists (a, b, c) : \gamma_{i,a} = \gamma'_{b,c}\}$.

Figure 3: DH-OPRF-based misclassification protocol DH-PDC₂

4.2 DH-PDC₂ Details

Recall from Section 3.2 that the DH-based OPRF is conceptually very simple. For PRF key K from sender S and blinding key r and input y from receiver R , R starts by sending a blinded $y' = y^r$ to S . Sender S replies with $y'' = (y')^K$, and R unblinds with $(y'')^{r^{-1}}$. Informally, this realizes $\text{PRF}_K(y) = y^K$.

Figure 3 presents technical details of the DH-based OPRF applied to our PDC₂ idea for PSI misclassification. We dub this protocol DH-PDC₂. The use of the DH-based OPRF in our idea is mostly straightforward, but there are two important peculiarities. First, for each $i \in [n]$, S collects all ℓ blinded inputs $y_i || \text{Prefix}_j(v_i)$ and shuffles them using a random permutation π_i before sending PRF outputs back, see Step (3(b)i) in Figure 3.

It is crucial that S shuffles the ℓ inputs of R for each i . Otherwise, if R later finds a match for one of the ℓ inputs, they would learn the bit position where u and v differ which is more leakage than in the ideal functionality. For the same reason, S also has to shuffle all of their PRF outputs $\gamma'_{i,j}$ before sending them to R , see Step 4. Note that S can shuffle all $\gamma'_{i,j}$ at once while the $h_{i,j}$ have to be shuffled per i such that R can later still determine the index of the element with a misclassification.

There is an optimization possible which we have omitted from Figure 3. As elements from \mathbb{G} are typically larger than λ , we can employ another hash functions $H' : \mathbb{G} \rightarrow \{0, 1\}^\lambda$ and hash the $\gamma'_{i,j}$ to smaller values. Our implementation in Section 5 uses this technique for increased efficiency.

4.3 Security Analysis of DH-PDC₂

Theorem 2. *Protocol DH-PDC₂ securely implements² functionality $\mathcal{F}^{\text{MCLASS}}$ in the semi-honest security model.*

Proof. Since protocol DH-PDC₂ does not leak we prove the existence of the following two simulators:

$$\begin{aligned} \text{Sim}_R(\mathcal{S}_R, \mathcal{S}_E) &\stackrel{c}{\equiv} \text{VIEW}_R^{\text{DH-PDC}_2} \\ \text{Sim}_S(\mathcal{S}_S) &\stackrel{c}{\equiv} \text{VIEW}_S^{\text{DHPDC}_2} \end{aligned}$$

$\text{Sim}_R(\mathcal{S}_R, \mathcal{S}_E)$: R receives $2n\ell$ PRF outputs under S 's key K . The simulator creates an empty sequence of size $2n\ell$. For each $y_i \in \mathcal{S}_E$, the simulator uniformly chooses a group element in \mathbb{G} and places it at two random

²Without leakage of the set intersection cardinality $|\mathcal{S}_I|$

positions in the sequence: between $2(i-1)\ell$ and $(2i-1)\ell-1$ and between $(2i-1)\ell$ and $2i\ell-1$. All other elements in the sequence are filled with uniformly chosen group elements. This sequence is computationally indistinguishable from R 's view in DH-PDC₂.

$\text{Sim}_S(\mathcal{S}_S)$: S receives $n\ell$ PRF outputs under R 's random number r . All of these are independently simulatable by uniformly chosen group elements. \square

4.4 KKRT-PDC₂ Details

While DH-PDC₂ features the lowest communication complexity of all techniques investigated in this paper, its main drawback is the need for $O(n\ell)$ public-key operations during OPRF evaluation. Thus in KKRT-PDC₂, we replace the DH-based OPRF by the KKRT-OPRF from Kolesnikov et al. [20]. This OPRF is based on OT extensions and, consequently, requires only λ public-key operations for an initial bootstrap. The $n\ell$ evaluations of the main OPRF functionality are then performed using only fast symmetric cryptography.

