

Lift-and-Shift: Obtaining Simulation Extractable Subversion and Updatable SNARKs Generically*

Behzad Abdolmaleki¹, Sebastian Ramacher², and Daniel Slamanig²

¹ University of Tartu, Estonia
behzad.abdolmaleki@ut.ee

² AIT Austrian Institute of Technology, Austria
{sebastian.ramacher, daniel.slamanig}@ait.ac.at

Abstract. Zero-knowledge proofs and in particular succinct non-interactive zero-knowledge proofs (so called zk-SNARKs) are getting increasingly used in real-world applications, with cryptocurrencies being the prime example. Simulation extractability (SE) is a strong security notion for zk-SNARKs which informally ensures non-malleability of proofs. The high importance of this property is acknowledged by leading companies in this field such as Zcash and underpinned by various attacks against the malleability of cryptographic primitives in the past. Another problematic issue for the practical use of zk-SNARKs is the requirement of a fully trusted setup, as especially for large-scale decentralized applications finding a trusted party that runs the setup is practically impossible. Quite recently, the study of approaches to relax or even remove the trust in the setup procedure, and in particular subversion as well as updatable zk-SNARKs (with latter being the most promising approach), has been initiated and received considerable attention since then. Unfortunately, so far SE-SNARKs with the aforementioned properties are only constructed in an ad-hoc manner and no generic techniques are available. In this paper, we are interested in such generic techniques and therefore firstly revisit the only available lifting technique due to Kosba et al. (called $C\emptyset C\emptyset$) to generically obtain SE-SNARKs. By exploring the design space of many recently proposed SNARK- and STARK-friendly symmetric-key primitives we thereby achieve significant improvements in the prover computation and proof size. Unfortunately, the $C\emptyset C\emptyset$ framework as well as our improved version (called $OC\emptyset C\emptyset$) is not compatible with updatable SNARKs. Consequently, we propose a novel generic lifting transformation called LAMASSU. It is built using different underlying ideas compared to $C\emptyset C\emptyset$ (and $OC\emptyset C\emptyset$). In contrast to $C\emptyset C\emptyset$ it only requires key-homomorphic signatures (which allow to shift keys) covering well studied schemes such as Schnorr or ECDSA. This makes LAMASSU highly interesting, as by using the novel concept of so called updatable signatures, which we introduce in this paper, we can prove that LAMASSU preserves the subversion and in particular updatable properties of the underlying zk-SNARK. This makes LAMASSU the first technique to

* This is the full version of a paper which appears in 2020 ACM SIGSAC Conference on Computer and Communications Security (CCS '20), November 9–13, 2020, Virtual Event, USA, ACM.

also generically obtain SE subversion and updatable SNARKs. As its performance compares favorably to $\text{OC}\emptyset\text{C}\emptyset$, LAMASSU is an attractive alternative that in contrast to $\text{OC}\emptyset\text{C}\emptyset$ is only based on well established cryptographic assumptions.

Keywords: Zero-knowledge; simulation extractability; SNARK; updatable SNARK; subversion SNARK

1 Introduction

Zero-knowledge (ZK) proofs were introduced by Goldwasser, Micali, and Rackoff [GMR85] more than 3 decades ago. They represent a cryptographic protocol between two parties called the prover and the verifier, with the goal that the prover convinces the verifier of the membership of a word x in any language in NP without revealing any information about the witness w certifying language membership of word x . Besides this zero-knowledge property, such a system needs to provide soundness, i.e., it must be infeasible for the prover to provide proofs for words outside of the language. While ZK proofs, in general, may require many rounds of interaction, a variant highly relevant to practical applications are non-interactive zero-knowledge (NIZK) proofs [BFM88]. They require only a single round, i.e., the prover outputs a proof which can then be verified by anybody. (NI)ZK plays a central role in the theory of cryptography and meanwhile increasingly finds its way into practice.^{3,4,5} Important applications are electronic voting [SK95, DGS03, Gro10b], anonymous credentials [Cha86, CL01, CL03, CL04, BCC⁺09, CKL⁺16, FHS19], and group signatures [Cv91, ACJT00, BBS04, DP06, BCC⁺16, DS18], including widely deployed schemes such as direct anonymous attestation (DAA) [BCC04, CCD⁺17] used in the Trusted Platform Module (TPM) or Intel’s Enhanced Privacy ID (EPID) [BL09], as well as many other applications that require balancing privacy and integrity (cf. [FPS⁺18]). They are also a core building block of verifiable computation [GGP10, GGPR13, PHGR13, BCG⁺18] and in the increasingly popular domain of privacy-respecting cryptocurrencies [BCG⁺14, CGL⁺17], smart contracts [KMS⁺16] and self-sovereign identity systems [MGGM18]. Latter arguably represent the most popular real-world applications of zero-knowledge nowadays, where it sees deployments in systems such as Zcash, Ethereum, or Sovrin.

A challenging issue, particularly important in the context of blockchains, is that users need to download and verify the state of the chain. Thus, small proof

³ ZKProof (<https://zkproof.org/>) being the most notable industry and academic initiative towards a common framework and standards in the field of zero-knowledge has been founded in 2018.

⁴ Zero-knowledge proofs are *on the rise* in Gartner’s Hype Cycle for Privacy 2019, cf. <https://www.gartner.com/en/documents/3947373/hype-cycle-for-privacy-2019>.

⁵ MIT technology review named zk-SNARKS as one of the “10 Breakthrough Technologies of 2018” cf. <https://www.technologyreview.com/lists/technologies/2018/>.

sizes and fast verification are important criteria for the practical use of ZK proofs. These desired features are provided by zero-knowledge Succinct Non-interactive ARGuments of Knowledge (zk-SNARKs)⁶, which are NIZK proofs in which proofs as well as the computation of the verifier are succinct and ideally represent a small constant amount of space and computation respectively. Additionally, they satisfy a stronger notion of soundness called knowledge soundness, which guarantees that if an adversarial prover comes up with a proof that is accepted by the verifier, then there exists an efficient extractor which given some secret information can extract the witness. A combined effort of a large number of recent research works [Gro10a, Lip12, GGPR13, PHGR13, Lip13, DFGK14, Gro16] (to only mention a few) has made it possible to construct very efficient zk-SNARKs for both the Boolean and the Arithmetic CIRCUIT-SAT and thus for NP. The most efficient known approach for constructing zk-SNARKs for the Arithmetic CIRCUIT-SAT is based on Quadratic Arithmetic Programs (QAPs) [GGPR13], where the prover builds a set of polynomial equations that are then checked by the verifier by using a small number of pairings. The current interest in zk-SNARKs is significant and recently first modular frameworks to efficiently compose zk-SNARKs [CFQ19] and also first important steps towards realizing zk-SNARKs from conjectured post-quantum secure assumptions have been made [GMNO18, BBC⁺18]. We note that in this work we do not consider recent NIZK proofs that allow larger proof sizes, e.g., logarithmic in the witness size, such as Bulletproofs [BBB⁺18] or STARKs [BBHR19] but do not require a trusted setup. The currently most efficient zk-SNARK for Arithmetic CIRCUIT-SAT was proposed by Groth [Gro16], who proved it to be knowledge-sound in the generic bilinear group model. In Groth’s zk-SNARK, a proof consists of only 3 bilinear group elements and the verifier has to check a single pairing equation.

Strong security for zk-SNARKs. For practical applications of NIZKs even stronger security notions than soundness and knowledge soundness, called simulation soundness (SS) and simulation knowledge soundness (or simply simulation extractability or SE) [Sah99, Sah01]), are required. Informally, these notions require soundness and knowledge soundness respectively to hold even if an adversary is allowed to see an arbitrary number of simulated proofs (which she can obtain adaptively on words of her choice). Firstly, these properties are important if NIZKs are used within larger cryptographic protocols and in particular if they are modeled and analyzed in the universal composability (UC) framework [Can01], as frequently used in blockchain-related protocols (e.g., [JKS16, CDD17, KKKZ19, FMMO19] to name a few). Secondly, NIZKs not satisfying this strong security may face severe threats when used in applications. Therefore, let us informally recall what this property does. It guarantees that proofs are non-malleable in a way that one can neither obtain another valid proof for the same word nor a new proof for a potentially related word from a given proof. Now, let us assume the typical blockchain setting where proofs are incor-

⁶ We note that we might drop the zk and simply write SNARK occasionally, though we are always talking about zk-SNARKs.

porated into the state of the blockchain via transactions (e.g., as in Zcash). This means that anyone could take a proof π and obtain from it another new proof π' for the same word and could advertise it as its own proof (as $\pi' \neq \pi$). This is what is often called man-in-the-middle attacks in the context of NIZKs and SNARKs (cf. [GM17]). Even worse, it might be possible to obtain from a proof π another proof π' for another word $x' \neq x$ (in the same language). For example, if π proves that 100\$ are transferred from A to B with transaction $ID = id$, π' might transfer 1000\$ from A to B with new $ID = id'$, which can be a devastating attack in systems deployed in the real-world. In fact, malleability of ECDSA signatures already enabled an attack on Bitcoin that is suspected to have caused a loss of over \$ 30 million.⁷ Therefore, to avoid such attacks in zk-SNARKs based cryptocurrencies, non-malleability of the proofs is of utmost importance and all these problems are mitigated by the use of simulation-extractable (SE) zk-SNARKs.

Simulation soundness and simulation extractability can be added generically to any NIZK. Compilers for the former are usually inspired by [Sah01, Gro06] and basically use the idea of extending the language to an OR language where the trapdoor for the OR part can be used to simulate proofs. Extractability can be obtained by additionally encrypting the witness under a public key in the common reference string (CRS) and to additionally prove correct encryption [DP92]. The most prominent compiler that exactly follows the ideas outlined before is the C0C0 framework [KZM⁺15] (e.g., used in [AB19, Bag19] and most prominently in the celebrated Hawk paper [KMS⁺16]). In addition to generic compilers, Groth and Maller in [GM17] initiated the study of ad-hoc constructions of SE zk-SNARKs. This work continued in [BG18] by extending Groth’s zk-SNARK [Gro16] in a non black-box way to obtain SE. There is also other recent work in this direction proposing other ad-hoc zk-SNARKs with these properties (cf. [Lip19]). Beyond the C0C0 framework, which, given the progress in the field of SNARKs (such as universal CRS) and SNARK-friendly primitives, is already quite outdated, there is no work towards lifting zk-SNARKs to SE zk-SNARKs generically.

Trust in CRS generation. Another important aspect for practical applications of zk-SNARKs is the question of the generation of the required common reference string (CRS) [BFM88], a structured random string available to the prover and the verifier. While the CRS model is widely accepted, one has to be very careful to ensure that the CRS has been created honestly, meaning that no one knows the associated trapdoor which allows to break zero-knowledge or soundness. In theory, it is simply assumed that some trusted party will perform the CRS generation, but such a party is hard to find in the real-world. After the Snowden revelations, there has been a recent surge of interest in constructing cryptographic primitives and protocols secure against active subversion and the CRS generation is exactly one of those processes where subversion can happen.

⁷ <https://www.coindesk.com/study-finds-mt-gox-lost-386-bitcoins-due-transaction-malleability>

In [BFS16], Bellare, Fuchsbauer, and Scafuro tackled this problem for NIZK proofs by studying how much security one can still achieve when the CRS generator cannot be trusted. They proved several negative and positive results. In particular, they showed that it is impossible to achieve subversion soundness and (even non-subversion) zero knowledge simultaneously. However, subversion zero-knowledge can be achieved. Later, this notion has also been considered for SNARKs [ABLZ17, Fuc18, ALSZ20] and used within practical applications in cryptocurrencies [CGGN17, Fuc19]. For deployed systems such as Zcash and Ethereum, instead of building them on top of subversion-resistant zk-SNARKs, they followed an alternative route to reduce the trust put in the CRS generation. For instance, the CRS for Pinocchio zk-SNARKs [PHGR13] was generated in a large “ceremony” [BGG19] by applying a generic method implementing the generation within a secure multi-party computation (MPC) protocol [BCG⁺15]. Coincidentally, they end up with a subversion-resistant zk-SNARK with a polynomial error even in the case where all parties are corrupted, and subversion soundness as long as at least one party is honest. While this is a significant achievement, MPC protocols for such tasks are complicated and expensive procedures in practice and require much effort besides the technical realization. Thus, more practical solutions are desirable.

Quite recently, to overcome this problem Groth et al. [GKM⁺18] proposed the notion of a so-called updatable CRS, where everyone can update a CRS and there is a way to check the correctness of an update. Here, zero-knowledge holds in the face of a malicious CRS generator and the verifier can trust the CRS (soundness holds) as long as one operation, either the creation of the CRS or one update, has been performed honestly. So the verifier could perform this update operation on its own and then send the CRS to the prover. This updatable setting thus seems much more practical than using MPC protocols, it is more promising than the subversion setting (as it overcomes the impossibility of subversion soundness), and has increasingly found interest recently (cf. [MBKM19, GR19, CHM⁺20]).

1.1 Our Contributions

Below we summarize the contributions of our work.

Revisiting C0C0. We revisit the C0C0 lifting technique [KZM⁺15] to generically obtain SE-SNARKs from SNARKs, which is prominently used within Hawk [KMS⁺16]. First, we discuss the concrete instantiation in [KZM⁺15] and point to efficiency problems and problems regarding provable security of this instantiation. Then, we extensively investigate alternative provably secure instantiations of their techniques by exploring the design space of many recently proposed SNARK- and STARK-friendly symmetric primitives including the most recent proposals Poseidon [GKK⁺19] as well as Vision and Rescue [AABS⁺19]. As these primitives are, however, all very recent and their cryptanalysis either still needs to start or has only recently started [ACG⁺19, LP19, Bon19, BBUV20, BSG20], confidence in their proposed security is far from certain. Nevertheless, we provide concrete recommendations for the selection of primitives and

provide lower bounds for their efficiency based on the currently available parameters. Additionally, we also propose a more conservative instantiation based on LowMC [ARS⁺15], which is the oldest of these proposals and has already received independent cryptanalysis [DEM16, BDD⁺15, DLMW15, RST18]. In comparison to the original C0C0 framework, with our revisited C0C0 framework (dubbed OC0C0) we obtain an improvement by a factor 10.4x in the number of rank-1 constraints with a conservative choice of symmetric primitives, whereas the most aggressive choice yields an improvement by up to a factor 55.4x.

A new framework. As the symmetric primitives underlying the efficiency gain of OC0C0 are very recent and the confidence in them might not yet be strong enough, we propose an alternative framework for lifting SNARKs to SE-SNARKs that is based on completely different cryptographic primitives. In particular, it is based on the ideas of Derler and Slamanig [DS19] using the notion of key-homomorphic signatures and thus only requires signature schemes. Our compiler, which we dub LAMASSU, allows instantiations based on well studied and widely used signature schemes such as ECDSA or EC-Schnorr (or simply Schnorr for short). Also for LAMASSU we provide concrete choices for the primitives and an extensive comparison with ad-hoc constructions. We show that LAMASSU yields efficient instantiations that compared to OC0C0 only needs 200 rank-1 constraints more than the most aggressive choice using the most efficient SNARK-friendly primitive Poseidon in this setting. For all other choices of SNARK-friendly symmetric-key primitives, LAMASSU beats them in the number of constraints and outperforms OC0C0 by a factor of up to 4.2x. Considering that Schnorr and ECDSA signatures are well established primitives, and that the confidence in their security is far bigger than all the recent SNARK/STARK-friendly primitives, this additional confidence comes at only a very small cost and makes LAMASSU an attractive alternative to (O)C0C0.

Subversion and updatable CRS. C0C0 as well as OC0C0 do not support lifting of subversion or updatable CRS zk-SNARKs to SE subversion or updatable SNARKs. While for the case of subversion zero-knowledge, Bagheri in independent work [Bag19] shows that using a part of the C0C0 framework (without the encryption of the witness) it is possible to preserve the subversion zero-knowledge property, the case of zk-SNARKs with updatable CRS is more problematic. In particular, the C0C0 and OC0C0 frameworks cannot be easily made updatable due to the missing algebraic structure in the used primitives, i.e., (hash) commitments.⁸ Fortunately, LAMASSU does not suffer from this problem and we can show that when basing LAMASSU on the notion of updatable signatures, an extension of key-homomorphic signatures introduced in this paper, instead of key-homomorphic signatures, we are able to prove that the property of updatability is preserved if the underlying zk-SNARK possesses this property, i.e., is updatable. Updatable signatures can be constructed from widely used signa-

⁸ Even using the C0C0 framework with commitments having enough algebraic structure, i.e., exponential ElGamal or Pedersen commitments, does not seem to yield updatability. And even if it would work, it would be less efficient than LAMASSU.

tures such as Schnorr signatures when instantiated in bilinear groups. Moreover, we also prove that LAMASSU preserves subversion of the underlying SNARK. Consequently, when starting from an subversion/updatable zk-SNARK, LAMASSU yields SE subversion/updatable SNARKs. This makes LAMASSU the first framework that allows to generically lift updatable zk-SNARKs to SE updatable SNARKs, a notion for which we introduce a natural definition in this work, using well established cryptographic primitives. Actually, it yields the first known SE updatable SNARK.

2 Preliminaries

Let PPT denote probabilistic polynomial-time. Let $\lambda \in \mathbb{N}$ be the security parameter. All adversaries will be stateful. By $y \leftarrow \mathcal{A}(x; \omega)$ we denote the fact that \mathcal{A} , given an input x and random coins ω , outputs y . By $x \leftarrow_s \mathcal{D}$ we denote that x is sampled according to distribution \mathcal{D} or uniformly randomly if \mathcal{D} is a set. Let $\text{RND}(\mathcal{A})$ denote the random tape of \mathcal{A} , and let $\omega \leftarrow_s \text{RND}(\mathcal{A})$ denote the random choice of the random coins ω from $\text{RND}(\mathcal{A})$. We denote by $\text{negl}(\lambda)$ an arbitrary negligible function. We write $a \approx_\lambda b$ if $|a - b| \leq \text{negl}(\lambda)$. A bilinear group generator $\text{Pgen}(1^\lambda)$ returns $\text{BG} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e)$, where $\mathbb{G}_1, \mathbb{G}_2$, and \mathbb{G}_T are three cyclic groups of prime order p , and $e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$ is a non-degenerate efficiently computable bilinear map (pairing).

2.1 X-SNARK

In the following we provide a formal definition of SNARKs (cf. Appendix A.2 for the basic definition of NIZK proofs).

Definition 1 (SNARK). *A non-interactive system Π is a succinct non-interactive argument of knowledge (SNARK) for relation generator RGen if it is complete and knowledge sound, and moreover succinct, meaning that for all λ , all $(\mathcal{R}, \text{aux}_{\mathcal{R}}) \in \text{image}(\text{RGen}(1^\lambda))$, all $\text{crs} \leftarrow \text{KGen}(\mathcal{R}, \text{aux}_{\mathcal{R}})$, all $(x, w) \in \mathcal{R}$ and all proofs $\pi \leftarrow \text{P}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, x, w)$ we have $|\pi| = \text{poly}(\lambda)$ and $\text{V}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, x, \pi)$ runs in time polynomial in $\lambda + |x|$. Π is a zk-SNARK if it additionally satisfies zero-knowledge and an SE (zk-)SNARK if instead of knowledge soundness it provides strong simulation extractability.*

We adopt the (SE) X-SNARK definitions from [ABLZ17, Fuc18, GKM⁺18] where $X \in \{\text{trusted}, \text{subverted}, \text{updatable}\}$. In other words, besides considering the standard setting with a trusted CRS generation, we also capture the subversion and updatable CRS setting. Trusted means generated by a trusted third party, or even a more general MPC protocol, subverted means that the setup generator gets the CRS from the adversary and uses it after checking that it is well formed, and, updatable means that an adversary can adaptively generate sequences of CRSs and arbitrarily interleave its own malicious updates into them. The only constraints on the final CRS are that it is well formed and that

at least one honest participant has contributed to it by providing an update (or the initial creation).

A X -SNARK $\Pi = (\text{KGen}, \text{Ucrs}, \text{Vcrs}, \text{P}, \text{V})$ for RGen consists of the following PPT algorithms (it contains Vcrs when $X = \text{subverted}$ and contains Ucrs and Vcrs when $X = \text{update}$):

$\text{KGen}_{\text{crs}}(\mathcal{R}, \text{aux}_{\mathcal{R}})$: On input $(\mathcal{R}, \text{aux}_{\mathcal{R}}) \in \text{image}(\text{RGen}(1^\lambda))$, outputs CRS crs , a trapdoor tc , and a proof ζ .⁹

$\text{Ucrs}(\mathcal{R}, \text{crs}, \zeta)$: On input $(\mathcal{R}, \text{crs}, \zeta)$ outputs $(\text{crs}_{\text{up}}, \zeta_{\text{up}})$ where crs_{up} is the updated CRS and ζ_{up} is a proof for the correctness of the updating procedure.

