

Omniring: Scaling Up Private Payments Without Trusted Setup

Formal Foundations and Constructions of Ring Confidential Transactions with Log-size Proofs

Russell W. F. Lai^{*}, Viktoria Ronge^{*}, Tim Ruffing[†],
Dominique Schröder^{*}, Sri Aravinda Krishnan Thyagarajan^{*}, and Jiafan Wang[‡]

^{*}Friedrich-Alexander University Erlangen-Nürnberg, Germany

[†]Blockstream

[‡]The Chinese University of Hong Kong, Hong Kong

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Abstract

Monero is the largest cryptocurrency with built-in cryptographic privacy features. The transactions are authenticated using spend proofs, which provide a certain level of anonymity by hiding the source accounts from which the funds are sent among a set (known as a ring) of other accounts. Due to its similarities to ring signatures, this core cryptographic component is called Ring Confidential Transactions (RingCT). Because of its practical relevance, several works attempt to analyze the security of RingCT. However, due to the complexity of RingCT they are either informal, miss fundamental functionalities, or introduce undesirable trusted setup assumptions. Regarding efficiency, Monero currently deploys a scheme in which the size of the spend proof is linear in the ring size. This limits the ring size to only a few accounts, which in turn limits the acquired anonymity significantly and facilitates de-anonymization attacks.

As a solution to these problems, we present the first complete rigorous formalization of RingCT as a cryptographic primitive. We then propose a generic construction of RingCT and prove it secure in our formal security model. By instantiating our generic construction with new efficient zero-knowledge proofs we obtain *Omniring*, a fully-fledged RingCT scheme in the discrete logarithm setting that provides the highest concrete and asymptotic efficiency as of today. *Omniring* is the first RingCT scheme which 1) does not require a trusted setup or pairing-friendly elliptic curves, 2) has a proof size *logarithmic* in the size of the ring, and 3) allows to share the same ring between all source accounts in a transaction, thereby enabling significantly improved privacy level without sacrificing performance. Our zero-knowledge proofs rely on novel enhancements to the Bulletproofs framework (S&P 2018), which we believe are of independent interest.

1 Introduction

Cryptocurrencies such as Bitcoin and Monero eliminate the need for a trusted central party, giving rise to a fully decentralized and publicly verifiable currency systems. A cryptocurrency typically consists of two components: (1) a public ledger, *e.g.*, realized by a blockchain protocol, to publish transactions, and (2) a transaction scheme which specifies the format and validity of transactions. For example, we can think of the simplest transaction type in Bitcoin as simply requiring a signed statement of the form “Pseudonym i pays amount a to pseudonym j ”. This allows for easy public verification of the ledger but on the flip side, the inherently public nature of cryptocurrencies is a threat for the individual privacy of their users. At first glance, the usage of pseudonyms may give the impression that users are anonymous, but a long list of literature demonstrates that different pseudonyms belonging to the same user can be linked by simple as well as sophisticated heuristics when observing transaction on the public blockchain [3, 6, 31, 40, 41, 53, 59]. As a consequence, transactions can be traced and users can be re-identified.

In order to improve this situation, many privacy-enhancing technologies have been proposed by the academic and the cryptocurrency community [9, 11, 26, 27, 30, 36, 37, 42, 54, 61, 62, 65], and multiple cryptocurrencies [44, 51, 63, 64] with a special focus on privacy have emerged. Monero [44] (with a market capitalization of \$720M at the time of writing [20]) is the largest such privacy-focused cryptocurrency. In contrast to other approaches, most notably Zerocash [9], which relies on a trusted cryptographic setup to be able to scale to very large anonymity sets, one of the main design goals of Monero is avoid any form of trusted setup. This approach is arguably much closer to the original spirit of cryptocurrencies whose point is to avoid decentralization as much as possible.

For privacy, Monero uses *Ring Confidential Transactions* (RingCT) proposed by Noether *et al.* [47] as its cryptographic core component. The idea behind RingCT is an ad-hoc approach to integrate three privacy-enhancing technologies:

Its first component is similar in spirit to linkable ring signatures which guarantee some form of *anonymity* and enable to detect double-spends simultaneously. The second component of RingCT is Confidential Transactions (CT) [36], which hides the monetary amounts of transactions in homomorphic commitments while still being able to verify that the *balance* of a transaction, *i.e.*, the sum of the amounts sent to target accounts does not exceed the sum of amounts available in the source accounts. The third component is Stealth Addresses (SA), which provides a form of *receiver anonymity*. Given just a single long-term public key of a receiver (called stealth address), a spender can derive an arbitrary number of seemingly unrelated public keys owned by this receiver, which avoids that two transactions paying the same receiver can be linked.

Despite its practical importance, there is a lack of theoretical foundation of ring confidential transactions (RingCT) and satisfactory constructions. In this work, we provide the first complete formalization of RingCT. We also put forward a new construction of RingCT which is significantly better than existing ones in terms of supported features and practical efficiency. The efficiency improvements allow implementations to choose larger parameters that would strengthen privacy without sacrificing performance.

1.1 Our Contributions

Our main contributions are summarized as follows.

1.1.1 Rigorous Formalization of RingCT

The necessity of *precise* security models for cryptographic primitives cannot be overstated, as they concisely point out security guarantees, allow comparison, and serve as a guideline for protocol designs. An example which highlights this necessity is the denial-of-spending attack [55] on Zerocoin [26, 42], a different cryptographic approach for privacy in cryptocurrencies. This attack was not captured by the insufficient security model of Zerocoin and lead to vulnerabilities in multiple cryptocurrencies allowing an attacker to destroy funds of honest users. Notably, denial-of-spending attacks are not considered in the RingCT proposal by Noether *et al.* [47] either, even though they generally apply to the RingCT setting as well [55]. While the concrete RingCT scheme by Noether *et al.* [47] seems not susceptible to these attacks, the fact that the attacks have apparently been overlooked in [47] underlines the importance of a rigorous and thorough security model.

Sun *et al.* proposed the first formalization of RingCT. Unfortunately, their definition and their scheme do not cover stealth addresses, which is one of the central properties, and their security definitions are too weak, as they fail to cover some realistic attacks. Below, we highlight the strength of our model and defer the comparison with [60] to Appendix A.

Capturing Stealth Addresses Our model is the first one that captures stealth addresses. This is a critical component of the overall security model because it directly relates to receiver anonymity, it also affects all functionalities of the primitive itself, and all other security properties.

Non-reliance on External Communication Channels Our model only assumes a public ledger onto which transactions can be published and does not rely on any external secure channels.

Stronger Security Guarantees We provide stronger security definitions for balance and spender anonymity. In contrast to prior work, our definition requires balance if all accounts in the transaction are maliciously generated and in case of spender anonymity, we allow some of the source accounts to be corrupt and still require the other non-corrupt accounts to be anonymous.

Unified Ring for All Source Accounts All previous RingCT schemes use separate rings for each source accounts. This means that transactions that spend from multiple source accounts (as is common in cryptocurrencies), each source account is anonymous in a *separate* anonymity set. In our model, all source accounts of a single transaction share one ring, hence the name “Omniring”. This approach does not only improve efficiency, but it also improves the level of anonymity: Let us consider the case of spending from k source accounts. In the separated-rings approach, each source account is hidden within a different ring of some size n , meaning that each of the k source accounts has at most 1-out-of- n anonymity. On the other hand, in the unified ring approach of ours, having a ring of size kn offers up to k -out-of- kn anonymity.

Now consider for instance the case that one of the real source accounts used for spending is de-anonymized. In the unified ring approach, the other real source accounts now still have $(k-1)$ -out-of- $(kn-1)$ anonymity, *i.e.*, all other accounts in the unified ring still count towards the crowd to hide in. However, in the separated-ring approach, the entire ring containing the de-anonymized account would be useless for anonymity after de-anonymization.

1.1.2 Efficient Construction

We propose a new construction of RingCT, Omniring, whose spend proof size is only $O(\log(|\mathcal{R}||\mathcal{S}|+\beta|\mathcal{T}|))$, where $|\mathcal{R}|$, $|\mathcal{S}|$, and $|\mathcal{T}|$ are the size of the ring, the set of source accounts, and the set of target accounts respectively, and 2^β is the maximum allowed amount to be transferred. It is the first scheme that does not require a trusted setup or pairings, supports stealth addresses, and has a logarithmic spend proof size.

Our construction follows the high-level idea of combining a ring signature scheme with a range proof system. We first propose a generic construction of RingCT from signatures of knowledge (SoK) of a certain language. In the second step, we develop techniques to extend the Bulletproof framework [16] into an argument system for proving knowledge of a discrete logarithm representation, where the exponents in the representation satisfies an arbitrary arithmetic circuit. The statements of the desired language can be expressed in a format which is optimized for the extended Bulletproof framework, leading to a very efficient RingCT construction. We argue that this extension of ours has potentially far reaching consequences: It leads to a natural construction of logarithmic-size ring signatures, which is $\sim 20\%$ shorter than the state of the art [14, 15, 26]. The technique can also be generalized to proving general (bilinear) group arithmetic relations [4], which makes it a topic of independent interest.

We defer the overview of the techniques enabling this construction to Section 1.3.

1.1.3 Adaption to Monero

Our main instantiation presented in Section 4.4 is designed to simplify the language that it induces, and is not directly applicable to Monero due to the difference in format of the tags (or the so called “key images”). To tackle this issue, we formally detail in Appendix G how our Omniring construction can be adapted and readily deployed in Monero. The adaption retains essentially the same efficiency as our main instantiation except for a slightly higher computational effort for the verifier. We refer to Appendix G for more details.

1.2 On Ring Selection

Our formalization follows the spirit of ring signatures and does not cover how rings are sampled. We believe that this question of formalizing what “good” ring sampler is, is orthogonal to the properties of a RingCT scheme. The study of ring sampling strategies also does not seem to be a cryptographic, as the ring sampler does not involve any cryptographic keys. We believe that understanding the strategies of ring selection, and hence maximizing the non-deanonymized subsets of sampled rings, are important questions that deserve an in-depth investigation in an independent paper. Our view is motivated by recent attacks against the anonymity provided by cryptocurrency

based on RingCT as discussed by Möser *et al.* [45] and Kumar *et al.* [32]. These works show that the ring sampling strategies of the spenders are critical.