4.4.1 Technical Challenges

Ideally, we would like to simply exchange the DH-based OPRF building block in DH-OPRF-based protocol DH-PDC₂ by the KKRT-OPRF to benefit from its significantly higher efficiency. Yet, usage of the KKRT OPRF in our scenario brings new technical challenges. The KKRT OPRF has the special property that it generates a separate key for each invocation. That is, for the j^{th} input y_j from receiver R , R receives $\text{PRF}_{K_j}(y_j)$, and sender S receives a random K_j . For the next input y_{j+1} , another key K_{j+1} is used, such that R receives $\text{PRF}_{K_{j+1}}(y_{j+1})$, and S receives K_{j+1} . So, in contrast to the DH-based OPRF before, the KKRT OPRF outputs depend on the order of inputs, and the above standard approach to compute set intersection or even misclassification cannot be applied. Even if both parties hold the same inputs $x_i = y_j$, PRF outputs of x_i and y_i will match only if they have been computed using the same key. More specifically, if R uses y_j as their j^{th} input, they will obtain $\text{PRF}_{K_j}(y_j)$. Now, S has to somehow send $\text{PRF}_{K_j}(x_i)$, even if x_i is not the j^{th} input of S .

For standard PSI, a well-known trick to enforce that parties use the same order for their inputs is for R to hash their inputs into a (stash-less) Cuckoo hash table [3, 10, 20, 21, 29–32]. R iterates over all buckets in their Cuckoo table and, for the b^{th} bucket containing element y_j , runs an OPRF with S to receive $\text{PRF}_{K_b}(y_j)$, i.e., the PRF output using the b^{th} key. When using Cuckoo hashing with η hash functions, S knows all η possible buckets b_1, \dots, b_η where each of their inputs x_i could have been placed by R . So, S knows all η possible keys and sends $\text{PRF}_{K_{b_1}}(x_i), \dots, \text{PRF}_{K_{b_\eta}}(x_i)$ to R which can then compare PRF outputs and compute the intersection.

While this is a valid technique to employ the KKRT OPRF in a standard PSI protocol, it does not apply to our misclassification scenario. Recall that we have to run ℓ OPRFs for each of the n inputs pairs. Assume R would hash all $n\ell$ inputs $\langle y_j, \text{Prefix}_k(v_j) \oplus 1 \rangle_{j \in [n], k \in [\ell]}$ into a Cuckoo hash table, run an OPRF with $y_j || \text{Prefix}_k(v_j) \oplus 1$ as input, and receive $\text{PRF}_{K_b}(y_j || \text{Prefix}_k(v_j) \oplus 1)$ as output. Later, R would also receive PRF outputs for inputs $\langle x_i, \text{Prefix}_k(u_i) \oplus 1 \rangle_{i \in [n], k \in [\ell]}$ from S . Then R , would be able to learn for which prefix label u_i would differ from v_j .

To remedy, S would need to shuffle the ℓ OPRF outputs $\text{PRF}_{K_b}(y_j || \text{Prefix}_1(v_j) \oplus 1), \dots, \text{PRF}_{K_b}(y_j || \text{Prefix}_\ell(v_j) \oplus 1)$ for each b before sending them. Unfortunately, this shuffling is impossible with the KKRT OPRF.

4.4.2 Main Protocol

Figure 4 formalizes protocol KKRT-PDC₂, an application of the KKRT OPRF to the PDC₂ idea. This protocol comprises the following main steps which we explain in detail.

Step 1. First, both S and R hash their input data elements x_i and y_j into stash-less, m -bucket, 3-hash-function Cuckoo tables \mathcal{T}_S and \mathcal{T}_R .