$\text{Vcrs}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \zeta)$: On input $(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \zeta)$, returns either 0 (the CRS is ill-formed) or 1 (the CRS is well-formed).

$\text{P}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \mathbf{x}, \mathbf{w})$: On input $(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \mathbf{x}, \mathbf{w})$, where $(\mathbf{x}, \mathbf{w}) \in \mathcal{R}$, output a proof π .

$\text{V}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \mathbf{x}, \pi)$: On input $(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \mathbf{x}, \pi)$, returns either 0 (reject) or 1 (accept).

$\text{Sim}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \text{tc}, \mathbf{x})$: On input $(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \text{tc}, \mathbf{x})$, outputs a simulated proof π .

We may omit 1^λ , \mathcal{R} and $\text{aux}_{\mathcal{R}}$ as inputs in the following and assume that they are implicitly available.

Definition 2. Let $\Pi = (\text{KGen}_{\text{crs}}, \text{Ucrs}, \text{Vcrs}, \text{P}, \text{V})$ be a non-interactive argument for the relation \mathcal{R} . Then the argument Π is X -secure for $X \in \{\text{trusted}, \text{subverted}, \text{updatable}\}$, if it satisfies the following properties:

X -Completeness. Π is complete for RGen , if for all λ , $(\mathbf{x}, \mathbf{w}) \in \mathcal{R}$, and PPT algorithms A ,

$$\Pr \left[\begin{array}{l} (\mathcal{R}, \text{aux}_{\mathcal{R}}) \leftarrow \text{RGen}(1^\lambda), (\text{crs}, \text{tc}, \zeta) \leftarrow A(\mathcal{R}, \text{aux}_{\mathcal{R}}), \\ 1 \leftarrow \text{Vcrs}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \zeta): \\ \text{V}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \mathbf{x}, \text{P}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \mathbf{x}, \mathbf{w})) = 1 \end{array} \right] = 1.$$

Where ζ is a proof for the correctness of the generation (or updating) of the CRS. If $X = \text{trusted}$ then A is KGen_{crs} and $\zeta = \perp$ and A is adversary A otherwise.

X -Strong simulation extractability. For $X \in \{\text{trusted}, \text{subverted}\}$, Π is strong simulation extractable for RGen , if for every PPT A , there exists a PPT extractor Ext_A ,

$$\Pr \left[\begin{array}{l} (\mathcal{R}, \text{aux}_{\mathcal{R}}) \leftarrow \text{RGen}(1^\lambda), \\ (\text{crs}, \text{tc}, \zeta) \leftarrow \text{KGen}_{\text{crs}}(\mathcal{R}, \text{aux}_{\mathcal{R}}), \omega_{\mathcal{A}} \leftarrow \text{RND}(\mathcal{A}), \\ (\mathbf{x}, \pi) \leftarrow \mathcal{A}^{O(\cdot)}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}; \omega_{\mathcal{A}}), \\ \mathbf{w} \leftarrow \text{Ext}_A(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}; \omega_{\mathcal{A}}): \\ (\mathbf{x}, \pi) \notin \mathcal{Q} \wedge (\mathbf{x}, \mathbf{w}) \notin \mathcal{R} \wedge \\ \text{V}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \mathbf{x}, \pi) = 1 \end{array} \right] \approx_{\lambda} 0.$$

⁹ If $X = \text{trusted}$, then $\zeta = \perp$ and we may omit it.

Here, $\mathcal{O}(\mathbf{x})$ returns $\pi := \text{Sim}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \text{tc}, \mathbf{x})$ and keeps track of all queries and the result, (\mathbf{x}, π) , via \mathcal{Q} . For $X = \text{updatable}$, Π is strong simulation extractable for RGen , if for every PPT \mathcal{A} and any subverter Z , there exists a PPT extractor $\text{Ext}_{\mathcal{A}}$,

$$\Pr \left[\begin{array}{l} (\mathcal{R}, \text{aux}_{\mathcal{R}}) \leftarrow \text{RGen}(1^\lambda), \\ (\text{crs}, \text{tc}, \zeta) \leftarrow \text{KGen}_{\text{crs}}(\mathcal{R}, \text{aux}_{\mathcal{R}}), \omega_Z \leftarrow_{\$} \text{RND}(Z), \\ (\text{crs}_{\text{up}}, \zeta_{\text{up}}, \text{aux}_Z) \leftarrow Z(\text{crs}, \{\zeta_i\}_{i=1}^{i=n}, \omega_Z), \\ \text{if } \text{Vcrs}(\text{crs}_{\text{up}}, \zeta_{\text{up}}) = 0 \text{ then return } 0, \\ \omega_{\mathcal{A}} \leftarrow_{\$} \text{RND}(\mathcal{A}), \\ (\mathbf{x}, \pi) \leftarrow \mathcal{A}^{\mathcal{O}(\cdot)}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}_{\text{up}}, \text{aux}_Z; \omega_{\mathcal{A}}), \\ \mathbf{w} \leftarrow \text{Ext}_{\mathcal{A}}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}_{\text{up}}, \text{aux}_Z; \omega_{\mathcal{A}}): \\ (\mathbf{x}, \pi) \notin \mathcal{Q} \wedge (\mathbf{x}, \mathbf{w}) \notin \mathcal{R} \wedge \\ \text{V}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}_{\text{up}}, \mathbf{x}, \pi) = 1 \end{array} \right] \approx_{\lambda} 0.$$

Here $\text{RND}(Z) = \text{RND}(\mathcal{A})$ and $\{\zeta_i\}_{i=1}^{i=n}$ for $n \in \mathbb{N}$ is a number proofs for the correctness of the updating procedure. The oracle $\mathcal{O}(\cdot)$ represents two oracles $\mathcal{O}_1(\cdot)$ and $\mathcal{O}_2(\cdot)$ which return $\pi := \text{Sim}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \text{tc}, \mathbf{x})$ and $\pi := \text{Sim}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}_{\text{up}}, \text{tc}_{\text{up}}, \mathbf{x})$ respectively. $\mathcal{O}(\cdot)$ keeps track of all queried (\mathbf{x}, π) via \mathcal{Q} . Note that Z can also first generate crs and then an honest updater updates it and outputs crs_{up} . In the latter case, $\mathcal{O}(\cdot) = \mathcal{O}_2(\cdot)$. The elements $\{\zeta_i\}_{i=1}^{i=n}$ of the input of Z may be empty when all updating (but one which is honestly computed) are done by Z .

X-Zero-knowledge. For $X = \text{trusted}$, Π is statistically unbounded ZK for RGen [Gro06], if for all $(\mathcal{R}, \text{aux}_{\mathcal{R}}) \in \text{im}(\text{RGen}(1^\lambda))$, and all computationally unbounded \mathcal{A} , $\varepsilon_0^{\text{unb}} \approx_{\lambda} \varepsilon_1^{\text{unb}}$, where

$$\varepsilon_b^{\text{unb}} = \Pr \left[(\text{crs}, \text{tc}, \zeta) \leftarrow \text{KGen}_{\text{crs}}(\mathcal{R}, \text{aux}_{\mathcal{R}}) : \mathcal{A}^{\mathcal{O}_b(\cdot, \cdot)}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}) = 1 \right].$$

Here, oracle $\mathcal{O}_0(\mathbf{x}, \mathbf{w})$ returns \perp (reject) if $(\mathbf{x}, \mathbf{w}) \notin \mathcal{R}$, and otherwise it returns $\text{P}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \mathbf{x}, \mathbf{w})$. Similarly, $\mathcal{O}_1(\mathbf{x}, \mathbf{w})$ returns \perp (reject) if $(\mathbf{x}, \mathbf{w}) \notin \mathcal{R}$, and otherwise it returns $\text{Sim}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \text{tc}, \mathbf{x})$. Π is perfectly unbounded ZK for RGen if we require $\varepsilon_0^{\text{unb}} = \varepsilon_1^{\text{unb}}$.

For $X \in \{\text{subverted}, \text{updatable}\}$, Π is statistically unbounded X-ZK for RGen [ABLZ17, Fuc18, GKM⁺18], if for any PPT Z there exists a PPT Ext_Z , such that for all $(\mathcal{R}, \text{aux}_{\mathcal{R}}) \in \text{im}(\text{RGen}(1^\lambda))$, and computationally unbounded \mathcal{A} , $\varepsilon_0^{\text{unb}} \approx_{\lambda} \varepsilon_1^{\text{unb}}$, where

$$\varepsilon_b^{\text{unb}} = \Pr \left[\begin{array}{l} \omega_Z \leftarrow_{\$} \text{RND}(Z), (\text{crs}, \zeta, \text{aux}_Z) \leftarrow Z(\mathcal{R}, \text{aux}_{\mathcal{R}}; \omega_Z), \\ \text{tc} \leftarrow \text{Ext}_Z(\mathcal{R}, \text{aux}_{\mathcal{R}}; \omega_Z): \\ \text{Vcrs}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \zeta) = 1 \wedge \mathcal{A}^{\mathcal{O}_b(\cdot, \cdot)}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \text{tc}, \text{aux}_Z) = 1 \end{array} \right].$$

Here $\text{RND}(Z) = \text{RND}(\mathcal{A})$, the oracle $\mathcal{O}_0(\mathbf{x}, \mathbf{w})$ returns \perp (reject) if $(\mathbf{x}, \mathbf{w}) \notin \mathcal{R}$, and otherwise it returns $\text{P}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \mathbf{x}, \mathbf{w})$. Similarly, $\mathcal{O}_1(\mathbf{x}, \mathbf{w})$ returns \perp (reject) if $(\mathbf{x}, \mathbf{w}) \notin \mathcal{R}$, and otherwise it returns $\text{Sim}(\mathcal{R}, \text{aux}_{\mathcal{R}}, \text{crs}, \text{tc}, \mathbf{x})$. Π is perfectly unbounded X-ZK for RGen if one requires that $\varepsilon_0^{\text{unb}} = \varepsilon_1^{\text{unb}}$.

In all of the above properties, $\text{aux}_{\mathcal{R}}$ can be seen as a common auxiliary input to algorithm \mathbf{A} that is generated by using a benign [BCPR14] relation generator; we recall that we just think of $\text{aux}_{\mathcal{R}}$ as being the description of a bilinear group.

We note that strong simulation-sound extractability in this work (for consistency with [KZM⁺15]) is often called simulation-sound extractability (e.g., in [DS19] which will be the basis for the LAMASSU framework). For completeness, quadratic arithmetic programs and rank 1 constraint systems are discussed in Appendix A.3

2.2 Key-Homomorphic Signatures

We recall relevant parts of the definitional framework of key-homomorphic signatures as introduced in [DS19]. Therefore, let $\Sigma = (\text{KGen}, \text{Sign}, \text{Verify})$ be a signature scheme (cf. Appendix A.5) and the secret and public key elements live in groups $(\mathbb{H}, +)$ and (\mathbb{E}, \cdot) , respectively. For these two groups it is required that group operations, inversions, membership testing as well as sampling from the uniform distribution are efficient.

Definition 3 (Secret Key to Public Key Homomorphism). *A signature scheme Σ provides a secret key to public key homomorphism, if there exists an efficiently computable map $\mu : \mathbb{H} \rightarrow \mathbb{E}$ such that for all $\text{sk}, \text{sk}' \in \mathbb{H}$ it holds that $\mu(\text{sk} + \text{sk}') = \mu(\text{sk}) \cdot \mu(\text{sk}')$, and for all $(\text{sk}, \text{pk}) \leftarrow \text{KGen}$, it holds that $\text{pk} = \mu(\text{sk})$.*

In the discrete logarithm setting, it is usually the case $\text{sk} \leftarrow \mathbb{Z}_p$ and $\text{pk} = g^{\text{sk}}$ with g being the generator of some group \mathbb{G} of prime order p , e.g., for ECDSA or Schnorr signatures (cf. [DS19]).

Definition 4 (Key-Homomorphic Signatures). *A signature scheme is called key-homomorphic, if it provides a secret key to public key homomorphism and an additional PPT algorithm Adapt , defined as:*

$\text{Adapt}(\text{pk}, m, \sigma, \Delta)$: *Given a public key pk , a message m , a signature σ , and a shift amount Δ outputs a public key pk' and a signature σ' ,*

such that for all $\Delta \in \mathbb{H}$ and all $(\text{pk}, \text{sk}) \leftarrow \text{KGen}(1^\lambda)$, all messages $m \in \mathcal{M}$ and all $\sigma \leftarrow \text{Sign}(\text{sk}, m)$ and $(\text{pk}', \sigma') \leftarrow \text{Adapt}(\text{pk}, m, \sigma, \Delta)$ it holds that

$$\Pr[\text{Verify}(\text{pk}', m, \sigma') = 1] = 1 \quad \wedge \quad \text{pk}' = \mu(\Delta) \cdot \text{pk}.$$

The following notion covers whether adapted signatures look like freshly generated signatures, where we do not need the strongest notion in [DS19], which requires this to hold even if the initial signature used in Adapt is known.

Definition 5 (Adaptability of Signatures). *A key-homomorphic signature scheme provides adaptability of signatures, if for every $\lambda \in \mathbb{N}$ and every message $m \in \mathcal{M}$, it holds that*

$$[(\text{sk}, \text{pk}), \text{Adapt}(\text{pk}, m, \text{Sign}(\text{sk}, m), \Delta)],$$

where $(\text{sk}, \text{pk}) \leftarrow \text{KGen}(1^\lambda)$, $\Delta \leftarrow \mathbb{H}$, and

$$[(\text{sk}, \mu(\text{sk})), (\mu(\text{sk}) \cdot \mu(\Delta), \text{Sign}(\text{sk} + \Delta, m))],$$

where $\text{sk} \leftarrow \mathbb{H}$, $\Delta \leftarrow \mathbb{H}$, are identically distributed.

For illustration purposes we will use the Schnorr signature scheme [Sch90], which is very popular in the blockchain and distributed ledger domain, and whose adaption notion we discuss in Appendix A.6.

2.3 The C0C0 Framework

Kosba et al. [KZM⁺15] proposed lifting transformations for SNARKs in three different versions basic, improved lifting, and the strengthened lifting. We only consider the strongest version which lifts a SNARK to a strongly simulation extractable (SE) SNARK. In particular, their construction, which we recall in Fig. 1, transforms any NIZK Π to one that satisfies SE. Given a language \mathcal{L} with NP relation $\mathcal{R}_{\mathcal{L}}$, let \mathcal{L}' be s.t. $\{(\mathbf{x}, \mathbf{c}, \mu, \text{pk}_{\text{OT}}, \text{pk}_e, \rho), (\mathbf{w}, r_1, r_0, s_0)\} \in \mathcal{R}_{\mathcal{L}'}$ iff:

$$\begin{aligned} \mathbf{c} &= \Omega.\text{Enc}(\text{pk}_e, \mathbf{w}; r_1) \wedge ((\mathbf{x}, \mathbf{w}) \in \mathcal{R}_{\mathcal{L}} \vee \\ &(\mu = f_{s_0}(\text{pk}_{\text{OT}}) \wedge \rho = \text{Commit}(s_0; r_0))) \end{aligned} \quad (1)$$

where pk_e is the public key of a public key encryption scheme Ω (cf. Appendix A.4) and pk_{OT} is the verification key of a strong OTS scheme Σ_{OT} (cf. Appendix A.5). Note that extraction is defined here with respect to a black-box extractor (i.e., decrypting \mathbf{c} to obtain the witness \mathbf{w}), which Kosba et al. [KZM⁺15] do to support UC-security. If this is not required, as in our case, then one can use the non black-box extractor of the underlying SNARK and simplify the language \mathcal{L}' by removing the part in the gray box, which we will do subsequently (cf. [Bag19] for a formal treatment). In this case, C0C0 retains subversion resistance of the underlying SNARK.

3 Lifting Transformations for SE (Subversion/Updatable) SNARKs

In this section, we first revisit the C0C0 framework and then present a different novel transformation which we call LAMASSU.

3.1 Revisiting the C0C0 Framework

We will now revisit the most efficient version of the C0C0 framework based on a commitment and PRF evaluation (Equation (1) without the gray box). Kosba et al. [KZM⁺15] proposed to instantiate the commitment and the PRF using hash functions, and in particular SHA-256. Similarly, the commitment is instantiated as a hash commitment using the same hash function. With the development of SNARK/STARK-friendly primitives soon after the introduction of

$\text{KGen}_{\text{crs}}(\mathcal{R}, \text{aux}_{\mathcal{R}})$ <hr/> <ul style="list-style-type: none"> - $\Pi.\text{crs} \leftarrow \Pi.\text{KGen}(\mathcal{R}, \text{aux}_{\mathcal{R}}); (\text{pk}_e, \text{sk}_e) \leftarrow \Omega.\text{KGen}(1^\lambda);$ - $\text{tc} := (s_0, r_0) \leftarrow_s \{0, 1\}^\lambda; \rho \leftarrow \text{Commit}(s_0; r_0);$ - return $(\text{crs} := (\Pi.\text{crs}, \text{pk}_e, \rho), \text{tc}_{\text{ext}} := \text{sk}_e)$
$\text{P}(\text{crs}, \mathbf{x}, \mathbf{w})$ <hr/> <ul style="list-style-type: none"> - $(\text{pk}_{\text{OT}}, \text{sk}_{\text{OT}}) \leftarrow \Sigma_{\text{OT}}.\text{KGen}(1^\lambda); r_1, z_0, z_1, z_2 \leftarrow_s \{0, 1\}^\lambda;$ - $\mathbf{c} = \Omega.\text{Enc}(\text{pk}_e, \mathbf{w}; r_1); \mu \leftarrow z_0;$ - $\pi_\Pi \leftarrow \Pi.\text{P}(\Pi.\text{crs}, (\mathbf{x}, \mathbf{c}, \text{pk}_e, \text{pk}_{\text{OT}}, \mu, \rho), (\mathbf{w}, r_1, z_1, z_2));$ - $\sigma_{\text{OT}} \leftarrow \Sigma_{\text{OT}}.\text{Sign}(\text{sk}_{\text{OT}}, (\mathbf{x}, \mathbf{c}, \mu, \pi_\Pi));$ - return $\pi := (\mathbf{c}, \mu, \pi_\Pi, \text{pk}_{\text{OT}}, \sigma_{\text{OT}});$
$\text{V}(\text{crs}, \mathbf{x}, \pi)$ <hr/> <ul style="list-style-type: none"> - if $\Sigma_{\text{OT}}.\text{Verify}(\text{pk}_{\text{OT}}, (\mathbf{x}, \mathbf{c}, \mu, \pi_\Pi, \sigma_{\text{OT}})) = 0$ - $\vee \Pi.\text{V}(\Pi.\text{crs}, \mathbf{x}, \mathbf{c}, \mu, \text{pk}_e, \text{pk}_{\text{OT}}, \rho, \pi_\Pi) = 0$ - then return 0 else return 1;
$\text{Sim}(\text{crs}, \mathbf{x}, \text{tc})$ <hr/> <ul style="list-style-type: none"> - $(\text{pk}_{\text{OT}}, \text{sk}_{\text{OT}}) \leftarrow \Sigma_{\text{OT}}.\text{KGen}(1^\lambda); \mu = f_{s_0}(\text{pk}_{\text{OT}});$ - $r_1, z_3 \leftarrow_s \{0, 1\}^\lambda; \mathbf{c} \leftarrow \Omega.\text{Enc}(\text{pk}_e, z_3; r_1); \mathbf{w} \leftarrow z_3;$ - $\pi_\Pi \leftarrow \Pi.\text{P}(\Pi.\text{crs}, (\mathbf{x}, \mathbf{c}, \text{pk}_e, \text{pk}_{\text{OT}}, \mu, \rho), (\mathbf{w}, r_1, r_0, s_0));$ - $\sigma_{\text{OT}} \leftarrow \Sigma_{\text{OT}}.\text{Sign}(\text{sk}_{\text{OT}}, (\mathbf{x}, \mathbf{c}, \mu, \pi_\Pi));$ - return $\pi = (\mathbf{c}, \mu, \pi_\Pi, \text{pk}_{\text{OT}}, \sigma_{\text{OT}});$
$\text{Ext}(\text{crs}, \mathbf{x}, \pi, \text{tc}_{\text{ext}})$ <hr/> <ul style="list-style-type: none"> - return $\mathbf{w} \leftarrow \Omega.\text{Dec}(\text{tc}_{\text{ext}}, \mathbf{c});$

Fig. 1. The strong version of the $\text{C}\emptyset\text{C}\emptyset$ transformation.

the $\text{C}\emptyset\text{C}\emptyset$ framework, we observe that this choice is non-optimal from an efficiency point of view. Moreover, the choice of the commitment is also problematic in a different sense, because the specific commitment used in $\text{C}\emptyset\text{C}\emptyset$ is secure in the random oracle model (ROM). Since this implies that statements need to be proven with respect to the preimage of a random oracle, instantiating the framework in a provable secure way is not possible. Consequently, we discuss an alternative approach to commit to the PRF key. Our approach can be instantiated in a provably secure way and, on top of that, is also more efficient while still relying on symmetric-key primitives only.

