

Verifiable Functional Encryption using Intel SGX*

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Abstract

Most functional encryption schemes implicitly assume that inputs to decryption algorithms, i.e., secret keys and ciphertexts, are generated honestly. However, they may be tampered by malicious adversaries. Thus, verifiable functional encryption (VFE) was proposed by Badrinarayanan et al. in ASIACRYPT 2016 where anyone can publicly check the validity of secret keys and ciphertexts. They employed indistinguishability-based (IND-based) security due to an impossibility result of simulation-based (SIM-based) VFE even though SIM-based security is more desirable. In this paper, we propose a SIM-based VFE scheme. To bypass the impossibility result, we introduce a trusted setup assumption. Although it appears to be a strong assumption, we demonstrate that it is reasonable in a hardware-based construction, e.g., Fisch et al. in ACM CCS 2017. Our construction is based on a verifiable public-key encryption scheme (Nieto et al. in SCN 2012), a signature scheme, and a secure hardware scheme, which we refer to as VFE-HW. Finally, we discuss our implementation of VFE-HW using Intel Software Guard Extensions (Intel SGX).

1 Introduction

Functional Encryption: Cloud computing has gained increasing attention since it support several functionalities, e.g., data analysis. However, sensitive user data must be secured, and protected. Thus, functional encryption [9], an extension of Public-Key Encryption (PKE), has been proposed. functional encryption allows clients to flexibly access sensitive data toward usual “all or nothing” decryption procedure. Briefly, a Trusted Authority (TA) first generates a master public key mpk and a master secret key msk . A client sends the information of function P to the TA. Generally, P can enforce sophisticated functions, e.g., access control etc. The TA generates a secret key sk_P using the msk , and gives it to the client. A plaintext msg is encrypted by the mpk , where CT is ciphertext. Finally, the client obtains $P(\text{msg})$ by decrypting the CT using sk_P .

The security of functional encryption is guaranteed by indistinguishability-based (IND-based) or simulation-based (SIM-based) notions. IND-based security guarantees that no adversary can distinguish, which plaintext was encrypted. IND-based functional encryption schemes have been proposed for the class of all (polynomial-sized) functionalities under inefficient assumptions, e.g.,

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Table 1: Comparison of Verifiable Functional Encryption

	Security	Functionality	Verifiability	Secure HW	Trusted Setup
Fisch et al. [18] (Functional Encryption)	SIM-based	Any	Not Considered	Yes	Yes ¹
Badrinarayanan et al. [7]	IND-based	Limited	Normal	No	No
Soroush et al. [30]	IND-based	Limited	Normal	No	No
Our VFE scheme	SIM-based	Any	Weak	Yes	Yes

multi-linear maps, or indistinguishability obfuscation [10,20,21,32]. Consequently, Abdalla et al. [2] proposed an IND-based functional encryption scheme that supports inner products under simple assumptions, and several works followed this direction [14, 16, 17, 31]. However, Boneh et al. [9] and O’Neil [28] demonstrated that IND-based functional encryption yields insufficient security. For example, an adversary is allowed to obtain secret keys for a function P selected by the adversary with the restriction $P(\text{msg}_0^*) = P(\text{msg}_1^*)$ where msg_0^* and msg_1^* are challenge plaintexts with the condition $\text{msg}_0^* \neq \text{msg}_1^*$. Thus, the class of P remains restricted, e.g., we cannot specify a cryptographic hash function as P due to collision resistance. Thus, SIM-based security is more desirable. Several SIM-based functional encryption schemes [3–5, 9, 13, 28] have been proposed recently. However, several works [3, 4, 9, 13] have shown that achieving SIM-based functional encryption that supports all (polynomial-sized) functionalities is impossible.

Functional Encryption using Intel SGX: To overcome this impossibility result, Fisch et al. [18] proposed IRON, an SIM-based functional encryption scheme that uses Intel SGX [6, 24–26]. Intel SGX is a hardware protection set that protects sensitive data (e.g. medical data) from malicious adversaries by storing them in enclaves generated as isolated spaces in an application.

Briefly, IRON is described as follows. The TA generates a public key pk and a decryption key dk for the PKE scheme, as well as a verification key vk and a signing key sk for the signature scheme (SIG). Then, the TA generates a secret key sk_P , where P is a function for the client. The TA generates a signature of P as a secret key sk_P using sk in a Key Manager Enclave (KME), and sends it to the client. Let CT be the ciphertext of a plaintext msg under pk . In the decryption procedure, if sk_P is a valid signature, CT is decrypted inside an enclave, and $P(\text{msg})$ is output.

Verifiable Functional Encryption: Most functional encryption schemes implicitly assume that inputs to decryption algorithm, i.e., sk_P and CT , are generated honestly according to the algorithmic procedures. However, they may be tampered by malicious adversaries. Badrinarayanan et al. [7] proposed Verifiable Functional Encryption (VFE). With VFE, anyone can publicly check the validity of sk_P and CT . If verification of sk_P and CT passes, the decryption algorithm of VFE correctly outputs $P(\text{msg})$. Badrinarayanan et al. insisted that VFE are useful for some applications, e.g., storing encrypted images [9] and audits [23]. As a drawback, they demonstrated that SIM-based VFE implies the existence of one message zero-knowledge proof systems for **NP** in the plain model. This implication contradicts the impossibility result (Section 3) shown by Goldreich et al. [22]. We emphasize that IRON does not help us to bypass this impossibility result. As a result, they employed IND-based security as shown in Table 1. A VFE proposed by Soroush et al. [30], which supports inner products, employs the same IND-based security definition. Thus, no SIM-based VFE has been proposed so far.

Our Contribution: We propose a SIM-based VFE scheme that supports any (polynomial-sized)

¹The HW.Setup algorithm in the pre-processing phase is required to be honestly run by the TA.

functionality. To support such functionality, we employ the hardware-based construction given in IRON [18], and, to achieve SIM-based security, we relax the verifiability of the definition given by Badrinarayanan et al. without losing the practicability. Intuitively, we assume that mpk and msk are generated honestly whereas those can be arbitrary values in the definition given by Badrinarayanan et al. Due to this trusted setup assumption, mpk can be considered a common reference string (CRS) in the one message zero-knowledge context [8]. One may think that this trusted setup assumption is unreasonable and too strong in practice. However, this is not the case in the hardware-based construction. We will explain it in detail in Section 3.

In addition to provide a security definition that bypasses the impossibility result, we also give a SIM-based VFE construction. The original IRON has supported public verifiability of secret keys (because these are signatures), thus we focus on how to support public verifiability for ciphertexts. Therefore, we employ (publicly) Verifiable PKE (VPKE) [27] proposed by Nieto et al. in addition to the ingredients of IRON (PKE, SIG, and HW). We employ HW as in IRON, thus we refer to proposed system as VFE-HW. Note that publicly executable computations should be run outside of memory-constrained enclaves as much as possible. Simultaneously, as in IRON, ciphertexts input to enclaves require to be non-malleable, and thus the underlying (V)PKE scheme needs to be CCA-secure. Consequently, we modify the definition of VPKE (Section 2).

Finally, we give our implementation of the proposed VFE-HW scheme for a cryptographic hash function H as the function P , i.e., the decryption algorithm for a ciphertext of msg outputs $H(\text{msg})$. Due to the nonlinearity of the hash function, the functionality seems hard to be supported by functional encryption with linear computations, e.g., inner products. Moreover, the IND-based VFE scheme does not support the function due to the key generation query restriction. In addition to these theoretical perspectives, it seems meaningful to support this functionality in practice, e.g., a password PW is encrypted and $H(PW)$ can be computed without revealing PW . Here, we employ the Pairing-Based Cryptography (PBC) library [1] to implement the VPKE scheme proposed by Nieto et al. Briefly, the encryption algorithm runs in 0.11845 sec, the verification algorithm for ciphertexts runs in 0.12329 sec, the verification algorithm for secret keys runs in 0.00057 sec, and the decryption algorithm runs in 0.06164 sec.

2 Preliminaries

Here, we define PKE, VPKE, SIG, and HW. When x is selected uniformly from set S , we denote this as $x \xleftarrow{\$} S$, and $y \leftarrow A(x)$ represents that y is the output of an algorithm A with an input x .

First, we define PKE as follows. Here, let \mathcal{M}_{pke} be a plaintext space of PKE. The setup algorithm $\text{PKE.KeyGen}(1^\lambda)$ generates $(\text{pk}_{\text{pke}}, \text{dk}_{\text{pke}})$, the encryption algorithm $\text{PKE.Enc}(\text{pk}_{\text{pke}}, \text{msg})$ outputs CT, and the decryption algorithm $\text{PKE.Dec}(\text{dk}_{\text{pke}}, \text{CT})$ outputs msg or \perp . We require that the PKE provides the indistinguishability against chosen ciphertext attack (IND-CCA) security, which is defined in Appendix A.1.

Next, we define SIG as follows. Here, let \mathcal{M}_{sig} be a message space. The key generation algorithm $\text{SIG.KeyGen}(1^\lambda)$ generates $(\text{sk}_{\text{sig}}, \text{vk}_{\text{sig}})$, the signing algorithm $\text{SIG.Sign}(\text{sk}_{\text{sig}}, \text{msg})$ outputs σ , the verification algorithm $\text{SIG.Verify}(\text{vk}_{\text{sig}}, \text{msg}, \sigma)$ outputs 0 or 1. We require that the SIG provides the existential unforgeability against chosen message attack (EUF-CMA) security, which is defined in Appendix A.2.

Next, we introduce VPKE as defined by Nieto et al. [27]. VPKE provides public verifiability, where anyone can check the validity of ciphertexts without using any secret value. They defined the decryption algorithm VPKE.Dec using two algorithms, i.e., the verification algorithm VPKE.Ver and the decryption algorithm for converted ciphertext $\text{VPKE.Dec}'$. VPKE.Ver verifies ciphertext CT

and converts CT to CT' if CT is valid. $VPKE.Dec'$ decrypts CT' , and outputs msg . In this paper, we further decompose $VPKE.Ver$ into two algorithms, i.e., $VPKE.Ver$ and $VPKE.Conv$, which will be explained later. The verification algorithm $VPKE.Ver$ verifies CT and the conversion algorithm $VPKE.Conv$ converts CT into CT' .

Next, we define $VPKE$. Here, let \mathcal{M}_{vpke} be a plaintext space of $VPKE$.

Definition 1 (Syntax of $VPKE$).

$VPKE.PGen(1^\lambda)$: This public parameter generation algorithm takes the security parameter $\lambda \in \mathbb{N}$ as input, and returns a public parameter $pars$.

$VPKE.KeyGen(pars)$: This key generation algorithm takes $pars$ as input, and returns a public key pk_{vpke} and a secret key dk_{vpke} .

$VPKE.Enc(pars, pk_{vpke}, msg)$: This encryption algorithm takes $pars$, pk_{vpke} and a plaintext $msg \in \mathcal{M}_{vpke}$ as input, and returns a ciphertext CT .

$VPKE.Dec(pars, pk_{vpke}, dk_{vpke}, CT)$: This decryption algorithm takes $pars$, pk_{vpke} , dk_{vpke} and CT as input, and returns a plaintext msg or reject symbol \perp . Internally the algorithm runs $VPKE.Ver$, $VPKE.Conv$, and $VPKE.Dec'$, which are defined as follows.

$VPKE.Ver(pars, pk_{vpke}, CT)$: This verification algorithm takes $pars$, pk_{vpke} and CT as input, and returns 1 or 0.

$VPKE.Conv(pars, pk_{vpke}, CT)$: This conversion algorithm takes $pars$, pk_{vpke} and CT as input, and returns a ciphertext CT' .

$VPKE.Dec'(pars, pk_{vpke}, dk_{vpke}, CT')$: This decryption algorithm takes $pars$, pk_{vpke} , dk_{vpke} and CT' as input, and returns a plaintext msg .