Step 2. For each bucket in \mathcal{T}_S , S and R run ℓ OPRFs with S being the receiver. The inputs are the $x_i || \text{Prefix}_k(u_i)$ for the data element x_i mapped to that bucket. As a result, S obtains PRF outputs $z_{j,k}$. Then, parties run another ℓ OPRFs for each bucket of \mathcal{T}_R such that R obtains PRF outputs $z'_{j,k}$ for their inputs $y_j || \text{Prefix}_k(v_j)$. Additionally, R obtains PRF keys $K_{R,b,k}$ used for all buckets b in \mathcal{T}_S , and S obtains PRF keys $K_{S,b,k}$ used for all buckets b in \mathcal{T}_R .

INPUT OF S : n pairs $\langle (x_1, u_1), \dots, (x_n, u_n) \rangle, x_i \in \{0, 1\}^*, u_i \in \{0, 1\}^\ell$
INPUT OF R : n pairs $\langle (y_1, v_1), \dots, (y_n, v_n) \rangle, y_i \in \{0, 1\}^*, v_i \in \{0, 1\}^\ell$, key $K_{R,*}$
PARAMETERS: Security parameter λ , Cuckoo table size $m = 1.27n$, hash functions $H : \{0, 1\}^* \rightarrow \{0, 1\}^\lambda$,
 $H_1, H_2, H_3 : \{0, 1\}^* \rightarrow [m]$, OPRF functionality $\mathcal{F}^{\text{OPRF}}$

PROTOCOL:

1. S creates an m -bucket Cuckoo hash table \mathcal{T}_S using the x_i as keys and hash functions H_1, H_2 , and H_3 .
 R creates an m -bucket Cuckoo hash table \mathcal{T}_R using the y_i as keys and hash functions H_1, H_2 , and H_3 .

2. S and R iterate over the m buckets of their Cuckoo hash tables.

For $j \in [m]$,

- (a) Let x_i be mapped into bucket $\mathcal{T}_S[j]$. S and R run ℓ instances of $\mathcal{F}^{\text{OPRF}}$ with S being the receiver and R the sender. During the k^{th} instance of $\mathcal{F}^{\text{OPRF}}$, the input of S is $h_{j,k} = H(x_i || \text{Prefix}_k(u_i))$. The output of R is a key $K_{R,j,k}$, and the output of S is $z_{j,k} = \text{PRF}_{K_{R,j,k}}(h_{j,k})$.

If no element is mapped into $\mathcal{T}_S[j]$, S runs the ℓ instances of $\mathcal{F}^{\text{OPRF}}$ with random bit strings as input.

- (b) Let y_i be mapped into $\mathcal{T}_R[j]$. S and R run ℓ instances of $\mathcal{F}^{\text{OPRF}}$ with R being the receiver and S the sender. During the k^{th} instance of $\mathcal{F}^{\text{OPRF}}$, the input of R is $h'_{j,k} = H(y_i || \text{Prefix}_k(v_i) \oplus 1)$. The output of S is a key $K_{S,j,k}$, and the output of R is $z'_{j,k} = \text{PRF}_{K_{S,j,k}}(h'_{j,k})$.

If no element is mapped into $\mathcal{T}_R[j]$, R runs the ℓ instances of $\mathcal{F}^{\text{OPRF}}$ with random bit strings as input.

3. R creates an m -bucket table \mathcal{T}^* . Each bucket $\mathcal{T}^*[j]$ comprises ℓ slots $\langle T^*[j][k] \rangle_{k \in [\ell]}$, $|T^*[j][k]| = 3\lambda$ Bit. R fills \mathcal{T}^* as follows.