The problem in the symmetric setting is to find efficient binding commitments. The signature scheme construction in [DOR⁺16] based on the Bellare-Goldwasser paradigm [BG90] also needs to “commit” to a PRF key. There, signatures consist of a simulation extractable NIZK proof of a PRF key, where the PRF is built from symmetric-key primitives. The standard notion of PRF security, however, does not immediately imply any binding property on the key. Therefore, the construction relies on a *computational fixed-valued-key-binding* PRF [CMR98, Fis99], i.e., a PRF f with the additional property that there ex-

ists a β such that for a PRF key s and given $y = f_s(\beta)$ it is hard to provide a second PRF key s' , $s \neq s'$, satisfying $y = f_{s'}(\beta)$:

Definition 6 (Computational Fixed-Value-Key-Binding PRF). *A PRF family $f: \mathcal{S} \times D \rightarrow \mathbb{R}$ is computationally key-binding if there exists a special value $\beta \in D$ so that it holds for all adversaries \mathcal{A} that:*

$$\Pr \left[s \leftarrow_s \mathcal{S}, s' \leftarrow \mathcal{A}^{f_s(\cdot)}(f_s(\beta), \beta) : f_{s'}(\beta) = f_s(\beta) \right] \approx_\lambda 0.$$

Extending the public key with the PRF evaluation at β and proving its well-formedness is then sufficient to “commit” to the PRF key.¹⁰

For $\mathcal{C}\mathcal{O}\mathcal{C}\mathcal{O}$, we can apply the same idea: we replace the commitment to the PRF key with the evaluation of the PRF at β and adapt the language accordingly. That is, for the construction depicted in Fig. 1¹¹, let the language \mathcal{L}' be such that $\{(\mathbf{x}, \mu, \mathbf{pk}_{\text{OT}}, \rho, \beta), (\mathbf{w}, s_0)\} \in \mathcal{R}_{\mathcal{L}'}$ if and only if:

$$\{(\mathbf{x}, \mathbf{w}) \in \mathcal{R}_{\mathcal{L}} \vee (\mu = f_{s_0}(\mathbf{pk}_{\text{OT}}) \wedge \rho = f_{s_0}(\beta))\}.$$

We denote the $\mathcal{C}\mathcal{O}\mathcal{C}\mathcal{O}$ framework using the language \mathcal{L}' as Optimized $\mathcal{C}\mathcal{O}\mathcal{C}\mathcal{O}$, or $\text{OC}\mathcal{O}\mathcal{C}\mathcal{O}$ for short. For the security proofs (Theorem 2 and Theorem 3 in [Bag19]), we note for each game change based on computational hiding of the commitment, we now use the PRF property and replace them with the evaluation of a random function (Lemma 4). For the step relying on the commitment scheme’s binding property (Lemma 2), one can now argue the uniqueness of the PRF key using the fixed-value-key-binding property of the PRF. Therefore, we obtain the following corollary:

Corollary 1. *If the underlying SNARK satisfies perfect completeness, knowledge soundness, subversion zero-knowledge, the PRF is secure and computationally fixed-value-key-binding, and the one-time signature is sEUF-CMA secure, then $\text{OC}\mathcal{O}\mathcal{C}\mathcal{O}$ yields a SNARK satisfying perfect completeness, subversion zero-knowledge, and strong simulation extractability.*

Instantiating the $\text{OC}\mathcal{O}\mathcal{C}\mathcal{O}$ Framework When instantiating the original $\mathcal{C}\mathcal{O}\mathcal{C}\mathcal{O}$ framework or $\text{OC}\mathcal{O}\mathcal{C}\mathcal{O}$ based on our modifications, SHA-256 as well as any other variant of the SHA2 family or the SHA3 family are non-optimal choices from an efficiency point of view. Indeed, representing the SHA-256 compression function as R1CS requires 22,272 constraints [CGGN17]. The permutation used in SHA3 is even more expensive with 38,400 constraints [AGR⁺16]. Recent lines of work specifically designed block ciphers and hash functions that work especially well in the context of SNARKs. These include MiMC [AGR⁺16], GMiMC [AGP⁺19], Poseidon [GKK⁺19], Friday [AD18], Vision and Rescue [AABS⁺19], which all were specifically designed with SNARK/STARK-based applications in mind. We

¹⁰ Similarly, [DRS18] employs the same idea to commit to a PRF key.

¹¹ Now, one will use the non black-box extractor of the underlying SNARK instead of the black-box extractor Ext from Fig. 1.

however want to note that these designs are all relatively young and were not available at the time $C\emptyset C\emptyset$ was proposed.

Since those designs are all very recent, their cryptanalysis is still ongoing. Friday suffers from a Gröbner-basis attack [ACG⁺19], the key schedule of some variants of MiMC can be attacked using an interpolation attack [LP19] and they also suffer from a collision attack [Bon19], which can also be applied to some variants of GMiMC. Notably, the designs also received some interest as part of a hash collision challenge for STARK-friendly designs,¹² where collisions have been found for low-security instances already. Therefore, we will only include instances in our evaluation that – to the best of our knowledge – have not been broken so far.

Even though these symmetric primitives are designed for SNARKs, they often run into practical problems. For instance, one of the popular choices for instantiating SNARKs is the pairing-friendly BLS12-381¹³ curve. However, its group order q does not match MiMC’s and GMiMC’s requirement coming from the choice of $x \mapsto x^3$ as Sbox that $\gcd(q-1, 3) = 1$. Additionally, MiMC operates in large prime fields, requiring one to emulate the required fields on top of \mathbb{F}_q . The latter issue is solved by GMiMC working over smaller fields, but the order requirement is still an issue. Poseidon, which allows one to choose $x \mapsto x^5$ as Sbox meaning that $\gcd(q-1, 5) = 1$ needs to be satisfied, fixes both problems and can be directly implemented in \mathbb{F}_q arithmetic. Similarly, Rescue faces similar issues as the Sboxes used there are $x \mapsto x^\alpha$ and $x \mapsto x^{1/\alpha}$. Hence, for the specific choice of BLS12-381 this would imply $\alpha = 5$. Vision, on the other hand, is specified over a binary field and can thus also not be directly implemented in \mathbb{F}_q arithmetic.

Additionally, the signature scheme PICNIC [CDG⁺17] demonstrated that LowMC [ARS⁺15], initially designed for MPC and FHE, performs well enough in the context of NIZKs. We consider LowMC in our evaluation as the conservative choice of SNARK-friendly primitives, since it has seen some rounds of cryptanalysis [DEM16, DLMW15] and corresponding updates to the round formula [RST18], and additionally gained some attention in terms of efficient implementations [DKP⁺19].

Evaluation In Table 1 we evaluate a variety of SNARK-friendly primitives together with the SHA2 and SHA3 families of hash functions. Our evaluation focuses on the provable secure version using fixed-value-key-binding PRFs as discussed above with a PRF having 256 bit keys, inputs, and outputs. The number of constraints is computed according to the formulas given in the respective works. We consider MiMC- (N, R) , GMiMC- (N, t, R) with the expanding round function (ERF) construction, POSEIDON- (N, t, R_f, R_p) with $x \mapsto x^5$ as SBox, RESCUE- (N, t, R) with $x \mapsto x^5$ and $x \mapsto x^{1/5}$, VISION- (N, t, R) , and LOWMC- (N, k, m, R) , where N denotes the block size, t the number of branches, R the number of rounds, R_f and R_p the number of full and partial rounds, k the key size and m the number of Sboxes. Where possible, we selected instances

¹² <https://starkware.co/hash-challenge/>

¹³ <https://electriccoin.co/blog/new-snark-curve/>

Table 1. Number of constraints required for $C\emptyset C\emptyset$ and $OC\emptyset C\emptyset$.

Framework	Symmetric primitive	PRF / Commitment	Probably secure	# of constraints PRF / Com.	Σ
$C\emptyset C\emptyset$	SHA256	HMAC PRF + hash com.	\times	111360 + 44544	244992
	SHA256	HMAC PRF	\checkmark	111360	222720
	MIMC-(1025, 646)	TLS 1.2 PRF	\checkmark	230400	460800
	GMIMC-(1024, 4, 332)	Sponge PRF	\checkmark	646	1292
	POSEIDON-(1536, 2, 10, 114)	Sponge PRF	\checkmark	999	1998
$OC\emptyset C\emptyset$	VISTON-(1778, 14, 10)	Sponge PRF	\checkmark	402	804
	RESCUE-(1750, 14, 10)	Sponge PRF	\checkmark	1400	2800
	LOWMC-(256, 256, 1, 537)	feed-forward PRF	\checkmark	840	1680
	LOWMC-(1024, 256, 1, 1027)	Sponge PRF	\checkmark	1074	2148
			\checkmark	2144	4288

compatible with the field induced by BLS12-381, i.e., for Poseidon and Rescue. The table also provides various different PRF constructions. Where possible, we use a Sponge-based approach [BDPV08] akin to SHAKE256. For comparison, we also consider a feed-forward PRF built as $f_s(x) = \mathcal{E}(s, x) \oplus x$ from LowMC where \mathcal{E} denotes the encryption of a block. While we expect this construction to be secure in the ideal cipher model, no security analysis of LowMC or any of the other SNARK-friendly ciphers is available for this type of construction.¹⁴ In the case of SHA256, we consider three variants that can partly also be observed in practice – directly using the HMAC output as PRF and the one from TLS 1.2 [DR08]. Regardless of the concrete choice, even the rather expensive SHAKE256 PRF is a better choice than any of the SHA256-based ones.

We stress that the numbers in Table 1 should be treated as lower bounds. On the one hand, as the security analysis of these primitives evolves, the rather aggressive choice of round numbers may need to be increased. Considering that the STARK-friendly hash challenge was almost immediately solved for the low security instances of MiMC, GMiMC, and Poseidon, we expect those numbers to grow. Indeed, the numbers for the recommended Rescue instance [BSGL20] are higher. On the other hand, for some of the instantiations, it might not be immediately clear if they actually provide the fixed-value-key-binding property. For a very conservative instantiation, one could fallback to the tree-based approach by Fischlin [Fis99], which would be even more expensive, since then every PRF evaluation would internally call the PRF multiple times.

Other Important Remarks Furthermore, besides more efficient instantiations than within the original $C\emptyset C\emptyset$ framework, our approach based on fixed-value-key-binding PRFs also circumvents another issue in concrete instantiations. Hash commitments can only be proven secure in the ROM, which would require to prove preimages of a random oracle. Hence, the construction is impossible to properly instantiate with provable security guarantees. In any case, the choice of a commitment based on symmetric primitive comes with other drawbacks as well. Since such a commitment lacks any useful algebraic structure, it is not obvious how to obtain SE updatable SNARKs.

Regarding the choice of strongly unforgeable one-time signature schemes, Groth’s sOTS (as discussed in this paper) or Boneh-Boyen signatures [BB04] (as proposed in other instantiations of $C\emptyset C\emptyset$ [AB19, Bag19]) would be natural choices, especially when considering the underlying SNARKs already rely on discrete logarithm assumptions (in bilinear groups). Alternatively, any strong EUF-CMA secure signature such as Schnorr would fit as well. We note, however, while this choice would avoid the need for a pairing evaluation for signature ver-

¹⁴ This construction is similar to Davies-Meyer hash function for a fixed input size which is secure in the ideal cipher model [BRS02]. If the PRF is fixed-value key-binding, then the so obtained hash function is collision resistant with respect to a fixed postfix.

ification (in the case of Boneh-Boyer) and the proof overhead would be slightly smaller, Schnorr provides the necessary security guarantees only in the ROM.¹⁵

Putting everything together, instantiating the $C\emptyset C\emptyset$ or $OC\emptyset C\emptyset$ framework with concrete symmetric primitives is non-trivial and comes with some limitations. Subsequently, we will propose an alternative framework LAMASSU, which comes with the same cost as the most aggressive choice of symmetric-key primitive and in contrast to $C\emptyset C\emptyset$ also provides SE updatable SNARKs.

3.2 The LAMASSU Framework

Now, we introduce the LAMASSU framework, which builds upon the recent compiler to obtain SE-NIZK proposed in [DS19]. However, we want to stress that we cannot directly use their compiler in order to construct SE updatable SNARKs and this requires non-trivial changes. The ingredients of their construction are to use a combination of an EUF-CMA secure adaptable key-homomorphic signature scheme Σ (Schnorr or ECDSA are prime candidate for pairing based SNARKs) and a strongly unforgeable one-time signature (sOTS) scheme Σ_{OT} (Groth’s sOTS under the discrete logarithm assumption is a prime candidate) to add the required non-malleability guarantees to the underlying knowledge sound NIZK proof system Π together with the folklore OR-trick to add simulation soundness. The distinguishing feature of this transformation is that in the proof computation one computes a signature to certify a public key of OTS using freshly sampled signing key sk of Σ in plain and thus does not need to encrypt a signature and prove that it verifies with a verification key in the CRS (e.g., as done in [Gro06]). Consequently, in the OR part of the proof one just needs to prove that one knows the shift csk (which is the trapdoor of the CRS) to adapt signatures from pk to ones valid under verification key cpk in the CRS. As it turns out, this feature lays the foundation for being able to support updatability. Now, given any language \mathcal{L} with NP relation $\mathcal{R}_{\mathcal{L}}$, the language obtained via the compiler is \mathcal{L}' s.t. $\{(x, cpk, pk), (w, csk - sk)\} \in \mathcal{R}_{\mathcal{L}'}$ iff:

$$\{(x, w) \in \mathcal{R}_{\mathcal{L}} \vee cpk = pk \cdot \mu(csk - sk)\}.$$

More precisely, in every proof computation one uses Σ to “certify” the public key of a newly generated key pair of Σ_{OT} . The secret key of Σ_{OT} is then used to sign the parts of the proof which must be non-malleable. Adaptability of Σ makes it possible to also use newly generated keys of Σ upon each proof computation. In particular, the relation associated with \mathcal{L}' is designed so that the additional clause introduced via the OR-trick is the “shift amount” required to shift such signatures to signatures under a key cpk of Σ in the CRS. A proof for $x \in \mathcal{L}$ is easy to compute when given w such that $(x, w) \in \mathcal{L}'$. One does not need a

¹⁵ In private correspondance, A. Kosba confirmed that their implementation used a non-malleable variant of ECDSA for benchmarking. To the best of our knowledge, this variant is suspected to be strongly unforgeable without proof so far. Thus we consider Schnorr as a candidate. Performance and overhead are expected to be the same.

satisfying assignment for the second clause in the OR statement, and can thus compute all signatures under newly generated keys. To simulate proofs, however, one can set up CRS in a way that we know csk corresponding to cpk , compute the “shift amount” and use it as a satisfying witness for the other clause in the OR statement. We recall the construction in Fig. 2 and for completeness recall the Theorem given in [DS19] below.¹⁶ We note that for non black-box extraction as it is the case with SNARKs, the trapdoor $\text{tc}_{\text{ext}} = \perp$ and one simply uses the non black-box extractor of the underlying SNARK.

Theorem 1 ([DS19]). *Let Π be a complete, witness indistinguishable non-interactive argument of knowledge system for the language \mathcal{L} , let Σ be an EUF-CMA secure signature scheme that adapts signatures, and let Σ_{OT} be a strongly unforgeable one-time signature scheme, then the argument system Π' is a complete and strong simulation extractable argument system for language \mathcal{L}' .*

Note that the theorem clearly applies to any proof system that is zero-knowledge, as this implies the weaker notion of witness-indistinguishability.

Applying [DS19] to NIZKs without knowledge soundness We now argue that, although we do not require it in context of SNARKs, analogous to the folklore compiler used in [KZM⁺15], we can also start from any NIZK that is only sound instead of knowledge sound. Then, using the compiler in [DS19] we still can obtain SE-NIZK when starting from any conventional NIZK. More precisely, the by now folklore compiler [DP92] to obtain knowledge soundness for any sound NIZK is to put a public key pk_e of any perfectly correct IND-CPA secure public key encryption scheme into the CRS, where the extraction trapdoor tc_{Ext} is the corresponding secret key, and extend the language such that it contains an encryption of the witness of the original language. We will capture this in the following corollary, where starting from a NIZK for \mathcal{L} with NP relation $\mathcal{R}_{\mathcal{L}}$, we obtain a knowledge sound NIZK by extending the language to \mathcal{L}' such that $\{(\mathbf{x}, c), (\mathbf{w}, \omega)\} \in \mathcal{R}_{\mathcal{L}'}$ iff:

$$\{(\mathbf{x}, \mathbf{w}) \in \mathcal{R}_{\mathcal{L}} \wedge c = \text{Enc}(\text{pk}_e, \mathbf{w}; \omega)\}.$$

Corollary 2. *Let NIZK for language \mathcal{L} be complete, sound, and zero-knowledge, the public key encryption scheme be perfectly correct and IND-CPA secure, then the NIZK for language \mathcal{L}' is complete, knowledge-sound and zero-knowledge.*

The proof exactly follows the argumentation in [KZM⁺15] and is thus omitted. We stress that if we base the compiler of [DS19] on a NIZK that is based on standard or falsifiable assumptions that is only sound, then we require this additional encryption of the witness \mathbf{w} . However, when we are relying on knowledge assumption, as it is the case within SNARKs used in this paper, then we do not need the language extension in Corollary 2 and simply use the non black-box extractor of the underlying SNARK.

¹⁶ We note that what is called simulation sound extractable in [DS19] is called strong simulation extractable in this paper in order to be aligned with the notation used in the $\mathcal{C}\emptyset\mathcal{C}\emptyset$ framework.

$\text{KGen}_{\text{crs}}(\mathcal{R}, \text{aux}_{\mathcal{R}})$ <hr/> <ul style="list-style-type: none"> - $(\text{crs}_{\Pi}, \text{tc}_{\Pi}, \text{tc}_{\text{ext}}) \leftarrow \Pi.\text{KGen}(\mathcal{R}, \text{aux}_{\mathcal{R}});$ - $(\text{csk}, \text{cpk}) \leftarrow \Sigma.\text{KGen}(1^{\lambda});$ - $\text{crs} := (\text{crs}_{\Pi}, \text{cpk}), \text{tc} := (\text{tc}_{\Pi}, \text{csk});$ return $\text{crs}.$
$\text{P}(\text{crs}, \mathbf{x}, \mathbf{w})$ <hr/> <ul style="list-style-type: none"> - $(\text{sk}, \text{pk}) \leftarrow \Sigma.\text{KGen}(1^{\lambda});$ - $(\text{sk}_{\text{OT}}, \text{pk}_{\text{OT}}) \leftarrow \Sigma_{\text{OT}}.\text{KGen}(1^{\lambda});$ - $\pi_{\Pi} \leftarrow \Pi.\text{P}(\text{crs}, \mathbf{x}, (\mathbf{w}, \perp)); \sigma \leftarrow \Sigma.\text{Sign}(\text{sk}, \text{pk}_{\text{OT}});$ - $\sigma_{\text{OT}} \leftarrow \Sigma_{\text{OT}}.\text{Sign}(\text{sk}_{\text{OT}}, \pi_{\Pi} \ \mathbf{x}\ \ \text{pk}\ \ \sigma\);$ return $\pi := (\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}}).$
$\text{V}(\text{crs}, \mathbf{x}, \pi)$ <hr/> <ul style="list-style-type: none"> - Parse π as $(\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}});$ - if $\Pi.\text{V}(\text{crs}_{\Pi}, \mathbf{x}, \pi_{\Pi}) = 0$ $\vee \Sigma.\text{Verify}(\text{pk}, \text{pk}_{\text{OT}}, \sigma) = 0$ $\vee \Sigma_{\text{OT}}.\text{Verify}(\text{pk}_{\text{OT}}, \pi_{\Pi} \ \mathbf{x}\ \ \text{pk}\ \ \sigma, \sigma_{\text{OT}}) = 0$ then return 0; else return 1.
$\text{Sim}(\text{crs}, \mathbf{x}, \text{tc})$ <hr/> <ul style="list-style-type: none"> - $(\text{sk}, \text{pk}) \leftarrow \Sigma.\text{KGen}(1^{\lambda}); (\text{sk}_{\text{OT}}, \text{pk}_{\text{OT}}) \leftarrow \Sigma_{\text{OT}}.\text{KGen}(1^{\lambda});$ - $\pi_{\Pi} \leftarrow \Pi.\text{P}(\text{crs}, \mathbf{x}, (\perp, \text{csk} - \text{sk}); \sigma \leftarrow \Sigma.\text{Sign}(\text{sk}, \text{pk}_{\text{OT}});$ - $\sigma_{\text{OT}} \leftarrow \Sigma_{\text{OT}}.\text{Sign}(\text{sk}_{\text{OT}}, \pi_{\Pi} \ \mathbf{x}\ \ \text{pk}\ \ \sigma\);$ - return $\pi := (\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}}).$
$\text{Ext}(\text{crs}, \mathbf{x}, \pi, \text{tc}_{\text{ext}})$ <hr/> <ul style="list-style-type: none"> - $(\mathbf{w}, \perp) \leftarrow \Pi.\text{Ext}(\text{crs}, \mathbf{x}, \pi, \text{tc}_{\text{ext}});$ return $\mathbf{w}.$

Fig. 2. The generic SE-NIZK compiler from [DS19].