What we do formalize is the intuition that, given a ring of accounts selected by some external mechanisms, the source accounts of a transaction are hidden within the *non-deanonimized* subset of the ring.

1.3 Technical Overview

Recall that the statements to be proven in our RingCT construction use signature of knowledge and consists of two parts. The first part corresponds to knowing the secret key of one of the ring accounts and second part guarantees that the amount being transferred lies within a certain range. The first natural idea to construct a RingCT is to combine a state-of-the-art ring signature scheme (*e.g.*, by Groth and Kohlweiss [26] or Bootle and Groth [12]) with the most efficient range proof to date, namely Bulletproof [16] with a logarithmic size in the number of bits in the range. However, since both works use significantly different techniques, combining them naïvely yields a RingCT scheme with signature size asymptotically equal to that of the proposed one, but with worse concrete efficiency.

We then explore the possibility of building a ring signature scheme using the techniques of [16], so that it can be combined natively with all Bulletproof range proofs into one single Bulletproof of a single combined statement, leveraging the logarithmic size of Bulletproofs as much as possible. In order to understand the challenge that we tackled while exploring this path, recall that Bulletproof is a framework for proving “inner product relations” between the exponents in a discrete logarithm representation. Although the Bulletproof framework is expressive enough for capturing arithmetic circuits satisfiability, it is particularly optimized for range proofs. For instance, proving that a committed integer a lies within a range $[0, 2^\beta - 1]$ yields a proof whose size is only $O(\log \beta)$.

In Bulletproof range proof, the prover encodes the binary representation of a in the vector $\vec{\mathbf{a}}$ and sets $\vec{\mathbf{b}} = \vec{\mathbf{a}} - \vec{\mathbf{1}}^{|\vec{\mathbf{a}}|}$. It commits to $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ as $A = h^\alpha \vec{\mathbf{g}}^{\vec{\mathbf{a}}} \vec{\mathbf{h}}^{\vec{\mathbf{b}}}$ with randomness α and public (vector of) group elements h , $\vec{\mathbf{g}}$, and $\vec{\mathbf{h}}$. It then proves that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ satisfy the Hadamard product $\vec{\mathbf{a}} \circ \vec{\mathbf{b}} = \vec{\mathbf{0}}^{|\vec{\mathbf{a}}|}$, $\vec{\mathbf{a}} - \vec{\mathbf{b}} = \vec{\mathbf{1}}^{|\vec{\mathbf{a}}|}$, and $\langle \vec{\mathbf{a}}, \vec{\mathbf{2}}^{|\vec{\mathbf{a}}|} \rangle = a$. These relations guarantee that $\vec{\mathbf{a}}$ is a valid binary representation of a . Moreover, the length $|\vec{\mathbf{a}}|$ guarantees that a must be between 0 and $2^{|\vec{\mathbf{a}}|} - 1$. The extractability of the proof crucially depends on the assumption that non-trivial discrete logarithm representations of the identity element with respect to base $(h \parallel \vec{\mathbf{g}} \parallel \vec{\mathbf{h}})$ are unknown to the prover. Under such an assumption, one can extract the exponents $(\alpha, \vec{\mathbf{a}}, \vec{\mathbf{b}})$ in the discrete logarithm representation of A .

With the basics above, we can highlight the technical difficulties one would encounter, when attempting to construct ring membership proofs using the Bulletproof techniques.

Let the vector of group elements $\vec{\mathbf{R}} = (R_1 \parallel \dots \parallel R_n)$ consist of the public keys of the ring members. In a ring membership proof, one would like to prove the knowledge of a tuple (i, x_i) such that $R_i = H^{x_i}$ for a public generator H . Equivalently, the prover would prove its knowledge of a unit vector $\vec{\mathbf{e}}_i$ (whose i -th entry is 1 and zero everywhere else) and an integer $-x_i$ such that $I = H^{-x_i} \vec{\mathbf{R}}^{\vec{\mathbf{e}}_i}$ where I is the identity element; we call this relation the *main equality*.

An idea of using the Bulletproof technique for ring membership proof is to somehow embed the expression $H^{-x_i} \vec{\mathbf{R}}^{\vec{\mathbf{e}}_i}$ into a part of the commitment A , show that $\vec{\mathbf{e}}_i$ is indeed a unit vector by defining certain inner product relations, and at the same time show that the main equality holds. Finding a way to embed the expression in A is surely one of the main challenges. Added to the challenge, regardless of how the expression is embedded, is that the prover might know about the discrete logarithms between the elements in $\vec{\mathbf{R}}$, which forbids one to argue about soundness as done for Bulletproof. We propose a general technique to embed the main equality, or in general any representation of the identity element, into the commitment A , and avoid the above problem regarding soundness.

First, we observe that if $\vec{\mathbf{p}}$ is a vector of group elements chosen randomly and independently of $\vec{\mathbf{R}}$, then for any $w \in \mathbb{Z}_q$, the discrete logarithm representation problem base $\vec{\mathbf{g}}_w := (H \parallel \vec{\mathbf{R}})^w \circ \vec{\mathbf{p}}$ is equivalent to the standard discrete logarithm problem.

Second, let $\vec{\mathbf{a}} = (-x_i \parallel \vec{\mathbf{e}}_i)$. Note that $\vec{\mathbf{g}}_w^{\vec{\mathbf{a}}} = \vec{\mathbf{g}}_w^{\vec{\mathbf{a}}}$, for any $w, w' \in \mathbb{Z}_q$ due to the main equality (introduced above). Therefore, if $A = h^\alpha \vec{\mathbf{g}}_w^{\vec{\mathbf{a}}} \vec{\mathbf{h}}^{\vec{\mathbf{b}}}$ for some $w \in \mathbb{Z}_q$ and some vector of group elements $\vec{\mathbf{b}}$, then for any other $w' \in \mathbb{Z}_q$ it also holds that $A = h^\alpha \vec{\mathbf{g}}_{w'}^{\vec{\mathbf{a}}} \vec{\mathbf{h}}^{\vec{\mathbf{b}}}$.

With the above observations, we let the prover run a Bulletproof-like protocol twice on A with respect to two different bases, *i.e.*, $(h \parallel \vec{\mathbf{g}}_w \parallel \vec{\mathbf{h}})$ and $(h \parallel \vec{\mathbf{g}}_{w'} \parallel \vec{\mathbf{h}})$. If the prover is able to convince the verifier in both executions, then we can construct an extractor which extracts the exponents $(\alpha, \vec{\mathbf{a}}, \vec{\mathbf{b}})$ such that $A = h^\alpha \vec{\mathbf{g}}_w^{\vec{\mathbf{a}}} \vec{\mathbf{h}}^{\vec{\mathbf{b}}} = h^\alpha \vec{\mathbf{g}}_{w'}^{\vec{\mathbf{a}}} \vec{\mathbf{h}}^{\vec{\mathbf{b}}}$. Dividing the two representations of A yields the main equality.

At this point, we have already obtained a “Bulletproof-friendly” protocol for ring membership proofs, which can be combined with the range proofs of [16] to construct a very efficient spend algorithm in RingCT. However, this approach requires to execute the Bulletproof protocol twice, which blows up the proof size by a factor of 2. Our third observation allows to compress the two Bulletproof executions into one. Recall that in the second observation we have $A = h^\alpha \bar{\mathbf{g}}_w^{\mathbf{a}} \bar{\mathbf{h}}^{\mathbf{b}}$ for all $w \in \mathbb{Z}_q$. Therefore the prover can first compute $A = h^\alpha \bar{\mathbf{g}}_0^{\mathbf{a}} \bar{\mathbf{h}}^{\mathbf{b}}$ (i.e., $w=0$) without knowing w , and then obtain a random w as a challenge by hashing A . If the prover is able to convince the verifier with a randomly chosen w , then in the proof of extractability we can run the prover twice on w and w' respectively, and then apply the analysis mentioned above.

2 Formalizing RingCT

We present a complete formalization of ring confidential transactions (RingCT), which in particular incorporates the stealth address feature. We first describe the intended use of each algorithm along with some conventions that we adopt (Section 2.1), then provide a formalization of the core syntax (Section 2.2). In Appendix C, we further extend the syntax to tracking and viewing features, which enable a user to delegate detection and decoding of incoming transactions.

2.1 Overview

We overview the core functionality of RingCT.

Setup and Joining A RingCT scheme is initialized by running the **Setup** algorithm. Anyone can join the system by generating a key-tuple (mpk, msk) using **SAKGen**, where mpk is the master public key, msk is the master secret key. The master public key is also called a stealth address as it allows the derivation of one-time target accounts for receiving funds.

Transaction A transaction tx consists of a set of ring accounts $\{\text{acc}_i^{\mathcal{R}}\}_{i=1}^{|\mathcal{R}|}$, tags $\{\text{tag}_i\}_{i=1}^{|\mathcal{S}|}$, target accounts $\{\text{acc}_i^{\mathcal{T}}\}_{i=1}^{|\mathcal{T}|}$, and some message μ . The “(linkability) tag” is known in the context of cryptocurrencies as “serial number” (ZCash) or “key image” (Monero), but we follow the name convention of linkable ring signatures. The public key is bound to a secret key sk , while the coin is bound to a secret coin key ck and a secret amount a .

Spending Let

$$\mathcal{R} = \{\text{acc}_i^{\mathcal{R}}\}_{i=1}^{|\mathcal{R}|} \text{ and } \mathcal{S} = \{(j_i, \text{ck}_i^{\mathcal{S}}, a_i^{\mathcal{S}}, \text{sk}_i, \text{tag}_i)\}_{i=1}^{|\mathcal{S}|},$$

where \mathcal{R} is a set of ring accounts sampled by some external mechanism, such that a user knows a set of indices, secret keys, coin keys, amounts and tags \mathcal{S} corresponding to the source accounts $\{\text{acc}_j^{\mathcal{R}}\}_{j=1}^{|\mathcal{S}|}$. The user can transfer a batch of amounts $\{a_i^{\mathcal{T}}\}_{i=1}^{|\mathcal{T}|}$ to the owners of the stealth addresses $\{\text{mpk}_i\}_{i=1}^{|\mathcal{T}|}$ as follows. It first executes **OTAccGen** on each $(\text{mpk}_i, a_i^{\mathcal{T}})$ tuple to generate a one-time target account $\text{acc}_i^{\mathcal{T}}$, and a coin key ck_i . This process is sometimes known as minting. The account $\text{acc}_i^{\mathcal{T}}$ will be made publicly available, while the coin key ck_i is kept secret by the spender. Let $\mathcal{T} = \{(\text{ck}_i^{\mathcal{T}}, a_i^{\mathcal{T}}, \text{acc}_i^{\mathcal{T}})\}_{i=1}^{|\mathcal{T}|}$ specifying the target accounts and other relevant information, and μ be some additional message that the user wishes to include as part of the transaction. The user runs **Spend** on $(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu)$ to create a proof σ that the transaction tx (defined by $(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu)$) is valid. The transaction tx and the proof σ are then published (e.g., in a public ledger) for verification. In practice, there may be parts of the source and target accounts, e.g. trapdoor information intended to be used by the receiver, that are not necessary for public verification. They can instead be sent to the receiver off-chain; we do not model this explicitly.