Correctness is defined as follows: For all $pars \leftarrow VPKE.PGen(1^\lambda)$, all $(pk_{vpke}, dk_{vpke}) \leftarrow VPKE.KeyGen(pars)$, all $msg \in \mathcal{M}_{vpke}$, $VPKE.Dec'(pars, pk_{vpke}, dk_{vpke}, VPKE.Conv(pars, pk_{vpke}, CT)) = msg$ holds, where $CT \leftarrow VPKE.Enc(pars, pk_{vpke}, msg)$ and $VPKE.Ver(pars, pk_{vpke}, CT) = 1$.

Next, we define strictly non-trivial public verification. Condition 1 requires that the decryption of a ciphertext CT succeeds if and only if its verification outputs 1, and Condition 2 excludes CCA-secure schemes where the decryption algorithm does not output \perp .

Definition 2 (Strictly Non-Trivial Public Verification). For any PPT adversary \mathcal{A} and the security parameter $\lambda \in \mathbb{N}$, let $pars \leftarrow VPKE.PGen(1^\lambda)$. We define the $VPKE.Ver$ algorithm is strictly non-trivial public verifiable if (1) $(pk_{vpke}, dk_{vpke}) \leftarrow VPKE.KeyGen(pars)$, and $VPKE.Ver(pars, pk_{vpke}, CT) = 0 \iff VPKE.Dec(pars, pk_{vpke}, dk_{vpke}, CT) = \perp$ for all CT , and (2) there exists a ciphertext CT for which $VPKE.Dec(pars, pk_{vpke}, dk_{vpke}, CT) = \perp$ are provided.

Next, we define IND-CCA as follows.

Definition 3 (IND-CCA). For any PPT adversary \mathcal{A} and the security parameter $\lambda \in \mathbb{N}$, we define the experiment $\text{Exp}_{VPKE, \mathcal{A}}^{\text{IND-CCA}}(\lambda)$ as follows. Here, $state$ is state information that an adversary \mathcal{A} can

preserve any information, and state is used for transferring state information to the other stage.

$\text{Exp}_{\text{VPKE}, \mathcal{A}}^{\text{IND-CCA}}(\lambda)$:

$\text{pars} \leftarrow \text{VPKE.PGen}(1^\lambda)$; $(\text{pk}_{\text{vpke}}, \text{dk}_{\text{vpke}}) \leftarrow \text{VPKE.KeyGen}(\text{pars})$

$(\text{msg}_0^*, \text{msg}_1^*, \text{state}) \leftarrow \mathcal{A}^{\text{VPKE.DEC}}(\text{find}, \text{pars}, \text{pk}_{\text{vpke}})$

$\text{msg}_0^*, \text{msg}_1^* \in \mathcal{M}_{\text{vpke}}$; $|\text{msg}_0^*| = |\text{msg}_1^*|$

$\mu \xleftarrow{\$} \{0, 1\}$; $\text{CT}^* \leftarrow \text{VPKE.Enc}(\text{pars}, \text{pk}_{\text{vpke}}, \text{msg}_\mu^*)$

$\mu' \leftarrow \mathcal{A}^{\text{VPKE.DEC}}(\text{guess}, \text{CT}^*, \text{state})$

If $\mu = \mu'$ then output 1, and 0 otherwise

- VPKE.DEC : This decryption oracle takes a ciphertext $\text{CT} \neq \text{CT}^*$ as input. If $\text{VPKE.Ver}(\text{pars}, \text{pk}_{\text{vpke}}, \text{CT}) = 0$, output \perp . Otherwise, compute $\text{CT}' \leftarrow \text{VPKE.Conv}(\text{pars}, \text{pk}_{\text{vpke}}, \text{CT})$, and return msg by running the $\text{VPKE.Dec}'(\text{pars}, \text{pk}_{\text{vpke}}, \text{dk}_{\text{vpke}}, \text{CT}')$ algorithm.

We say that VPKE is IND-CCA secure if the advantage $\text{Adv}_{\text{VPKE}, \mathcal{A}}^{\text{IND-CCA}}(\lambda) := |\Pr[\text{Exp}_{\text{VPKE}, \mathcal{A}}^{\text{IND-CCA}}(\lambda) = 1] - 1/2|$ is negligible for any PPT adversary \mathcal{A} .

For clarity, we describe the VPKE scheme proposed by Nieto et al. in Appendix B. This scheme is used in our implementation.

Next, we define the secure hardware scheme (HW scheme) [18]. In this paper, the hardware instance HW denotes an oracle that provides the functionalities given in Definition 8. Furthermore, the hardware oracle $\text{HW}(\cdot)$ denotes an interaction with other local secure hardware in addition to HW , and the Key Manager oracle $\text{KM}(\cdot)$ denotes an interaction with a remote secure hardware over an untrusted channel.

Definition 4 (Syntax of HW Scheme). A HW scheme for a set of probabilistic programs \mathcal{Q} comprises the following seven algorithms. HW has variables $\text{HW.sk}_{\text{report}}$, $\text{HW.sk}_{\text{quote}}$, and a table T . Here, $\text{HW.sk}_{\text{report}}$ and $\text{HW.sk}_{\text{quote}}$ are leveraged to store keys, and the table T is leveraged to manage the internal state of loaded enclave programs.

- $\text{HW.Setup}(1^\lambda)$: This hardware setup algorithm takes the security parameter $\lambda \in \mathbb{N}$ as input, and returns a public parameters params . This algorithm also generates the secret keys $\text{sk}_{\text{report}}$ and sk_{quote} , and stores these keys in the $\text{HW.sk}_{\text{report}}$ and $\text{HW.sk}_{\text{quote}}$ variables respectively.
- $\text{HW.Load}(\text{params}, \text{Q})$: This loading program algorithm takes params and a program $\text{Q} \in \mathcal{Q}$ as input, and returns a handle hdl . Intuitively, this algorithm loads the stateful program into the enclave to be launched. Here, hdl is leveraged to identify the enclave running Q .
- $\text{HW.Run}(\text{hdl}, \text{in})$: This running program algorithm takes hdl and a symbol in as input, and returns out corresponding to an enclave running a designated program Q . Intuitively, this algorithm runs Q at state $\text{T}[\text{hdl}]$ with in , and records out .
- $\text{HW.Run\&Report}_{\text{sk}_{\text{report}}}(\text{hdl}, \text{in})$: This running program and generating report algorithm, which can be verified by an enclave program on the same hardware platform for a local attestation, takes hdl and in as input, and returns a report $\text{report} := (\text{md}_{\text{hdl}}, \text{tag}_{\text{Q}}, \text{in}, \text{out}, \text{mac})$, where md_{hdl} is a metadata relative enclave, tag_{Q} is a program tag that identifies the program running inside an enclave, and mac is a message authentication code produced using $\text{sk}_{\text{report}}$ for $(\text{md}_{\text{hdl}}, \text{tag}_{\text{Q}}, \text{in}, \text{out})$.

- $\text{HW.Run\&Quote}_{\text{sk}_{\text{quote}}}(\text{hdl}, \text{in})$: This running program and generating quote algorithm, which can be publicly verified different hardware platform for a remote attestation, takes hdl and in as input, and returns a quote $\text{quote} := (\text{md}_{\text{hdl}}, \text{tag}_{\text{Q}}, \text{in}, \text{out}, \sigma)$, where md_{hdl} is a metadata relative enclave, tag_{Q} is a program tag that identifies the program running inside an enclave, and σ is a signature produced using sk_{quote} for $(\text{md}_{\text{hdl}}, \text{tag}_{\text{Q}}, \text{in}, \text{out})$.
- $\text{HW.ReportVerify}_{\text{sk}_{\text{report}}}(\text{hdl}, \text{report})$: This report verification algorithm takes hdl and report as input, and uses $\text{sk}_{\text{report}}$ to verify mac . If mac is valid, then the algorithm outputs 1 and adds a tuple $(1, \text{report})$ to $\text{T}[\text{hdl}]$. Otherwise, the algorithm outputs 0 and adds tuple $(0, \text{report})$ to $\text{T}[\text{hdl}]$.
- $\text{HW.QuoteVerify}(\text{params}, \text{quote})$: This quote verification algorithm, takes params and quote as input. This algorithm verifies σ . If the verification of σ succeeds, then the algorithm outputs 1. Otherwise, 0 is output.

We require that the HW provides local attestation unforgeability (LOC-ATT-UNF) and remote attestation unforgeability (REM-ATT-UNF) security, which are defined in Appendix A.3.

3 Impossibility Result of VFE and Our Solution

In this section, we introduce the impossibility result of VFE demonstrated by Badrinarayanan et al. [7]. This impossibility is caused by the verifiability of VFE. Thus, they indicated that the impossibility of VFE remains even if the impossibility of the SIM-based security given by Agrawal et al. [3] is bypassed.

Their VFE syntax differs from our VFE-HW. Thus, we first introduce their syntax. The setup algorithm $\text{VFE.Setup}(1^\lambda)$ generates (mpk, msk) , the key-generation algorithm $\text{VFE.KeyGen}(\text{mpk}, \text{msk}, \text{P})$ outputs sk_{P} , the encryption algorithm $\text{VFE.Enc}(\text{mpk}, \text{msg})$ outputs CT , and the decryption algorithm $\text{VFE.Dec}(\text{mpk}, \text{P}, \text{sk}_{\text{P}}, \text{CT})$ outputs $\text{P}(\text{msg})$ or \perp . In addition, VFE supports two verification algorithms, i.e., ciphertext verification algorithm $\text{VFE.VerifyCT}(\text{mpk}, \text{CT})$ outputs 0 or 1, and the secret key verification algorithm $\text{VFE.VerifyK}(\text{mpk}, \text{P}, \text{sk}_{\text{P}})$ outputs 0 or 1.

Next, we introduce verifiability as defined by them. Here, verifiability guarantees that if ciphertexts and secret keys are verified by the respective algorithms, then each ciphertext should be associated with a unique message msg , and the decryption result is $\text{P}(\text{msg})$. Note that it holds even under a possibly maliciously generated mpk . Let \mathcal{P}_{VFE} and \mathcal{M}_{VFE} be a family of function for VFE and a plaintext space of VFE respectively.

Definition 5 (Verifiability). *For all security parameters $\lambda \in \mathbb{N}$, $\text{mpk} \in \{0, 1\}^*$, and all $\text{CT} \in \{0, 1\}^*$, there exists $\text{msg} \in \mathcal{M}_{\text{VFE}}$ such that for all $\text{P} \in \mathcal{P}_{\text{VFE}}$ and $\text{sk}_{\text{P}} \in \{0, 1\}^*$, if $\text{VFE.VerifyCT}(\text{mpk}, \text{CT}) = 1$ and $\text{VFE.VerifyK}(\text{mpk}, \text{P}, \text{sk}_{\text{P}}) = 1$, then $\Pr[\text{VFE.Dec}(\text{mpk}, \text{P}, \text{sk}_{\text{P}}, \text{CT}) = \text{P}(\text{msg})] = 1$ holds.*

The probability that the VFE.Dec algorithm will output $\text{P}(\text{msg})$ is exactly 1 if CT and sk_{P} are valid. Thus, Badrinarayanan et al. assumed that perfect correctness holds (otherwise, a non-uniform malicious authority can sample ciphertexts/keys from the space where it fails to be correct). Note that the probability being exactly 1 yields perfect soundness for all adversaries when a proof system is constructed from VFE.

Next, we describe the impossibility result.