4 Instantiations of LAMASSU

Now we are going to investigate instantiations of the LAMASSU framework in the malicious setting where the CRS could be subverted. We show how to instantiate the LAMASSU framework for subversion zk-SNARKs (Sub-zk-SNARK) (i.e., [ABLZ17, Fuc18]) and for updatable zk-SNARKs (i.e., [GKM⁺18]), and obtain SE Sub-zk-SNARK and SE updatable zk-SNARK constructions. While the former case can directly be obtained from LAMASSU as introduced, for the latter case we need to introduce the novel notion of updatable signatures and use the LAMASSU framework with updatable signatures instead of key-homomorphic ones.

Our subversion definition is adapted from Abdolmaleki et al. [ABLZ17, ALSZ20], and that of update security is adapted from Groth et al. [GKM⁺18].

$\text{KGen}_{\text{crs}}(\mathcal{R}, \text{aux}_{\mathcal{R}})$ <hr/> <ul style="list-style-type: none"> - $(\text{crs}_{\Pi}, \text{tc}_{\Pi}) \leftarrow \Pi.\text{KGen}(\mathcal{R}, \text{aux}_{\mathcal{R}})$; - $(\text{csk}, \text{cpk}) \leftarrow \Sigma.\text{KGen}(1^{\lambda})$; - $\text{crs} := (\text{crs}_{\Pi}, \text{cpk}), \text{tc} := (\text{tc}_{\Pi}, \text{csk})$; return crs.
$\text{Vcrs}(\text{crs}, \zeta)$ <hr/> <ul style="list-style-type: none"> - Parse crs as $(\text{crs}_{\Pi}, \text{cpk})$; - if $\text{Vcrs}(\text{crs}_{\Pi}, \zeta_{\Pi}) = 0$ then return 0; else return 1.
$\text{P}(\text{crs}, \mathbf{x}, \bar{\mathbf{w}})$ <hr/> <ul style="list-style-type: none"> - $(\text{sk}, \text{pk}) \leftarrow \Sigma.\text{KGen}(1^{\lambda})$; - $(\text{sk}_{\text{OT}}, \text{pk}_{\text{OT}}) \leftarrow \Sigma_{\text{OT}}.\text{KGen}(1^{\lambda})$; - $\pi_{\Pi} \leftarrow \Pi.\text{P}(\text{crs}, \mathbf{x}, (\bar{\mathbf{w}}, \perp), \perp)$; $\sigma \leftarrow \Sigma.\text{Sign}(\text{sk}, \text{pk}_{\text{OT}})$; - $\sigma_{\text{OT}} \leftarrow \Sigma_{\text{OT}}.\text{Sign}(\text{sk}_{\text{OT}}, \pi_{\Pi} \parallel \mathbf{x} \parallel \text{pk} \parallel \sigma)$; return $\pi := (\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}})$.
$\text{V}(\text{crs}, \mathbf{x}, \pi)$ <hr/> <ul style="list-style-type: none"> - Parse π as $(\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}})$; - if $\Pi.\text{V}(\text{crs}, \mathbf{x}, \pi_{\Pi}) = 0 \vee \Sigma.\text{Verify}(\text{pk}, \text{pk}_{\text{OT}}, \sigma) = 0$ $\vee \Sigma_{\text{OT}}.\text{Verify}(\text{pk}_{\text{OT}}, \pi_{\Pi} \parallel \mathbf{x} \parallel \text{pk} \parallel \sigma, \sigma_{\text{OT}}) = 0$ then return 0; else return 1.
$\text{Sim}(\text{crs}, \mathbf{x}, \text{tc})$ <hr/> <ul style="list-style-type: none"> - $(\text{sk}, \text{pk}) \leftarrow \Sigma.\text{KGen}(1^{\lambda})$; $(\text{sk}_{\text{OT}}, \text{pk}_{\text{OT}}) \leftarrow \Sigma_{\text{OT}}.\text{KGen}(1^{\lambda})$; - $\pi_{\text{Sim}} \leftarrow \Pi.\text{Sim}(\text{crs}, \mathbf{x}, (\bar{\mathbf{w}}, \text{tc}_{\Pi}), \perp)$; $\sigma \leftarrow \Sigma.\text{Sign}(\text{sk}, \text{pk}_{\text{OT}})$; - $\sigma_{\text{OT}} \leftarrow \Sigma_{\text{OT}}.\text{Sign}(\text{sk}_{\text{OT}}, \pi_{\text{Sim}} \parallel \mathbf{x} \parallel \text{pk} \parallel \sigma)$; return $\pi := (\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}})$.

Fig. 3. The SE Sub-zk-SNARKs from LAMASSU.

4.1 Subversion SNARK Instantiation

Consider a Sub-zk-SNARK (e.g., [ABLZ17, Fuc18]) for $\mathcal{R}_{\mathcal{L}}$ which consists of PPT algorithms $(\text{KGen}_{\text{crs}}, \text{Vcrs}, \text{P}, \text{V}, \text{Sim})$ and provides knowledge soundness. Let $\Sigma_{\text{OT}} = (\text{KGen}_{\text{OT}}, \text{Sign}_{\text{OT}}, \text{Verify}_{\text{OT}})$ be a strongly unforgeable one-time signature scheme and Σ be an adaptable EUF-CMA secure signature scheme (like Schnorr or ECDSA). Using LAMASSU in Section 3.2, given the language \mathcal{L} with NP relation $\mathcal{R}_{\mathcal{L}}$, one can extend it to the new \mathcal{L}' language proposed in Section 3.2, such that $\{(\mathbf{x}, \text{cpk}, \text{pk}), (\bar{\mathbf{w}}, \text{csk} - \text{sk})\} \in \mathcal{R}_{\mathcal{L}'}$ iff:

$$\{(\mathbf{x}, \bar{\mathbf{w}}) \in \mathcal{R}_{\mathcal{L}} \vee \text{cpk} = \text{pk} \cdot \mu(\text{csk} - \text{sk})\}.$$

We present the construction of SE Sub-zk-SNARKs in Fig. 3. And for LAMASSU we can prove the following:

Theorem 2. *Let the underlying Sub-zk-SNARK scheme satisfy perfect completeness, knowledge soundness, subversion zero-knowledge. Let Σ be an EUF-CMA secure adaptable key-homomorphic signature scheme and Σ_{OT} a strongly*

unforgeable one-time signature scheme. Then the Sub-zk-SNARK from Fig. 3 is (i) perfectly complete, (ii) subversion zero-knowledge, and (iii) strongly simulation extractable.

Completeness is straightforward. For strong simulation extractability, note that in Sub-zk-SNARKs we assume that the CRS generator is trusted by the verifier. Consequently, the proof of strong simulation extractability directly follows from *Theorem 1*. The idea for proving subversion zero-knowledge is to use the extractor of the underlying SNARK to extract the simulation trapdoor which can then be used to simulate proofs. If the CRS verification succeeds, this extractor exists following from the knowledge assumption of the underlying SNARK. We present the full proof in Appendix B.1.

4.2 Updatable Signature Schemes

Before discussing how to achieve SE updatable zk-SNARKs from updatable SNARKS using the LAMASSU framework, we need to introduce the new notation of *updatable signature schemes*, which are an extension of key-homomorphic signatures. We stress that in contrast to subversion-resilient signatures [AMV15], where the signing algorithm may be subverted, here, we allow updates on the key and want to have unforgeability guarantees as long as either the initial key generation or at least one of the updates was performed honestly. However, signing is performed honestly. We note that like in Groth et al. [GKM⁺18] for updatable CRS (using Lemma 6), we model only a single update as a single adversarial update implies updatable signatures with arbitrary many updates.

Definition 7 (Updatable signature schemes). *An updatable signature scheme $\Sigma = (\text{KGen}, \text{Upk}, \text{Vpk}, \text{Sign}, \text{Verify})$ is a key-homomorphic signature scheme¹⁷ and consists of the following PPT algorithms:*

$\text{KGen}(1^\lambda)$: *Given a security parameter λ it outputs a signing key sk , a verification key pk and a proof ζ with message space \mathcal{M} .*

$\text{Upk}(\text{pk})$: *Given a verification key pk it outputs an updated verification key pk_{up} with associated secret updating key up_{sk} , and a proof ζ .*

$\text{Vpk}(\text{pk}, \text{pk}_{\text{up}}, \zeta)$: *Given a verification key pk , a potentially updated verification key pk_{up} , and the proof ζ it checks if pk_{up} has been updated correctly.*

$\text{Sign}(\text{sk}, m)$: *Given potentially updated secret key sk and a message $m \in \mathcal{M}$ it outputs a signature σ .*

$\text{Verify}(\text{pk}, m, \sigma)$: *Given potentially updated public key pk , a message $m \in \mathcal{M}$ and a signature σ it outputs a bit $b \in \{0, 1\}$.*

¹⁷ We do not require to make the Adapt algorithm explicit.

Definition 8 (Updatable correctness). *A signature scheme Σ is updatable correct, if for all $m \in \mathcal{M}$*

$$\Pr \left[\begin{array}{l} (\text{sk}, \text{pk}, \zeta) \leftarrow \text{KGen}(1^\lambda), (\text{up}_{\text{sk}}, \text{pk}_{\text{up}}, \zeta_{\text{up}}) \leftarrow \text{Upk}(\text{pk}), \\ \text{Vpk}(\text{pk}, \text{pk}_{\text{up}}, \zeta_{\text{up}}) = 1: \\ \text{Verify}(\text{pk}, m, \text{Sign}(\text{sk}, m)) = 1 \wedge \\ \text{Verify}(\text{pk}_{\text{up}}, m, \text{Sign}(\text{sk} + \text{up}_{\text{sk}}, m)) = 1 \end{array} \right] = 1,$$

Definition 9 (Updatable strong key hiding). *We have that for $(\text{sk}, \text{pk}) \leftarrow \text{KGen}(1^\lambda)$ and $(\text{up}_{\text{sk}}, \text{pk}_{\text{up}}, \zeta_{\text{up}}) \leftarrow \text{Upk}(\text{pk})$ it holds that $(\text{sk}, \text{pk}) \approx_\lambda (\text{sk} + \text{up}_{\text{sk}}, \text{pk}_{\text{up}}) \in \text{KGen}(1^\lambda)$ if one of the following setting holds,*

- *the original pk was honestly generated and the key-update verifies: $(\text{sk}, \text{pk}) \leftarrow \text{KGen}(1^\lambda)$ and $\text{Vpk}(\text{pk}, \text{pk}_{\text{up}}, \zeta_{\text{up}}) = 1$.*
- *the original pk verifies and the key-update was honest: $\text{Vpk}(\text{pk}, \text{pk}, \zeta) = 1$, and $(\text{up}_{\text{sk}}, \text{pk}_{\text{up}}, \zeta_{\text{up}}) \leftarrow \text{Upk}(\text{pk})$.*

Now, we present the updatable EUF-CMA security notion.

Definition 10 (Updatable EUF-CMA). *A signature scheme Σ is updatable EUF-CMA secure, if for all PPT subverter Z , there exists a PPT extractor Ext_Z , s.t. for all PPT adversaries \mathcal{A}*

$$\Pr \left[\begin{array}{l} (\text{sk}, \text{pk}, \zeta) \leftarrow \text{KGen}(1^\lambda), \\ \omega_Z \leftarrow_{\mathfrak{s}} \text{RND}(Z), (\text{pk}_{\text{up}}, \zeta_{\text{up}}, \text{aux}_Z) \leftarrow Z(\text{pk}; \omega_Z), \\ \text{up}_{\text{sk}} \leftarrow \text{Ext}_Z(\text{pk}, \omega_Z), \\ (m^*, \sigma^*) \leftarrow \mathcal{A}^{\text{Sign}(\text{sk}, \cdot), \text{Sign}(\text{sk} + \text{up}_{\text{sk}}, \cdot)}(\text{pk}_{\text{up}}, \text{aux}_Z): \\ \text{Vpk}(\text{pk}, \text{pk}_{\text{up}}, \zeta_{\text{up}}) = 1 \wedge \text{pk}_{\text{up}} = \text{pk} \cdot \mu(\text{up}_{\text{sk}}) \wedge \\ \text{Verify}(\text{pk}_{\text{up}}, m^*, \sigma^*) = 1 \wedge m^* \notin \mathcal{Q}^{\text{Sign}} \end{array} \right] \approx_\lambda 0,$$

where the environment keeps track of the queries to the signing oracles via $\mathcal{Q}^{\text{Sign}}$. Note that Z can also generate the initial pk and an honest updater Upk updates it and outputs pk_{up} , up_{sk} , and the proof ζ_{up} . Then we require that $\text{Vpk}(\text{pk}, \text{pk}, \zeta) = 1$ and we extract sk from Ext_Z .

We now prove the following theorem yielding a generic way to construct updatable signature schemes and note that we call an updatable signature adaptable if the underlying key-homomorphic signature scheme is adaptable.

Theorem 3. *Every correct and EUF-CMA secure key-homomorphic signature scheme Σ that is adaptable according to Definition 5 and provides an efficient extractor Ext_Z satisfies updatable correctness, updatable strong key hiding and updatable EUF-CMA security.*

Proof. We first discuss correctness. Therefore let the Upk and Vpk algorithms be as follows:

$\text{Upk}(\text{pk})$: Choose $\Delta \leftarrow_{\mathfrak{s}} \mathbb{H}$, set $\text{up}_{\text{sk}} := \Delta$, $\text{pk}_{\text{up}} := \text{pk} \cdot \mu(\Delta)$ and $\zeta_{\text{up}} := \mu(\Delta)$ and return $(\text{up}_{\text{sk}}, \text{pk}_{\text{up}}, \zeta_{\text{up}})$.
 $\text{Vpk}(\text{pk}, \text{pk}_{\text{up}}, \zeta_{\text{up}})$: Return 1 if either $\text{pk} = \text{pk}_{\text{up}}$ or $\text{pk}_{\text{up}} := \text{pk} \cdot \zeta_{\text{up}}$ and 0 otherwise.

It is easy to see that $\text{sk}_{\text{up}} := \text{sk} + \Delta$ and thus updatable correctness follows from the correctness of Σ .

Updatable strong key hiding directly follows from the key-homomorphic property of Σ and the algorithms Upk and Vpk introduced above.

Now, we prove updatable EUF-CMA security by a reduction to the EUF-CMA security of Σ . Let pk be the verification key from the challenger of Σ and $(\text{pk}_{\text{up}}, \zeta_{\text{up}}, \text{aux}_z)$ the output of \mathcal{A} on pk . Now, we can use Ext_z to obtain up_{sk} and we know that $\text{Vpk}(\text{pk}, \text{pk}_{\text{up}}, \zeta_{\text{up}}) = 1$ and $\text{pk}_{\text{up}} := \text{pk} \cdot \mu(\text{up}_{\text{sk}})$. Consequently, on every signature query for some message m from \mathcal{A} , we query the signing oracle of Σ and when given σ in return we return $(\cdot, \sigma') \leftarrow \text{Adapt}(\text{pk}, m, \sigma, \text{up}_{\text{sk}})$ to \mathcal{A} . When \mathcal{A} outputs a valid forgery (m^*, σ^*) under pk_{up} , we output σ'^* to the challenger of Σ where $(\cdot, \sigma'^*) \leftarrow \text{Adapt}(\text{pk}_{\text{up}}, m^*, \sigma^*, -\text{up}_{\text{sk}})$ and win with the same probability as \mathcal{A} wins. We note that the case where the initial pk is subverted and the update is honest can be shown analogously and is thus omitted.

Example of Updatable Signatures Now, we show that Schnorr signatures (cf. Appendix A.6) instantiated in a bilinear group $\text{BG} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g, \hat{g})$, where in contrast to conventional Schnorr signatures the public key consists of pairs (g^x, \hat{g}^x) , represent an updatable signature scheme. Therefore, we first discuss the required algorithms and will then show an efficient extractor Ext_z . We start with the algorithms:

Upk : Set $\text{up}_{\text{sk}} := x' \leftarrow_{\mathfrak{s}} \mathbb{Z}_p$, $\text{pk}_{\text{up}} := (w \cdot g^{x'}, \hat{w} \cdot \hat{g}^{x'})$, $\zeta_{\text{up}} := (g^{x'}, \hat{g}^{x'})$ and return $(\text{up}_{\text{sk}}, \text{pk}_{\text{up}}, \zeta_{\text{up}})$.
 Vpk : Parse $\text{pk} = (w, \hat{w})$, $\text{pk}_{\text{up}} = (w', \hat{w}')$ and $\zeta_{\text{up}} = (z, \hat{z}')$. If $w = w'$ and $\hat{w} = \hat{w}'$ check if $e(w, \hat{g}) = e(g, \hat{w}')$ and $e(g, \hat{w}) = e(w', \hat{g})$. Otherwise check if $e(w \cdot z, \hat{g}) = e(g, \hat{w}')$ and $e(g, \hat{w} \cdot \hat{z}) = e(w', \hat{g})$ holds. If the check holds return 1 and 0 otherwise.

Finally, let us present an efficient extractor Ext_z which exists assuming the BDH knowledge assumption (cf. Appendix A.8) holds. Therefore, note that if Vpk returns 1 on any input $(\text{pk}, \text{pk}_{\text{up}}, \zeta_{\text{up}})$ by BDH we have an extractor that from this algorithm extracts $\text{up}_{\text{sk}} := x'$ from z and \hat{z} s.t. $\text{sk}_{\text{up}} = \text{sk} + \text{up}_{\text{sk}}$ and $\text{pk}_{\text{up}} = \text{pk} \cdot \mu(\text{up}_{\text{sk}})$ (componentwise).

4.3 Updatable SNARK Instantiation

Now, we demonstrate the main advantage of LAMASSU in that one can use it to generically construct SE updatable zk-SNARKs. In the following, we present our generic construction using the definitional framework in [GKM⁺18]. Roughly

speaking, Groth et. al relaxed the CRS model by allowing the adversary to either fully generate the CRS itself, or at least contribute to its computation as one of the parties performing updates. In other words, we can think of this as having the adversary interact with the KGen_{crs} algorithm. An updatable SNARK has the following additional PPT algorithms on top of $(\text{KGen}_{\text{crs}}, \text{P}, \text{V}, \text{Sim})$. After running $(\text{crs}, \text{tc}, \zeta) \leftarrow \text{KGen}_{\text{crs}}$, where ζ is a proof of correctness of crs .

$\text{Ucrs}(1^\lambda, \text{crs}, \{\zeta_i\}_{i=1}^{i=n})$. Takes as input the security parameter λ , a CRS crs , and a list of update proofs for the CRS. It outputs an updated CRS crs_{up} and a proof ζ_{up} of the correctness of the update.

$\text{Vcrs}(1^\lambda, \text{crs}, \{\zeta_i\}_{i=1}^{i=n})$. Given the security parameter λ , a CRS crs , and a list of proofs ζ_i . It outputs a bit indicating accept ($b = 1$), or reject ($b = 0$).

The standard trusted setup can be considered as an updatable setup with $\zeta = \epsilon$ as the updated proof, in a way that the verification algorithm Vcrs accepts anything ζ . For a subversion resistant setup (Sub-zk-SNARKs), the proof ζ could be added as extra elements into the CRS solely to make the CRS verifiable.

We present the full construction of SE updatable SNARKs in Fig. 4. Notice that in the Fig. 4, the subverter Z could be either the algorithms $(\Pi.\text{KGen}, \Sigma.\text{KGen})$ or the updater Ucrs .

Theorem 4. *Let the underlying updatable SNARK scheme satisfy perfect completeness, updatable zero-knowledge, and updatable knowledge soundness. Let Σ be an EUF-CMA secure adaptable and updatable signature scheme and Σ_{OT} is a strongly unforgeable one-time signature scheme. Then, the SE updatable SNARKs argument system from Fig. 4, is (i) perfectly complete, (ii) updatable zero-knowledge, and (iii) updatable strong simulation extractable.*

We refer to Appendix B.2 for the full proof.