For $j \in [m]$,

- If $\mathcal{T}_R[j]$ is empty, then R sets $\mathcal{T}^*[j][k] \stackrel{\$}{\leftarrow} \{0, 1\}^{3\lambda}$ for all $k \in [\ell]$.
- If $\mathcal{T}_R[j]$ is not empty, then let $\mathcal{T}_R[j] = y_i$. Let $H_1(y_i), H_2(y_i), H_3(y_i)$ be the three possible positions where y_i can be mapped to in \mathcal{T}_S .
For $k \in [\ell]$, R computes

$$\begin{aligned} c_{j,k,1} &= z'_{j,k} \oplus \text{PRF}_{K_{R,H_1(y_i),k}}(h'_{j,k}) \oplus \text{PRF}_{K_{R,*}}(y_i) \\ c_{j,k,2} &= z'_{j,k} \oplus \text{PRF}_{K_{R,H_2(y_i),k}}(h'_{j,k}) \oplus \text{PRF}_{K_{R,*}}(y_i) \\ c_{j,k,3} &= z'_{j,k} \oplus \text{PRF}_{K_{R,H_3(y_i),k}}(h'_{j,k}) \oplus \text{PRF}_{K_{R,*}}(y_i). \end{aligned}$$

R sets $\mathcal{T}^*[j][k] = c_{j,k,1} || c_{j,k,2} || c_{j,k,3}$.

4. R sends \mathcal{T}^* to S .

5. S creates an empty set \mathcal{S} and fills it as follows.

For $i \in [n]$,

- Let $H_1(x_i), H_2(x_i), H_3(x_i)$ be the three possible positions where x_i can be mapped to in \mathcal{T}_R . Let $\text{IDX}(x_i) \in \{1, 2, 3\}$ be the index of the hash function that was used to eventually map x_i into \mathcal{T}_S during Step (1). Let $\text{cut} : \{0, 1\}^\lambda \times \{1, 2, 3\} \rightarrow \lambda$ be a function with two inputs. The first input to cut is a bit string L of length 3λ , and the second input is either 1, 2 or 3. Function cut outputs either the first, second or third λ bits of L .
- For $k \in [\ell]$, S computes

$$\begin{aligned} d_{i,k,1} &= z_{H_{\text{IDX}}(x_i),k} \oplus \text{PRF}_{K_{S,H_1(x_i),k}}(h_{H_{\text{IDX}}(x_i),k}) \oplus \text{cut}(\mathcal{T}^*[H_1(x_i)], \text{IDX}(x_i)) \\ d_{i,k,2} &= z_{H_{\text{IDX}}(x_i),k} \oplus \text{PRF}_{K_{S,H_2(x_i),k}}(h_{H_{\text{IDX}}(x_i),k}) \oplus \text{cut}(\mathcal{T}^*[H_2(x_i)], \text{IDX}(x_i)) \\ d_{i,k,3} &= z_{H_{\text{IDX}}(x_i),k} \oplus \text{PRF}_{K_{S,H_3(x_i),k}}(h_{H_{\text{IDX}}(x_i),k}) \oplus \text{cut}(\mathcal{T}^*[H_3(x_i)], \text{IDX}(x_i)) \end{aligned}$$

and appends $d_{i,k,1}, d_{i,k,2}, d_{i,k,3}$ to \mathcal{S} .

6. S shuffles \mathcal{S} and sends it to R .

7. For $i \in [n]$, if $\text{PRF}_{K_{R,*}}(y_i) \in \mathcal{S}$, then R outputs y_i .

PARAMETERS: Security parameter λ , pseudo-random function family $\text{PRF} : \{0, 1\}^\lambda \times \{0, 1\}^\lambda \rightarrow \{0, 1\}^\lambda$

1. Wait for input $x \in \{0, 1\}^\lambda$ from receiver R .
2. Choose $K \xleftarrow{\$} \{0, 1\}^\lambda$. Output $\text{PRF}_K(x)$ to R , and output K to S .

Figure 5: Ideal oblivious pseudo-random function (OPRF) functionality $\mathcal{F}^{\text{OPRF}}$

Steps 3 to 6. Let there be two elements $x_i = y_j$ from S and R , and let R have mapped y_j into bucket b_1 in \mathcal{T}_R . Due to Cuckoo hashing, R does not know where S has mapped x_i in \mathcal{T}_S , but they now the three potential buckets b_1, b_2, b_3 where S could have mapped x_i to.