Verification Once a tuple (tx, σ) is published, all parties can run the Vf algorithm to verify its validity. Roughly speaking, a spend proof σ is considered valid for a transaction tx if it demonstrates that the total amounts in the source accounts are equal to that in the target accounts. The infeasibility of forging a proof on an invalid transaction will be formally modeled as the balance property defined in Section 3.1.

In a cryptocurrency system, the verifiers need to perform two additional checks other than the validity of the proof to verify the validity of the transaction in the context of the ledger. First, they need to check that none of the tags in $\{\text{tag}_i\}_{i=1}^{|\mathcal{S}|}$ appeared in a previous transaction (double-spending); the balance property will guarantee this check is sufficient to detect double-spending. Second, they need to check that each of the ring accounts $\{\text{acc}_i^{\mathcal{R}}\}_{i=1}^{|\mathcal{R}|}$ in the transaction tx has been a target account in some previous transaction, or is a “coinbase” account, *i.e.*, an account with which newly mined coins are generated. In the latter case, the miner runs OTAccGen to generate a new account with his own stealth address and publishes the created account together with the amount and coin key to claim newly mined coins in a coinbase account; then verifiers can verify the correctness of the claimed amount by CheckAmount .

Receiving A user can receive funds from a target account acc that it owns by running Receive on acc which was published (*e.g.*, via a public ledger) as part of a transaction. Using the output of Receive , which includes a secret key of the target account, the amount in the account and its tag, the receiver can later spend the received coin in another transaction.

In a cryptocurrency system, the receiver will need to check that he has not already received a different set of funds in an account with the same tag, because then only set of funds be spendable and the other will be considered a double-spend; this malicious reuse of the account by the spender is known as the faerie gold attack [46] or burning bug [21]. To ensure that this check is sufficient, the balance property will guarantee that tag output by Receive is indeed the correct tag for the given target account, as well as that the amount output by Receive is the amount which is actually stored in the account.

2.2 Formal Syntax

Definition 2.1. A ring confidential transaction (*RingCT*) scheme consists of a tuple of main PPT algorithms $(\text{Setup}, \text{SAKGen}, \text{OTAccGen}, \text{Receive}, \text{Spend}, \text{Vf})$, and a tuple of auxiliary PPT algorithms $(\text{CheckAmount}, \text{CheckTag})$ defined as follows.

$\text{pp} \leftarrow \text{Setup}(1^\lambda, 1^{\alpha}, 1^{\beta})$: The setup algorithm takes as input the security parameter 1^λ and integers $1^\alpha, 1^\beta$, where 1^α represents an upper bound 2^α on the number of outputs in a single transaction and 1^β an upper bound 2^β of amounts to be transferred in a transaction. It outputs the public parameter pp to be given to all algorithms implicitly.

$(\text{mpk}, \text{msk}) \leftarrow \text{SAKGen}(\text{pp})$: The stealth address / master key generation algorithm inputs the public parameter pp , and outputs a master public key mpk and a master secret key msk . The master public key mpk is also known as a stealth address.

$(\text{ck}, \text{acc}) \leftarrow \text{OTAccGen}(\text{mpk}, a)$: The one-time account generation algorithm takes as input a master public key mpk and an amount $a \in \{0, \dots, 2^\beta - 1\}$. It outputs a coin key ck and an account acc .

$(\text{ck}, a, \text{sk}, \text{tag}) \leftarrow \text{Receive}(\text{msk}, \text{acc})$: The receive algorithm takes as input a master secret key msk and an account acc . It outputs a coin key ck , an amount a , a secret key sk , and a tag.

$\sigma \leftarrow \text{Spend}(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu)$: The spend algorithm inputs the following:

- $\mathcal{R} = \{\text{acc}_i^{\mathcal{R}}\}_{i=1}^{|\mathcal{R}|}$: a set of ring accounts $\text{acc}_i^{\mathcal{R}}$
- $\mathcal{S} = \left\{ (j_i, \text{ck}_i^{\mathcal{S}}, a_i^{\mathcal{S}}, \text{sk}_i, \text{tag}_i) \right\}_{i=1}^{|\mathcal{S}|}$: a set of tuples consisting of an index $j_i \in [|\mathcal{R}|]$, a coin key $\text{ck}_i^{\mathcal{S}}$, a secret key sk_i , an amount $a_i^{\mathcal{S}}$, and a tag tag_i (of $\text{acc}_{j_i}^{\mathcal{R}}$)
- $\mathcal{T} = \left\{ (\text{ck}_i^{\mathcal{T}}, a_i^{\mathcal{T}}, \text{acc}_i^{\mathcal{T}}) \right\}_{i=1}^{|\mathcal{T}|}$: a set of tuples consisting of a coin key $\text{ck}_i^{\mathcal{T}}$, an amount $a_i^{\mathcal{T}}$, and a target account $\text{acc}_i^{\mathcal{T}}$
- μ : an additional message

It outputs a proof σ .

$b \leftarrow \text{Vf}(\text{tx}, \sigma)$: The verify algorithm inputs a transaction tx and a signature σ . It outputs a bit b indicating the validity of σ . A transaction tx defined as follows:

$$\text{tx}(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu) := \left(\left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_i \right\}_{i=1}^{|\mathcal{S}|}, \left\{ \text{acc}_i^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|}, \mu \right)$$

$b \leftarrow \text{CheckAmount}(\text{acc}, \text{ck}, a)$: The check amount algorithm inputs an account acc , a coin key ck , and an amount a . It outputs a bit b indicating the consistency of the inputs.

$b \leftarrow \text{CheckTag}(\text{acc}, \text{sk}, \text{tag})$: The check tag algorithm inputs an account acc , a secret key sk , and a tag tag . It outputs a bit b indicating the consistency of the inputs.

Definition 2.2 (Correctness). A RingCT scheme is said to be correct, if the following holds for all $\lambda, \alpha, \beta \in \mathbb{N}$, and all $\text{pp} \in \text{Setup}(1^\lambda, 1^\alpha, 1^\beta)$.

- *Honestly generated payments should be received correctly.* Concretely, for any $(\text{mpk}, \text{msk}) \in \text{SAKGen}(\text{pp})$, any amount $a \in \{0, \dots, 2^\beta - 1\}$, any $(\text{ck}, \text{acc}) \in \text{OTAccGen}(\text{mpk}, a)$, and any $(\text{ck}', a', \text{sk}, \text{tag}) \in \text{Receive}(\text{msk}, \text{acc})$, it holds that $(\text{ck}, a) = (\text{ck}', a')$.
- *Correctly received payments should have well-defined amounts and tags.* Concretely, for any $(\text{ck}, a, \text{sk}, \text{tag}) \in \text{Receive}(\text{msk}, \text{acc})$ ($\neq \perp$), it holds that $\text{CheckAmount}(\text{acc}, \text{ck}, a) = 1$ and $\text{CheckTag}(\text{acc}, \text{sk}, \text{tag}) = 1$.
- *Honestly generated transactions should be recognized as valid.* Concretely, for any tuple $(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu)$ with syntax defined in Definition 2.1 and satisfying the following conditions:

- $|\mathcal{T}| \leq 2^\alpha$
- for all $i \in [|\mathcal{T}|]$, $a_i^{\mathcal{T}} \in \{0, \dots, 2^\beta - 1\}$
- for all $i \in [|\mathcal{S}|]$, $\text{CheckTag}(\text{acc}_{j_i}^{\mathcal{R}}, \text{sk}_i, \text{tag}_i) = 1$
- for all $i \in [|\mathcal{S}|]$, $\text{CheckAmount}(\text{acc}_{j_i}^{\mathcal{R}}, \text{ck}_i^{\mathcal{S}}, a_i^{\mathcal{S}}) = 1$
- for all $i \in [|\mathcal{T}|]$, $\text{CheckAmount}(\text{acc}_i^{\mathcal{T}}, \text{ck}_i^{\mathcal{T}}, a_i^{\mathcal{T}}) = 1$
- $\sum_{i \in [|\mathcal{S}|]} a_i^{\mathcal{S}} = \sum_{i \in [|\mathcal{T}|]} a_i^{\mathcal{T}}$

and any proof $\sigma \in \text{Spend}(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu)$, it holds that $\text{Vf}(\text{tx}, \sigma) = 1$, where $\text{tx} = \text{tx}(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu)$.

3 Security of RingCT

We formalize the security notions of RingCT, which consist of balance, privacy, and non-slanderability. The oracles used in the security games are shown in Figure 1. Unlike prior works, our definitions take into account stealth address. The definitions are further extended to account for the tracking and viewing features in Appendix C.

3.1 Balance

Balance roughly means that a spender cannot double-spend, or spend more than what it possesses. The formal definition (Definition 3.1) is more complicated than one would initially expect because the amounts being transferred in a transaction are confidential. In more detail, we say that a RingCT scheme is balanced if the following two properties are satisfied.