Instantiation By taking updatable Schnorr signatures (cf. Section 4.2), using the LAMASSU framework we can now obtain an SE updatable SNARK by lifting the updatable SNARK in [GKM⁺18]. This, for instance, results in an overhead of $1\mathbb{G}_1 + 1\mathbb{G}_2$ elements in the CRS and $2\mathbb{G}_1 + 2\mathbb{G}_2 + 2\mathbb{Z}_q$ elements in the proofs (cf. Table 2).

5 Evaluation

For the evaluation of $\text{OC}\emptyset\text{C}\emptyset$ and LAMASSU, we focus on SNARKs built from the pairing-friendly elliptic curve BLS12-381, so we can leverage the Jubjub curve [HBHW19] used by Zcash for fast elliptic-curve arithmetic in the circuit. The Jubjub curve is a twisted Edwards curve defined over \mathbb{F}_q with q being the prime order of BLS12-381. Twisted Edwards curves enjoy complete addition laws and they naturally fit the requirements of Schnorr signatures.

The Sapling protocol uses the Jubjub curve to prove relations of the form $\text{rk} = \text{ak} \cdot g^\alpha$ and checking that α is in the correct range for the witness α . The first

Table 2. Comparison of SE-SNARKs. The given sizes for the CRS and proofs as well as the number of operations are overheads compared to the underlying SNARKs. For ad-hoc constructions the overhead is relative to Groth’s SNARK. n denotes the number of multiplication gates.

	Features		Overhead		V	
	generic	subversion updatable	crs	bits		π bits
$\mathcal{C}\mathcal{O}\mathcal{C}\emptyset$ [KZM ⁺ 15] [‡]	✓	✓*	1 λ	256	1 \mathbb{G} , 2 \mathbb{Z}_q , 1 λ 1016	2 $\mathbb{E}_{\mathbb{G}}$
$\mathcal{O}\mathcal{C}\mathcal{O}\emptyset$ [S]	✓	✓*	2 λ	512	1 \mathbb{G} , 2 \mathbb{Z}_q , 1 λ 1016	2 $\mathbb{E}_{\mathbb{G}}$
$\mathcal{O}\mathcal{C}\mathcal{O}\emptyset$ [G]	✓	✓*	2 λ	512	3 \mathbb{G} , 2 \mathbb{Z}_q , 1 λ 1528	3 $\mathbb{E}_{\mathbb{G}}$
LAMASSU[S,S]	✓	✓	1 \mathbb{G}	256	2 \mathbb{G} , 4 \mathbb{Z}_q 1520	4 $\mathbb{E}_{\mathbb{G}}$
LAMASSU[S,S]	✓	✓	1 \mathbb{G}_1 , 1 \mathbb{G}_2	1145	1 \mathbb{G}_1 , 1 \mathbb{G}_2 , 1 \mathbb{G} , 4 \mathbb{Z}_q 2415	2 $\mathbb{E}_{\mathbb{G}_1}$, 2 $\mathbb{E}_{\mathbb{G}}$
LAMASSU[S,G]	✓	✓	1 \mathbb{G}	256	4 \mathbb{G} , 4 \mathbb{Z}_q 2032	5 $\mathbb{E}_{\mathbb{G}}$
LAMASSU[S,G]	✓	✓	1 \mathbb{G}_1 , 1 \mathbb{G}_2	1145	1 \mathbb{G}_1 , 1 \mathbb{G}_2 , 3 \mathbb{G} , 4 \mathbb{Z}_q 2927	2 $\mathbb{E}_{\mathbb{G}_1}$, 3 $\mathbb{E}_{\mathbb{G}}$
LAMASSU[S,BB]	✓	✓	1 \mathbb{G}	256	1 \mathbb{G}_1 , 1 \mathbb{G}_2 , 1 \mathbb{G} , 2 \mathbb{Z}_q 1905	2 $\mathbb{E}_{\mathbb{G}}$, 1P
LAMASSU[S,BB]	✓	✓	1 \mathbb{G}_1 , 1 \mathbb{G}_2	1145	2 \mathbb{G}_1 , 2 \mathbb{G}_2 , 2 \mathbb{Z}_q 2800	2 $\mathbb{E}_{\mathbb{G}_1}$, 1P
Groth-Maller [GM17]	✗	✗	(2n + 5) \mathbb{G}_1 , n \mathbb{G}_2	1910 + 1527n	–	0
Bowe-Gabizon [BG18]	✗	✗	– [†]	0	1 \mathbb{G}_1 , 1 \mathbb{G}_2	1146
Lipmaa (S ^{se} _{gap}) [Lip19]	✗	✓	n \mathbb{G}_1	382n	1 \mathbb{G}_1	382
Atapoor-Baghery [AB19] [‡]	✗	✗	1 λ	256	1 \mathbb{G}_1 , 1 \mathbb{G}_2 , 1 λ	1401
Baghery [Bag19] [‡]	✗	✓	1 λ	256	1 \mathbb{G}_1 , 1 \mathbb{G}_2 , 1 λ	1401

[‡] Proves statements with respect to the evaluation of a random oracle (cf. Section 3.1).

[†] Achieves no crs overhead by additionally requiring random oracles.

* With the non-black box extractor, $\mathcal{C}\mathcal{O}\mathcal{C}\emptyset$ retains the subversion resistance of the underlying SNARK [Bag19].

$\text{KGen}_{\text{crs}}(\mathcal{R}, \text{aux}_{\mathcal{R}})$ <hr/> <ul style="list-style-type: none"> - $(\text{crs}_{\Pi}, \text{tc}_{\Pi}, \zeta_{\Pi}) \leftarrow \Pi.\text{KGen}(\mathcal{R}, \text{aux}_{\mathcal{R}}); (\text{csk}, \text{cpk}, \zeta_{\text{cpk}}) \leftarrow \Sigma.\text{KGen}(1^{\lambda});$ - $\text{crs} := (\text{crs}_{\Pi}, \text{cpk}), \text{tc} := (\text{tc}_{\Pi}, \text{csk}), \zeta := (\zeta_{\Pi}, \zeta_{\text{cpk}});$ - return $(\text{crs}, \text{tc}, \zeta).$
$\text{Ucrs}(\text{crs}, \{\zeta_i\}_{i=1}^{i=n})$ <hr/> <ul style="list-style-type: none"> - $(\text{crs}_{\Pi, \text{up}}, \zeta_{\Pi, \text{up}}) \leftarrow \Pi.\text{Ucrs}(1^{\lambda}, \text{crs}_{\Pi}, \{\zeta_{\Pi, i}\}_{i=1}^{i=n});$ - $(\text{cpk}_{\text{up}}, \zeta_{\text{cpk}, \text{up}}) \leftarrow \Sigma.\text{Ucrs}(\text{cpk}, \{\zeta_{\text{cpk}, i}\}_{i=1}^{i=n});$ - return $(\text{crs}_{\text{up}} = (\text{crs}_{\Pi, \text{up}}, \text{cpk}_{\text{up}}), \zeta_{\text{up}} = (\zeta_{\Pi, \text{up}}, \zeta_{\text{cpk}, \text{up}}))$
$\text{Vcrs}(\text{crs}, \{\zeta_i\}_{i=1}^{i=n})$ <hr/> <ul style="list-style-type: none"> - if $\forall \text{crs}_{\Pi}(1^{\lambda}, \text{crs}_{\Pi}, \{\zeta_{\Pi, i}\}_{i=1}^{i=n}) = 1 \wedge$ - $\Sigma.\forall \text{pk}(\text{pk}, \text{cpk}, \{\zeta_{\text{cpk}, i}\}_{i=1}^{i=n}) = 1$ then return 1; else return 0.
$\text{P}(\text{crs}_{\text{up}}, \mathbf{x}, \mathbf{w})$ <hr/> <ul style="list-style-type: none"> - $(\text{sk}, \text{pk}) \leftarrow \Sigma.\text{KGen}(1^{\lambda});$ - $(\text{sk}_{\text{OT}}, \text{pk}_{\text{OT}}) \leftarrow \Sigma_{\text{OT}}.\text{KGen}(1^{\lambda});$ - $\pi_{\Pi} \leftarrow \Pi.\text{P}(\text{crs}_{\text{up}}, \mathbf{x}, (\mathbf{w}, \perp), \perp); \sigma \leftarrow \Sigma.\text{Sign}(\text{sk}, \text{pk}_{\text{OT}});$ - $\sigma_{\text{OT}} \leftarrow \Sigma_{\text{OT}}.\text{Sign}(\text{sk}_{\text{OT}}, \pi_{\Pi} \ \mathbf{x}\ \ \text{pk}\ \ \sigma\);$ return $\pi := (\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}}).$
$\text{V}(\text{crs}_{\text{up}}, \mathbf{x}, \pi)$ <hr/> <ul style="list-style-type: none"> - Parse π as $(\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}});$ - if $\Pi.\text{V}(\text{crs}_{\text{up}}, \mathbf{x}, \pi_{\Pi}) = 0 \vee \Sigma.\text{Verify}(\text{pk}, \text{pk}_{\text{OT}}, \sigma) = 0$ $\vee \Sigma_{\text{OT}}.\text{Verify}(\text{pk}_{\text{OT}}, \pi_{\Pi} \ \mathbf{x}\ \ \text{pk}\ \ \sigma, \sigma_{\text{OT}}) = 0$ then return 0; else return 1.
$\text{Sim}(\text{crs}_{\text{up}}, \mathbf{x}, \text{tc})$ <hr/> <ul style="list-style-type: none"> - $(\text{sk}, \text{pk}) \leftarrow \Sigma.\text{KGen}(1^{\lambda}); (\text{sk}_{\text{OT}}, \text{pk}_{\text{OT}}) \leftarrow \Sigma_{\text{OT}}.\text{KGen}(1^{\lambda});$ - $\pi_{\text{Sim}} \leftarrow \Pi.\text{Sim}(\text{crs}_{\text{up}}, \mathbf{x}, (\perp, \text{tc}_{\Pi}), \perp);$ - $\sigma \leftarrow \Sigma.\text{Sign}(\text{sk}, \text{pk}_{\text{OT}});$ - $\sigma_{\text{OT}} \leftarrow \Sigma_{\text{OT}}.\text{Sign}(\text{sk}_{\text{OT}}, \pi_{\text{Sim}} \ \mathbf{x}\ \ \text{pk}\ \ \sigma\);$ return $\pi := (\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}}).$

Fig. 4. The SE updatable SNARKs from LAMASSU.

part of the relation can be expressed with 756 constraints, whereas the latter can be expressed with 252 constraints, so a total of 1008 constraints [HBHW19, Section A.4]. For the LAMASSU compiler, we extend the relation with a proof of the statement $\text{cpk} = \text{pk} \cdot \mu(\text{csk} - \text{sk})$ with the witness $\text{csk} - \text{sk}$. For Schnorr signatures (cf. Appendix A.6), but also other DLOG-based signature schemes such as ECDSA, the public key is a group element of the form g^{sk} and similarly, μ simply maps scalars to the corresponding group element, i.e., $\mu(x) = g^x$. Hence, the circuit for this relation also requires 1008 constraints. Compared to the OC0C0 framework instantiations (cf. Table 1), LAMASSU needs only 200 con-

straints more than the most aggressive choice using Poseidon and beats all others in the number of constraints. Considering that Schnorr and ECDSA signatures are well established primitives, and that the confidence in their security is far bigger than all the recent SNARK/STARK-friendly primitives, this additional confidence and the updatability feature come at a very small cost for the prover.

In terms of bandwidth overhead, we only need to compare the overhead induced by $\text{cpk} = \text{pk} \cdot \mu(\text{csk} - \text{sk})$ together with the signature and one-time signature in LAMASSU, and $\mu = f_{s_0}(\text{pk}_s) \wedge \rho = f_{s_0}(\beta_0)$ and the one-time signature in the case of OC0C0. For LAMASSU, the CRS is extended with a public key cpk of signature scheme Σ , i.e., when using Schnorr a point on the Jubjub curve which requires 510 bits (256 bits with point compression). For each proof, new Σ and Σ_{OT} keys are sampled. The proof then includes a Σ public key and signature, as well as Σ_{OT} public key and signature. The former amounts to 256 bits for the public key and 504 bits for the signature (2 integers modulo the group order), and the latter – when instantiated as Groth’s sOTS over Jubjub (or a curve of similar size) – amounts to 768 bits for the public key (3 group elements) and 504 bits for the signature (2 integers modulo the group order). In total, the size of the proof is increased by 2032 bits. The updatable version is similar, but Schnorr is performed in \mathbb{G}_1 with additional public key and update in \mathbb{G}_2 .

For C0C0, the CRS is extended with a SHA256 commitment. The proofs are extended with a freshly generated Σ_{OT} public key and a signature together with the evaluation of a PRF also instantiated with SHA256. Hence, the CRS grows by 256 bits and each proof grows by 1016 bits (with Schnorr over Jubjub). For our version, OC0C0, the CRS is extended with ρ and β , both 256 bits each. Each proof additionally contains μ and a fresh Σ_{OT} public key and signature. Using Groth’s sOTS, the proof grows by 1528 bits.

In Table 2 we present a comparison of SE-SNARKs including OC0C0 using Groth’s OTS, OC0C0[G], and Schnorr, OC0C0[S], LAMASSU using Schnorr, LAMASSU[S,S], Groth’s OTS, LAMASSU[S,G], and Boneh-Boyen signatures [BB04], LAMASSU[S,BB], both as non-updatable and updatable variant. The overhead is relative to the underlying SNARK (for the generic constructions) or the SNARK they are based on, e.g., relative to [Gro16]. In the table, n denotes the number of multiplication gates in the circuit, \mathbb{G}_1 and \mathbb{G}_2 the two source groups of a bilinear group, \mathbb{G} a group with prime order q , and λ the sizes of commitments and PRF evaluations. For concrete numbers, we followed the above choice of curves, namely Jubjub (\mathbb{G}) and BLS12-381 as a bilinear group $(q, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g, \hat{g})$, respectively. For commitments and PRFs, we assume 256 bit outputs. For the verifier overhead, we consider the most expensive operations. $E_{\mathbb{G}}$ denotes an exponentiation in \mathbb{G} and P a pairing evaluation (with P a factor 10 slower than $E_{\mathbb{G}}$).

(O)C0C0, and LAMASSU offer a trade-off between the size of CRS and proofs, and verifier overhead when comparing to the ad-hoc constructions. The verifier overhead is smaller than the ones for Groth-Maller [GM17], Bowe-Gabizon [BG18] and Lipmaa [Lip19] and is comparable to those of Atappoor-Baghery [AB19] and Baghery [Bag19], yet LAMASSU offers more features.

6 Conclusion

In this paper, we revisited the lifting technique of the $C\emptyset C\emptyset$ framework to obtain SE SNARKs. By refining the construction and selecting well-suited SNARK-friendly primitives, we obtained an improved version ($OC\emptyset C\emptyset$), which outperforms the original construction in both number of constraints as well as proof size significantly.

We then presented an alternative generic framework, dubbed LAMASSU, that lifts SNARKs to SE SNARKs and also preserves subversion resistance and updatability of the underlying SNARK. In particular, LAMASSU represents the first known framework to generically obtain SE updatable SNARKs and actually yields the first known SE updatable SNARK. It requires only signatures with certain key-homomorphic properties or updatable signatures, a novel primitive introduced in this paper, for SE updatable SNARKs. Moreover, LAMASSU compares favorably to $OC\emptyset C\emptyset$.

Acknowledgements. We thank anonymous reviewers and Ahmed Kosba for valuable feedback. This work was supported by the European Union’s Horizon 2020 research and innovation programme under grant agreements n°830929 (CyberSec4Europe) and n°871473 (KRAKEN), by the Austrian Science Fund (FWF) and netidee SCIENCE grant P31621-N38 (PROFET) and the Estonian Research Council grant PRG49.

References

- AABS⁺19. Abdelrahman Aly, Tomer Ashur, Eli Ben-Sasson, Siemen Dhooghe, and Alan Szepieniec. Design of symmetric-key primitives for advanced cryptographic protocols. Cryptology ePrint Archive, Report 2019/426, 2019. <https://eprint.iacr.org/2019/426>.
- AB19. Shahla Atapoor and Karim Bagheri. Simulation extractability in groth’s zk-snark. In Cristina Pérez-Solà, Guillermo Navarro-Arribas, Alex Biryukov, and Joaquín García-Alfaro, editors, *Data Privacy Management, Cryptocurrencies and Blockchain Technology - ESORICS 2019 International Workshops, DPM 2019 and CBT 2019, Luxembourg, September 26-27, 2019, Proceedings*, volume 11737 of *LNCS*, pages 336–354. Springer, 2019.
- ABLZ17. Behzad Abdolmaleki, Karim Bagheri, Helger Lipmaa, and Michal Zajac. A subversion-resistant SNARK. In Tsuyoshi Takagi and Thomas Peyrin, editors, *ASIACRYPT 2017, Part III*, volume 10626 of *LNCS*, pages 3–33. Springer, Heidelberg, December 2017.
- ACG⁺19. Martin R. Albrecht, Carlos Cid, Lorenzo Grassi, Dmitry Khovratovich, Reinhard Lüftenegger, Christian Rechberger, and Markus Schofnegger. Algebraic cryptanalysis of STARK-friendly designs: Application to MARVELLous and MiMC. In Steven D. Galbraith and Shiho Moriai, editors, *ASIACRYPT 2019, Part III*, volume 11923 of *LNCS*, pages 371–397. Springer, Heidelberg, December 2019.

- ACJT00. Giuseppe Ateniese, Jan Camenisch, Marc Joye, and Gene Tsudik. A practical and provably secure coalition-resistant group signature scheme. In Mihir Bellare, editor, *CRYPTO 2000*, volume 1880 of *LNCS*, pages 255–270. Springer, Heidelberg, August 2000.
- AD18. Tomer Ashur and Siemen Dhooghe. Marvellous: a stark-friendly family of cryptographic primitives. Cryptology ePrint Archive, Report 2018/1098, 2018. <https://eprint.iacr.org/2018/1098>.
- AGP⁺19. Martin R. Albrecht, Lorenzo Grassi, Léo Perrin, Sebastian Ramacher, Christian Rechberger, Dragos Rotaru, Arnab Roy, and Markus Schofnegger. Feistel structures for MPC, and more. In Kazue Sako, Steve Schneider, and Peter Y. A. Ryan, editors, *ESORICS 2019, Part II*, volume 11736 of *LNCS*, pages 151–171. Springer, Heidelberg, September 2019.
- AGR⁺16. Martin R. Albrecht, Lorenzo Grassi, Christian Rechberger, Arnab Roy, and Tyge Tiessen. MiMC: Efficient encryption and cryptographic hashing with minimal multiplicative complexity. In Jung Hee Cheon and Tsuyoshi Takagi, editors, *ASIACRYPT 2016, Part I*, volume 10031 of *LNCS*, pages 191–219. Springer, Heidelberg, December 2016.
- ALSZ20. Behzad Abdolmaleki, Helger Lipmaa, Janno Siim, and Michal Zajac. On subversion-resistant snarks. Cryptology ePrint Archive, Report 2020/668, 2020. <https://eprint.iacr.org/2020/668>.
- AMV15. Giuseppe Ateniese, Bernardo Magri, and Daniele Venturi. Subversion-resilient signature schemes. In Indrajit Ray, Ninghui Li, and Christopher Kruegel, editors, *ACM CCS 2015*, pages 364–375. ACM Press, October 2015.
- ARS⁺15. Martin R. Albrecht, Christian Rechberger, Thomas Schneider, Tyge Tiessen, and Michael Zohner. Ciphers for MPC and FHE. In Elisabeth Oswald and Marc Fischlin, editors, *EUROCRYPT 2015, Part I*, volume 9056 of *LNCS*, pages 430–454. Springer, Heidelberg, April 2015.
- Bag19. Karim Baghery. Subversion-resistant simulation (knowledge) sound NIZKs. In Martin Albrecht, editor, *17th IMA International Conference on Cryptography and Coding*, volume 11929 of *LNCS*, pages 42–63. Springer, Heidelberg, December 2019.
- BB04. Dan Boneh and Xavier Boyen. Short signatures without random oracles. In Christian Cachin and Jan Camenisch, editors, *EUROCRYPT 2004*, volume 3027 of *LNCS*, pages 56–73. Springer, Heidelberg, May 2004.
- BBB⁺18. Benedikt Bünz, Jonathan Bootle, Dan Boneh, Andrew Poelstra, Pieter Wuille, and Greg Maxwell. Bulletproofs: Short proofs for confidential transactions and more. In *2018 IEEE Symposium on Security and Privacy*, pages 315–334. IEEE Computer Society Press, May 2018.
- BBC⁺18. Carsten Baum, Jonathan Bootle, Andrea Cerulli, Rafaël del Pino, Jens Groth, and Vadim Lyubashevsky. Sub-linear lattice-based zero-knowledge arguments for arithmetic circuits. In Hovav Shacham and Alexandra Boldyreva, editors, *CRYPTO 2018, Part II*, volume 10992 of *LNCS*, pages 669–699. Springer, Heidelberg, August 2018.
- BBHR19. Eli Ben-Sasson, Iddo Bentov, Yinon Horesh, and Michael Riabzev. Scalable zero knowledge with no trusted setup. In Alexandra Boldyreva and Daniele Micciancio, editors, *CRYPTO 2019, Part III*, volume 11694 of *LNCS*, pages 701–732. Springer, Heidelberg, August 2019.
- BBS04. Dan Boneh, Xavier Boyen, and Hovav Shacham. Short group signatures. In Matthew Franklin, editor, *CRYPTO 2004*, volume 3152 of *LNCS*, pages 41–55. Springer, Heidelberg, August 2004.