Theorem 1 (Theorem 3 [7]) *There exists a family of functions, each of which can be represented as a polynomial sized circuit, for which there does not exist any simulation secure verifiable functional encryption scheme.*

To prove this theorem, Badrinarayanan et al. demonstrated that SIM-based VFE implies the existence of one message zero-knowledge proof system for **NP** in the plain model which is known to be impossible. Concretely, let L be an **NP** complete language and R be the relation of L that takes a string x and a polynomial sized (in the length of x) witness ω . $R(x, \omega)$ outputs 1 if and only if $x \in L$ and ω is its witness as input. Here, we denote $R(x, \cdot)$ for all $x \in \{0, 1\}^\lambda$. A one message zero-knowledge proof system $(\mathcal{P}, \mathcal{V})$ for the language L with relation R is constructed from VFE as follows. For (x, ω) , the prover \mathcal{P} runs $(\text{mpk}, \text{msk}) \leftarrow \text{VFE.Setup}(1^\lambda)$ where $\lambda = |x|$, computes $\text{CT} \leftarrow \text{VFE.Enc}(\text{mpk}, \omega)$ and $\text{sk}_{R(x, \cdot)} \leftarrow \text{VFE.KeyGen}(\text{mpk}, \text{msk}, R(x, \cdot))$, and outputs a proof $\pi = (\text{mpk}, \text{CT}, \text{sk}_{R(x, \cdot)})$. The verifier \mathcal{V} accepts π if $\text{VFE.Dec}(\text{mpk}, R(x, \cdot), \text{sk}_{R(x, \cdot)}, \text{CT}) = 1$. Obviously, the proof system is perfectly complete if the underlying VFE scheme is perfectly correct. In addition, due to the verifiability property, the system is perfectly sound, and, since the verifiability holds even for maliciously generated mpk , CT , and sk , no trusted setup is assumed. Due to the SIM-based security, i.e., the existence of a simulator that can produce a ciphertext from only $R(x, \omega)$ without knowing ω (here, $1 = R(x, \omega)$ in this case), the system provides computational zero knowledge.

Our Solution: To bypass the impossibility result, we introduce the trusted setup where (mpk, msk) is generated honestly, and mpk is considered as a CRS.² One may think that this trusted setup assumption is unreasonable and too strong in practice. However, this is not the case in the hardware-based construction. For example, a TA manages a TAP and setups the KME in the TAP. In our system, mpk and msk are generated by running a setup program in the KME, and it is implicitly assumed that the setup program is executed correctly (Q in our scheme). In other words, anyone can verify the description of the function. In addition, we assume that the program is hardcoded as the static data, and cannot be tampered using attestation. The remaining problem can be solved since we use trusted computer that correctly runs the program, which is widely assumed in the implementation of cryptographic protocols. Thus, we claim that the trusted assumption is reasonable, and we leave how to remove this assumption without losing the SIM-based security as a future work.

Even if one message zero-knowledge proof system in the CRS model can be constructed from SIM-based VFE, this does not bypass the impossibility result since the proof system in the plain model implies a proof system in the CRS model. We emphasize that the setup algorithm that generates (mpk, msk) must be run first since other algorithms take mpk or msk as input. Therefore, we can bypass the impossibility result of Badrinarayanan et al. since any VFE-based one message zero-knowledge proof system or argument need to run the Setup algorithm first, and then mpk can be considered as a CRS.

Regarding the CRS model, Badrinarayanan et al. have mentioned that VFE appears to be constructed from a functional encryption scheme with Non-Interactive Zero-Knowledge (NIZK) proof systems. However, the CRS may be generated maliciously, and then soundness does not hold. As a result, they gave up for employing NIZK proof systems and employed non-interactive witness indistinguishable proof (NIWI) systems as the ingredients. We introduce the trusted setup assumption. Thus, we may be able to construct VFE accordingly without employing a HW scheme. However, even then, another impossibility arises, where SIM-based functional encryption cannot support all (polynomial-sized) functionalities [3]. For bypassing the impossibility, we employ a HW scheme. Random oracles may be employed to avoid introducing the trusted setup assumption. However, as mentioned by Agrawal, Koppula, and Waters [4], there is an impossibility result of

²We note that we also relax the condition that the verifiability holds where the probability that the decryption algorithm outputs $\text{P}(\text{msg})$ is not exactly 1 (concretely $1 - \text{negl}(\lambda)$) in our definition. Because the underlying local or remote attestations require non-perfect correctness, this relaxation is reasonable. This relaxation provides the converted proof system to be an argument, i.e., soundness holds only for computationally bounded adversaries.

SIM-based security in the random oracle model. Thus, we do not further consider the random oracle in this paper.

4 Definitions of VFE-HW

In this section, we define VFE-HW. Here, let HW be a hardware instance that takes a handle hdl that identifies an enclave. If an algorithm is allowed to access HW, then the algorithm can use the secure hardware functionality given in Definition 4. Let HW(\cdot) (resp. KM(\cdot)) be a hardware (resp. a key manager) oracle that takes hdl and an authentication information (Report (resp. Quote) in our construction), interacts with other local enclave specified by hdl, and runs the function contained in the authentication information. Let $\mathcal{P}_{\text{VFE-HW}}$ and $\mathcal{M}_{\text{VFE-HW}}$ be a family of functions for VFE-HW and a plaintext space of VFE-HW respectively.

Definition 6 (Syntax of VFE-HW). *A VFE-HW scheme comprises the following seven algorithms:*

VFE-HW.Setup^{HW}(1^λ): *This setup algorithm takes the security parameter $\lambda \in \mathbb{N}$ as input, and returns a master public key mpk and a master secret key msk.*

VFE-HW.KeyGen^{HW}(msk, P): *This key generation algorithm takes msk and a function $P \in \mathcal{P}_{\text{VFE-HW}}$ as input, and returns a secret key sk_P for P.*

VFE-HW.Enc(mpk, msg): *This encryption algorithm takes mpk and a plaintext $\text{msg} \in \mathcal{M}_{\text{VFE-HW}}$ as input, and returns a ciphertext CT.*

VFE-HW.DecSetup^{HW, KM(\cdot)}(mpk): *This decryption node setup algorithm takes mpk as input, and returns a handle hdl.*

VFE-HW.VerifyCT(mpk, CT): *This ciphertext verification algorithm takes mpk and CT as input, and returns 1 or 0.*

VFE-HW.VerifyK(mpk, P, sk_P): *This secret key verification algorithm takes mpk, P, and sk_P as input, and returns 1 or 0.*

VFE-HW.Dec^{HW(\cdot)}(mpk, hdl, P, sk_P , CT): *This decryption algorithm takes mpk, hdl, sk_P , and CT as input, and returns a value P(msg) or a reject symbol \perp .*

Correctness is defined as follows: For all $P \in \mathcal{P}_{\text{VFE-HW}}$, all $(\text{mpk}, \text{msk}) \leftarrow \text{VFE-HW.Setup}^{\text{HW}}(1^\lambda)$, all $\text{sk}_P \leftarrow \text{VFE-HW.KeyGen}^{\text{HW}}(\text{msk}, P)$, all $\text{hdl} \leftarrow \text{VFE-HW.DecSetup}^{\text{HW, KM}(\cdot)}(\text{mpk})$, and all $\text{msg} \in \mathcal{M}_{\text{VFE-HW}}$, let $\text{CT} \leftarrow \text{VFE-HW.Enc}(\text{mpk}, \text{msg})$, then $\Pr[\text{VFE-HW.Dec}^{\text{HW}(\cdot)}(\text{mpk}, \text{hdl}, \text{sk}_P, \text{CT}) = P(\text{msg})] = 1 - \text{negl}(\lambda)$ holds.

Next we define weak verifiability. As mentioned in Section 3, we somewhat relax the original verifiability definition, i.e., we employ the trusted setup and the probability of verifiability is not exactly 1 due to the correctness of HW scheme. Thus, we call our definition weak verifiability. Weak verifiability guarantees that if ciphertexts and secret keys are verified by the respective algorithms, then each ciphertext should be associated with a unique message msg, and the decryption result is P(msg). Note that this holds only when mpk is generated honestly and hdl is non- \perp .

Definition 7 (Weak Verifiability). *For all security parameters $\lambda \in \mathbb{N}$, $(\text{mpk}, \text{msk}) \leftarrow \text{VFE-HW.Setup}^{\text{HW}}(1^\lambda)$, and $\text{hdl} \leftarrow \text{VFE-HW.DecSetup}^{\text{HW, KM}(\cdot)}(\text{mpk})$ where $\text{hdl} \neq \perp$, and all $\text{CT} \in \{0, 1\}^*$, there exists $\text{msg} \in \mathcal{M}_{\text{VFE-HW}}$ such that for all $P \in \mathcal{P}_{\text{VFE-HW}}$ and $\text{sk}_P \in \{0, 1\}^*$, if $\text{VFE-HW.VerifyCT}(\text{mpk}, \text{CT}) = 1$ and $\text{VFE-HW.VerifyK}(\text{mpk}, P, \text{sk}_P) = 1$, then $\Pr[\text{VFE-HW.Dec}^{\text{HW}(\cdot)}(\text{mpk}, \text{hdl}, P, \text{sk}_P, \text{CT}) = P(\text{msg})] = 1 - \text{negl}(\lambda)$ holds.*

Next we define the simulation security of VFE-HW as follows. This security guarantees that no adversary can distinguish REAL and IDEAL, where REAL represents the actual environment. Note that msk and the challenge plaintext msg^* are not explicitly used in IDEAL.

Definition 8 (Simulation security). *For a stateful PPT adversary \mathcal{A} , a stateful PPT simulator \mathcal{S} and the security parameter $\lambda \in \mathbb{N}$, we define the real experiment $\text{Exp}_{\text{VFE-HW}}^{\text{REAL}}(\lambda)$ and the ideal experiment $\text{Exp}_{\text{VFE-HW}}^{\text{IDEAL}}(\lambda)$ as follows. Here, let $\mathcal{U}_{\text{msg}}(\cdot)$ denote a universal oracle where $\mathcal{U}_{\text{msg}}(P) = \text{P}(\text{msg})$.*

$\text{Exp}_{\text{VFE-HW}}^{\text{REAL}}(\lambda)$:
 $(\text{mpk}, \text{msk}) \leftarrow \text{VFE-HW.Setup}^{\text{HW}}(1^\lambda)$; $\text{msg}^* \leftarrow \mathcal{A}^{\text{VFE-HW.KeyGen}^{\text{HW}}(\text{msk}, \cdot)}(\text{mpk})$
 $\text{CT}^* \leftarrow \text{VFE-HW.Enc}(\text{mpk}, \text{msg}^*)$; $\alpha \leftarrow \mathcal{A}^{\text{VFE-HW.KeyGen}^{\text{HW}}(\text{msk}, \cdot), \text{HW}(\cdot), \text{KM}(\cdot)}(\text{mpk}, \text{CT}^*)$
 Output (msg^*, α)

- HW: \mathcal{A} can access the instance as follows.
 - HW.LOAD: \mathcal{A} queries the instance as input params and Q , and the instance returns hdl by running the $\text{HW.Load}(\text{params}, \text{Q})$ algorithm.
 - HW.RUN: \mathcal{A} queries the instance as input hdl and in , and the instance returns out by running the $\text{HW.Run}(\text{hdl}, \text{in})$ algorithm.
- $\text{VFE-HW.KeyGen}^{\text{HW}}$: \mathcal{A} queries this key generation oracle as input msk and P . The oracle accesses HW.RUN as input $\text{hdl} = \text{msk}$ and $\text{in} = \text{P}$, and the oracle returns sk_P as out by running the $\text{HW.Run}(\text{hdl}, \text{in})$ algorithm.
- $\text{HW}(\cdot)$: \mathcal{A} can access HW.RUN\&REPORT in addition to HW as input hdl and in , and the oracle returns report by running the $\text{HW.Run\&Report}_{\text{sk}_{\text{report}}}(\text{hdl}, \text{in})$ algorithm.
- $\text{KM}(\cdot)$: \mathcal{A} can access HW.RUN\"E as input hdl and in , and the oracle returns quote by running the $\text{HW.Run\&Quote}_{\text{sk}_{\text{quote}}}(\text{hdl}, \text{in})$ algorithm.

$\text{Exp}_{\text{VFE-HW}}^{\text{IDEAL}}(\lambda)$:
 $\text{mpk} \leftarrow \mathcal{S}(1^\lambda)$; $\text{msg}^* \leftarrow \mathcal{A}^{\mathcal{S}(\cdot)}(\text{mpk})$
 $\text{CT}^* \leftarrow \mathcal{S}^{\mathcal{U}_{\text{msg}}(\cdot)}(1^\lambda, 1^{|\text{msg}^*|})$; $\alpha \leftarrow \mathcal{A}^{\mathcal{S}^{\mathcal{U}_{\text{msg}}(\cdot)}(\cdot)}(\text{mpk}, \text{CT}^*)$
 Output (msg^*, α)

- $\mathcal{S}(\cdot)$: \mathcal{S} simulates the HW , $\text{VFE-HW.KeyGen}^{\text{HW}}$, $\text{HW}(\cdot)$ and $\text{KM}(\cdot)$ oracles.
- $\mathcal{S}^{\mathcal{U}_{\text{msg}}(\cdot)}(\cdot)$: \mathcal{S} simulates the HW , the $\text{VFE-HW.KeyGen}^{\text{HW}}$, the $\text{HW}(\cdot)$ and the $\text{KM}(\cdot)$ oracles. Here, if \mathcal{A} queries this oracle as input CT^* and sk_P , \mathcal{S} outputs $\text{P}(\text{msg})$ using the universal oracle $\mathcal{U}_{\text{msg}}(\cdot)$ that inputs P queried in the $\text{VFE-HW.KeyGen}^{\text{HW}}$ oracle.