First, the predicates CheckTag and CheckAmount are required to be “binding” in a sense similar to a commitment scheme. The binding property of CheckTag ensures that a tag is computationally bound to a source account, which in turn ensures that checking for duplicate tags is sufficient to prevent double-spending. Similarly, the binding property of CheckAmount ensures that an amount is computationally bound to an account, which ensures that money cannot be “created out of thin air” by changing the amount of coins in a given account. This

<pre> InitOracles() // Initialize Lists MPK := MSK := Received := Wallet := ∅ // Initialize Sets Spent := Σ := ∅ SAKGenO() // Generate keys for a new honest user. (msk, mpk) ← SAKGen(pp) MPK := MPK mpk, MSK := MSK msk return mpk SpendO(I, R, S, T, μ) // Instruct honest spender(s) to generate a proof. // R and S are (incomplete) lists containing malicious // information. // I contains instructions for populating R and S with information of honest spenders. // For each (s_i, j_i, k_i, l_i) in I, fill in S[s_i] and R[j_i] // using data retrieved from Wallet[k_i][l_i]. parse I as {(s_i, j_i, k_i, l_i)}_{i=1}^{ I } for i ∈ [I] do (ck_i, a_i, sk_i, tag_i, acc_i) := Wallet[k_i][l_i] R[j_i] := acc_i S[s_i] := (j_i, ck_i, a_i, sk_i, tag_i) endfor tx := tx(R, S, T, μ) σ ← Spend(R, S, T, μ) Σ := Σ ∪ {(tx, σ)} if ∀(tx_t, σ_t) = 0 then return 0 Spent := Spent ∪ {tag_i}_{i=1}^{ I } return σ </pre>	<pre> ReceiveO(k, acc) // Instruct user k to receive from acc. // Store the output in Wallet[k] for SpendO. Received := Received (k, acc) if Receive(MSK[k], acc) = ⊥ then return "Fail" (ck, a, sk, tag) ← Receive(MSK[k], acc) Wallet[k] := Wallet[k] (ck, a, sk, tag, acc) return "Success" </pre>
--	---

Figure 1: Oracles for Security Experiments

formalization does not contradict the mining of new coins, because this is modeled by explicitly creating a new account (see Section 2.1). These binding properties make the balance experiment, defined below, meaningful.

The second property requires that, for any efficient adversary \mathcal{A} which produces a transaction with a proof, there exists an extractor $\mathcal{E}_{\mathcal{A}}$ such that, if the proof is valid (Event 0), then the probability of the following inconsistencies (Event 1 to 4) occurring is negligible.

- Event 1: The extractor $\mathcal{E}_{\mathcal{A}}$ extracted an index j_i and a secret key sk_i , yet sk_i is inconsistent with the j_i -th ring account $acc_{j_i}^{\mathcal{R}}$ and the i -th tag tag_i according to the predicate `CheckTag`.
- Event 2: The extractor $\mathcal{E}_{\mathcal{A}}$ extracted an index j_i , a coin key $ck_i^{\mathcal{S}}$, and an amount $a_i^{\mathcal{S}}$, yet $(ck_i^{\mathcal{S}}, a_i^{\mathcal{S}})$ is inconsistent with the j_i -th ring account $acc_{j_i}^{\mathcal{R}}$ according to the predicate `CheckAmount`.
- Event 3: The extractor $\mathcal{E}_{\mathcal{A}}$ extracted a coin key $ck_i^{\mathcal{T}}$, and an amount $a_i^{\mathcal{T}}$, yet $(ck_i^{\mathcal{T}}, a_i^{\mathcal{T}})$ is inconsistent with the i -th target account $acc_i^{\mathcal{T}}$ according to the predicate `CheckAmount`.
- Event 4: The amounts extracted by $\mathcal{E}_{\mathcal{A}}$ are not balanced, *i.e.*, the sum of the target amounts exceeds the sum of the source amounts.

The two properties can be interpreted in the following way. If a spender (*e.g.*, the adversary) is able to produce a transaction with a valid proof, then it must possess knowledge of balanced input and output amounts, as they

<div style="border-bottom: 1px solid black; margin-bottom: 10px;"> $\text{Balance}_{\Omega, \mathcal{A}, \mathcal{E}_{\mathcal{A}}}(1^\lambda, 1^\alpha, 1^\beta)$ </div> <p> $\text{pp} \leftarrow \text{Setup}(1^\lambda, 1^\alpha, 1^\beta)$ $(\text{tx}, \sigma) \leftarrow \mathcal{A}(\text{pp})$ $(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu) \leftarrow \mathcal{E}_{\mathcal{A}}(\text{pp}, \text{tx}, \sigma)$ parse \mathcal{R} as $\left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{ \mathcal{R} }$ parse \mathcal{S} as $\left\{ (j_i, \text{ck}_i^{\mathcal{S}}, a_i^{\mathcal{S}}, \text{sk}_i, \text{tag}_i) \right\}_{i=1}^{ \mathcal{S} }$ parse \mathcal{T} as $\left\{ (\text{ck}_i^{\mathcal{T}}, a_i^{\mathcal{T}}, \text{acc}_i^{\mathcal{T}}) \right\}_{i=1}^{ \mathcal{T} }$ $b_0 := \forall f(\text{tx}, \sigma)$ $b_1 := \left(\exists i \in [\mathcal{S}] \text{ s.t. } \text{CheckTag}(\text{acc}_{j_i}^{\mathcal{R}}, \text{sk}_i, \text{tag}_i) = 0 \right)$ $b_2 := \left(\exists i \in [\mathcal{S}] \text{ s.t. } \text{CheckAmount}(\text{acc}_{j_i}^{\mathcal{R}}, \text{ck}_i^{\mathcal{S}}, a_i^{\mathcal{S}}) = 0 \right)$ $b_3 := \left(\exists i \in [\mathcal{T}] \text{ s.t. } \text{CheckAmount}(\text{acc}_i^{\mathcal{T}}, \text{ck}_i^{\mathcal{T}}, a_i^{\mathcal{T}}) = 0 \right)$ $b_4 := \left(\sum_{i \in [\mathcal{S}]} a_i^{\mathcal{S}} < \sum_{i \in [\mathcal{T}]} a_i^{\mathcal{T}} \right)$ return $b_0 \wedge (b_1 \vee b_2 \vee b_3 \vee b_4)$ </p>
--

Figure 2: Balance Experiment

can be extracted by $\mathcal{E}_{\mathcal{A}}$. If the actual amounts of the source and target accounts are different from those extracted by the extractor, *e.g.*, the spender attempts to create money “out of thin air”, then one can break the binding property of **CheckAmount**. Therefore the amounts that the spender has in mind cannot be different from those extracted by $\mathcal{E}_{\mathcal{A}}$. Similarly, if the spender attempts to spend from the same account twice by producing different tags for the account, then with the spender and the extractor $\mathcal{E}_{\mathcal{A}}$ one can break the binding property of **CheckTag**. Therefore double-spending is infeasible.

```

Privacy $_{\Omega, \mathcal{A}}^b(1^\lambda, 1^\alpha, 1^\beta)$ 


---


pp  $\leftarrow$  Setup( $1^\lambda, 1^\alpha, 1^\beta$ ), InitOracles()
 $\mathcal{O} := \{\text{SAKGen}\mathcal{O}, \text{Spend}\mathcal{O}, \text{Receive}\mathcal{O}\}$ 
 $(I, J, \mathcal{R}, \mathcal{S}, \mathcal{T}, \mu) \leftarrow \mathcal{A}^{\mathcal{O}}(\text{pp})$ 
 $\mathcal{S}_0 := \mathcal{S}_1 := \mathcal{S}, \mathcal{T}_0 := \mathcal{T}_1 := \mathcal{T}$ 
// Preparing honest spenders as instructed by adversary.
parse  $I$  as  $\left\{ (s_i, \{j_{t,i}, k_{t,i}^{\mathcal{S}}, \ell_{t,i}^1\}_{t=0}^1) \right\}_{i=1}^{|I|}$ 
for  $i \in [|I|]$  do
  for  $t \in \{0, 1\}$  do
     $(\text{ck}_{t,i}^{\mathcal{S}}, \text{sk}_{t,i}^{\mathcal{S}}, a_{t,i}^{\mathcal{S}}, \text{tag}_{t,i}^{\mathcal{S}}, \text{acc}_{t,i}^{\mathcal{S}}) := \text{Wallet}[k_{t,i}^{\mathcal{S}}][\ell_{t,i}]$ 
     $\mathcal{R}[j_{t,i}] := \text{acc}_{t,i}^{\mathcal{S}}$ 
     $\mathcal{S}_t[s_i] := (j_{t,i}, \text{ck}_{t,i}^{\mathcal{S}}, \text{sk}_{t,i}^{\mathcal{S}}, a_{t,i}^{\mathcal{S}}, \text{tag}_{t,i}^{\mathcal{S}})$ 
  endfor
  if  $\text{tag}_{0,i} \neq \text{tag}_{1,i} \wedge \{\text{tag}_{0,i}, \text{tag}_{1,i}\} \cap \text{Spent} \neq \emptyset$  then return 0
endfor
// Preparing honest receivers as instructed by adversary.
parse  $J$  as  $\left\{ (d_j, \{k_{t,j}^{\mathcal{T}}, a_{t,j}^{\mathcal{T}}\}_{t=0}^1) \right\}_{j=1}^{|J|}$ 
for  $j \in [|J|]$  do
  for  $t \in \{0, 1\}$  do
     $(\text{ck}_{t,j}^{\mathcal{T}}, \text{acc}_{t,j}^{\mathcal{T}}) := \text{OTAccGen}(\text{MPK}[k_{t,j}^{\mathcal{T}}], a_{t,j}^{\mathcal{T}})$ 
     $\mathcal{T}_t[d_j] := (\text{ck}_{t,j}^{\mathcal{T}}, a_{t,j}^{\mathcal{T}}, \text{acc}_{t,j}^{\mathcal{T}})$ 
  endfor
endfor
for  $t \in \{0, 1\}$  do
   $\text{tx}_t := \text{tx}(\mathcal{R}, \mathcal{S}_t, \mathcal{T}_t, \mu)$ 
   $\sigma_t \leftarrow \text{Spend}(\mathcal{R}, \mathcal{S}_t, \mathcal{T}_t, \mu)$ 
  if  $\forall f(\text{tx}_t, \sigma_t) = 0$  then return 0
endfor
 $b_0 \leftarrow \mathcal{A}^{\mathcal{O}}(\text{tx}_b, \sigma_b)$ 
 $b_1 := \left( \left\{ k_{t,j}^{\mathcal{T}} : t \in \{0, 1\}, j \in [|J|] \right\} = \emptyset \right)$ 
 $b_2 := \left( (\text{Received}) \cap \left\{ (k_{t,j}^{\mathcal{T}}, \text{acc}_{b,j}^{\mathcal{T}}) : t \in \{0, 1\}, j \in [|J|] \right\} = \emptyset \right)$ 
return  $b_0 \wedge b_1 \wedge b_2$ 