- BBUV20. Ward Beullens, Tim Beyne, Aleksei Udovenko, and Giuseppe Vitto. Cryptanalysis of the Legendre PRF and generalizations. *IACR Trans. Symm. Cryptol.*, 2020(1):313–330, 2020.
- BCC04. Ernest F. Brickell, Jan Camenisch, and Liqun Chen. Direct anonymous attestation. In Vijayalakshmi Atluri, Birgit Pfitzmann, and Patrick McDaniel, editors, *ACM CCS 2004*, pages 132–145. ACM Press, October 2004.
- BCC⁺09. Mira Belenkiy, Jan Camenisch, Melissa Chase, Markulf Kohlweiss, Anna Lysyanskaya, and Hovav Shacham. Randomizable proofs and delegatable anonymous credentials. In Shai Halevi, editor, *CRYPTO 2009*, volume 5677 of *LNCS*, pages 108–125. Springer, Heidelberg, August 2009.
- BCC⁺16. Jonathan Bootle, Andrea Cerulli, Pyrros Chaidos, Essam Ghadafi, and Jens Groth. Foundations of fully dynamic group signatures. In Mark Manulis, Ahmad-Reza Sadeghi, and Steve Schneider, editors, *ACNS 16*, volume 9696 of *LNCS*, pages 117–136. Springer, Heidelberg, June 2016.
- BCG⁺13. Eli Ben-Sasson, Alessandro Chiesa, Daniel Genkin, Eran Tromer, and Madars Virza. SNARKs for C: Verifying program executions succinctly and in zero knowledge. In Ran Canetti and Juan A. Garay, editors, *CRYPTO 2013, Part II*, volume 8043 of *LNCS*, pages 90–108. Springer, Heidelberg, August 2013.
- BCG⁺14. Eli Ben-Sasson, Alessandro Chiesa, Christina Garman, Matthew Green, Ian Miers, Eran Tromer, and Madars Virza. Zerocash: Decentralized anonymous payments from bitcoin. In *2014 IEEE Symposium on Security and Privacy*, pages 459–474. IEEE Computer Society Press, May 2014.
- BCG⁺15. Eli Ben-Sasson, Alessandro Chiesa, Matthew Green, Eran Tromer, and Madars Virza. Secure sampling of public parameters for succinct zero knowledge proofs. In *2015 IEEE Symposium on Security and Privacy*, pages 287–304. IEEE Computer Society Press, May 2015.
- BCG⁺18. Jonathan Bootle, Andrea Cerulli, Jens Groth, Sune K. Jakobsen, and Mary Maller. Arya: Nearly linear-time zero-knowledge proofs for correct program execution. In Thomas Peyrin and Steven Galbraith, editors, *ASIACRYPT 2018, Part I*, volume 11272 of *LNCS*, pages 595–626. Springer, Heidelberg, December 2018.
- BCPR14. Nir Bitansky, Ran Canetti, Omer Paneth, and Alon Rosen. On the existence of extractable one-way functions. In David B. Shmoys, editor, *46th ACM STOC*, pages 505–514. ACM Press, May / June 2014.
- BDD⁺15. Achiya Bar-On, Itai Dinur, Orr Dunkelman, Virginie Lallemand, Nathan Keller, and Boaz Tsaban. Cryptanalysis of SP networks with partial non-linear layers. In Elisabeth Oswald and Marc Fischlin, editors, *EUROCRYPT 2015, Part I*, volume 9056 of *LNCS*, pages 315–342. Springer, Heidelberg, April 2015.
- BDPV08. Guido Bertoni, Joan Daemen, Michaël Peeters, and Gilles Van Assche. On the indistinguishability of the sponge construction. In Nigel P. Smart, editor, *EUROCRYPT 2008*, volume 4965 of *LNCS*, pages 181–197. Springer, Heidelberg, April 2008.
- BFM88. Manuel Blum, Paul Feldman, and Silvio Micali. Non-interactive zero-knowledge and its applications (extended abstract). In *20th ACM STOC*, pages 103–112. ACM Press, May 1988.
- BFS16. Mihir Bellare, Georg Fuchsbauer, and Alessandra Scafuro. NIZKs with an untrusted CRS: Security in the face of parameter subversion. In Jung Hee Cheon and Tsuyoshi Takagi, editors, *ASIACRYPT 2016, Part II*, volume 10032 of *LNCS*, pages 777–804. Springer, Heidelberg, December 2016.

- BG90. Mihir Bellare and Shafi Goldwasser. New paradigms for digital signatures and message authentication based on non-interactive zero knowledge proofs. In Gilles Brassard, editor, *CRYPTO'89*, volume 435 of *LNCS*, pages 194–211. Springer, Heidelberg, August 1990.
- BG18. Sean Bowe and Ariel Gabizon. Making groth’s zk-SNARK simulation extractable in the random oracle model. Cryptology ePrint Archive, Report 2018/187, 2018. <https://eprint.iacr.org/2018/187>.
- BGG19. Sean Bowe, Ariel Gabizon, and Matthew D. Green. A multi-party protocol for constructing the public parameters of the pinocchio zk-SNARK. In Aviv Zohar, Ittay Eyal, Vanessa Teague, Jeremy Clark, Andrea Bracciali, Federico Pintore, and Massimiliano Sala, editors, *FC 2018 Workshops*, volume 10958 of *LNCS*, pages 64–77. Springer, Heidelberg, March 2019.
- BL09. Ernie Brickell and Jiangtao Li. Enhanced privacy ID from bilinear pairing. Cryptology ePrint Archive, Report 2009/095, 2009. <http://eprint.iacr.org/2009/095>.
- Bon19. Xavier Bonnetain. Collisions on feistel-mimc and univariate gmimc. Cryptology ePrint Archive, Report 2019/951, 2019. <https://eprint.iacr.org/2019/951>.
- BRS02. John Black, Phillip Rogaway, and Thomas Shrimpton. Black-box analysis of the block-cipher-based hash-function constructions from PGV. In Moti Yung, editor, *CRYPTO 2002*, volume 2442 of *LNCS*, pages 320–335. Springer, Heidelberg, August 2002.
- BSGL20. Eli Ben-Sasson, Lior Goldberg, and David Levit. Stark friendly hash – survey and recommendation. Cryptology ePrint Archive, Report 2020/948, 2020. <https://eprint.iacr.org/2020/948>.
- Can01. Ran Canetti. Universally composable security: A new paradigm for cryptographic protocols. In *42nd FOCS*, pages 136–145. IEEE Computer Society Press, October 2001.
- CCD⁺17. Jan Camenisch, Liqun Chen, Manu Drijvers, Anja Lehmann, David Novick, and Rainer Urian. One TPM to bind them all: Fixing TPM 2.0 for provably secure anonymous attestation. In *2017 IEEE Symposium on Security and Privacy*, pages 901–920. IEEE Computer Society Press, May 2017.
- CDD17. Jan Camenisch, Manu Drijvers, and Maria Dubovitskaya. Practical UC-secure delegatable credentials with attributes and their application to blockchain. In Bhavani M. Thuraisingham, David Evans, Tal Malkin, and Dongyan Xu, editors, *ACM CCS 2017*, pages 683–699. ACM Press, October / November 2017.
- CDG⁺17. Melissa Chase, David Derler, Steven Goldfeder, Claudio Orlandi, Sebastian Ramacher, Christian Rechberger, Daniel Slamanig, and Greg Zaverucha. Post-quantum zero-knowledge and signatures from symmetric-key primitives. In Bhavani M. Thuraisingham, David Evans, Tal Malkin, and Dongyan Xu, editors, *ACM CCS 2017*, pages 1825–1842. ACM Press, October / November 2017.
- CFQ19. Matteo Campanelli, Dario Fiore, and Anaïs Querol. LegoSNARK: Modular design and composition of succinct zero-knowledge proofs. In Lorenzo Cavallaro, Johannes Kinder, XiaoFeng Wang, and Jonathan Katz, editors, *ACM CCS 2019*, pages 2075–2092. ACM Press, November 2019.
- CGGN17. Matteo Campanelli, Rosario Gennaro, Steven Goldfeder, and Luca Nizardo. Zero-knowledge contingent payments revisited: Attacks and payments for services. In Bhavani M. Thuraisingham, David Evans, Tal

- Malkin, and Dongyan Xu, editors, *ACM CCS 2017*, pages 229–243. ACM Press, October / November 2017.
- CGL⁺17. Alessandro Chiesa, Matthew Green, Jingcheng Liu, Peihan Miao, Ian Miers, and Pratyush Mishra. Decentralized anonymous micropayments. In Jean-Sébastien Coron and Jesper Buus Nielsen, editors, *EUROCRYPT 2017, Part II*, volume 10211 of *LNCS*, pages 609–642. Springer, Heidelberg, April / May 2017.
- Cha86. David Chaum. Showing credentials without identification: Signatures transferred between unconditionally unlinkable pseudonyms. In Franz Pichler, editor, *EUROCRYPT'85*, volume 219 of *LNCS*, pages 241–244. Springer, Heidelberg, April 1986.
- CHM⁺20. Alessandro Chiesa, Yuncong Hu, Mary Maller, Pratyush Mishra, Noah Vesely, and Nicholas P. Ward. Marlin: Preprocessing zkSNARKs with universal and updatable SRS. In Anne Canteaut and Yuval Ishai, editors, *EUROCRYPT 2020, Part I*, volume 12105 of *LNCS*, pages 738–768. Springer, Heidelberg, May 2020.
- CKL⁺16. Jan Camenisch, Stephan Krenn, Anja Lehmann, Gert Læssøe Mikkelsen, Gregory Neven, and Michael Østergaard Pedersen. Formal treatment of privacy-enhancing credential systems. In Orr Dunkelman and Liam Keliher, editors, *SAC 2015*, volume 9566 of *LNCS*, pages 3–24. Springer, Heidelberg, August 2016.
- CL01. Jan Camenisch and Anna Lysyanskaya. An efficient system for non-transferable anonymous credentials with optional anonymity revocation. In Birgit Pfitzmann, editor, *EUROCRYPT 2001*, volume 2045 of *LNCS*, pages 93–118. Springer, Heidelberg, May 2001.
- CL03. Jan Camenisch and Anna Lysyanskaya. A signature scheme with efficient protocols. In Stelvio Cimato, Clemente Galdi, and Giuseppe Persiano, editors, *SCN 02*, volume 2576 of *LNCS*, pages 268–289. Springer, Heidelberg, September 2003.
- CL04. Jan Camenisch and Anna Lysyanskaya. Signature schemes and anonymous credentials from bilinear maps. In Matthew Franklin, editor, *CRYPTO 2004*, volume 3152 of *LNCS*, pages 56–72. Springer, Heidelberg, August 2004.
- CMR98. Ran Canetti, Daniele Micciancio, and Omer Reingold. Perfectly one-way probabilistic hash functions (preliminary version). In *30th ACM STOC*, pages 131–140. ACM Press, May 1998.
- Cv91. David Chaum and Eugène van Heyst. Group signatures. In Donald W. Davies, editor, *EUROCRYPT'91*, volume 547 of *LNCS*, pages 257–265. Springer, Heidelberg, April 1991.
- Dam92. Ivan Damgård. Towards practical public key systems secure against chosen ciphertext attacks. In Joan Feigenbaum, editor, *CRYPTO'91*, volume 576 of *LNCS*, pages 445–456. Springer, Heidelberg, August 1992.
- DEM16. Christoph Dobraunig, Maria Eichlseder, and Florian Mendel. Higher-order cryptanalysis of LowMC. In Soonhak Kwon and Aaram Yun, editors, *ICISC 15*, volume 9558 of *LNCS*, pages 87–101. Springer, Heidelberg, November 2016.
- DFGK14. George Danezis, Cédric Fournet, Jens Groth, and Markulf Kohlweiss. Square span programs with applications to succinct NIZK arguments. In Palash Sarkar and Tetsu Iwata, editors, *ASIACRYPT 2014, Part I*, volume 8873 of *LNCS*, pages 532–550. Springer, Heidelberg, December 2014.

- DGS03. Ivan Damgård, Jens Groth, and Gorm Salomonsen. The theory and implementation of an electronic voting system. In Dimitris Gritzalis, editor, *Secure Electronic Voting*, volume 7 of *Advances in Information Security*, pages 77–98. Springer, 2003.
- DKP⁺19. Itai Dinur, Daniel Kales, Angela Promitzer, Sebastian Ramacher, and Christian Rechberger. Linear equivalence of block ciphers with partial non-linear layers: Application to LowMC. In Yuval Ishai and Vincent Rijmen, editors, *EUROCRYPT 2019, Part I*, volume 11476 of *LNCS*, pages 343–372. Springer, Heidelberg, May 2019.
- DLMW15. Itai Dinur, Yunwen Liu, Willi Meier, and Qingju Wang. Optimized interpolation attacks on LowMC. In Tetsu Iwata and Jung Hee Cheon, editors, *ASIACRYPT 2015, Part II*, volume 9453 of *LNCS*, pages 535–560. Springer, Heidelberg, November / December 2015.
- DOR⁺16. David Derler, Claudio Orlandi, Sebastian Ramacher, Christian Rechberger, and Daniel Slamanig. Digital signatures from symmetric-key primitives. Cryptology ePrint Archive, Report 2016/1085, 2016. <http://eprint.iacr.org/2016/1085>.
- DP92. Alfredo De Santis and Giuseppe Persiano. Zero-knowledge proofs of knowledge without interaction (extended abstract). In *33rd FOCS*, pages 427–436. IEEE Computer Society Press, October 1992.
- DP06. Cécile Delerablée and David Pointcheval. Dynamic fully anonymous short group signatures. In Phong Q. Nguyen, editor, *Progress in Cryptology - VIETCRYPT 06*, volume 4341 of *LNCS*, pages 193–210. Springer, Heidelberg, September 2006.
- DR08. Tim Dierks and Eric Rescorla. The transport layer security (TLS) protocol version 1.2. *RFC*, 5246:1–104, 2008.
- DRS18. David Derler, Sebastian Ramacher, and Daniel Slamanig. Generic double-authentication preventing signatures and a post-quantum instantiation. In Joonsang Baek, Willy Susilo, and Jongkil Kim, editors, *ProvSec 2018*, volume 11192 of *LNCS*, pages 258–276. Springer, Heidelberg, October 2018.
- DS18. David Derler and Daniel Slamanig. Highly-efficient fully-anonymous dynamic group signatures. In Jong Kim, Gail-Joon Ahn, Seungjoo Kim, Yongdae Kim, Javier López, and Taesoo Kim, editors, *ASIACCS 18*, pages 551–565. ACM Press, April 2018.
- DS19. David Derler and Daniel Slamanig. Key-homomorphic signatures: definitions and applications to multiparty signatures and non-interactive zero-knowledge. *Des. Codes Cryptogr.*, 87(6):1373–1413, 2019.
- FHS19. Georg Fuchsbauer, Christian Hanser, and Daniel Slamanig. Structure-preserving signatures on equivalence classes and constant-size anonymous credentials. *Journal of Cryptology*, 32(2):498–546, April 2019.
- Fis99. Marc Fischlin. Pseudorandom function tribe ensembles based on one-way permutations: Improvements and applications. In Jacques Stern, editor, *EUROCRYPT’99*, volume 1592 of *LNCS*, pages 432–445. Springer, Heidelberg, May 1999.
- FMMO19. Prastudy Fauzi, Sarah Meiklejohn, Rebekah Mercer, and Claudio Orlandi. Quisquis: A new design for anonymous cryptocurrencies. In Steven D. Galbraith and Shiho Moriai, editors, *ASIACRYPT 2019, Part I*, volume 11921 of *LNCS*, pages 649–678. Springer, Heidelberg, December 2019.
- FPS⁺18. Jonathan Frankle, Sunoo Park, Daniel Shaar, Shafi Goldwasser, and Daniel J. Weitzner. Practical accountability of secret processes. In William

- Enck and Adrienne Porter Felt, editors, *USENIX Security 2018*, pages 657–674. USENIX Association, August 2018.
- Fuc18. Georg Fuchsbauer. Subversion-zero-knowledge SNARKs. In Michel Abdalla and Ricardo Dahab, editors, *PKC 2018, Part I*, volume 10769 of *LNCS*, pages 315–347. Springer, Heidelberg, March 2018.
- Fuc19. Georg Fuchsbauer. WI is not enough: Zero-knowledge contingent (service) payments revisited. In Lorenzo Cavallaro, Johannes Kinder, XiaoFeng Wang, and Jonathan Katz, editors, *ACM CCS 2019*, pages 49–62. ACM Press, November 2019.
- GGP10. Rosario Gennaro, Craig Gentry, and Bryan Parno. Non-interactive verifiable computing: Outsourcing computation to untrusted workers. In Tal Rabin, editor, *CRYPTO 2010*, volume 6223 of *LNCS*, pages 465–482. Springer, Heidelberg, August 2010.
- GGPR13. Rosario Gennaro, Craig Gentry, Bryan Parno, and Mariana Raykova. Quadratic span programs and succinct NIZKs without PCPs. In Thomas Johansson and Phong Q. Nguyen, editors, *EUROCRYPT 2013*, volume 7881 of *LNCS*, pages 626–645. Springer, Heidelberg, May 2013.
- GKK⁺19. Lorenzo Grassi, Daniel Kales, Dmitry Khovratovich, Arnab Roy, Christian Rechberger, and Markus Schofnegger. Starkad and poseidon: New hash functions for zero knowledge proof systems. Cryptology ePrint Archive, Report 2019/458, 2019. <https://eprint.iacr.org/2019/458>.
- GKM⁺18. Jens Groth, Markulf Kohlweiss, Mary Maller, Sarah Meiklejohn, and Ian Miers. Updatable and universal common reference strings with applications to zk-SNARKs. In Hovav Shacham and Alexandra Boldyreva, editors, *CRYPTO 2018, Part III*, volume 10993 of *LNCS*, pages 698–728. Springer, Heidelberg, August 2018.
- GM17. Jens Groth and Mary Maller. Snarky signatures: Minimal signatures of knowledge from simulation-extractable SNARKs. In Jonathan Katz and Hovav Shacham, editors, *CRYPTO 2017, Part II*, volume 10402 of *LNCS*, pages 581–612. Springer, Heidelberg, August 2017.
- GMNO18. Rosario Gennaro, Michele Minelli, Anca Nitulescu, and Michele Orrù. Lattice-based zk-SNARKs from square span programs. In David Lie, Mohammad Mannan, Michael Backes, and XiaoFeng Wang, editors, *ACM CCS 2018*, pages 556–573. ACM Press, October 2018.
- GMR85. Shafi Goldwasser, Silvio Micali, and Charles Rackoff. The knowledge complexity of interactive proof-systems (extended abstract). In *17th ACM STOC*, pages 291–304. ACM Press, May 1985.
- GR19. Alonso González and Carla Ràfols. Sublinear pairing-based arguments with updatable crs and weaker assumptions. *IACR Cryptology ePrint Archive*, 2019:326, 2019.
- Gro06. Jens Groth. Simulation-sound NIZK proofs for a practical language and constant size group signatures. In Xuejia Lai and Kefei Chen, editors, *ASIACRYPT 2006*, volume 4284 of *LNCS*, pages 444–459. Springer, Heidelberg, December 2006.
- Gro10a. Jens Groth. Short pairing-based non-interactive zero-knowledge arguments. In Masayuki Abe, editor, *ASIACRYPT 2010*, volume 6477 of *LNCS*, pages 321–340. Springer, Heidelberg, December 2010.
- Gro10b. Jens Groth. A verifiable secret shuffle of homomorphic encryptions. *Journal of Cryptology*, 23(4):546–579, October 2010.