If there exists a stateful simulator \mathcal{S} and $\text{Exp}_{\text{VFE-HW}}^{\text{REAL}}(\lambda)$ and $\text{Exp}_{\text{VFE-HW}}^{\text{IDEAL}}(\lambda)$ are computationally indistinguishable, then we say that the VFE-HW scheme is simulation secure against a stateful PPT adversary

5 Proposed Scheme

In this section, we describe the proposed VFE-HW scheme. The proposed scheme is constructed from IND-CCA secure and strictly non-trivial public verifiable VPKE, IND-CCA secure PKE, EUF-CMA secure SIG and REM-ATT-UNF, and LOC-ATT-UNF secure HW.

High-Level Description: Essentially, we follow the construction of IRON. IRON has supported public verifiability of secret keys (since these are signatures), we focus on supporting the public verifiability of ciphertexts. Therefore, we replace a PKE scheme employed to encrypt msg with a VPKE scheme.

In our VFE-HW scheme, the (function) enclave securely executes computations that require secret values, however, its computational power and memory are constrained. Thus, the verification part should be run outside of the enclave, and we employ the public verifiability of VFE. However, the ciphertext is converted if the original VPKE.Ver algorithm is employed. Thus, the converted ciphertext CT' is decrypted via VPKE.Dec' in the enclave. Although at least IND-CPA security is guaranteed if VPKE.Dec is replaced with VPKE.Dec' [27], the underlying VPKE scheme is required to be CCA-secure. Thus, we decompose VPKE.Ver to VPKE.Ver and VPKE.Conv, and run VPKE.Conv inside of the enclave.

We consider the following assumptions in the construction of the VFE-HW. The first two assumptions are the same as those of IRON, and we introduce the last assumption in this paper.

- **Pre-Processing:** The TA and a client need to complete the pre-processing phase before using VFE-HW scheme. In our construction, we consider that a manufacturer setups and initializes the secure hardware. A public parameter is generated by this phase independent of the VFE-HW algorithms, and this parameter is implicitly given to all algorithms.
- **Non-Interaction:** In VFE-HW, a plaintext is encrypted using a public key of a VPKE scheme, and thus the decryption of the ciphertext requires the corresponding decryption key, which differs from a secret key sk_P . To obtain the decryption key from the KME, we require a one-time hardware setup operation. The VFE-HW.DecSetup^{HW, KM(·)} algorithm interacts with the KME via the KM(·), and the VFE-HW.Dec^{HW(·)} algorithm is non-interactive.
- **Trusted Setup:** VFE-HW.Setup^{HW} and VFE-HW.DecSetup^{HW, KM(·)} are executed honestly. In short, mpk, msk and hdl are generated honestly.

The proposed scheme is given as follows. First, we describe the programs Q_{KME} (for the KME), Q_{DE} (for a Decryption Enclave DE) and Q_{FE} (for a Function Enclave FE). Q_{FE} is parameterized by a function P , and thus we denote $Q_{FE}(P)$. Let T be an internal state valuable, $tag_{Q_{DE}}$ be a measurement of Q_{DE} hardcoded in the static data of Q_{KME} , and $tag_{Q_{FE}(P)}$ be a measurement of $Q_{FE}(P)$.

Q_{KME} :

- On input (“init”, 1^λ):
 1. Run $pars \leftarrow VPKE.PGen(1^\lambda)$.
 2. Run $(pk_{vpke}, dk_{vpke}) \leftarrow VPKE.KeyGen(pars)$ and $(sk_{sign}, vk_{sign}) \leftarrow SIG.KeyGen(1^\lambda)$.
 3. Update T to $(dk_{vpke}, sk_{sign}, vk_{sign})$ and output $(pars, pk_{vpke}, vk_{sign})$.
- On input (“provision”, quote, params):
 1. Parse $quote = (md_{hdl_{DE}}, tag_{Q_{DE}}, in, out, \sigma)$. If $tag_{Q_{DE}}$ is not matched to tag hardcoded as static data, then output \perp .

2. Parse $in = (\text{"init setup"}, vk_{\text{sign}})$ and check if vk_{sign} matches with one in T .
3. Parse $out = (sid, pk_{ra})$ and run $b \leftarrow \text{HW.QuoteVerify}(params, quote)$. If $b = 0$ output \perp .
4. Retrieve dk_{vpke} from T and compute $ct_{dk} = \text{PKE.Enc}(pk_{ra}, dk_{vpke})$ and $\sigma_{dk} = \text{SIG.Sign}(sk_{\text{sign}}, (sid, dk_{vpke}))$, and output $(sid, ct_{dk}, \sigma_{dk})$.

- On input $(\text{"sign"}, msg)$: Compute $sig \leftarrow \text{SIG.Sign}(sk_{\text{sign}}, msg)$ and output sig .

Q_{DE} :

- On input $(\text{"init setup"}, vk_{\text{sign}})$:
 1. Run $(pk_{ra}, dk_{ra}) \leftarrow \text{PKE.KeyGen}(1^\lambda)$.
 2. Generate a session ID, $sid \leftarrow \{0, 1\}^\lambda$.
 3. Update T to $(sid, dk_{ra}, vk_{\text{sign}})$ and output (sid, pk_{ra}) .
- On input $(\text{"complete setup"}, pk_{ra}, sid, ct_{dk}, \sigma_{dk})$:
 1. Look up T to obtain the entry $(sid, dk_{ra}, vk_{\text{sign}})$. If no entry exists for sid , output \perp .
 2. If $\text{SIG.Verify}(vk_{\text{sign}}, (sid, ct_{dk}), \sigma_{dk}) = 0$, output \perp . Otherwise, run $dk_{vpke} \leftarrow \text{PKE.Dec}(dk_{ra}, ct_{dk})$.
 3. Add the tuple $(dk_{vpke}, vk_{\text{sign}})$ to T .
- On input $(\text{"provision"}, report, sig)$:
 1. Check to see that the setup has been completed, i.e. T contains the tuple $(dk_{vpke}, vk_{\text{sign}})$. If not, output \perp .
 2. Check to see that the report has been verified, i.e. T contains the tuple $(1, report)$. If not, output \perp .
 3. Parse $report = (md_{\text{hdl}_P}, tag_{Q_{FE}(P)}, in, out, mac)$ and parse $out = (sid, pk_{la})$.
 4. If $\text{SIG.Verify}(vk_{\text{sign}}, tag_{Q_{FE}(P)}, sig) = 0$, then output \perp . Otherwise, output $(sid, ct_{key} = \text{PKE.Enc}(pk_{la}, dk_{vpke}))$.

$Q_{FE}(P)$:

- On input $(\text{"init"}, sig)$:
 1. Run $(pk_{la}, dk_{la}) \leftarrow \text{PKE.KeyGen}(1^\lambda)$.
 2. Generate a session ID, $sid \leftarrow \{0, 1\}^\lambda$.
 3. Update T to (sid, dk_{la}) and output (sid, pk_{la}) .
- On input $(\text{"run"}, pars, params, mpk, pk_{la}, report_{dk}, CT)$:
 1. Parse $mpk = (pk_{vpke}, vk_{\text{sign}})$.
 2. Check to see that the report has been verified, i.e. T contains the tuple $(1, report_{dk})$. If not, output \perp .
 3. Parse $report_{dk} = (md_{\text{hdl}_{DE}}, tag_{Q_{DE}}, in, out, mac)$. Parse $out = (sid, ct_{key})$.
 4. Look up T to obtain the entry (sid, dk_{la}, sk_P) . If no entry exists for sid , output \perp .
 5. Compute $dk_{vpke} \leftarrow \text{PKE.Dec}(dk_{la}, ct_{key})$.
 6. Compute $CT' \leftarrow \text{VPKE.Conv}(pars, pk_{vpke}, CT)$.

7. Compute $\text{msg} \leftarrow \text{VPKE.Dec}'(\text{pars}, \text{pk}_{\text{vpke}}, \text{dk}_{\text{vpke}}, \text{CT}')$.
8. Evaluate P on msg using sk_P and record the output $\text{out} := P(\text{msg})$. Output out .

Next, we describe the proposed scheme as follows. Here, without loss of generality, prior to running VFE-HW.Dec , we assume that a ciphertext CT is verified by VFE-HW.VerifyCT , and a secret key sk_P is verified by VFE-HW.VerifyK . Then, CT and sk_P are input to VFE-HW.Dec only when these are valid, and VFE-HW.Dec does not check their validity. This assumption is natural because we consider public verifiability for both CT and sk_P .

Proposed scheme:

Pre-Processing phase : The trusted authority platform and decryption node run respectively.

1. Call $\text{params} \leftarrow \text{HW.Setup}(1^\lambda)$, and output params .

$\text{VFE-HW.Setup}^{\text{HW}}(1^\lambda)$:

1. Call $\text{hdl}_{\text{KME}} \leftarrow \text{HW.Load}(\text{params}, \text{Q}_{\text{KME}})$.
2. Call $(\text{pars}, \text{pk}_{\text{vpke}}, \text{vk}_{\text{sign}}) \leftarrow \text{HW.Run}(\text{hdl}_{\text{KME}}, (\text{"init"}, 1^\lambda))$.
3. Output $\text{mpk} = (\text{pars}, \text{pk}_{\text{vpke}}, \text{vk}_{\text{sign}})$, $\text{msk} = \text{hdl}_{\text{KME}}$.

$\text{VFE-HW.Keygen}^{\text{HW}}(\text{msk}, P)$:

1. Parse $\text{msk} = \text{hdl}_{\text{KME}}$.
2. Compute tag_P by using a function P .
3. Call $\text{sig} \leftarrow \text{HW.Run}(\text{hdl}_{\text{KME}}, (\text{"sign"}, \text{tag}_P))$.
4. Output $\text{sk}_P = \text{sig}$.

$\text{VFE-HW.Enc}(\text{mpk}, \text{msg})$:

1. Parse $\text{mpk} = (\text{pars}, \text{pk}_{\text{vpke}}, \text{vk}_{\text{sign}})$.
2. Compute $\text{CT} \leftarrow \text{VPKE.Enc}(\text{pars}, \text{pk}_{\text{vpke}}, \text{msg})$.

$\text{VFE-HW.DecSetup}^{\text{HW}, \text{KM}(\cdot)}(\text{mpk})$:

1. Call $\text{hdl}_{\text{DE}} \leftarrow \text{HW.Load}(\text{params}, \text{Q}_{\text{DE}})$.
2. Parse $\text{mpk} = (\text{pars}, \text{pk}_{\text{vpke}}, \text{vk}_{\text{sign}})$.
3. Call $\text{quote} \leftarrow \text{HW.Run}\&\text{Quote}_{\text{sk}_{\text{quote}}}(\text{hdl}_{\text{DE}}, (\text{"init setup"}, \text{vk}_{\text{sign}}))$.
4. Call $\text{KM}(\text{quote})$ which internally run $(\text{sid}, \text{ct}_{\text{dk}}, \sigma_{\text{dk}}) \leftarrow \text{HW.Run}(\text{hdl}_{\text{KME}}, (\text{"provision"}, \text{quote}, \text{params}))$.

$\text{VFE-HW.VerifyCT}(\text{mpk}, \text{CT})$:

1. Parse $\text{mpk} = (\text{pars}, \text{pk}_{\text{vpke}}, \text{vk}_{\text{sign}})$.
2. If $\text{VPKE.Ver}(\text{pars}, \text{pk}_{\text{vpke}}, \text{CT}) = \perp$, then output 0. Otherwise, output 1.