```

Figure 3: Privacy Experiment

```

NSland $_{\Omega, \mathcal{A}}(1^\lambda, 1^\alpha, 1^\beta)$ 


---


pp  $\leftarrow$  Setup( $1^\lambda, 1^\alpha, 1^\beta$ ), InitOracles()
 $(\text{tx}^*, \sigma^*) \leftarrow \mathcal{A}^{\text{SAKGen}\mathcal{O}, \text{Receive}\mathcal{O}, \text{Spend}\mathcal{O}}(\text{pp})$ 
parse  $\text{tx}^*$  as  $\left( \left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_i \right\}_{i=1}^{|\mathcal{S}|}, \left\{ \text{acc}_i^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|}, \mu \right)$ 
 $b_0 := \forall f(\text{tx}^*, \sigma^*)$ 
 $b_1 := ((\text{tx}^*, \sigma^*) \notin \Sigma)$ 
 $b_2 := \left( \left\{ \text{tag}_i \right\}_{i=1}^{|\mathcal{S}|} \cap \text{Spent} \neq \emptyset \right)$ 
return  $b_0 \wedge b_1 \wedge b_2$ 

```

Figure 4: Non-slanderability Experiment

Definition 3.1 (Balance). A RingCT scheme Ω is balanced if:

1. CheckTag and CheckAmount are binding. That is, for any PPT adversary \mathcal{A} , for all positive integers $\alpha, \beta \in \text{poly}(\lambda)$,

$$\Pr \left[\begin{array}{l} \text{CheckTag}(\text{acc}, \text{sk}, \text{tag}) = 1 \\ \text{CheckTag}(\text{acc}, \text{sk}', \text{tag}') = 1 \\ (\text{sk}, \text{tag}) \neq (\text{sk}', \text{tag}') \end{array} \middle| \begin{array}{l} \text{pp} \leftarrow \text{Setup}(1^\lambda, 1^\alpha, 1^\beta) \\ (\text{acc}, \text{sk}, \text{tag}, \\ \text{sk}', \text{tag}') \leftarrow \mathcal{A}(\text{pp}) \end{array} \right] \leq \text{negl}(\lambda), \text{ and}$$

$$\Pr \left[\begin{array}{l} \text{CheckAmount}(\text{acc}, \text{ck}, a) = 1 \\ \text{CheckAmount}(\text{acc}, \text{ck}', a') = 1 \\ (\text{ck}, a) \neq (\text{ck}', a') \end{array} \middle| \begin{array}{l} \text{pp} \leftarrow \text{Setup}(1^\lambda, 1^\alpha, 1^\beta) \\ (\text{acc}, \text{ck}, a, \\ \text{ck}', a') \leftarrow \mathcal{A}(\text{pp}) \end{array} \right] \leq \text{negl}(\lambda)$$

2. For all PPT adversaries \mathcal{A} , and all positive integers $\alpha, \beta \in \text{poly}(\lambda)$, there exists a PPT extractor $\mathcal{E}_{\mathcal{A}}$ such that

$$\Pr[\text{Balance}_{\Omega, \mathcal{A}, \mathcal{E}_{\mathcal{A}}}(1^\lambda, 1^\alpha, 1^\beta) = 1] \leq \text{negl}(\lambda)$$

where $\text{Balance}_{\Omega, \mathcal{A}, \mathcal{E}_{\mathcal{A}}}(1^\lambda, 1^\alpha, 1^\beta)$ is defined in Figure 2.

3.2 Privacy

Privacy captures the anonymity for both the spenders and the receivers, and the confidentiality of the amounts being transferred. The formalization of privacy is inspired by that of anonymity in ring signatures, key-privacy of encryption, and hiding in commitments. While closely related to the anonymity of ring signatures, the spender anonymity aspect of RingCT is significantly more difficult to capture as it must still hold in the presence of stealth addresses, a concept that does not exist for ring signatures.

Roughly speaking, privacy means that an adversary should not be able to distinguish two transactions with the same ring and their proofs, even if the majority of the ring is corrupt and the adversary has prior knowledge about the identities of the spenders and receivers and the amounts being transferred. In more detail, the adversary is allowed to specify a ring with arbitrarily many corrupt accounts, and two honest subsets of the ring which are the potential spenders. The adversary also specify two sets of receivers and the amounts that they are supposed to receive. A transaction is then created using one of the two specifications of the adversary, who should not be able to tell which specification is used to create the transaction.

More concretely, we model privacy in the security experiments Privacy^b for $b \in \{0, 1\}$. Let \mathcal{A} be a PPT adversary who, after several queries to the oracles, produces an *incomplete* input $(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu)$ to the **Spend** algorithm (Figure 1), along with two instructions I and J . The incomplete input $(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu)$ corresponds to all malicious information that will be used to generate a transaction and its proof, while the instructions I and J specify how the sets \mathcal{R} , \mathcal{S} , and \mathcal{T} should be populated by information held by the honest users. Following the adversary's instructions, the experiment duplicates $(\mathcal{S}, \mathcal{T})$ into $(\mathcal{S}_0, \mathcal{T}_0)$ and $(\mathcal{S}_1, \mathcal{T}_1)$, and populates $(\mathcal{R}, \mathcal{S}_t, \mathcal{T}_t)$ for $t \in \{0, 1\}$ as follows.

The instruction I is of the form $\{(s_i, \{j_{t,i}, k_{t,i}^{\mathcal{S}}, \ell_{t,i}\}_{t=0}^1\})_{i=1}^{|I|}$. The experiment retrieves the $\ell_{t,i}$ -th entry from the wallet of user $k_{t,i}^{\mathcal{S}}$, which consists of an account $\text{acc}_{t,i}^{\mathcal{S}}$ and the information required to spend from this account. It sets $\mathcal{R}[j_{t,i}]$ to this account, and $\mathcal{S}_t[s_i]$ to the retrieved spender information.

Similarly, the instruction J is of the form $\{(d_j, \{k_{t,j}^{\mathcal{T}}, a_{t,j}^{\mathcal{T}}\}_{t=0}^1\})_{j=1}^{|J|}$. The experiment creates a one-time account using the master public key of user $k_{t,j}^{\mathcal{T}}$ and the amount $a_{t,j}^{\mathcal{T}}$, and set $\mathcal{T}_t[d_j]$ to the appropriate receiver information.

The experiment then proceeds to create the proofs σ_0 and σ_1 for both transactions $\text{tx}(\mathcal{R}, \mathcal{S}_0, \mathcal{T}_0, \mu)$ and $\text{tx}(\mathcal{R}, \mathcal{S}_1, \mathcal{T}_1, \mu)$ respectively. If both proofs are valid, meaning in particular that both sets of inputs created as instructed by the adversary are well-formed, the experiment sends σ_b to the adversary, where b is the parameter of the experiment Privacy^b .

Definition 3.2 (Privacy). A RingCT scheme Ω is private if for all PPT adversaries \mathcal{A} and all positive integers $\alpha, \beta \in \text{poly}(\lambda)$,

$$\left| \Pr[\text{Privacy}_{\Omega, \mathcal{A}}^0(1^\lambda, 1^\alpha, 1^\beta) = 1] - \Pr[\text{Privacy}_{\Omega, \mathcal{A}}^1(1^\lambda, 1^\alpha, 1^\beta) = 1] \right| \leq \text{negl}(\lambda)$$

where $\text{Privacy}_{\Omega, \mathcal{A}}^b$ is defined in Figure 3.

3.3 Non-Slanderability (and Unforgeability)

In the context of RingCT, slandering is an act of producing a valid proof on behalf of another user. Note that a proof authenticates a transaction which specifies a set of tags bound to a set of source accounts. If the owner of one of the source accounts later attempts to spend from the account, the proof will not be accepted as the tag corresponding to the account has already been published in the slandering transaction. Slandering thus effectively causes the owners of these source accounts to lose money. Non-slanderability is a property that prevents the above attack, which is known as denial-of-spending attack in the literature [55].

Formally, we model non-slanderability by defining a security experiment in which the adversary produces a transaction-proof tuple, after several queries to the oracles (Figure 1). The adversary is successful if the tuple is valid and not produced by the spend oracle, and some of the tags specified in the slandering transaction collide with those that are signed by the spend oracle.

Definition 3.3 (Non-slanderability). *A RingCT scheme Ω is non-slanderable if for all PPT adversaries \mathcal{A} and all $\alpha, \beta \in \text{poly}(\lambda)$,*

$$\Pr[\text{NSland}_{\Omega, \mathcal{A}}(1^\lambda, 1^\alpha, 1^\beta) = 1] \leq \text{negl}(\lambda)$$

where the experiment $\text{NSland}_{\Omega, \mathcal{A}}$ is defined in Figure 4.

Since a tag is computationally bound to a unique account (as required by the balance property), non-slanderability (which states that no adversary can forge under a tag of a honest user account), naturally captures that no adversary can forge spend proofs for honest accounts. As a consequence, we do not need to define an unforgeability property explicitly.

4 RingCT Construction

We present a generic construction of RingCT schemes and an efficient instantiation.

4.1 Tagging Scheme

Our generic construction depends on a new primitive called tagging schemes. Roughly speaking, a tagging scheme is a one-way permutation over group elements.

Formally, a tagging scheme $\text{Tag} = (\text{TagSetup}, \text{TagEval})$ consists of a PPT setup algorithm TagSetup and an efficient *deterministic* algorithm TagEval . TagSetup inputs the security parameter 1^λ and outputs the public parameter pp , which defines a input space $(\chi, +)$, which is a group equipped with the operation $+$, and a tag space ψ . TagEval inputs $x \in \chi$ and outputs a tag $\text{tag} \in \psi$.

We require a tagging scheme which satisfies (related-input) one-wayness and pseudorandomness, defined as follows.

Definition 4.1 (Security of Tagging Schemes). *A tagging scheme Tag is said to be related-input one-way if for any PPT adversary \mathcal{A} ,*

$$\Pr[\text{OneWay}_{\text{Tag}, \mathcal{A}}(1^\lambda) = 1] \leq \text{negl}(\lambda).$$

It is related-input pseudorandom if for any PPT adversary \mathcal{A} ,

$$\left| \Pr[\text{PR}_{\text{Tag}, \mathcal{A}}^0(1^\lambda) = 1] - \Pr[\text{PR}_{\text{Tag}, \mathcal{A}}^1(1^\lambda) = 1] \right| \leq \text{negl}(\lambda),$$

where $\text{PR}_{\text{Tag}, \mathcal{A}}^b$ and $\text{OneWay}_{\text{Tag}, \mathcal{A}}$ are defined in Figure 5.