In Step 3, R creates another m -bucket table \mathcal{T}^* . As R has mapped y_j into b_1 of \mathcal{T}_R , they put into bucket b_1 of \mathcal{T}^* the corresponding $z'_{b_1,k}$ PRF output they have received for bucket b_1 during Step 2.

Assume R would send such a table \mathcal{T}^* to S . As S also knows possible bucket indexes b_1, b_2, b_3 where R could have mapped x_i in \mathcal{T}_R , S would check whether one of the buckets b_1, b_2, b_3 contains $\text{PRF}_{K_{S,b,k}}(x_i \parallel \text{Prefix}_k(u_i) \oplus 1)$. This is bad, because S would learn the index of the bucket which leaks information about R 's Cuckoo table. Moreover, R would also learn which of the prefixes is different between u_i and v_j .

To avoid that, R does not put $z'_{b_1,k}$ into bucket b_1 of \mathcal{T}^* , but three copies of $z'_{b_1,k}$, each one 1) additionally masked with the PRF output of $y_i \parallel \text{Prefix}_k(v_i) \oplus 1$ using the three possible keys $K_{R,b_1,k}, K_{R,b_2,k}, K_{R,b_3,k}$, and 2) masked by a random ID representing y_i . Now, S can “peel off” the first mask from *one* of these copies only if has received the correct $z_{b_1,k}$ for input $x_i \parallel \text{Prefix}_k(u_i)$ in bucket b_1 in Step 2.

Thus, for each bucket b_1, b_2, b_3 from \mathcal{T}^* , S has computed a candidate bit string $d_{i,j}$. At most one of these bit strings will be equal to R 's random ID. Finally, S sends back all candidate bit strings in shuffled order. Again, shuffling is crucial to avoid that R learns at which index the prefixes of u_i and v_j differ.

Step 7. In the shuffled set of candidate strings, R searches for their random IDs. For each random ID found, R knows that the corresponding tuple (y_j, v_j) represents a misclassification.

4.4.3 Discussion

To ease our exposition above, we have omitted several important technicalities.

When parties are performing Cuckoo hashing using the three hash functions H_1, H_2, H_3 , we must make sure that no hash collisions for parties' data elements occur. Otherwise, this leads to a security issue in Step 3. There, two (or even all three) values $c_{j,k,1}$ would be the same, telling S something about the input of R . There exist several techniques to avoid collisions among three hash functions for Cuckoo hashing, but we choose the following straightforward one.

Let all hash functions H_i come from a set of possible hash functions \mathcal{H} that all share the property of mapping into bit strings of length λ . Let S and R share a seed s for a PRG. S and R pseudo-randomly choose $H_1, H_2, H_3 \xleftarrow{\$} \mathcal{H}$ using the PRG. Whenever party S (or R) observes a collision for one of their input data elements x_i (or y_j), S (or R) hashes this data element x_i (or y_j) with the next three hash functions $H_4, H_5, H_6 \xleftarrow{\$} \mathcal{H}$ into their Cuckoo table. Again H_4, H_5, H_6 are chosen pseudo-randomly using the PRG. If there is still a collision for x_i (or y_j), H_7, H_8, H_9 are chosen and so on. The next data element x_{i+1} (or y_{j+1}) will be hashed into the Cuckoo table starting again with hash functions H_1, H_2, H_3 . Using this trick, we make sure that the same inputs $x_i = y_j$ will always be hashed with the same three hash functions. Note that the Cuckoo hash table maintains its capacity properties, since the set of hash bins for each element remains independently, identically distributed.

The formal description of protocol KKRT-PDC_2 in Figure 4 is in the $\mathcal{F}^{\text{OPRF}}$ -hybrid model. Both parties are allowed to make ideal calls to a trusted party implementing the OPRF functionality $\mathcal{F}^{\text{OPRF}}$ as defined in Figure 5. Note that the OPRF by Kolesnikov et al. [20] implements the $\mathcal{F}^{\text{OPRF}}$ functionality, and we use it for our implementation and evaluation.