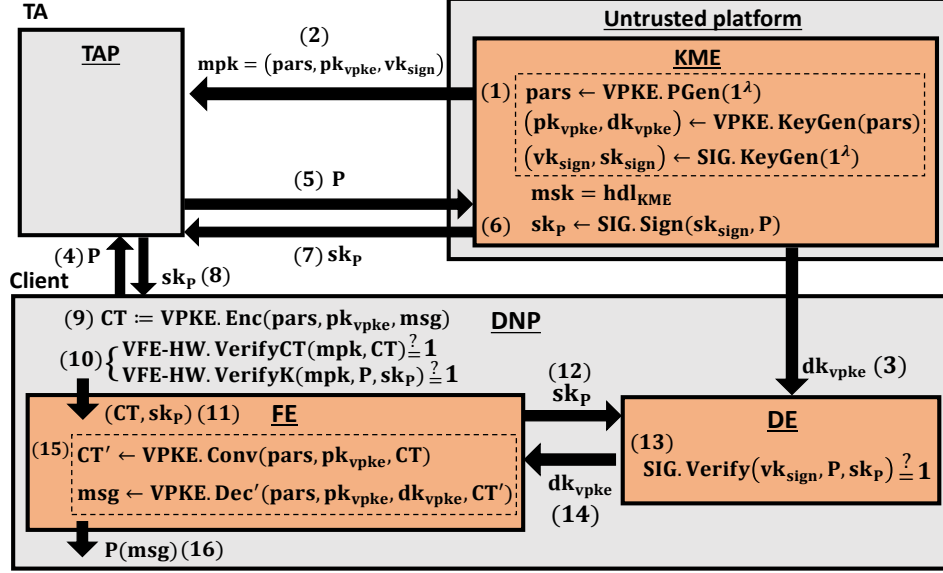


Figure 1: Protocol flow. Steps (1) and (2) specify VFE-HW.Setup, step (3) specifies VFE-HW.DecSetup, steps (4), (5), (6), (7) and (8) specify VFE-HW.KeyGen, step (9) specifies VFE-HW.Enc, steps (10) and (11) specify VFE-HW.VerifyK and VFE-HW.VerifyCT, and steps (12), (13), (14), (15) and (16) specify VFE-HW.Dec.

VFE-HW.VerifyK(mpk, P, sk_P):

1. Parse $mpk = (\text{pars}, pk_{vpke}, vk_{sign})$, and $sk_P = \text{sig}$.
2. If $\text{SIG.Verify}(vk_{sign}, sk_P, P) = 0$, then output 0. Otherwise, output 1.

VFE-HW.Dec^{HW}(mpk, hdl, P, sk_P, CT):

1. Parse $mpk = (\text{pars}, pk_{vpke}, vk_{sign})$, $hdl = hdl_{DE}$, $sk_P = \text{sig}$.
2. Call $hdl_{FE}(P) \leftarrow \text{HW.Load}(\text{params}, Q_{FE}(P))$.
3. Call $\text{report} \leftarrow \text{HW.Run\&Report}_{sk_{report}}(hdl_{FE}(P), (\text{"init"}, \text{sig}))$.
4. If $\text{HW.ReportVerify}_{sk_{report}}(hdl_{DE}, \text{report}) = 0$, then output \perp . Otherwise, call $\text{report}_{dk} \leftarrow \text{HW.Run\&Report}_{sk_{report}}(hdl_{DE}, (\text{"provision"}, \text{report}, \text{sig}))$.
5. If $\text{HW.ReportVerify}_{sk_{report}}(hdl_{FE}(P), \text{report}_{dk}) = 0$, then output \perp . Otherwise, call $\text{out} \leftarrow \text{HW.Run}(hdl_P, (\text{"run"}, \text{pars}, \text{params}, mpk, pk_{Ia}, \text{report}_{dk}, CT))$, and output out .

Obviously, correctness holds if VPKE, PKE, SIG, and HW are correct. For clarity, we describe the protocol flow of VFE-HW using Figure 1, where the gray areas represent the untrusted space of each platform, orange areas represent the trusted space of each platform, and the procedures inside dashed boxes are run within enclaves. For example, the TA manages the Trusted Authority Platform (TAP), and setups the KME in the TAP. A client manages a Decryption Node Platform (DNP), and setups a DE in the DNP. The TA generates a public key pk_{vpke} and a secret key dk_{vpke} , as well as a signing key sk_{sign} and a verification key vk_{sign} as step (1) within KME. Here, mpk generated by the VFE-HW.Setup^{HW} algorithm consists of pars , pk_{vpke} and vk_{sign} as step (2). Furthermore, msk

generated by the $\text{VFE-HW.Setup}^{\text{HW}}$ algorithm is a handle hdl_{KME} used to confirm the KME. Next, the client preserves dk_{vpke} into the DE via a remote attestation as step (3). Next, the client gets the secret key sk_{P} of the $\text{VFE-HW.KeyGen}^{\text{HW}}$ algorithm which KME issues as a signature on a function P via a secure channel as step (4) to (8). Here, let CT be a ciphertext of a plaintext msg under pk using the VFE-HW.Enc algorithm as step (9). If an external encryptor generates CT , it is sent to the client. Note that we omit this procedure in Figure 1. In the decryption procedure, the client setups a FE parameterized P in the DNP. Then, the client checks the validity of sk_{P} and CT using the VFE-HW.VerifyK and VFE-HW.VerifyCT algorithms respectively as step (10). If sk_{P} and CT are valid, the client inputs them into the FE via hardware invocation as step (11). If the DNP is managed remotely by the client, then a remote attestation is employed in this case. Next, the FE transfers sk_{P} to the DE via a local attestation as step (12). The validity of sk_{P} is confirmed by using the SIG.Verify algorithm as step (13). If sk_{P} is valid, the DE transfers dk_{vpke} to FE via a local attestation as step (14). The FE decrypts CT as step (15) using the aVPKE.Conv and $\text{VPKE.Dec}'$ algorithms. Finally, the client obtains $\text{P}(\text{msg})$ as step (16).

6 Security Analysis

We provide two proofs to demonstrate that the proposed scheme provides weak verifiability and simulation security.

6.1 Weak Verifiability

In this section, we prove the weak verifiability of VFE-HW. Essentially, we employ the strictly non-trivial public verifiability of VPKE. To do so, we need to guarantee that dk_{vpke} used in the VPKE.Dec algorithm is generated correctly by the VPKE.KeyGen algorithm. We guarantee this using the correctness of HW. Formally, the following theorem holds.

Theorem 2 *VFE-HW is weak verifiable if VPKE is strictly non-trivial public verifiable, and HW is correct.*

Proof. According to our trusted setup assumption, $\text{VFE-HW.Setup}^{\text{HW}}$ and $\text{VFE-HW.DecSetup}^{\text{HW,KM}(\cdot)}$ algorithms were honestly run which means that dk_{vpke} was correctly generated, and sent from the KME to a DE. Moreover, $\text{VFE-HW.VerifyCT}(\text{mpk}, \text{CT}) = 1$ and $\text{VFE-HW.VerifyK}(\text{mpk}, \text{P}, \text{sk}_{\text{P}}) = 1$ hold. Now, we need to guarantee that dk_{vpke} is correctly sent from the DE to a FE in the $\text{VFE-HW.Dec}^{\text{HW}(\cdot)}$ algorithm. This holds with probability $1 - \text{negl}(\lambda)$ due to the correctness of HW. Next, by using this dk_{vpke} , $\text{VPKE.Ver}(\text{pars}, \text{pk}_{\text{vpke}}, \text{CT}) = 1 \Rightarrow \text{VPKE.Dec}(\text{pars}, \text{pk}_{\text{vpke}}, \text{dk}_{\text{vpke}}, \text{CT}) \neq \perp$ holds due to the strictly non-trivial public verifiability of VPKE. Thus, decryption result of CT is determined to be unique since the VPKE.Dec algorithm is deterministic algorithm. Let the decryption result denote msg . Then, the VFE-HW.Dec algorithm outputs $\text{P}(\text{msg})$ from P and msg .

6.2 Simulation Security

Here, we prove the simulation security of the VFE-HW scheme. We replace the PKE scheme of IRON with a VPKE scheme. In this case, we primarily consider whether the SIM-based security is preserved after the replacement. In other words, an adversary \mathcal{A} can check the validity of ciphertexts and it may use for distinguishing REAL and IDEAL. For example, if the challenge ciphertext is changed as a random number (typically employed to provide key privacy/anonymity in the PKE/IBE context), then the public verifiability helps \mathcal{A} to distinguish REAL and IDEAL, and the proof fails. Fortunately, the security proof of IRON does not employ the step, and hence we can replace the PKE scheme with the VPKE scheme.

Theorem 3 VFE-HW is simulation secure if VPKE is IND-CCA secure, PKE is IND-CCA secure, SIG is EUF-CMA secure, and HW is a secure hardware scheme.

Proof. We construct a simulator \mathcal{S} . First, \mathcal{S} needs to simulate the Pre-Processing phase as REAL. \mathcal{S} runs $\text{HW.Setup}(1^\lambda)$ and records $(\text{sk}_{\text{report}}, \text{sk}_{\text{quote}})$. \mathcal{S} measures the designated program Q_{DE} , and stores the program tag $\text{tag}_{Q_{\text{DE}}}$. Finally, \mathcal{S} creates seven empty lists $\mathcal{L}_K, \mathcal{L}_R, \mathcal{L}_D, \mathcal{L}_{KM}, \mathcal{L}_{DE}, \mathcal{L}_{DE2}$, and \mathcal{L}_{FE} .

We use sequences of games $\text{Game}_0, \dots, \text{Game}_7$ to prove that adversary \mathcal{A} cannot computationally distinguish between REAL and IDEAL as follows.

$\boxed{\text{Game}_0}$ \mathcal{S} runs REAL.

$\boxed{\text{Game}_1}$ \mathcal{S} runs as Game_0 with the following exceptions

- $\text{HW.LOAD}(\text{params}, Q_{\text{DE}})$: If \mathcal{A} queries this oracle as input params and Q_{DE} , \mathcal{S} responds hdl_{DE} by running the $\text{HW.Load}(\text{params}, Q_{\text{DE}})$ algorithm, and storing it in \mathcal{L}_D .
- $\text{HW.LOAD}(\text{params}, Q_{\text{FE}}(P))$: If \mathcal{A} queries this oracle as input params and $Q_{\text{FE}}(P)$, \mathcal{S} responds hdl_P by running the $\text{HW.Load}(\text{params}, Q_{\text{FE}}(P))$ algorithm, and storing it in \mathcal{L}_K . If $\text{tag}_{Q_{\text{FE}}(P)} \notin \mathcal{L}_K$, then \mathcal{S} stores $(0, \text{tag}_{Q_{\text{FE}}(P)}, \text{hdl}_{\text{FE}}(P))$ in \mathcal{L}_K .
- $\text{HW.RUN}(\text{hdl}, \text{in})$: If \mathcal{A} queries this oracle as input hdl and in , \mathcal{S} responds out by running the $\text{HW.Run}(\text{hdl}, \text{in})$ algorithm. If vk_{sign} , which is queried by \mathcal{A} as the $\text{HW.Run}(\text{hdl}_{\text{DE}}, \text{in} = (\text{“provision”}, \text{vk}_{\text{sign}}))$ algorithm, is not the same as that of mpk , \mathcal{S} removes hdl_{DE} from \mathcal{L}_D .
- $\text{VFE-HW.KeyGen}^{\text{HW}}(\text{msk}, P)$: If \mathcal{A} queries to this oracle as input P , \mathcal{S} responds sk_P by running the $\text{HW.Run}(\text{hdl}, \text{in})$ algorithm as follows. Parse $\text{msk} = \text{hdl}_{\text{KME}}$. \mathcal{S} computes $\text{tag}_{Q_{\text{FE}}(P)}$, calls $\text{sig} \leftarrow \text{HW.Run}(\text{hdl}_{\text{KME}}, (\text{“sign”}, \text{tag}_{Q_{\text{FE}}(P)}))$, and outputs $\text{sk}_P := \text{sig}$. If $\text{tag}_{Q_{\text{FE}}(P)}$ already has an entry in \mathcal{L}_K , \mathcal{S} creates the first entry 1 (we call “honest-bit” for the first entry in \mathcal{L}_K); otherwise, \mathcal{S} adds the tuple $(1, \text{tag}_{Q_{\text{FE}}(P)}, \{\})$ to \mathcal{L}_K .
- $\text{VFE-HW.Enc}(\text{mpk}, \text{msg})$: If \mathcal{A} queries this encryption algorithm as input msg , \mathcal{S} responds CT by running the $\text{VPKE.Enc}(\text{pars}, \text{pk}_{\text{vpke}}, \text{msg})$ algorithm. If msg is a challenge plaintext msg^* , \mathcal{S} responds CT^* by running the algorithm, and stores it in \mathcal{L}_R .