<p>OneWay_{Tag, A}(1^λ)</p> <pre> pp ← TagSetup(1^λ) x, s* ← χ tag* ← TagEval(x + s*) x' ← A^{TagOx}(pp, s*, tag*) return (TagEval(x') = tag*) PR_{Tag, A}^b(1^λ) pp ← TagSetup(1^λ) x ← χ (s*, tag*) ← Ch_{b, x}() b' ← A^{TagOx}(pp, s*, tag*) return b'</pre>	<p>Tag_{Ox}()</p> <pre> s ← χ return (s, TagEval(x + s)) Ch_{x, b}() tag* ← ψ s* ← χ if b = 0 then tag* ← TagEval(x + s*) endif return (s*, tag*)</pre>
--	---

Figure 5: One-Wayness and Pseudorandomness Experiments for Tagging Schemes

<p>Setup(1^λ, 1^α, 1^β)</p> <pre> pp_{PKE} ← PKESetup(1^λ) pp_{HC} ← HCSetup(1^λ) pp_{Tag} ← TagSetup(1^λ) pp_{SoK} ← SoKSetup(1^λ, (pp_{HC}, pp_{Tag})) pp := (β, pp_{PKE}, pp_{HC}, pp_{Tag}, pp_{SoK}) return pp SAKGen(pp) (tpk, tsk) ← KGen(pp_{PKE}) (vpk, vsk) ← KGen(pp_{PKE}) x ← χ mpk := (tpk, vpk, Com(0; x)) msk := (tsk, vsk, x) return (mpk, msk)</pre>	<p>OTAccGen(mpk, a)</p> <pre> parse mpk as (tpk, vpk, X) ek ← s{0, 1}^λ, ck := r ← χ s := H(mpk, ek) pk := X · Com(0; s), co := Com(a; r) ēk ← Enc(tpk, (pk, co), ek) c̄k ← Enc(vpk, (pk, co), (a, r)) acc := (pk, co, ēk, c̄k) return (ck, acc) Vf(tx, σ) parse tx as ({acc_i^R }_{i=1}^R, {tag_i }_{i=1}^S, {acc_i^T }_{i=1}^T, μ) if T > 2^α then return 0 return b := SoKVf(stmt(tx), σ, tx)</pre>	<p>Spend($\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu$)</p> <pre> tx := tx($\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu$) return σ ← SoKSig(stmt(tx), wit(\mathcal{S}, \mathcal{T}), tx) Receive(msk, acc) parse msk as (tsk, vsk, x) parse acc as (pk, co, ēk, c̄k) ek ← Dec(tsk, (pk, co), ēk) (a, r) ← Dec(vsk, (pk, co), c̄k) s := H(mpk, ek), x' := x + s if (pk, co) ≠ (Com(0; x'), Com(a; r)) then return ⊥ tag := TagEval(x') return (r, a, x', tag)</pre>
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Figure 6: RingCT Construction (Core Components).

<p>CheckAmount(acc, ck, a)</p> <pre> parse acc as (pk, co, ēk, c̄k) return (co = Com(a; ck))</pre>	<p>CheckTag(acc, sk, tag)</p> <pre> parse acc as (pk, co, ēk, c̄k) return (tag = TagEval(sk) ∧ pk = Com(0, sk))</pre>
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Figure 7: RingCT Construction (Auxiliary Algorithms).

4.2 Scheme Description

Let $\beta \in \mathbb{N}$. Let $\text{PKE} = (\text{PKESetup}, \text{KGen}, \text{Enc}, \text{Dec})$ be a (labeled) public-key encryption scheme, $\text{HC} = (\text{HCSetup}, \text{Com})$ be a homomorphic commitment scheme with message space $(\mathcal{M}, +)$ where $\{0, 1, \dots, 2^{\alpha+\beta} - 1\} \subseteq \mathcal{M}$ and randomness space $(\chi, +)$, $\text{Tag} = (\text{TagSetup}, \text{TagEval})$ be a tagging scheme with input space $(\chi, +)$ and tag space ψ , and $\text{SoK} = (\text{SoKSetup}, \text{SoKSig}, \text{SoKVf})$ be a signature of knowledge scheme for the language $\mathcal{L}[\text{pp}_{\text{HC}}, \text{pp}_{\text{Tag}}]$ (parameterized by the public parameters of HC and Tag) to be defined below; we recall the definitions of these well-known primitives in Appendix B. Let $H: \{0, 1\}^* \rightarrow \chi$ be a hash function modeled as a random oracle. We give a generic

construction of RingCT in Figure 6. An overview of the construction is as follows.

Recall that given the sets \mathcal{R} , \mathcal{S} , and \mathcal{T} , and a message μ , where

$$\begin{aligned}\mathcal{R} &= \{\text{acc}_i^{\mathcal{R}}\}_{i=1}^{|\mathcal{R}|} = \left\{ (\text{pk}_i^{\mathcal{R}}, \text{co}_i^{\mathcal{R}}, \tilde{\text{ek}}_i^{\mathcal{R}}, \tilde{\text{ck}}_i^{\mathcal{R}}) \right\}_{i=1}^{|\mathcal{R}|}, \\ \mathcal{S} &= \left\{ (j_i, \text{ck}_i^{\mathcal{S}}, a_i^{\mathcal{S}}, \text{sk}_i, \text{tag}_i) \right\}_{i=1}^{|\mathcal{S}|} = \left\{ (j_i, r_i^{\mathcal{S}}, a_i^{\mathcal{S}}, x_i, \text{tag}_i) \right\}_{i=1}^{|\mathcal{S}|}, \\ \mathcal{T} &= \left\{ (\text{ck}_i^{\mathcal{T}}, a_i^{\mathcal{T}}, \text{acc}_i^{\mathcal{T}}) \right\}_{i=1}^{|\mathcal{T}|} = \left\{ (r_i^{\mathcal{T}}, a_i^{\mathcal{T}}, (\text{pk}_i^{\mathcal{T}}, \text{co}_i^{\mathcal{T}}, \tilde{\text{ek}}_i^{\mathcal{T}}, \tilde{\text{ck}}_i^{\mathcal{T}})) \right\}_{i=1}^{|\mathcal{T}|},\end{aligned}$$

we defined the corresponding transaction to be

$$\text{tx}(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu) := \left(\left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_i \right\}_{i=1}^{|\mathcal{S}|}, \left\{ (\text{acc}_i^{\mathcal{T}}) \right\}_{i=1}^{|\mathcal{T}|}, \mu \right).$$

Given $\text{tx} = \text{tx}(\mathcal{R}, \mathcal{S}, \mathcal{T}, \mu)$, we further define the statement

$$\text{stmt}(\text{tx}) := \left(\left\{ \text{pk}_i^{\mathcal{R}}, \text{co}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_i \right\}_{i=1}^{|\mathcal{S}|}, \left\{ \text{co}_i^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|} \right).$$

The witness to the above statement is defined as

$$\text{wit}(\mathcal{S}, \mathcal{T}) := \left(\left\{ (j_i, x_i, a_i^{\mathcal{S}}, r_i^{\mathcal{S}}) \right\}_{i=1}^{|\mathcal{S}|}, \left\{ (a_i^{\mathcal{T}}, r_i^{\mathcal{T}}) \right\}_{i=1}^{|\mathcal{T}|} \right).$$

Setup The setup algorithm generates the public parameters of all the underlying primitives.

Stealth Address Generation The master public key mpk consists of the two PKE public keys tpk and vpk , and a commitment $X = \text{Com}(0; x)$ to 0 with randomness x . The master secret key msk consists of the two PKE secret keys tsk and vsk , and the value x . The key tsk also serves as the tracing secret key.

One-Time Account Generation The algorithm commits to the amount a as $\text{co} := \text{Com}(a; r)$ with some randomness r . It then generates a random bit-string as an *ephemeral key* ek and hashes it with mpk to get a random exponent $s \in \chi$. A one-time public key pk is then derived as $\text{pk} = X \cdot \text{Com}(0; s)$. Next it encrypts ek and ck using the appropriate instances of PKE and obtains $\tilde{\text{ek}}$ and $\tilde{\text{ck}}$ respectively as ciphertexts. This for elements are assembled to the *account* $\text{acc} = (\text{pk}, \text{co}, \tilde{\text{ek}}, \tilde{\text{ck}})$ and output together with the coin key, *i.e.* randomness r .

Receiving The algorithm decrypts both ciphertexts $\tilde{\text{ek}}$ and $\tilde{\text{ck}}$ in $\text{acc} = (\text{pk}, \text{co}, \tilde{\text{ek}}, \tilde{\text{ck}})$ to obtain ek and (a, r) , checks if $\text{co} = \text{Com}(a; r)$, derives the (one-time) secret key of the account as $x' = x + s$, and checks if $\text{pk} = \text{Com}(0; x')$. It also generates the *tag* of the account as $\text{tag} := \text{TagEval}(x')$.

Spending The algorithm derives the transaction tx , the statement stmt and the witness wit , and creates a signature of knowledge of the statement $\text{stmt} \in \mathcal{L}[\text{pp}_{\text{HC}}, \text{pp}_{\text{Tag}}]$ with message tx , where

$$\begin{aligned} & \mathcal{L}[\text{pp}_{\text{HC}}, \text{pp}_{\text{Tag}}] \\ := & \left\{ \begin{array}{l} \text{stmt} = \left(\left\{ \text{pk}_i^{\mathcal{R}}, \text{co}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_i \right\}_{i=1}^{|\mathcal{S}|}, \left\{ \text{co}_i^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|} \right) : \\ \exists \text{wit} = \left(\left\{ (j_i, x_i, a_i^{\mathcal{S}}, r_i^{\mathcal{S}}) \right\}_{i=1}^{|\mathcal{S}|}, \left\{ (a_i^{\mathcal{T}}, r_i^{\mathcal{T}}) \right\}_{i=1}^{|\mathcal{T}|} \right) \text{ s.t.} \\ \forall i \in [|\mathcal{S}|], \begin{cases} \text{pk}_{j_i}^{\mathcal{R}} = \text{Com}(0; x_i) \\ \text{co}_{j_i}^{\mathcal{R}} = \text{Com}(a_i^{\mathcal{S}}; r_i^{\mathcal{S}}) \\ \text{tag}_i = \text{TagEval}(x_i) \end{cases} \\ \forall i \in [|\mathcal{T}|], \begin{cases} \text{co}_i^{\mathcal{T}} = \text{Com}(a_i^{\mathcal{T}}; r_i^{\mathcal{T}}) \\ a_i^{\mathcal{T}} \in \{0, \dots, 2^\beta - 1\} \end{cases} \\ \sum_{i \in [|\mathcal{S}|]} a_i^{\mathcal{S}} = \sum_{i \in [|\mathcal{T}|]} a_i^{\mathcal{T}} \end{array} \right. \end{aligned}$$

Verify Given a transaction tx and a proof σ , the verifier derives the statement stmt from tx and verifies if σ is a valid signature of knowledge of stmt with message tx .