We choose our parameters for stash-less Cuckoo hashing ($m = 1.27n$ buckets, 3 hash functions) following the previous analysis by Pinkas et al. [30].

4.5 Security Analysis

Theorem 3. *Protocol KKRT-PDC₂ securely implements functionality $\mathcal{F}^{\text{MCLASS}}$ in the semi-honest security model.*

Proof. $\text{Sim}_R(\mathcal{S}_R, \mathcal{S}_E)$: In the first round, R receives m PRF outputs under S 's keys. These can be simulated by uniformly chosen numbers in $\{0, 1\}^\lambda$ according to functionality $\mathcal{F}^{\text{OPRF}}$ (see also Kolesnikov et al. [20] for details).

In the second round, R receives $3n$ numbers. For each element $y_i \in \mathcal{S}_E$, the simulator outputs the PRF of y_i under R 's key $K_{R,*}$. For the other $3n - |\mathcal{S}_E|$ elements, the simulator outputs uniformly chosen numbers in $\{0, 1\}^\lambda$. The simulator outputs these $3n$ numbers in random order. The uniformly chosen numbers in $\{0, 1\}^\lambda$ are indistinguishable from the view, because either the corresponding prefix is not in S 's set and hence the PRF output under S 's key is indistinguishable to R or the prefix is in the set, but R only has access to one PRF output for the corresponding bucket in \mathcal{T}_R .

$\text{Sim}_S(\mathcal{S}_S)$: In the first round, S also receives m PRF outputs but under R 's keys. These can be simulated by uniformly chosen numbers in $\{0, 1\}^\lambda$.

In the second round, S receives $3m$ numbers. All of these can be simulated by independently, uniformly chosen numbers in $\{0, 1\}^\lambda$. The uniformly chosen numbers are indistinguishable from the view, because either the corresponding prefix is not in S 's set and hence the PRF output under R 's key is indistinguishable to R or the prefix is in the set, but S does not have access to the PRF output for the corresponding bucket in \mathcal{T}_S or it is the PRF output of y_i under R 's key $K_{r,*}$. \square

5 Evaluation

We have implemented our protocol variations PDC₁, DH-PDC₂, and KKRT-PDC₂ and evaluated them in different settings, varying input size n , label length ℓ , and network bandwidth. The goal of our evaluation is to demonstrate the real-world practicality of our constructions and to point out their individual advantages depending on the setting. We have also compared our construction to an implementation of the strawman circuit-PSI approach from Section 1.3. This strawman protocol serves as a baseline, and we will see that our constructions deliver better overall performance in all settings.

Our implementation is done in C++, and we will publish the source code upon publication of the paper. Both DH-based PRF and Elgamal encryption in PDC₁ as well as the DH-based OPRF in DH-PDC₂ use point operations on curve ristretto255 [7, 35], implemented in libSodium [24]. Our code for protocol KKRT-PDC₂ integrates the KKRT OPRF implementation by Rindal [33].

To benchmark the circuit-PSI strawman approach, we use the implementation of the circuit-PSI by Chandran et al. from the original authors [3, 37]. To compute the actual misclassification, we follow the *Associated Payload* technique described in Section 6 of Pinkas et al. [31] (and also Section 5.1 of Chandran et al. [3]). That is, we run two instances of circuit-PSI and then implement a Boolean circuit on top of resulting circuit-PSI shares. We evaluate this circuit with 2PC using the EMP-Toolkit [39]. The input bit length of the data elements is set to the optimal $\log n + \sigma$ for both the circuit-PSI part and the 2PC part, and we only vary label lengths ℓ for the 2PC part.