$\boxed{\text{Game}_2}$ \mathcal{S} runs as Game_1 with the following exceptions.

$\text{HW.RUN\&REPORT}(\text{hdl}, \text{in})$: If \mathcal{A} queries this oracle as input $\text{hdl} = \text{hdl}_{\text{DE}}$ and $\text{in} = (\text{“provision”}, \text{report}, \text{sig})$, then \mathcal{S} responds $\text{report}_{\text{dk}}$ by running the $\text{HW.Run\&Report}_{\text{sk}_{\text{report}}}(\text{hdl}_{\text{DE}}, (\text{“provision”}, \text{report}, \text{sig}))$ algorithm. If $\text{tag}_{Q_{\text{FE}}(P)}$ in report is not contained as a component of an honest-bit tuple in \mathcal{L}_K , \mathcal{S} outputs \perp .

Here, we consider a case where the $\text{HW.RUN\&REPORT}(\text{hdl}_{\text{DE}}, (\text{“provision”}, \text{report}, \text{sig}))$ algorithm outputs non \perp even if $\text{tag}_{Q_{\text{FE}}(P)}$ is not contained as an honest-bit tuple in \mathcal{L}_K . If \mathcal{A} can make a query while ensuring this case, we can break the existentially unforgeability for SIG with non-negligible probability. The following Lemma is the same as Lemma C.1 of IRON.

Lemma 1 If the signature scheme SIG is EUF-CMA secure, then Game_2 is indistinguishable from Game_1 .

$\boxed{\text{Game}_{3.0}}$ \mathcal{S} runs as Game_2 with the following exceptions.

1. $\text{HW.RUN\"E}(\text{hdl}, \text{in})$: If \mathcal{A} queries this oracle as input $\text{hdl} = \text{hdl}_{\text{DE}}$ and $\text{in} = (\text{"init setup"}, \text{vk}_{\text{sign}})$, \mathcal{S} responds quote by running the $\text{HW.Run\&Quote}_{\text{sk}_{\text{quote}}}(\text{hdl}_{\text{DE}}, (\text{"init setup"}, \text{vk}_{\text{sign}}))$ algorithm, and stores $\text{out} = (\text{sid}, \text{pk}_{\text{ra}})$ as a component of quote in \mathcal{L}_{DE2} .
2. $\text{HW.RUN}(\text{hdl}, \text{in})$: If \mathcal{A} queries this oracle as input $\text{hdl} = \text{hdl}_{\text{KME}}$ and $\text{in} = (\text{"provision"}, \text{quote}, \text{params})$, \mathcal{S} responds $(\text{sid}, \text{ct}_{\text{dk}}, \sigma_{\text{dk}})$ by running the $\text{HW.Run}(\text{hdl}_{\text{KME}}, (\text{"provision"}, \text{quote}, \text{params}))$ algorithm. If $(\text{sid}, \text{pk}_{\text{ra}}) \notin \mathcal{L}_{\text{DE2}}$, then \mathcal{S} outputs \perp .

Here, we consider a case where the $\text{HW.RUN}(\text{hdl}_{\text{KME}}, (\text{"provision"}, \text{quote}, \text{params}))$ algorithm outputs non \perp even if $(\text{sid}, \text{pk}_{\text{ra}}) \notin \mathcal{L}_{\text{DE2}}$. Here, if \mathcal{A} can make a query while ensuring this case, then we can break the remote attestation unforgeability for HW with non-negligible probability. The following Lemma is the same as Lemma C.4 of IRON.

Lemma 2 *If the secure hardware scheme HW is REM-ATT-UNF secure, then $\text{Game}_{3,0}$ is indistinguishable from Game_2 .*

$\boxed{\text{Game}_{3,1}}$ \mathcal{S} runs as $\text{Game}_{3,0}$ with the following exceptions.

1. $\text{HW.RUN\&REPORT}(\text{hdl}, \text{in})$: If \mathcal{A} queries this oracle as input $\text{hdl} = \text{hdl}_{\text{FE}}(\text{P})$ and $\text{in} = \text{"init"}$, then \mathcal{S} responds report by running the $\text{HW.Run\&Report}_{\text{sk}_{\text{report}}}(\text{hdl}_{\text{FE}}(\text{P}), \text{"init"})$ algorithm, and storing $\text{out} = (\text{sid}, \text{pk}_{\text{la}})$ as a component of report in \mathcal{L}_{FE} .
2. $\text{HW.RUN}(\text{hdl}, \text{in})$: If \mathcal{A} queries this oracle as input $\text{hdl} = \text{hdl}_{\text{DE}}$ and $\text{in} = (\text{"provision"}, \text{report}, \text{sig})$, \mathcal{S} responds $\text{report}_{\text{dk}}$ by running the $\text{HW.Run}(\text{hdl}_{\text{DE}}, (\text{"provision"}, \text{report}, \text{sig}))$ algorithm. If $(\text{sid}, \text{pk}_{\text{la}}) \notin \mathcal{L}_{\text{FE}}$, \mathcal{S} outputs \perp .

Here, we consider a case where the $\text{HW.RUN\&REPORT}(\text{hdl}_{\text{DE}}, (\text{"provision"}, \text{report}, \text{sig}))$ algorithm outputs non \perp even if $(\text{sid}, \text{pk}_{\text{la}}) \notin \mathcal{L}_{\text{FE}}$. If \mathcal{A} can make a query while ensuring this case, we can break the local attestation unforgeability for HW with non-negligible probability. The following Lemma is the same as Lemma C.5 of IRON.

Lemma 3 *If the secure hardware scheme HW is LOC-ATT-UNF secure, $\text{Game}_{3,1}$ is indistinguishable from $\text{Game}_{3,0}$.*

$\boxed{\text{Game}_{4,0}}$ \mathcal{S} runs as $\text{Game}_{3,1}$ with the following exceptions.

$\text{HW.RUN}(\text{hdl}, \text{in})$:

1. If \mathcal{A} queries this oracle as input $\text{hdl} = \text{hdl}_{\text{KME}}$ and $\text{in} = (\text{"provision"}, \text{quote}, \text{params})$, \mathcal{S} responds $(\text{sid}, \text{ct}_{\text{dk}})$ by running the $\text{HW.Run}(\text{hdl}_{\text{KME}}, (\text{"provision"}, \text{quote}, \text{params}))$ algorithm, and storing it in \mathcal{L}_{KM} .
2. If \mathcal{A} queries this oracle as input $\text{hdl} = \text{hdl}_{\text{DE}}$ and $\text{in} = (\text{"complete setup"}, \text{sid}, \text{ct}_{\text{dk}}, \sigma_{\text{dk}})$, \mathcal{S} runs the $\text{HW.Run}(\text{hdl}_{\text{DE}}, (\text{"complete setup"}, \text{sid}, \text{ct}_{\text{dk}}))$ algorithm. If $(\text{sid}, \text{ct}_{\text{dk}}) \notin \mathcal{L}_{\text{KM}}$, then \mathcal{S} outputs \perp .

Here, we consider a case that the $\text{HW.RUN}(\text{hdl}_{\text{DE}}, (\text{"complete setup"}, \text{sid}, \text{ct}_{\text{dk}}, \sigma_{\text{dk}}))$ algorithm outputs non \perp even if $(\text{sid}, \text{ct}_{\text{dk}}) \notin \mathcal{L}_{\text{KM}}$. If \mathcal{A} can make a query while ensuring this case, we can break the existentially unforgeability for SIG with non-negligible probability. The following Lemma is the same as Lemma C.2 of IRON.

Lemma 4 *If the signature scheme SIG is EUF-CMA secure, $\text{Game}_{4.0}$ is indistinguishable from $\text{Game}_{3.1}$.*

$\boxed{\text{Game}_{4.1}}$ \mathcal{S} runs as $\text{Game}_{4.0}$ with the following exceptions.

1. $\text{HW.RUN\&REPORT}(\text{hdl}, \text{in})$: If \mathcal{A} queries this oracle as input $\text{hdl} = \text{hdl}_{\text{DE}}$ and $\text{in} = (\text{“provision”}, \text{report}, \text{sig})$, \mathcal{S} responds $\text{report}_{\text{dk}}$ by running the $\text{HW.Run\&Report}_{\text{sk}_{\text{report}}}(\text{hdl}_{\text{DE}}, (\text{“provision”}, \text{report}, \text{sig}))$ algorithm, and storing $\text{out} = (\text{sid}, \text{ct}_{\text{key}})$ as a component of $\text{report}_{\text{dk}}$ in \mathcal{L}_{DE} .
2. $\text{HW.RUN}(\text{hdl}, \text{in})$: If \mathcal{A} queries this oracle as input $\text{hdl} = \text{hdl}_{\text{FE}}(\text{P})$ and $\text{in} = (\text{“run”}, \text{params}, \text{mpk}, \text{pk}_{\text{la}}, \text{report}_{\text{dk}}, \text{CT})$, \mathcal{S} responds $\text{P}(\text{msg})$ by running the $\text{HW.Run}(\text{hdl}_{\text{FE}}(\text{P}), (\text{“run”}, \text{params}, \text{mpk}, \text{pk}_{\text{la}}, \text{report}_{\text{dk}}, \text{CT}))$ algorithm. If $(\text{sid}, \text{ct}_{\text{key}}) \notin \mathcal{L}_{\text{DE}}$, \mathcal{S} outputs \perp .

Here, we consider a case where the $\text{HW.RUN}(\text{hdl}_{\text{P}}, (\text{“run”}, \text{params}, \text{mpk}, \text{pk}_{\text{la}}, \text{report}_{\text{dk}}, \text{CT}))$ algorithm outputs non \perp even if $(\text{sid}, \text{ct}_{\text{key}}) \notin \mathcal{L}_{\text{DE}}$. If \mathcal{A} can make a query while ensuring this case, we can break the local attestation unforgeability for HW with non-negligible probability. The following Lemma is the same as Lemma C.3 of IRON.

Lemma 5 *If the secure hardware scheme HW is LOC-ATT-UNF secure, $\text{Game}_{4.1}$ is indistinguishable from $\text{Game}_{4.0}$.*

$\boxed{\text{Game}_5}$ \mathcal{S} runs as $\text{Game}_{4.1}$ with the following exceptions.

$\text{HW.RUN}(\text{hdl}, \text{in})$: If \mathcal{A} queries this oracle as input $\text{hdl} = \text{hdl}_{\text{FE}}(\text{P})$ and $\text{in} = (\text{“run”}, \text{params}, \text{mpk}, \text{pk}_{\text{la}}, \text{report}_{\text{dk}}, \text{CT})$, \mathcal{S} evaluates CT as follows.

- If $\text{CT} \notin \mathcal{L}_{\text{R}}$, \mathcal{S} retrieves dk_{vpke} from ct_{key} , and computes $\text{msg} \leftarrow \text{VPKE.Dec}(\text{pars}, \text{pk}_{\text{vpke}}, \text{dk}_{\text{vpke}}, \text{CT})$. Finally, \mathcal{S} evaluates P on msg, and outputs $\text{out} := \text{P}(\text{msg})$
- If $\text{CT} \in \mathcal{L}_{\text{R}}$, \mathcal{S} uses the $\text{U}_{\text{msg}^*}(\text{P})$ oracle, and responds with $\text{P}(\text{msg}^*)$.

$\boxed{\text{Game}_6}$ \mathcal{S} runs as Game_5 with the following exceptions.