4.3 Analysis

The correctness of the construction is obvious. Below we state the security results.

In Appendix C we further extend the construction to tracking and viewing features. The security games are updated accordingly and the following theorems are proven for the extended versions.

Theorem 4.2 (Balance). *If HC is computationally binding, and SoK is extractable, then Ω is balanced.*

Theorem 4.3 (Privacy). *If HC is perfectly hiding and computationally binding, PKE is IK-CCA-secure and IND-CCA-secure, H is modeled as a random oracle, SoK is simulatable, and Tag is related-input pseudorandom, then Ω is private.*

Theorem 4.4 (Non-slanderability). *If SoK is extractable and simulatable, Tag is related-input one-way, and H is modeled as a random oracle, then Ω is non-slanderable.*

The proofs of the above theorems are deferred to Appendix D.

4.4 Concrete Instantiation: Omniring (\mathcal{U})

We propose the following concrete instantiation of our generic construction (Ω) called Omniring denoted by \mathcal{U} . We instantiate PKE with a (labeled variant of) ECIES [56], HC with the Pedersen commitment [50], and Tag with the pseudorandom function of Dodis and Yampolsky [23] in a non-black-box manner, which we denote by $\text{Tag}_{\mathcal{U}}$.

Concretely, let $\mathcal{G} = (\mathbb{G}, q, G)$ be the description of a cyclic group \mathbb{G} of prime order $q \geq 2^{\alpha+\beta}$ with generator G , where certain Diffie-Hellman-types assumptions hold (see Appendix B for details). Let $H \in \mathbb{G}$ be another random generator of \mathbb{G} . We set $\text{pp}_{\text{HC}} := (\mathbb{G}, q, G, H)$ which defines $\mathcal{M} := \mathbb{Z}_q$ and $\chi := \mathbb{Z}_q$. We also set $\text{pp}_{\text{Tag}} := (\mathbb{G}, q, G)$ so that the tag space of the tagging scheme Tag is $\psi := \mathbb{G}$. For $a, r \in \mathbb{Z}_q$, we define $\text{Com}(a; r) := G^a H^r$. For $x \in \mathbb{Z}_q^*$, we define $\text{TagEval}(x) := G^{\frac{1}{x}}$. More details of these constructions can be found in Appendices B and F.

With the above choices of HC and Tag fixed, we introduce the following notation for describing the language $\mathcal{L}[\text{pp}_{\text{HC}}, \text{pp}_{\text{Tag}}]$ more conveniently. Given the statement and witness

$$\begin{aligned} \text{stmt} &= \left(\left\{ \text{pk}_i^{\mathcal{R}}, \text{co}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_i \right\}_{i=1}^{|\mathcal{S}|}, \left\{ \text{co}_i^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|} \right), \\ \text{wit} &= \left(\left\{ (j_i, x_i, a_i^{\mathcal{S}}, r_i^{\mathcal{S}}) \right\}_{i=1}^{|\mathcal{S}|}, \left\{ (a_i^{\mathcal{T}}, r_i^{\mathcal{T}}) \right\}_{i=1}^{|\mathcal{T}|} \right), \end{aligned}$$

we define the following notation:

$$\begin{aligned} \vec{\mathbf{R}} &:= (\text{pk}_1^{\mathcal{R}}, \dots, \text{pk}_{|\mathcal{R}|}^{\mathcal{R}}) & \vec{\mathbf{C}}_{\mathcal{R}} &:= (\text{co}_1^{\mathcal{R}}, \dots, \text{co}_{|\mathcal{R}|}^{\mathcal{R}}) \\ \vec{\mathbf{T}} &:= (\text{tag}_1, \dots, \text{tag}_{|\mathcal{S}|}) & \vec{\mathbf{C}}_{\mathcal{T}} &:= (\text{co}_1^{\mathcal{T}}, \dots, \text{co}_{|\mathcal{T}|}^{\mathcal{T}}) \\ \vec{\mathbf{x}} &:= (x_1, \dots, x_{|\mathcal{S}|}) & \vec{\mathbf{a}}^{\mathcal{S}} &:= (a_1^{\mathcal{S}}, \dots, a_{|\mathcal{S}|}^{\mathcal{S}}) \\ \vec{\mathbf{x}}^{\circ-1} &:= (x_1^{-1}, \dots, x_{|\mathcal{S}|}^{-1}) & \vec{\mathbf{a}}^{\mathcal{T}} &:= (a_1^{\mathcal{T}}, \dots, a_{|\mathcal{T}|}^{\mathcal{T}}) \\ \vec{\mathbf{r}}^{\mathcal{S}} &:= (r_1^{\mathcal{S}} \parallel \dots \parallel r_{|\mathcal{S}|}^{\mathcal{S}}) & \vec{\mathbf{r}}^{\mathcal{T}} &:= (r_1^{\mathcal{T}} \parallel \dots \parallel r_{|\mathcal{T}|}^{\mathcal{T}}). \end{aligned}$$

Furthermore, let $\vec{\mathbf{e}}_i$ be the $|\mathcal{R}|$ -dimensional unit vector with 1 at the j_i -th position and 0 everywhere else, and let $\vec{\mathbf{b}}_i$ be the binary representation of $\vec{\mathbf{a}}_i^{\mathcal{T}}$. We define their concatenations as the matrices \mathbf{E} and \mathbf{B} respectively. That is, the i -th row of \mathbf{E} and \mathbf{B} are $\vec{\mathbf{e}}_i$ and $\vec{\mathbf{b}}_i$ respectively. We write the (row) vectorizations of \mathbf{E} and \mathbf{B} as

$$\text{vec}(\mathbf{E}) := (\vec{\mathbf{e}}_1, \dots, \vec{\mathbf{e}}_{|\mathcal{S}|}) \quad \text{vec}(\mathbf{B}) := (\vec{\mathbf{b}}_1, \dots, \vec{\mathbf{b}}_{|\mathcal{T}|}).$$

The language becomes:

$$\mathcal{L}_{\mathcal{U}}[\mathbb{G},q,G,H] := \left\{ \begin{array}{l} \text{stmt} = (\vec{\mathbf{R}}, \vec{\mathbf{C}}_{\mathcal{R}}, \vec{\mathbf{T}}, \vec{\mathbf{C}}_{\mathcal{T}}) : \\ \exists \text{wit} = (\mathbf{E}, \vec{\mathbf{x}}, \vec{\mathbf{a}}^{\mathcal{S}}, \vec{\mathbf{r}}^{\mathcal{S}}, \mathbf{B}, \vec{\mathbf{a}}^{\mathcal{T}}, \vec{\mathbf{r}}^{\mathcal{T}}) \text{ s.t.} \\ \\ \forall i \in [|\mathcal{S}|], \left\{ \begin{array}{l} \vec{\mathbf{e}}_i \text{ is a unit vector of length } |\mathcal{R}| \\ \vec{\mathbf{R}}^{\vec{\mathbf{e}}_i} = H^{x_i} \\ \vec{\mathbf{C}}_{\mathcal{R}}^{\vec{\mathbf{e}}_i} = G^{a_i^{\mathcal{S}}} H^{r_i^{\mathcal{S}}} \\ \text{tag}_i = G^{x_i^{-1}} \end{array} \right. \\ \\ \forall i \in [|\mathcal{T}|], \left\{ \begin{array}{l} \vec{\mathbf{b}}_i \text{ is the binary rep. of } a_i^{\mathcal{T}} \text{ of length } \beta \\ \text{co}_i^{\mathcal{T}} = G^{a_i^{\mathcal{T}}} H^{r_i^{\mathcal{T}}} \end{array} \right. \\ \\ \sum_{i \in [|\mathcal{S}|]} a_i^{\mathcal{S}} = \sum_{i \in [|\mathcal{T}|]} a_i^{\mathcal{T}} \end{array} \right.$$

Finally, we instantiate SoK for the language $\mathcal{L}[\mathbb{G},q,G,H]$ by applying the Fiat-Shamir transform [24] to the argument of knowledge scheme for $\mathcal{L}[\mathbb{G},q,G,H]$ to be constructed in Section 5.

We remark that with the above instantiations, all public parameters can be generated using *public coins*, i.e., without trusted setup.

5 Argument of Knowledge

Below we construct a logarithmic-round argument of knowledge scheme for the language $\mathcal{L}_{\mathcal{U}}[\mathbb{G},q,G,H]$. In the basic protocol described below, the total size of the messages sent by the prover is bounded by $O(|\mathcal{R}||\mathcal{S}| + \beta|\mathcal{T}|)$. We can then replace part of the protocol with the argument of knowledge for inner product relations \mathcal{L}_{IP} (defined below) of [16] as a black-box, in the same fashion of their “range proof” construction. This squashes the communication to $O(\log(|\mathcal{R}||\mathcal{S}| + \beta|\mathcal{T}|))$.

5.1 Our Basic Protocol

Below we describe our basic protocol $\Pi_{\mathcal{U}} = (\text{Setup}, \langle \mathcal{P}, \mathcal{V} \rangle)$ and state its security properties. The notation used within is defined in Section 4.4, Figures 9 to 11 and Figure 8.

$\text{Setup}(1^\lambda, \mathcal{L}_{\mathcal{U}})$:

Recall that $\mathcal{L}_{\mathcal{U}}$ is specified by a tuple (\mathbb{G}, q, G, H) . Output $\text{crs} = (\mathbb{G}, q, G, H)$.