- Gro16. Jens Groth. On the size of pairing-based non-interactive arguments. In Marc Fischlin and Jean-Sébastien Coron, editors, *EUROCRYPT 2016, Part II*, volume 9666 of *LNCS*, pages 305–326. Springer, Heidelberg, May 2016.
- HBHW19. Daira Hopwood, Sean Bowe, Taylor Hornby, and Nathan Wilcox. Zcash protocol specification: Version 2019.0.6 [overwinter+sapling], 2019.
- JKS16. Ari Juels, Ahmed E. Kosba, and Elaine Shi. The ring of Gyges: Investigating the future of criminal smart contracts. In Edgar R. Weippl, Stefan Katzenbeisser, Christopher Kruegel, Andrew C. Myers, and Shai Halevi, editors, *ACM CCS 2016*, pages 283–295. ACM Press, October 2016.
- KKKZ19. Thomas Kerber, Aggelos Kiayias, Markulf Kohlweiss, and Vassilis Zikas. Ouroboros cryptsinous: Privacy-preserving proof-of-stake. In *2019 IEEE Symposium on Security and Privacy*, pages 157–174. IEEE Computer Society Press, May 2019.
- KMP16. Eike Kiltz, Daniel Masny, and Jiaxin Pan. Optimal security proofs for signatures from identification schemes. In Matthew Robshaw and Jonathan Katz, editors, *CRYPTO 2016, Part II*, volume 9815 of *LNCS*, pages 33–61. Springer, Heidelberg, August 2016.
- KMS⁺16. Ahmed E. Kosba, Andrew Miller, Elaine Shi, Zikai Wen, and Charalampos Papamanthou. Hawk: The blockchain model of cryptography and privacy-preserving smart contracts. In *2016 IEEE Symposium on Security and Privacy*, pages 839–858. IEEE Computer Society Press, May 2016.
- KZM⁺15. Ahmed Kosba, Zhichao Zhao, Andrew Miller, Yi Qian, Hubert Chan, Charalampos Papamanthou, Rafael Pass, abhi shelat, and Elaine Shi. $C0c0$: A framework for building composable zero-knowledge proofs. Cryptology ePrint Archive, Report 2015/1093, 2015. <https://eprint.iacr.org/2015/1093>.
- Lip12. Helger Lipmaa. Progression-free sets and sublinear pairing-based non-interactive zero-knowledge arguments. In Ronald Cramer, editor, *TCC 2012*, volume 7194 of *LNCS*, pages 169–189. Springer, Heidelberg, March 2012.
- Lip13. Helger Lipmaa. Succinct non-interactive zero knowledge arguments from span programs and linear error-correcting codes. In Kazue Sako and Palash Sarkar, editors, *ASIACRYPT 2013, Part I*, volume 8269 of *LNCS*, pages 41–60. Springer, Heidelberg, December 2013.
- Lip19. Helger Lipmaa. Simulation-extractable snarks revisited. Cryptology ePrint Archive, Report 2019/612, 2019. <https://eprint.iacr.org/2019/612>.
- LP19. Chaoyun Li and Bart Preneel. Improved interpolation attacks on cryptographic primitives of low algebraic degree. In Kenneth G. Paterson and Douglas Stebila, editors, *SAC 2019*, volume 11959 of *LNCS*, pages 171–193. Springer, Heidelberg, August 2019.
- MBKM19. Mary Maller, Sean Bowe, Markulf Kohlweiss, and Sarah Meiklejohn. Sonic: Zero-knowledge SNARKs from linear-size universal and updatable structured reference strings. In Lorenzo Cavallaro, Johannes Kinder, XiaoFeng Wang, and Jonathan Katz, editors, *ACM CCS 2019*, pages 2111–2128. ACM Press, November 2019.
- MGGM18. Alexander Mühle, Andreas Grüner, Tatiana Gayvoronskaya, and Christoph Meinel. A survey on essential components of a self-sovereign identity. *Computer Science Review*, 30:80–86, 2018.

- PHGR13. Bryan Parno, Jon Howell, Craig Gentry, and Mariana Raykova. Pinocchio: Nearly practical verifiable computation. In *2013 IEEE Symposium on Security and Privacy*, pages 238–252. IEEE Computer Society Press, May 2013.
- PS96. David Pointcheval and Jacques Stern. Security proofs for signature schemes. In Ueli M. Maurer, editor, *EUROCRYPT'96*, volume 1070 of *LNCS*, pages 387–398. Springer, Heidelberg, May 1996.
- RST18. Christian Rechberger, Hadi Soleimany, and Tyge Tiessen. Cryptanalysis of low-data instances of full LowMCv2. *IACR Trans. Symm. Cryptol.*, 2018(3):163–181, 2018.
- Sah99. Amit Sahai. Non-malleable non-interactive zero knowledge and adaptive chosen-ciphertext security. In *40th FOCS*, pages 543–553. IEEE Computer Society Press, October 1999.
- Sah01. Amit Sahai. Simulation-sound non-interactive zero knowledge. Technical report, IBM RESEARCH REPORT RZ 3076, 2001.
- Sch90. Claus-Peter Schnorr. Efficient identification and signatures for smart cards. In Gilles Brassard, editor, *CRYPTO'89*, volume 435 of *LNCS*, pages 239–252. Springer, Heidelberg, August 1990.
- SK95. Kazuo Sako and Joe Kilian. Receipt-free mix-type voting scheme - a practical solution to the implementation of a voting booth. In Louis C. Guillou and Jean-Jacques Quisquater, editors, *EUROCRYPT'95*, volume 921 of *LNCS*, pages 393–403. Springer, Heidelberg, May 1995.

A Omitted Primitives

A.1 Pseudorandom Functions

We recall the standard notion of pseudorandom functions.

Definition 11 (PRF). Let $f: \mathcal{S} \times \mathcal{D} \rightarrow \mathcal{R}$ be a family of functions and let Γ be the set of all functions $\mathcal{D} \rightarrow \mathcal{R}$. f is a pseudorandom function (PRF) (family) if it is efficiently computable and for all PPT distinguishers \mathcal{D} such that

$$\left| \Pr \left[s \leftarrow_s \mathcal{S}, \mathcal{D}^{f_s(\cdot)}(1^\lambda) \right] - \Pr \left[g \leftarrow_s \Gamma, \mathcal{D}^{g(\cdot)}(1^\lambda) \right] \right| \approx_\lambda 0.$$

A.2 Non-Interactive Zero-Knowledge

Let RGen be a relation generator, such that $\text{RGen}(1^\lambda)$ returns a polynomial-time decidable binary relation $\mathcal{R} = \{(\mathbf{x}, \mathbf{w})\}$. Here, \mathbf{x} is the statement and \mathbf{w} is the witness. We assume that λ is explicitly deducible from the description of \mathcal{R} . The relation generator also outputs auxiliary information $\text{aux}_{\mathcal{R}}$ that will be given to the honest parties and the adversary. Let $\mathcal{L}_{\mathcal{R}} = \{\mathbf{x} : \exists \mathbf{w}, (\mathbf{x}, \mathbf{w}) \in \mathcal{R}\}$ be an NP-language. Non-interactive zero-knowledge (NIZK) proofs and arguments in the CRS model consist of algorithms $(\text{KGen}_{\text{crs}}, \text{P}, \text{V}, \text{Sim})$, and satisfy the following properties: completeness (for all common reference strings crs generated by KGen_{crs} and $(\mathbf{x}, \mathbf{w}) \in \mathcal{R}$, we have that $\text{V}(\text{crs}, \mathbf{x}, \text{P}(\text{crs}, \mathbf{x}, \mathbf{w})) = 1$), zero-knowledge (there exists a simulator Sim that outputs a simulated proof such

that an adversary cannot distinguish it from proofs computed by $P(\text{crs}, \mathbf{x}, \mathbf{w})$, soundness (an adversary cannot output a proof π and an instance $\mathbf{x} \notin \mathcal{L}_{\mathcal{R}}$ such that $V(\text{crs}, \mathbf{x}, \pi) = 1$. Moreover, knowledge soundness steps further and says that for any prover generating a valid proof there is an extractor Ext that can extract a valid witness.

A.3 QAPs and R1CS

Quadratic Arithmetic Programs (QAPs) have been introduced by Gennaro et al. [GGPR13] as a language where for an input \mathbf{x} and witness \mathbf{w} , $(\mathbf{x}, \mathbf{w}) \in \mathcal{R}$ can be verified by using a parallel quadratic check, and that has an efficient reduction from a well-known language (either Boolean or Arithmetic) CIRCUIT-SAT.

Definition 12 (QAP). *A quadratic arithmetic program over a field \mathbb{F} is a tuple of the form*

$$\left(\mathbb{F}, n, \{ \mathbf{A}_i(X), \mathbf{B}_i(X), \mathbf{C}_i(X) \}_{i=0}^{i=m}; \mathbf{D}(X) \right)$$

where $\mathbf{A}_i(X), \mathbf{B}_i(X), \mathbf{C}_i(X), \mathbf{D}(X) \in \mathbb{F}[X]$, define a language of statements $(s_1, \dots, s_n) \in \mathbb{F}$ and witnesses $(s_{n+1}, \dots, s_m) \in \mathbb{F}^{m-n}$ such that

$$\left(\sum_{i=0}^m s_i \mathbf{A}_i(X) \right) \cdot \left(\sum_{i=0}^m s_i \mathbf{B}_i(X) \right) = \left(\sum_{i=0}^m s_i \mathbf{C}_i(X) \right) + \mathbf{H}(X) \cdot \mathbf{D}(X) \quad (2)$$

where $s_0 = 1$ and for some degree- $(d-2)$ quotient polynomial $\mathbf{H}(X)$, where d is the degree of $\mathbf{D}(X)$. Let the degrees of all $\mathbf{A}_i(X), \mathbf{B}_i(X)$ and $\mathbf{C}_i(X)$ are at most $d-1$.

We note that all the considered SNARK constructions are for QAPs defined over a bilinear group. Thus we consider relation generators RGen of the following form:

Definition 13 (QAP relation). *A QAP relation generator RGen is a PT algorithm that on input λ returns a relation description $\mathcal{R} = (\text{pars}, n, (\mathbf{A}, \mathbf{B}, \mathbf{C}) \in \mathbb{F}^{(d-1)}[X]^{m-1}, \mathbf{D} \in \mathbb{F}^{(d)}[X])$ where pars is a bilinear group whose order p defines $\mathbb{F} := \mathbb{Z}_p$ and $n \leq m$. Fix $\mathbf{x} \in \mathbb{F}^n$ and $\mathbf{w} \in \mathbb{F}^{m-n}$, we define $\mathcal{R}(\mathbf{x}, \mathbf{w}) = 1$ if there exists $\mathbf{H}(X) \in \mathbb{F}[X]$ so that Eq. (2) holds for $\mathbf{x} = (s_1, \dots, s_n)$ and $\mathbf{w} = (s_{n+1}, \dots, s_m)$.*

For reducing arithmetic circuits to QAP relations, circuits can first be transformed into a system of rank-1 quadratic equations (R1CS) which is latter transformed into a QAP [BCG⁺13]. The R1CS relation over a field \mathbb{F} consists of instance-witness pairs $((A, B, C, \mathbf{v}), \mathbf{w})$ with matrices $A, B, C \in \mathbb{F}^{n \times m}$ and vectors \mathbf{v}, \mathbf{w} such that $(A\mathbf{z}) \circ (B\mathbf{z}) = C\mathbf{z}$ with $\mathbf{z} = (1, \mathbf{v}, \mathbf{w}) \in \mathbb{F}^m$ where \circ denotes the entry-wise product. For capturing arithmetic circuit satisfaction, A, B, C represent the gates, v the public inputs, and w the private inputs and wire values.

A.4 Public-key Encryption

Definition 14. A public key encryption scheme $\Omega = (\text{KGen}, \text{Enc}, \text{Dec})$ consists of the following PPT algorithms:

$\text{KGen}(1^\lambda)$: Given a security parameter λ it outputs the secret key sk and public key pk with message space \mathcal{M} .

$\text{Enc}(\text{pk}, m)$: Given a public key pk and a message $m \in \mathcal{M}$ it outputs a ciphertext c .

$\text{Dec}(\text{sk}, C)$: Given a secret key sk and a ciphertext c it outputs a message $m \in \mathcal{M} \cup \{\perp\}$.

We say that an encryption scheme Ω is perfectly correct if for all $\lambda \in \mathbb{N}$, for all $(\text{sk}, \text{pk}) \leftarrow \text{KGen}(1^\lambda)$ and for all $m \in \mathcal{M}$ it holds that $\text{Dec}(\text{sk}, \text{Enc}(\text{pk}, m)) = m$. Below, we recall the standard notion of indistinguishability under chosen plaintext attacks (IND-CPA security).

Definition 15 (IND-CPA). A public key encryption scheme Ω is IND-CPA secure, if for all PPT adversaries \mathcal{A} it holds that

$$\Pr \left[\begin{array}{l} (\text{sk}, \text{pk}) \leftarrow \text{KGen}(1^\lambda), b \leftarrow_{\$} \{0, 1\}, \\ (m_0, m_1, \text{st}) \leftarrow \mathcal{A}(\text{pk}), b^* \leftarrow \mathcal{A}(\text{Enc}(\text{pk}, m_b), \text{st}) : \\ b = b^* \end{array} \right] \approx_{\lambda} \frac{1}{2}.$$

A.5 Signature Schemes

A signature scheme $\Sigma = (\text{KGen}, \text{Sign}, \text{Verify})$ consists of the following PPT algorithms:

$\text{KGen}(1^\lambda)$: On input security parameter λ it outputs a signing key sk and a verification key pk with associated message space \mathcal{M} .

$\text{Sign}(\text{sk}, m)$: On input a secret key sk and a message $m \in \mathcal{M}$ it outputs a signature σ .

$\text{Verify}(\text{pk}, m, \sigma)$: On input a public key pk , a message $m \in \mathcal{M}$ and a signature σ it outputs a bit $b \in \{0, 1\}$.

We note that for a signature scheme many independently generated public keys may be with respect to the same parameters pp , e.g., some elliptic curve group parameters. In such a case we use an additional algorithm PGen and $\text{pp} \leftarrow \text{PGen}(1^\lambda)$ is then given to KGen . We assume that a signature scheme satisfies the usual (perfect) correctness notion. Now we recall the standard EUF-CMA security notion of signatures.

Definition 16 (EUF-CMA). A signature scheme Σ is EUF-CMA secure, if for all PPT adversaries \mathcal{A}

$$\Pr \left[\begin{array}{l} (\text{sk}, \text{pk}) \leftarrow \text{KGen}(1^\lambda), (m^*, \sigma^*) \leftarrow \mathcal{A}^{\text{Sign}(\text{sk}, \cdot)}(\text{pk}) : \\ \text{Verify}(\text{pk}, m^*, \sigma^*) = 1 \wedge m^* \notin \mathcal{Q}^{\text{Sign}} \end{array} \right] \approx_{\lambda} 0,$$

where the environment keeps track of the queries to the signing oracle via $\mathcal{Q}^{\text{Sign}}$.

For our compiler we also require one-time signature schemes that are sEUF-CMA secure (also called sOTS schemes).

Definition 17 (Strong One-Time Signature Scheme). *A strong one-time signature scheme Σ_{OT} is a signature scheme Σ which satisfies the following unforgeability notion: For all PPT adversaries \mathcal{A}*

$$\Pr \left[\begin{array}{l} (\text{sk}, \text{pk}) \leftarrow \text{KGen}(1^\lambda), (m^*, \sigma^*) \leftarrow \mathcal{A}^{\text{Sign}(\text{sk}, \cdot)}(\text{pk}) : \\ \text{Verify}(\text{pk}, m^*, \sigma^*) = 1 \wedge (m^*, \sigma^*) \notin \mathcal{Q}^{\text{Sign}} \end{array} \right] \approx_\lambda 0,$$

where the oracle $\text{Sign}(\text{sk}, m) := \Sigma.\text{Sign}(\text{sk}, m)$ can only be called once.

A.6 Schnorr Signatures

We recall the Schnorr signature scheme [Sch90] together with the required Adapt algorithm (cf. [DS19]) in Fig. 5. It can be shown to provide EUF-CMA security in the random oracle model (ROM) under the DLP in \mathbb{G} by using the now popular rewinding technique [PS96] (cf. also [KMP16] for a recent treatment on tightness and optimality of such reductions). In the following we present Schnorr signatures with respect to a common setup, i.e., $\mathcal{PP} \leftarrow \text{PGen}(1^\lambda)$ are given to all instances of KGen and let GGen be a group generator that on input 1^λ outputs the description of a prime order group $\mathcal{G} = (\mathbb{G}, g, p)$ with order p s.t. $\lambda = \log_2 p$ and generator g . Recall, that in addition Schnorr requires a collision resistant hash function $H : \mathbb{G} \times \mathcal{M} \rightarrow \mathbb{Z}_p$ (formally sampled uniformly at random from a family $\{H_k\}_{k \in \mathcal{K}}$ of hash functions) and thus we have $\mathcal{PP} := (\mathcal{G}, H)$ (which we assume to be an implicit input to all algorithms). We recall a lemma from [DS19] showing that Schnorr signatures using the Adapt algorithm in Fig. 5 satisfies the signature adaption notion in Definition 5.

Lemma 1 ([DS19]). *Schnorr signatures are adaptable according to Definition 5.*

A.7 Groth's Strong One-Time Signatures

In Fig. 6 we recall the strong one-time signature scheme from Groth [Gro06] and its security below:

Theorem 5 ([Gro06]). *Assuming hardness of computing discrete logarithms and collision-resistance of the hash function, the scheme $(\text{PGen}_{\text{ots}}, \text{KGen}_{\text{ots}}, \text{Sign}_{\text{ots}}, \text{Verify}_{\text{ots}})$ described in Fig. 6 is a strong one-time signature scheme for signing messages $m \in \{0, 1\}^*$ with perfect correctness.*

A.8 BDH Knowledge Assumption

Let BGen be a PPT algorithm that, on input a security parameter λ , outputs $\text{BG} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g, \hat{g})$ for generators g and \hat{g} of \mathbb{G}_1 and \mathbb{G}_2 , respectively, and $\Theta(\lambda)$ -bit prime p .

$\text{PGen}(1^\lambda)$ <hr/> <ul style="list-style-type: none"> - $\mathcal{G} \leftarrow \text{GGen}(1^\lambda)$; $H \leftarrow_{\\$} \{H_k\}_{k \in \mathcal{K}}$; - return $\text{PP} := (\mathcal{G}, H)$;
$\text{KGen}(\text{PP})$: <hr/> <ul style="list-style-type: none"> - Parse $\text{PP} = ((\mathbb{G}, g, p), H)$; - $x \leftarrow_{\\$} \mathbb{Z}_p$; - return $(\text{sk}, \text{pk}) := (x, g^x)$.
$\text{Sign}(\text{sk}, m)$: <hr/> <ul style="list-style-type: none"> - Parse $\text{sk} = x$; - $r \leftarrow_{\\$} \mathbb{Z}_p$; $R := g^r$; $c := H(R m)$; $y := r + x \cdot c \bmod p$ - return $\sigma := (c, y)$.
$\text{Verify}(\text{pk}, m, \sigma)$: <hr/> <ul style="list-style-type: none"> - Parse $\text{pk} = g^x$; $\sigma = (c, y)$; - if $c = H((g^x)^{-c} g^y, m)$ return 1 else return 0.
$\text{Adapt}(\text{pk}, m, \sigma, \Delta)$: <hr/> <ul style="list-style-type: none"> - Parse $\text{pk} = g^x$; $\sigma = (c, y)$; $\Delta \in \mathbb{Z}_p$; - $\text{pk}' := g^x \cdot g^\Delta$; $y' := y + c \cdot \Delta \bmod p$; - return $\sigma' := (c, y')$.

Fig. 5. Schnorr signatures.

Assumption 1 (BDH-Knowledge Assumption [ABLZ17]) *We say that BGen is BDH-KE secure for \mathcal{R} if for any λ , $(\mathcal{R}, \text{aux}_{\mathcal{R}}) \in \text{im}(\mathcal{R}(1^\lambda))$, and PPT adversary \mathcal{A} there exists a PPT extractor $\text{Ext}_{\mathcal{A}}^{\text{BDH}}$, such that*

$$\Pr \left[\begin{array}{l} \omega_{\mathcal{A}} \leftarrow_r \text{RND}(\mathcal{A}), \\ (V, \hat{V}||a) \leftarrow (\mathcal{A}||\text{Ext}_{\mathcal{A}}^{\text{BDH}})(\mathcal{R}, \text{aux}_{\mathcal{R}}; \omega_{\mathcal{A}}) : \\ e(V, \hat{g}) = e(g, \hat{V}) \wedge g^a \neq V \end{array} \right] \approx_{\lambda} 0.$$

Note that the BDH assumption can be considered as a simple case of the PKE assumption of [DFGK14] (where \mathcal{A} is given as an input the tuple $\{(g^{x^i}, \hat{g}^{x^i})\}_{i=0}^n$ for some $n \geq 0$, and assumed that if \mathcal{A} outputs (V, \hat{V}) then she knows (a_0, a_1, \dots, a_n) , such that $V = g^{\sum_{i=0}^n a_i x^i}$ as used in the case of asymmetric pairings in [DFGK14]. Thus, BDH can be seen as an asymmetric-pairing version of the original and by now well established KoE assumption due to Damgård [Dam92].