$\text{KM}(\text{quote})$: If \mathcal{A} queries this oracle as input $\text{quote} = (\text{md}_{\text{hdl}_{\text{DE}}}, \text{tag}_{\text{QDE}}, \text{in} = (\text{“run”}, \text{vk}_{\text{sign}}), \text{out} = (\text{sid}, \text{pk}_{\text{ra}}, \sigma))$, \mathcal{S} runs the $\text{HW.Run}(\text{hdl}_{\text{KME}}, (\text{“provision”}, \text{quote}, \text{params}))$ algorithm, which internally runs $\text{ct}_{\text{dk}} \leftarrow \text{PKE.Enc}(\text{pk}_{\text{ra}}, 0^{|\text{dk}_{\text{vpke}}|})$, and outputs $(\text{sid}, \text{ct}_{\text{dk}}, \sigma_{\text{dk}})$.

The following Lemma is the same as Lemma C.6 of IRON.

Lemma 6 *If the public key encryption scheme PKE is IND-CCA secure, Game_6 is indistinguishable from Game_5 .*

$\boxed{\text{Game}_7}$ \mathcal{S} runs as Game_6 with the following exceptions.

$\text{VFE-HW.Enc}(\text{mpk}, 0^{|\text{msg}^*|})$: If \mathcal{A} queries this algorithm as input msg, \mathcal{S} responds CT by running $\text{VPKE.Enc}(\text{pars}, \text{pk}_{\text{vpke}}, 0^{|\text{msg}^*|})$. If msg is a challenge plaintext msg^* , \mathcal{S} responds CT^* by running the algorithm, and storing it in \mathcal{L}_{R} .

Here, no step replaces a valid ciphertext with an invalid ciphertext, e.g., a random number; therefore, the public verifiability does not affect the security proof.

Lemma 7 *If the verifiable public key encryption scheme VPKE is IND-CCA secure, Game₇ is indistinguishable from Game₆.*

Proof. Let \mathcal{A} be an adversary who distinguishes between Game₆ and Game₇, and let \mathcal{C} be the challenger of IND-CCA security. We construct an algorithm \mathcal{B} that breaks IND-CCA as follows. First, \mathcal{C} runs $\text{pars} \leftarrow \text{VPKE.PGen}(1^\lambda)$, then $(\text{pk}_{\text{vpke}}, \text{dk}_{\text{vpke}}) \leftarrow \text{VPKE.KeyGen}(\text{pars})$, and gives pars and pk_{vpke} to \mathcal{B} . \mathcal{B} runs $(\text{sk}_{\text{sign}}, \text{vk}_{\text{sign}}) \leftarrow \text{SIG.KeyGen}(1^\lambda)$ and $\text{params} \leftarrow \text{HW.Setup}(1^\lambda)$, and gives params and $\text{mpk} = (\text{pars}, \text{pk}_{\text{vpke}}, \text{vk}_{\text{sign}})$ to \mathcal{A} .

For key generation query P , \mathcal{B} derives $\text{tag}_{\text{QFE}(P)}$ from P , and calls $\text{sig} \leftarrow \text{HW.Run}(\text{hdl}_{\text{KME}}, (\text{"sign"}, \text{tag}_{\text{QFE}(P)}))$. Then, \mathcal{B} sends $\text{sk}_P := \text{sig}$ to \mathcal{A} , and stores $\text{tag}_{\text{QFE}(P)}$ in \mathcal{L}_K .

For run query $(\text{hdl}_{\text{FE}(P)}, (\text{"run"}, \text{params}, \text{mpk}, \text{pk}_{\text{la}}, \text{report}_{\text{dk}}, \text{CT}))$ where $\text{report}_{\text{dk}}$ is valid and $\text{hdl}_{\text{FE}(P)} \in \mathcal{L}_K$ with honest-bit, \mathcal{B} forwards CT to \mathcal{C} as a decryption query. \mathcal{C} returns msg by running the $\text{VPKE.Dec}(\text{pars}, \text{pk}_{\text{vpke}}, \text{dk}_{\text{vpke}}, \text{CT})$ algorithm to \mathcal{B} . If $\text{msg} = \perp$, \mathcal{B} outputs \perp ; otherwise, \mathcal{B} runs P on msg , and sends $P(\text{msg})$ to \mathcal{A} .

In the challenge phase, \mathcal{A} sends $(\text{msg}^*, 0^{|\text{msg}^*|})$ to \mathcal{B} . \mathcal{B} sets $\text{msg}^* = M_0^*$ and $0^{|\text{msg}^*|} = M_1^*$, and sends (M_0^*, M_1^*) to \mathcal{C} . \mathcal{C} computes challenge ciphertext $\text{CT}^* = \text{VPKE.Enc}(\text{pars}, \text{pk}_{\text{vpke}}, M_\mu^*)$ where $\mu \in \{0, 1\}$, and sends CT^* to \mathcal{B} . \mathcal{B} sends CT^* to \mathcal{A} , and stores CT^* in \mathcal{L}_R .

For key generation query P , \mathcal{B} derives $\text{tag}_{\text{QFE}(P)}$ from P , and calls $\text{sig} \leftarrow \text{HW.Run}(\text{hdl}_{\text{KME}}, (\text{"sign"}, \text{tag}_{\text{QFE}(P)}))$. \mathcal{B} sends $\text{sk}_P := \text{sig}$ to \mathcal{A} , and stores $\text{tag}_{\text{QFE}(P)}$ in \mathcal{L}_K .

For run query $(\text{hdl}_{\text{FE}(P)}, (\text{"run"}, \text{params}, \text{mpk}, \text{pk}_{\text{la}}, \text{report}_{\text{dk}}, \text{CT}))$ where $\text{report}_{\text{dk}}$ is valid and $\text{hdl}_P \in \mathcal{L}_K$ with honest-bit:

- $\text{CT} \in \mathcal{L}_R$: \mathcal{B} uses the universal oracle $U_{\text{msg}^*}(P)$, and sends $P(\text{msg}^*)$ to \mathcal{A} .
- $\text{CT} \notin \mathcal{L}_R$: \mathcal{B} forwards CT to \mathcal{C} as a decryption query. \mathcal{C} returns msg by running the $\text{VPKE.Dec}(\text{pars}, \text{pk}_{\text{vpke}}, \text{dk}_{\text{vpke}}, \text{CT})$ algorithm to \mathcal{B} . If $\text{msg} = \perp$, \mathcal{B} outputs \perp ; otherwise, \mathcal{B} runs P on msg , and sends $P(\text{msg})$ to \mathcal{A} .

Finally, \mathcal{A} outputs $\mu' \in \{0, 1\}$. \mathcal{B} outputs μ' , and breaks IND-CCA security.

7 Implementation

In this section, we give an implementation result when we employ a cryptographic hash function H as a function P , i.e., the decryption algorithm outputs $H(\text{msg})$. As mentioned before, theoretically the function is not realized in the IND-based VFE scheme [7], and practically the function seems attractive when we compute a hashed value for a sensitive data such as a password. This system can be achieved by IRON, however no verifiability is guaranteed. On the other hand, in our scheme the server can verify the ciphertext, and can delegate the verification to other server as an option. Considering a concrete application is left as future work of this paper.

We measured the average times and standard deviations of the VFE-HW.Enc, VFE-HW.VerifyCT, VFE-HW.VerifyK and VFE-HW.Dec algorithms because we estimate the runtime of the algorithms related to msg for the proposed scheme. Here, except for the VFE-HW.Dec algorithm, all algorithms were run outside enclaves. In the VFE-HW.Dec algorithm, the FE runs the VPKE.Conv and VPKE.Dec' algorithms, and evaluates H on msg . We employ the VPKE scheme [27] (see Appendix B), ECDSA as SIG, and SHA-256 as H .

The VPKE.Ver algorithm checks whether (part of) the ciphertext is a DDH tuple, we employed symmetric pairings even though asymmetric pairings are desirable for efficient implementation [19]. We used the PBC library [1], which supports the symmetric pairings. We generated parameters for a Type-A curve with 128-bit security, defined over the field \mathbb{F}_p with a 256-bit prime p , where the

Table 2: Implementation results of VFE-HW scheme

Algorithms	Average Running Time (sec)	Standard Deviation
VFE-HW.Enc	0.11828	0.00228
VFE-HW.VerifyCT	0.12329	0.00252
VFE-HW.VerifyK	0.00054	0.00001
VFE-HW.Dec	0.06164	0.00145

order is a 1536-bit prime, using a function called `pbk_param_init_a_gen`. The parameter is given in Appendix C.

For running the PBC library in enclaves, we employed the PBC for SGX given by Conti et al. [15]. In our implementation, we set input-output of enclaves is as an array of unsigned char values regarding a valuable of PBC. We transformed the binary data into an element of elliptic curves using the `element_from_bytes` function supported by PBC within enclaves.

Our implementation environment includes the CPU: Intel(R) Core(TM) i3-7100U (2.40GHz), and the libraries openssl 1.0.2g, Intel SGX 1.5 Linux Driver, Intel SGX SDK, Intel SGX PSW, GMP, PBC, and PBC for SGX [15]. Note that, some vulnerabilities of Intel SGX have been reported [11, 12, 29]. It is possible to use a different secure HW.

We show the average times and the standard deviations of those algorithms over 10000 samples in Table 2. The average times of all algorithms (except the VFE-HW.VerifyK algorithm) were approximately 0.1 sec. The average time of the VFE-HW.VerifyK algorithm was 0.00054 sec.

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A Security Definitions

Here, we introduce the security definition of PKE, SIG and HW.

A.1 Security Definition of PKE

First, we introduce IND-CCA security of PKE as follows.

Definition 9 (IND-CCA). *For any probabilistic polynomial-time (PPT) adversary \mathcal{A} and the security parameter $\lambda \in \mathbb{N}$, we define the experiment $\text{Exp}_{\text{PKE}, \mathcal{A}}^{\text{IND-CCA}}(\lambda)$ as follows. Here, **state** is state information that an adversary \mathcal{A} can preserve any information, and **state** is used for transferring state information to the other stage.*

$$\begin{aligned}
 & \text{Exp}_{\text{PKE}, \mathcal{A}}^{\text{IND-CCA}}(\lambda) : \\
 & (\text{pk}_{\text{pke}}, \text{dk}_{\text{pke}}) \leftarrow \text{PKE.KeyGen}(1^\lambda) \\
 & (\text{msg}_0^*, \text{msg}_1^*, \text{state}) \leftarrow \mathcal{A}^{\text{PKE.DEC}}(\text{find}, \text{pk}_{\text{pke}}) \\
 & \text{msg}_0^*, \text{msg}_1^* \in \mathcal{M}_{\text{pke}}; |\text{msg}_0^*| = |\text{msg}_1^*| \\
 & \mu \xleftarrow{\$} \{0, 1\}; \text{CT}^* \leftarrow \text{PKE.Enc}(\text{pk}_{\text{pke}}, \text{msg}_\mu^*) \\
 & \mu' \leftarrow \mathcal{A}^{\text{PKE.DEC}}(\text{guess}, \text{CT}^*, \text{state}) \\
 & \text{If } \mu = \mu' \text{ then output 1, and 0 otherwise}
 \end{aligned}$$

- **PKE.DEC**: This decryption oracle takes as input a ciphertext $CT \neq CT^*$ and returns msg by running the $PKE.Dec(dk_{pke}, CT)$ algorithm.

We say that PKE is IND-CCA secure if the advantage

$$\text{Adv}_{\text{PKE}, \mathcal{A}}^{\text{IND-CCA}}(\lambda) := |\Pr[\text{Exp}_{\text{PKE}, \mathcal{A}}^{\text{IND-CCA}}(\lambda) = 1] - 1/2|$$

is negligible for any PPT adversary \mathcal{A} .

A.2 Security Definition of SIG

Next, we introduce EUF-CMA security of SIG as follows.

Definition 10 (EUF-CMA). For any PPT adversary \mathcal{A} and the security parameter $\lambda \in \mathbb{N}$, we define the experiment $\text{Exp}_{\text{SIG}, \mathcal{A}}^{\text{EUF-CMA}}(\lambda)$ as follows.