We have conducted our evaluation on a machine with 3.2 GHz Intel Xeon(R) W-1290 CPU and 64 GByte RAM. To emulate different network scenarios and precisely control network bandwidth, we use `WonderShaper` [15]. The evaluation results are shown in Table 2.

We vary input sizes n from 2^{16} to 2^{20} . For label length ℓ , we focus on practical values 1) $\ell = 1$, e.g., for binary classification, linear regression, medical diagnosis (cancer, no cancer), 2) $\ell = 4$, e.g., for MNIST CIFAR-10 image classification [23], 3) $\ell = 10$, e.g., for ImageNet classification [8] or types of malware, 4) $\ell = 20$, e.g., for supporting a huge number of classes such as with GPT-3 tokens [2]. Yet, we also evaluate arbitrarily long labels, marked * in Table 2, by setting $\ell = \log n + \sigma$.

We vary network bandwidth to emulate the effect of fast LAN networks with 1 GBit/s, inter-continental WANs (100 MBit/s), and even slower cell phone networks (10 MBit/s).

We measure total runtime for all schemes, i.e., from the time receiver R starts until they output misclassifications. The communication complexity comprises all data sent or received by R . For all benchmarks, we set $\lambda = 128$ and $\sigma = 40$.

Table 2: Total runtime. Times in s , communication in MByte, *: arbitrary length labels ($\ell = \log n + \sigma$)

n	ℓ	Bandwidth	Circuit-PSI		PDC ₁		DH-PDC ₂		KKRT-PDC ₂		
			Time	Comm	Time	Comm	Time	Comm	Time	Comm	
2^{16}	1	1 GBit/s	4.8		6.8		1.7		0.2		
		100 MBit/s	29.6	322	8.6	28	2.2	5	1.2	13	
		10 MBit/s	279.9		30.1		5.2		11.4		
	4	1 GBit/s	4.8		6.8		6.9		0.7		
		100 MBit/s	30.1	328	8.6	28	7.3	18	4.8	53	
		10 MBit/s	285.1		30.1		20.2		45.5		
	10	1 GBit/s	4.9		6.8		18.0		1.6		
		100 MBit/s	31.1	340	8.6	28	19.1	45	11.8	131	
		10 MBit/s	295.5		30.1		50.5		113.8		
	20	1 GBit/s	5.1		6.8		35.9		3.2		
		100 MBit/s	32.9	360	8.6	28	39.5	90	23.7	263	
		10 MBit/s	312.8		30.1		102.4		227.7		
	*	1 GBit/s	5.7		6.8		99.7		8.8		
		100 MBit/s	39.1	432	8.6	28	113.5	252	66.1	736	
		10 MBit/s	374.9		30.1		299.3		637.6		
	2^{18}	1	1 GBit/s	18.1		29.3		7.2		0.7	
			100 MBit/s	117.7	1301	35.9	112	7.8	18	5.0	54
			10 MBit/s	1127.5		121.0		20.2		47.0	
4		1 GBit/s	18.3		29.3		28.8		2.8		
		100 MBit/s	119.8	1325	35.9	112	32.2	72	19.7	217	
		10 MBit/s	1148.2		121.0		81.2		188.2		
10		1 GBit/s	18.7		29.3		71.3		6.6		
		100 MBit/s	123.9	1373	35.9	112	81.5	180	48.8	543	
		10 MBit/s	1189.7		121.0		210.9		470.3		
20		1 GBit/s	19.4		29.3		142.8		13.1		
		100 MBit/s	130.9	1453	35.9	112	165.8	360	97.5	1085	
		10 MBit/s	1258.7		121.0		432.6		940.5		
*		1 GBit/s	22.0		29.3		416.6		37.3		
		100 MBit/s	157.1	1757	35.9	112	491.9	1044	282.3	3147	
		10 MBit/s	1521.2		121.0		1290.1		2727.0		
2^{20}		1	1 GBit/s	71.0		119.3		28.8		3.1	
			100 MBit/s	475.7	5264	149.0	448	32.3	72	20.0	217
			10 MBit/s	4356.3		489.7		81.1		188.6	
	4	1 GBit/s	71.8		119.3		114.7		11.5		
		100 MBit/s	484.0	5360	149.0	448	132.4	288	79.0	868	
		10 MBit/s	4439.1		489.7		343.1		753.3		
	10	1 GBit/s	73.4		119.3		286.4		26.8		
		100 MBit/s	500.6	5552	149.0	448	336.5	720	195.8	2170	
		10 MBit/s	4604.8		489.7		884.6		1881.8		
	20	1 GBit/s	76.2		119.3		573.5		52.3		
		100 MBit/s	528.2	5872	149.0	448	678.9	1440	390.4	4341	
		10 MBit/s	4881.0		489.7		1786.8		3762.5		
	*	1 GBit/s	87.0		119.3		1733.4				
		100 MBit/s	638.7	7152	149.0	448	2065.2	4320		Out of RAM	
		10 MBit/s	5985.9		489.7		5412.2				