B Omitted Proofs

B.1 Proof of Theorem 2

Proof. (i: Completeness): This is straight forward from the construction.

$\text{PGen}_{\text{ots}}(1^\lambda)$ <hr style="border: 0.5px solid black;"/> <ul style="list-style-type: none"> - $\mathcal{G} \leftarrow \text{GGen}(1^\lambda)$; $H \leftarrow_{\\$} \{H_k\}_{k \in \mathcal{K}}$; - return $\text{PP} := (\mathcal{G}, H)$;
$\text{KGen}_{\text{ots}}(\text{PP})$: <hr style="border: 0.5px solid black;"/> <ul style="list-style-type: none"> - Parse $\text{PP} = ((\mathbb{G}, g, p), H)$; - $x_s, y_s, r_s, s_s \leftarrow_{\\$} \mathbb{Z}_p$; - $f_s := g^{x_s}$; $h_s := g^{y_s}$; $c_s := g^{r_s} \cdot h_s^{s_s}$; - return $(\text{sk}, \text{pk}) := ((x_s, y_s, r_s, s_s), (f_s, h_s, c_s))$.
$\text{Sign}_{\text{ots}}(\text{sk}, m)$: <hr style="border: 0.5px solid black;"/> <ul style="list-style-type: none"> - Parse $\text{sk} = (x_s, y_s)$; - $r \leftarrow_{\\$} \mathbb{Z}_p$; $z := x_s(r_s - r) + y_s \cdot s_s - H(m) \cdot y_s^{-1} \bmod p$ - return $\sigma := (r, z)$.
$\text{Verify}_{\text{ots}}(\text{pk}, m, \sigma)$: <hr style="border: 0.5px solid black;"/> <ul style="list-style-type: none"> - Parse $\text{pk} = (f_s, h_s, c_s)$; $\sigma = (r, z)$; - if $c_s = g^{H(m)} \cdot f_s^r \cdot h_s^z$ return 1 else return 0.

Fig. 6. Groth’s strong one-time signature scheme.

(ii: Subversion zero-knowledge): The intuition of proving Sub-ZK is that, since here the prover (and consequently the simulator) does not trust to the CRS generator, so relying on the knowledge assumption of the underlying SNARK, if $\text{Vcrs}(\text{crs}, \zeta) = 1$ (or more precisely $\text{Vcrs}(\text{crs}_\Pi, \zeta_\Pi) = 1$) then there is an extractor which can extract the trapdoor tc_Π similar to [ABLZ17] (under the BDH assumption) and [Fuc18] (under the SKE assumption Def. 2.15). Then the simulator $\Pi.\text{Sim}$ takes tc_Π together with crs_Π and \mathbf{x} , and simulates π_{Sim} , which is the simulated proof in the original Sub-zk-SNARK.

Let the knowledge assumption (depending on the underlying SNARK) hold. Let Z be a subverter that computes crs so as to break the Sub-ZK property. That is, $Z(1^\lambda, \omega_Z)$ outputs $(\text{crs}, \zeta, \text{aux}_Z)$. Let \mathcal{A} be the adversary from Fig. 7. Note that $\text{RND}(\mathcal{A}) = \text{RND}(Z)$. Under the knowledge assumption, there exists an extractor Ext_Z , such that if $\Pi.\text{Vcrs}(\text{crs}_\Pi, \zeta_\Pi) = 1$ then $\text{Ext}_Z(1^\lambda, \omega_Z)$ outputs tc_Π , such that $\pi_\Pi = \pi_{\text{Sim}}$. Note that π_Π is the real proof in the Sub-zk-SNARK.

$\mathcal{A}(\text{crs}; \omega_Z)$ <hr style="border: 0.5px solid black;"/> $(\text{crs}, \text{aux}_Z) \leftarrow Z(1^\lambda; \omega_Z)$; return pk ;	$\text{Ext}_Z(1^\lambda, \omega_Z)$ <hr style="border: 0.5px solid black;"/> return tc_Π ;
---	--

Fig. 7. The extractor and the constructed adversary \mathcal{A} from the Sub-ZK proof.

Fix concrete values of λ , $(\mathbf{x}, \mathbf{w}) \in \mathcal{R}_{\mathcal{L}}$, $\omega_Z \in \text{RND}(Z)$, and run $\text{Ext}_Z(1^\lambda, \omega_Z)$ to obtain \mathbf{tc}_Π . Thus, it suffices to show that $\text{Vcrs}(\text{crs}_\Pi, \zeta_\Pi) = 1$ and $(\mathbf{x}, \mathbf{w}) \in \mathcal{R}$ implies that

$$\begin{aligned} \mathcal{O}_0(\mathbf{x}, \mathbf{w}) &= \text{P}(\text{crs}, \mathbf{x}, \mathbf{w}) = \pi_\Pi, \\ \mathcal{O}_1(\mathbf{x}, \mathbf{w}) &= \text{Sim}(\text{crs}, \mathbf{x}, \mathbf{tc}_\Pi) = \pi_{\text{Sim}} \end{aligned}$$

have the same distribution. This holds since based on the Sub-ZK of the underlying SNARK (e.g., [ABLZ17, Fuc18]) if $\text{Vcrs}(\text{crs}_\Pi, \zeta_\Pi) = 1$ and $(\mathbf{x}, \mathbf{w}) \in \mathcal{R}_{\mathcal{L}}$, then π_Π and π_{Sim} have the same distribution. Hence, \mathcal{O}_0 and \mathcal{O}_1 have the same distribution and thus, π is Sub-ZK (under BDH [ABLZ17] or SKE [Fuc18] assumption).

(iii: Strong simulation extractability): This is straight forward from the Theorem 1.

B.2 Proof of Theorem 4

Proof. (i: Completeness): This is straight forward from the construction of SE updatable SNARKs in Fig. 4. If $(\text{crs}, (\zeta_i)_{i=1}^{i=n}, \mathbf{x}, \mathbf{w}) \leftarrow \mathcal{A}(1^\lambda)$ and $\text{Vcrs}(1^\lambda, \text{crs}, (\zeta_i)_{i=1}^{i=n}) = 1 \wedge (\mathbf{x}, \mathbf{w}) \in \mathcal{R}$, then $\text{V}(\text{crs}, \mathbf{x}, \text{P}(\text{crs}, \mathbf{x}, \mathbf{w})) = 1$.

(ii: Updatable zero-knowledge): Underlying the subvertible CRSs property of updatable SNARKs (i.e., the trapdoor extraction for subvertible CRSs in Lemma 4 of [GKM⁺18]), suppose that there exists a PPT subverter Z that outputs a crs and ζ such that $\text{Vcrs}(1^\lambda, \text{crs}, \zeta) = 1$ (or more precisely $\text{Vcrs}(1^\lambda, \text{crs}_\Pi, \zeta_\Pi) = 1$) with non-negligible probability. Then, by using a proper knowledge assumption (i.e., the 0-MK assumption that is equivalent to the B-KEA assumption in [GKM⁺18]) there exists a PPT extractor Ext_Z that, given the random tape ω_Z of Z as input, outputs \mathbf{tc}_Π . In this case adversary \mathcal{A} is the adversary from Fig. 7 and $\text{RND}(\mathcal{A}) = \text{RND}(Z)$.

Also from the extractability property of the updating procedure (i.e., the trapdoor extraction for the updatable CRS in Lemma 5 of [GKM⁺18]) if Z outputs crs_{up} and ζ_{up} , then under the knowledge assumption there exists a PPT extractor Ext_Z that, given the randomness of Z as input, outputs \mathbf{tc}_Π (i.e., under the q-MK and the q-MC assumptions of [GKM⁺18]). For this case adversary \mathcal{A} is the adversary from Fig. 8 and $\text{RND}(\mathcal{A}) = \text{RND}(Z)$. Now to

$\mathcal{A}(\text{crs}; \omega_Z)$	$\text{Ext}_Z(\text{crs}; \omega_Z)$
$(\text{crs}, \text{aux}_Z) \leftarrow Z(\text{crs}; \omega_Z); \text{return pk};$	return \mathbf{tc}_Π ;

Fig. 8. The extractor and the constructed adversary \mathcal{A} from the Sub-ZK proof.

prove updatable zero-knowledge, we use the extractor Ext_Z and $\Pi.\text{Sim}$ algorithm that produces proofs π_{Sim} when provided the extracted trapdoor, such that any

proof π_{Sim} has the same distribution as a real proof π_{Π} (i.e., for the existence of such extractor $\text{Ext}_{\mathcal{Z}}$ and $\Pi.\text{Sim}$ algorithms, one can use the ones in Theorem 3 of [GKM⁺18]). Finally $\Pi.\text{Sim}$ can generate locally $(\text{sk}, \text{pk}) \leftarrow \Sigma.\text{KGen}(1^\lambda)$; $(\text{sk}_{\text{OT}}, \text{pk}_{\text{OT}}) \leftarrow \Sigma_{\text{OT}}.\text{KGen}(1^\lambda)$ and then compute $\sigma_{\text{OT}} \leftarrow \Sigma_{\text{OT}}.\text{Sign}(\text{sk}_{\text{OT}}, \pi_{\Pi} \| x \| \text{pk} \| \sigma)$ such that $\pi = (\pi_{\text{Sim}}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}})$ has the same distribution as a real proof $\pi = (\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}})$. Note that π_{Sim} is the simulated proof and π_{Π} is the real proof in the original updatable SNARK.

$\text{Exp}^{\text{up-se}}(\mathcal{A}, \lambda)$

```

1 :  $\omega_{\mathcal{Z}} \leftarrow \text{RND}(\mathcal{Z})$ ;  $(\text{crs} = (\text{crs}_{\Pi}, \text{c}_{\text{pk}}), \{\zeta_i\}_{i=1}^{i=n}, \text{aux}_{\mathcal{Z}}) \leftarrow \mathcal{Z}(1^\lambda, \omega_{\mathcal{Z}})$ ;
2 :  $(\text{crs}_{\text{up}}, \zeta_{\text{up}}) \leftarrow \text{Ucrs}(1^\lambda, \text{crs}, \{\zeta_i\}_{i=1}^{i=n})$ ;
3 : if  $\forall \text{crs}, \{\zeta_i\}_{i=1}^{i=n} = 0$  then return 0
4 :  $\text{tc}_{\text{c}_{\text{pk}}}^{\text{up}} \leftarrow \text{Ext}_{\mathcal{Z}}(1^\lambda, \text{crs}, \text{crs}_{\text{up}}, \{\zeta_i\}_{i=1}^{i=n}, \omega_{\mathcal{Z}})$ ;
5 :  $\omega_{\mathcal{A}} \leftarrow \text{RND}(\mathcal{A})$ ;  $(x, \pi) \leftarrow \mathcal{A}^{\text{O}(\text{crs}, \text{tc}, \cdot)}(\text{crs}, \text{crs}_{\text{up}}, \text{aux}_{\mathcal{Z}}, \omega_{\mathcal{A}})$ ;
6 :  $\text{Parse } \pi := (\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}})$ ;
7 :  $w \leftarrow \text{Ext}_{\mathcal{A}}(\text{crs}, \text{crs}_{\text{up}}, \omega_{\mathcal{A}})$ ;
8 : if  $(x, \pi) \notin \mathcal{Q} \wedge \mathcal{V}(\text{crs}_{\text{up}}, x, \pi) = 1 \wedge (x, w) \notin \mathcal{R}$  return 1.
9 : else return 0.

```

$\text{O}(\text{crs}, \text{tc}, x)$

```

1 :  $(\text{sk}, \text{pk}) \leftarrow \Sigma.\text{KGen}(1^\lambda)$ ;  $(\text{sk}_{\text{OT}}, \text{pk}_{\text{OT}}) \leftarrow \Sigma_{\text{OT}}.\text{KGen}(1^\lambda)$ ;
2 :  $\pi_{\Pi} \leftarrow \Pi.\text{Sim}(\text{crs}_{\text{up}}, x, (\perp, \perp); \text{tc}_{\text{c}_{\text{pk}}}^{\text{up}})$ ;  $\sigma \leftarrow \Sigma.\text{Sign}(\text{sk}, \text{pk}_{\text{OT}})$ ;
3 :  $\sigma_{\text{OT}} \leftarrow \Sigma_{\text{OT}}.\text{Sign}(\text{sk}_{\text{OT}}, \pi_{\Pi} \| x \| \text{pk} \| \sigma)$ ;
4 :  $\pi := (\pi_{\Pi}, \text{pk}, \sigma, \text{pk}_{\text{OT}}, \sigma_{\text{OT}})$ ;
5 :  $\mathcal{Q} := \mathcal{Q} \cup \{(x, \pi)\}$ ;  $\mathcal{T} := \mathcal{T} \cup \{\text{pk}_{\text{OT}}\}$ ;
6 : return  $\pi$ ;

```

Fig. 9. Experiment $\text{Exp}^{\text{up-se}}(\mathcal{A}, \lambda)$ for SE updatable SNARKs from LAMASSU.

(iii: Updatable strong simulation extractability): For the sake of simplicity, let the subverter \mathcal{Z} make only a single update after an honest setup or he first generates the CRS and after that, we have only a single update by an honest updater (this can easily be generalized by using Lemma 6 of [GKM⁺18], i.e., single adversarial updates imply full updatable SE).

We remind that based on the subvertible CRSs of the updatable SNARKs (i.e., the trapdoor extraction for subvertible CRSs in Lemma 4 in [GKM⁺18]), it is possible to extract the adversary's contribution to the trapdoor when the adversary generates the CRS itself. Also from the updatable property of the updatable SNARKs (i.e., the trapdoor extraction for updatable CRSs in Lemma 5 of [GKM⁺18]), it is possible to extract it when the adversary updates an honest CRS. To collapse chains of honest updates into an honest setup it is convenient that the trapdoor contributions of the setup and update commute in our scheme. As the trapdoor in our scheme consists of all the randomness used by these algorithms, we will from now on refer to chains of honest updates and (single) honest setups interchangeably. Note that in updatable SNARKs, the proof ζ

depends only on the relation and the randomness of the update algorithm and is independent of the CRS being updated.

Our proof is based on Theorem 1 where we replace the underlying NIZK with an updatable SNARK and also use simulation trapdoors of the SNARK to simulate proofs. Based on the updatability property, if \mathcal{A} outputs crs_{up} and ζ_{up} , then by the respective knowledge assumption of the SNARK (i.e., the \mathfrak{q} -MK and the \mathfrak{q} -MC assumptions in Lemma 5 of [GKM⁺18]) and the one of the updatable signature scheme implies that there exists a PPT extractor $\text{Ext}_{\mathcal{A}}$, that, given the randomness of \mathcal{A} as input, outputs $\text{tc} = (\text{tc}_{\Pi}, \text{tc}_{\text{cpk}})$. We note that the SE adversary \mathcal{A} in the updatable case besides seeing a pair (crs, π) may even already did update the crs . Thus, here \mathcal{A} has more power than the SE adversary against Sub-zk SNARK in Section 4.1 and the one in Theorem 1. To make the proof more precise, we use the subverter Z for updating the crs and the adversary \mathcal{A} against the SE property. Note that Z and \mathcal{A} can communicate to each other and $\text{RND}(Z) = \text{RND}(\mathcal{A})$.

We recall the experiment for updatable SE in Fig. 9 and we highlight changes by pointing to the line numbers in the experiment or the oracle.

Game₀ This is the original experiment in Fig. 9.

Game₁ This game is the same as **Game₀**, but **Sim** uses $\text{tc}_{\Pi}^{\text{up}}$ and generates the simulated proof π_{Π} .

Exp: 5: $\text{tc}^{\text{up}} \leftarrow \text{Ext}_Z(1^\lambda, \text{crs}, \text{crs}_{\text{up}}, \{\zeta_i\}_{i=1}^{i=n}, \omega_Z)$;
O: 2: $\pi_{\Pi} \leftarrow \text{Sim}(\text{crs}_{\text{up}}, \mathbf{x}, (\perp, \text{tc}_{\Pi}^{\text{up}}); \perp)$;

Winning condition: Let Q be the set of (\mathbf{x}, π) pairs, let T be the set of verification keys generated by **O**. The game outputs 1 iff: $(\mathbf{x}, \pi) \notin Q \wedge \forall (\text{crs}_{\text{up}}, \mathbf{x}, \pi) = 1 \wedge \text{pk}_{\text{OT}} \notin T \wedge \text{cpk} = \text{pk} \cdot \mu(\text{csk} - \text{sk})$.

Game₀ → Game₁ If the underlying one-time signature scheme is strongly unforgeable, and that the underlying updatable SNARK is knowledge sound, and the zero-knowledge property of the updatable SNARK holds, then we have $\Pr[\text{Game}_0] \leq \Pr[\text{Game}_1] + \text{negl}(\lambda)$.

The reason is that if $(\mathbf{x}, \mathbf{w}) \notin Q$ and pk_{OT} has been generated by **O**, then the $(\mathbf{x}, \pi_{\Pi}, \text{pk})$ is a valid message/signature pair. Hence by the unforgeability of the σ_{OT} signature scheme, we know that the case $(\mathbf{x}, \mathbf{w}) \notin Q$ and pk_{OT} generated by **O**, happens with negligible probability, which allows us to focus on $\text{pk}_{\text{OT}} \notin T$. The extracted \mathbf{w} is unique for all valid witnesses. Further, if some witness is valid for \mathcal{L} and that $(\mathbf{x}, \mathbf{w}) \notin \mathcal{R}$, we know it must be the case that due to the zero-knowledge property of the updatable SNARK and the property of the updating procedure that if Vcrs output 1, then there is an extractor that extracts the tc (i.e., the trapdoor extraction for subvertible CRSs in Lemma 4 of [GKM⁺18] and the one of the updatable signature scheme implies that it is possible to extract the trapdoor when the adversary generates the CRS itself), there exists some $\text{tc}_{\Pi}^{\text{up}}$ and $\text{tc}_{\text{cpk}}^{\text{up}}$ such that one can simulate the proof in a way that no polynomial-time algorithm can distinguish them.

Game₂ This game is the same as **Game₁**, but the only difference is that \mathcal{A} updates the crs .

Exp: 1: $(\text{crs}_\Pi, \text{tc}_\Pi, \zeta_\Pi) \leftarrow \Pi.\text{KGen}(1^\lambda); (\text{csk}, \text{cpk}, \zeta_{\text{cpk}}) \leftarrow \Sigma.\text{KGen}(1^\lambda); \text{crs} := (\text{crs}_\Pi, \text{cpk}), \text{tc} := (\text{tc}_\Pi, \text{csk}), \zeta := (\zeta_\Pi, \zeta_{\text{cpk}}); \text{return } (\text{crs}, \zeta);$

Exp: 2: $\omega_Z \leftarrow_{\mathcal{S}} \text{RND}(\mathbb{Z}); (\text{crs}_{\text{up}}, \zeta_{\text{up}}, \text{aux}_Z) \leftarrow \mathbb{Z}(1^\lambda, \text{crs}, \{\zeta_i\}_{i=1}^{i=n}, \omega_Z)$

Game₁ \rightarrow Game₂ This is straightforward from the property of the updating procedure that if $\mathbb{V}\text{crs}$ output 1, then there is an extractor that extracts the tc (i.e the trapdoor extraction for updatable CRS in Lemma 5 of [GKM⁺18] and the knowledge assumption of the updatable signature scheme, that it is possible to extract it when the adversary updates an honest CRS) and the zero-knowledge property of the updatable SNARK. Thus we have $\Pr[\text{Game}_0] \leq \Pr[\text{Game}_1] + \text{negl}(\lambda)$.

Game₃ This game is the same as **Game₂**, but $\Delta \leftarrow_{\mathcal{S}} \mathbb{H}$ is replaced in $\text{cpk} = \mu(\Delta) \cdot \text{pk}$.

Exp: 1: $\Delta \leftarrow_{\mathcal{S}} \mathbb{H};$

Exp: 2: $\text{crs} := (\text{crs}_\Pi, \text{cpk} \cdot \mu(\Delta)), \text{tc} := (\text{tc}_\Pi, \text{csk});$

Winning condition: Let Q be the set of (\mathbf{x}, π) pairs, let T be the set of verification keys generated by the \mathbb{O} . The game outputs 1 iff: $(\mathbf{x}, \pi) \notin Q \wedge \mathbb{V}(\text{crs}_{\text{up}}, \mathbf{x}, \pi) = 1 \wedge \text{pk}_{\mathbb{O}\top} \notin T \wedge \text{cpk} \cdot \mu(\Delta) = \text{pk} \cdot \mu(\Delta) \cdot \mu(\text{csk} - \text{sk})$.

Game₂ \rightarrow Game₃ It is straightforward from the Theorem 3, adaptable and updatable EUF-CMA property of Σ .