$\text{Exp}_{\text{SIG}, \mathcal{A}}^{\text{EUF-CMA}}(1^\lambda)$:

$(sk_{\text{sign}}, vk_{\text{sign}}) \leftarrow \text{SIG.KeyGen}(1^\lambda)$; QUERY := \emptyset

$(msg^*, \sigma^*) \leftarrow \mathcal{A}^{\text{SIG.SIGN}}(vk_{\text{sign}})$

If $\text{SIG.Verify}(vk_{\text{sign}}, msg^*, \sigma^*) = 1$ and $msg^* \notin \text{QUERY}$

then output 1, and 0 otherwise

- **SIG.SIGN**: This signing oracle takes as input a message msg , and returns σ by running the $\text{SIG.Sign}(sk_{\text{sign}}, msg)$ algorithm. Finally, the challenger stores msg in QUERY.

We say that SIG is EUF-CMA secure if the advantage

$$\text{Adv}_{\text{SIG}, \mathcal{A}}^{\text{EUF-CMA}}(\lambda) := \Pr[\text{Exp}_{\text{SIG}, \mathcal{A}}^{\text{EUF-CMA}}(\lambda) = 1]$$

is negligible for any PPT adversary \mathcal{A} .

A.3 Security Definition of HW

Next, we introduce LOC-ATT-UNF of HW as follows. This security guarantees that no adversary that does not have sk_{report} can produce a valid report.

Definition 11 (LOC-ATT-UNF) For any PPT adversary \mathcal{A} and the security parameter $\lambda \in \mathbb{N}$, we define the experiment $\text{Exp}_{\text{HW}, \mathcal{A}}^{\text{LOC-ATT-UNF}}(\lambda)$ as follows.

$\text{Exp}_{\text{HW}, \mathcal{A}}^{\text{LOC-ATT-UNF}}(\lambda)$:

$(\text{params}, sk_{\text{report}}, sk_{\text{quote}}, \text{state}) \leftarrow \text{HW.Setup}(1^\lambda)$; QUERY := \emptyset

$(hdl^*, \text{report}^*) \leftarrow \mathcal{A}^{\text{HW}, \text{HW}(\cdot)}(\text{params})$

If $\text{HW.ReportVerify}_{sk_{\text{report}}}(hdl^*, \text{report}^*) = 1$ where

$\text{report}^* = (md_{hdl}^*, tag_Q^*, in^*, out^*, mac^*)$ and

$(md_{hdl}^*, tag_Q^*, in^*, out^*) \notin \text{QUERY}$

then output 1, and 0 otherwise

- **HW**: \mathcal{A} can access the instance as follows.

- HW.LOAD: \mathcal{A} queries the instance as input params and \mathbf{Q} , and the instance returns the handle hdl by running the $\text{HW.Load}(\text{params}, \mathbf{Q})$ algorithm.
- HW.REPORTVERIFY: \mathcal{A} queries the instance as input hdl and report , and the instance returns the result by running the $\text{HW.ReportVerify}_{\text{sk}_{\text{report}}}(\text{hdl}, \text{report})$ algorithm.
- $\text{HW}(\cdot)$: \mathcal{A} can access the oracle as follows.
 - HW.RUN&REPORT : \mathcal{A} queries the oracle as input hdl and in , and the oracle returns $\text{report} := (\text{md}_{\text{hdl}}, \text{tag}_{\mathbf{Q}}, \text{in}, \text{out}, \text{mac})$ by running the $\text{HW.Run\&Report}_{\text{sk}_{\text{report}}}(\text{hdl}, \text{in})$ algorithm. Finally, the oracle stores $(\text{md}_{\text{hdl}}, \text{tag}_{\mathbf{Q}}, \text{in}, \text{out})$ in QUERY .

We say that HW is *LOC-ATT-UNF* secure if the advantage

$$\text{Adv}_{\text{HW}, \mathcal{A}}^{\text{LOC-ATT-UNF}}(\lambda) := \Pr[\text{Exp}_{\text{HW}, \mathcal{A}}^{\text{LOC-ATT-UNF}}(\lambda) = 1]$$

is negligible for any PPT adversary \mathcal{A} .

Next, we define *REM-ATT-UNF* of HW as follows. This security guarantees that no adversary that does not have sk_{quote} can produce a valid quote.

Definition 12 (*REM-ATT-UNF*) For any PPT adversary \mathcal{A} and the security parameter $\lambda \in \mathbb{N}$, we define the experiment $\text{Exp}_{\text{HW}, \mathcal{A}}^{\text{REM-ATT-UNF}}(\lambda)$ as follows.

$\text{Exp}_{\text{HW}, \mathcal{A}}^{\text{REM-ATT-UNF}}(\lambda)$:

$(\text{params}, \text{sk}_{\text{report}}, \text{sk}_{\text{quote}}, \text{state}) \leftarrow \text{HW.Setup}(1^\lambda)$; $\text{QUERY} := \emptyset$
 $\text{quote}^* \leftarrow \mathcal{A}^{\text{HW}, \text{KM}(\cdot)}(\text{params})$
 If $\text{HW.QuoteVerify}(\text{params}, \text{quote}) = 1$ where
 $\text{quote}^* = (\text{md}_{\text{hdl}}^*, \text{tag}_{\mathbf{Q}}^*, \text{in}^*, \text{out}^*, \sigma)$ and
 $(\text{md}_{\text{hdl}}^*, \text{tag}_{\mathbf{Q}}^*, \text{in}^*, \text{out}^*) \notin \text{QUERY}$
 then output 1, and 0 otherwise

- HW : \mathcal{A} can access the instance as follows.
 - HW.LOAD: \mathcal{A} queries the instance as input params and \mathbf{Q} , and the instance returns the handle hdl by running the $\text{HW.Load}(\text{params}, \mathbf{Q})$ algorithm.
- $\text{KM}(\cdot)$: \mathcal{A} can access the oracle as follows.
 - HW.RUN"E: \mathcal{A} queries the oracle as input hdl and in , and the oracle returns $\text{quote} := (\text{md}_{\text{hdl}}, \text{tag}_{\mathbf{Q}}, \text{in}, \text{out}, \sigma)$ by running the $\text{HW.Run\&Quote}_{\text{sk}_{\text{quote}}}(\text{hdl}, \text{in})$ algorithm. Finally, the oracle stores $(\text{md}_{\text{hdl}}, \text{tag}_{\mathbf{Q}}, \text{in}, \text{out})$ in QUERY .

We say that HW is *REM-ATT-UNF* secure if the advantage

$$\text{Adv}_{\text{HW}, \mathcal{A}}^{\text{REM-ATT-UNF}}(\lambda) := \Pr[\text{Exp}_{\text{HW}, \mathcal{A}}^{\text{REM-ATT-UNF}}(\lambda) = 1]$$

is negligible for any PPT adversary \mathcal{A} .

B VPKE Scheme by Nieto et al.

Here, we introduce the Nieto et al. VPKE scheme proposed by Nieto et al. (Figure 4 [27]) as follows. For the underlying One-Time Signature (OTS) scheme, we employ the discrete-log-based Wee OTS scheme [33]. For the DDH test, we employ symmetric pairings whether $e(g, \pi)$ is the same as $e(c_1, u^t v)$ or not.

VPKE.PGen(1^λ): Choose $(p, e, g, \mathbb{G}, \mathbb{G}_T)$ where \mathbb{G} and \mathbb{G}_T are groups of λ -bit prime order p , $g \in \mathbb{G}$ is a generator, and $e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$ is a bilinear map. Let $H : \mathbb{G} \rightarrow \{0, 1\}^{\text{poly}(\lambda)}$, $H_{OTS} : \{0, 1\}^* \rightarrow \{0, 1\}^{\text{poly}(\lambda)}$, and $TCR : \mathbb{G} \times \{0, 1\} \rightarrow \mathbb{Z}_p$ be collision or target collision resistant hash functions where $\text{poly}(\lambda)$ is a polynomial in λ . Output $\text{pars} = (p, e, g, \mathbb{G}, \mathbb{G}_T, H, H_{OTS}, TCR)$.

VPKE.KeyGen(pars): Parse $\text{pars} = (p, e, g, \mathbb{G}, \mathbb{G}_T, H, H_{OTS}, TCR)$. Choose $x_1 \xleftarrow{\$} \mathbb{Z}_p^*$ and $v \xleftarrow{\$} \mathbb{G}$ and compute $u = g^{x_1}$. Output $\text{pk} = (u, v)$ and $\text{dk} = x_1$.

VPKE.Enc(pars, pk, msg): Parse $\text{pars} = (p, e, g, \mathbb{G}, \mathbb{G}_T, H, H_{OTS}, TCR)$ and $\text{pk} = (u, v)$. Choose $s_0, s_1, x_2, r, n \xleftarrow{\$} \mathbb{Z}_p^*$ and compute $u_0 = g^{s_0}$, $u_1 = g^{s_1}$, $c' = g^{x_2}$, $c_1 = g^r$, $t \leftarrow TCR(c_1, (u_0, u_1, c'))$, $K \leftarrow H(u^r)$ and $\pi \leftarrow (u^t v)^r$. Set $c_2 \leftarrow \text{msg} \oplus K$ and $c = (c_1, c_2, \pi)$. Compute $w \leftarrow x_2 + ns_0 + s_1(H_{OTS}(c) + n)$. Output $\text{CT} \leftarrow (c, (n, w), (u_0, u_1, c'))$

VPKE.Ver(pars, pk, CT): Parse $\text{pars} = (p, e, g, \mathbb{G}, \mathbb{G}_T, H, H_{OTS}, TCR)$, $\text{pk} = (u, v)$, $\text{CT} = (c, (n, w), (u_0, u_1, c'))$ and $c = (c_1, c_2, \pi)$. Compute $t \leftarrow TCR(c_1, (u_0, u_1, c'))$ and $\pi \leftarrow (u^t v)^r$. If $e(g, \pi) \neq e(c_1, u^t v)$ or $g^w \neq c' u_0^n \cdot u_1^{H_{OTS}(c) + n}$, then output 0. Otherwise, output 1.

VPKE.Conv: Parse $\text{pars} = (p, e, g, \mathbb{G}, \mathbb{G}_T, H, H_{OTS}, TCR)$, $\text{pk} = (u, v)$, $\text{CT} = (c, (n, w), (u_0, u_1, c'))$ and $c = (c_1, c_2, \pi)$. Output $\text{CT}' = (c_1, c_2)$.

VPKE.Dec'(pars, pk, dk, CT'): Parse $\text{pars} = (p, e, g, \mathbb{G}, \mathbb{G}_T, H, H_{OTS}, TCR)$, $\text{pk} = (u, v)$, $\text{dk} = x_1$ and $\text{CT}' = (c_1, c_2)$. Compute $K \leftarrow H(c_1^{x_1})$ and set $\text{msg} \leftarrow c_2 \oplus K$. Output msg .

C Type A Curve with 128-bit Security

Here, we indicate the parameters as shown in Table 3. h is defined as $h := (p + 1)/\text{Order}$ and is a multiple of 12, and sign0 , sign1 , exp1 , and exp2 are defined as $\text{Order} = 2^{\text{exp2}} + \text{sign1} \cdot 2^{\text{exp1}} + \text{sign0} \cdot 1$.

Table 3: Type A curve with 128-bit security

p	137829182137841914660939203166562778481072472868799212883736033373776389423 275856600849965727557905145379787147011573918838400696256791520969790954647 234026134149836279179970069912941702077185846892228741645147037546137834958 016993449032368771117716800854231045245128514829131301048171717614739196745 940412209360282518205988243325127502858859823618043686336864956271850425997 773219601256420082271109126943413847132693452774733004856610405223161761104 4807535038087
Order	578960446186580977117854925043439539266349923328202820197287920061555880755 21
h	238063209750643048886022474472094216560766062709758760649150166949046752384 245829423385367442267660654963459018826556642656137089040285666790582182002 598333807307620189224986606097900823156136453183171049170543365773619829534 386565283791806164145599669023668121875720159425971381043029195875236768247 182750347222425692281034022570346337224333818783563819554407177204040132394 72452603528
exp1	41
exp2	255
sign0	1
sign1	1