Discussion. Not surprisingly, our constructions outperform each other and circuit-PSI depending on the choice of ℓ .

For a realistic number of different labels of up to 1 million (2^{20}), KKRT-PDC₂ performs best, offering

the lowest total runtime. It is between 50% and 2400% faster than circuit-PSI, depending on the concrete choice of ℓ .

In scenarios where supporting arbitrary long labels ℓ are necessary and its additional leakage acceptable, PDC_1 offers the lowest communication complexity and, consequently, for slower networks the best overall runtime. For arbitrary ℓ , PDC_1 is 1200% faster than circuit-PSI which sends 1500% more data.

In scenarios where the amount of communication matters and dominates total cost [17, 18, 36], DH-PDC_2 outperforms all other schemes. Depending on the concrete choice of ℓ , circuit-PSI consumes between 65% to 6400% more communication than DH-PDC_2 .

6 Related Work

Private data cleaning (PDC) can be implemented using extended PSI, such as circuit-PSI, but we have shown that it also can be reduced to regular PSI. While the literature on PSI is too extensive to summarize it in this paper, we are not aware of related work investigating the connection between data cleaning and PSI or related work that considers mismatch (of labels) in PSI.

Labels associated with their data elements have been considered in the literature as labeled PSI [4, 5]. In labeled PSI, instead of a bit indicating inclusion in the intersection, the output is the label for each element in the intersection.

Circuit-PSI [3, 19, 31] which allows computing arbitrary circuits over the function also operates on “payloads”, which are similar to labels and we use to build our strawman protocol.

Privacy in data cleaning has so far been considered in the single database setting. It has previously been investigated how privacy impacts the querying party and how it can be improved by tailoring data cleaning methods [22]. It has also been previously investigated how privacy can be tailored for the data scientist performing the data cleaning [11].

Collaborative data cleaning without consideration of privacy has also been investigated [27]. However, the obvious privacy implications hinder deployment, and our paper addresses the problem in a systematic and formal manner.

7 Conclusion

In this paper, we have formalized the problem of private collaborative data cleaning (PDC) and investigated its connection to PSI. Private collaborative data cleaning is an important primitive in data science that leads to better data and hence better models with more accurate predictions. Just as PSI, it arises in many data science applications.

While PDC can be solved using circuit-PSI, we show that its efficiency does not scale to large data set sizes. We present a construction that has complexity independent of the number of possible labels, but has a small leakage, and we present a construction that reduces PDC to PSI at the expense of increasing the complexity by the size of the possible labels. However, when the PSI-based construction is implemented using (mostly) symmetric cryptography, its efficiency for current big data sizes is very practical. We combine the currently most efficient oblivious pseudo-random function by Kolesnikov et al. [20] with a new technique for shuffling its outputs in this second, most efficient protocol. As a result, we achieve a total runtime or communication improvement of up to two orders of magnitude over circuit-PSI.

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