$\langle \mathcal{P}(\text{crs}, \text{stmt}, \text{wit}), \mathcal{V}(\text{crs}, \text{stmt}) \rangle$:

\mathcal{V} :

1. $u, v \leftarrow \mathbb{Z}_q$
2. $h \leftarrow \mathbb{G}$, $\vec{\mathbf{p}} \leftarrow \mathbb{G}^{3+|\mathcal{R}|}$, $\vec{\mathbf{g}}' \leftarrow \mathbb{G}^{m-|\mathcal{R}|-3}$, $\vec{\mathbf{h}} \leftarrow \mathbb{G}^m$

$\mathcal{P} \leftarrow \mathcal{V} : u, v, h, \vec{\mathbf{p}}, \vec{\mathbf{g}}', \vec{\mathbf{h}}$

\mathcal{P}, \mathcal{V} :

1. $\hat{\mathbf{Y}} := \vec{\mathbf{R}} \circ \vec{\mathbf{C}}_{\mathcal{R}}^{\circ u}$
2. $\hat{\mathbf{T}} := \vec{\mathbf{T}}^{u^2 v^{|\mathcal{S}|}}$
3. For $w \in \mathbb{Z}_q$, denote

$$\vec{\mathbf{g}}_w := ((G \| H \| \hat{\mathbf{T}} \| \hat{\mathbf{Y}})^{\circ w} \circ \vec{\mathbf{p}} \| \vec{\mathbf{g}}') \quad (1)$$

\mathcal{P} :

1. $r_A \leftarrow \mathbb{Z}_q$

2. $A := h^{r_A} \vec{\mathbf{g}}_0^{\vec{\mathbf{c}}_L} \vec{\mathbf{h}}^{\vec{\mathbf{c}}_R}$

Note that $\vec{\mathbf{g}}_w^{\vec{\mathbf{c}}_L} = \vec{\mathbf{g}}_{w'}^{\vec{\mathbf{c}}_L}$ for all $w, w' \in \mathbb{Z}_q$ since $I = G^\xi H^\eta \hat{T} \hat{\mathbf{Y}}^{\hat{\mathbf{e}}}$. Thus $A = h^{r_A} \vec{\mathbf{g}}_w^{\vec{\mathbf{c}}_L} \vec{\mathbf{h}}^{\vec{\mathbf{c}}_R}$ for all $w \in \mathbb{Z}_q$.

$\mathcal{P} \rightarrow \mathcal{V}: A$

$\mathcal{V}: w \leftarrow \mathbb{Z}_q$

$\mathcal{P} \leftarrow \mathcal{V}: w$

\mathcal{P} :

1. $r_S \leftarrow \mathbb{Z}_q, \vec{\mathbf{s}}_L, \vec{\mathbf{s}}_R \leftarrow \mathbb{Z}_q^m$

2. $S := h^{r_S} \vec{\mathbf{g}}_w^{\vec{\mathbf{s}}_L} \vec{\mathbf{h}}^{\vec{\mathbf{s}}_R}$.

$\mathcal{P} \rightarrow \mathcal{V}: S$

$\mathcal{V}: y, z \leftarrow \mathbb{Z}_q$

$\mathcal{P} \leftarrow \mathcal{V}: y, z$

\mathcal{P} :

1. Define the following polynomials (in X):

$$l(X) := \vec{\mathbf{c}}_L + \vec{\alpha} + \vec{\mathbf{s}}_L \cdot X$$

$$r(X) := \vec{\theta} \circ (\vec{\mathbf{c}}_R + \vec{\mathbf{s}}_R \cdot X) + \vec{\mu}$$

$$t(X) := \langle l(X), r(X) \rangle = t_2 X^2 + t_1 X + t_0$$

for some $t_0, t_1, t_2 \in \mathbb{Z}_q$. In particular

$$t_0 = z^2 \cdot \langle \vec{\mathbf{a}}^{\mathcal{T}}, \vec{\mathbf{y}}^{|\mathcal{T}|} \rangle + \delta$$

2. $\tau_1, \tau_2 \leftarrow \mathbb{Z}_q$

3. $T_1 := g^{t_1} h^{\tau_1}, T_2 := g^{t_2} h^{\tau_2}$

$\mathcal{P} \rightarrow \mathcal{V}: T_1, T_2$

$\mathcal{V}: x \leftarrow \mathbb{Z}_q$

$\mathcal{P} \leftarrow \mathcal{V}: x$

\mathcal{P} :

1. $\tau := z^2 \cdot \langle \vec{\mathbf{r}}^{\mathcal{T}}, \vec{\mathbf{y}}^{|\mathcal{T}|} \rangle + \tau_1 x + \tau_2 x^2$

2. $r := r_A + r_S x$

3. $(\vec{l}, \vec{r}, t) := (l(x), r(x), t(x))$

$\mathcal{P} \rightarrow \mathcal{V}: \tau, r, \vec{l}, \vec{r}, t$

\mathcal{V} : Check if the following relations hold:

$$t = \langle \vec{l}, \vec{r} \rangle \tag{2}$$

$$h^r \vec{\mathbf{g}}_w^{\vec{l}} \vec{\mathbf{h}}^{\vec{\theta} \circ \vec{r} - 1} = A S^x \vec{\mathbf{g}}_w^{\vec{\alpha}} \vec{\mathbf{h}}^{\vec{\beta}} \tag{3}$$

$$G^t H^\tau = G^\delta \vec{\mathbf{C}}_T^{z^2 \cdot \vec{\mathbf{y}}^{|\mathcal{T}|}} T_1^x T_2^{x^2} \tag{4}$$

Theorem 5.1. *The verifier \mathcal{V} is public-coin. $\Pi_{\mathcal{U}}$ is constant-round, perfectly complete, and perfect special honest-verifier zero-knowledge.*

Theorem 5.2. *Assuming the discrete logarithm assumption holds over \mathcal{G} , $\Pi_{\mathcal{U}}$ has computational witness-extended emulation.*

The proofs of the above theorems can be found in Appendix E.

5.2 Inner Product Argument

We next recall the argument of knowledge for inner product relations in [16]. Formally, given a group description $\mathcal{G} = (\mathbb{G}, g, G)$, an integer $m \in \mathbb{N}$, and two vectors of group elements $\vec{\mathbf{g}}, \vec{\mathbf{h}} \in \mathbb{G}^m$, we define the following inner product relation:

$$\mathcal{L}_{\text{IP}}[\vec{\mathbf{g}}, \vec{\mathbf{h}}] := \left\{ \begin{array}{l} (P, t) \in \mathbb{G} \times \mathbb{Z}_q : \\ \exists \vec{l}, \vec{r} \in \mathbb{Z}_q^m \text{ s.t. } P = \vec{\mathbf{g}}^T \vec{\mathbf{h}}^{\vec{r}} \wedge \langle \vec{l}, \vec{r} \rangle = t \end{array} \right.$$

We denote the argument of knowledge protocol of [16] by Π_{IP} .

It is shown that if finding a non-trivial discrete logarithm representation of the identity element over (G) with base $\vec{\mathbf{g}} \parallel \vec{\mathbf{h}}$ is hard, then Π_{IP} has computational witness-extended emulation [16]. In their security proof, it is implicitly assumed that $\vec{\mathbf{g}}$ and $\vec{\mathbf{h}}$ are uniformly sampled from \mathbb{G}^m . For our purpose, we require a slightly stronger theorem which states that the argument of knowledge has computational witness-extended emulation even if the adversary has certain control over the values of $\vec{\mathbf{g}}$ and $\vec{\mathbf{h}}$.

Theorem 5.3 (Modified from [16]). *Let $\vec{\mathbf{g}}$ and $\vec{\mathbf{h}}$ be sampled as in Corollary 1, such that it is hard to find $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ with $I = \vec{\mathbf{g}}^{\vec{\mathbf{a}}} \vec{\mathbf{h}}^{\vec{\mathbf{b}}}$. Then Π_{IP} has computational witness-extended emulation.*

The proof of the above theorem is almost identical to that given in [16] and is therefore omitted.

5.3 Squashing Prover Communication

To squash the prover communication in our basic protocol from linear to logarithmic, we modify the protocol as follows. In the last message that \mathcal{P} sends to \mathcal{V} , instead of sending \vec{l} and \vec{r} in plain, \mathcal{P} commits to them as $P = \vec{\mathbf{g}}_w^T \vec{\mathbf{h}}^{\vec{\theta}^{\circ-1} \circ \vec{r}}$, where $\vec{\mathbf{g}}_w$ is defined in Equation (1), and sends P to \mathcal{V} . \mathcal{P} and \mathcal{V} then engage in the argument of knowledge Π_{IP} for inner product relation, which convinces \mathcal{V} that P is computed correctly and indeed $t = \langle \vec{l}, \vec{r} \rangle$. Finally, \mathcal{V} proceeds to check if $h^r P = AS^x \vec{\mathbf{g}}_w^{\vec{\mathbf{a}}} \vec{\mathbf{h}}^{\vec{\mathbf{b}}}$ and $G^t H^r = G^\delta \vec{\mathbf{C}}^{z^2 \cdot \vec{y}^{\lceil T \rceil}} T_1^x T_2^x$. As shown in [16], the following holds: \mathcal{V}_{IP} is public-coin; Π_{IP} has $\lceil \log_2 m \rceil$ rounds; and Π_{IP} is perfectly complete, perfect special honest-verifier zero-knowledge. As stated in Theorem 5.3, Π_{IP} has computational witness-extended emulation if finding non-trivial discrete logarithm relations among $\vec{\mathbf{g}}_w$ and $\vec{\mathbf{h}}$ is hard. Consequently, after the aforementioned changes, Theorem 5.1 and Theorem 5.2 still hold for the resulting protocol, except that the latter now consists of $\lceil \log_2 m \rceil + O(1)$ number of rounds.

6 Optimizations and Performance

We discuss several optimization techniques and compare the efficiency of Omniring with that of Monero.

6.1 Efficient (Batch) Verification

An Omniring transaction is computationally efficient to verify, as it can be reduced to a single multi-exponentiation of size $2m + \log(m) + O(1)$ using the technique of [16], where $m = 3 + |\mathcal{R}| + |\mathcal{R}||\mathcal{S}| + \beta|\mathcal{T}| + 3|\mathcal{S}|$. Since multi-exponentiations can be computed much more efficiently than performing an equivalent amount of individual exponentiations, they enable large savings. The cost of verification can further be lowered by batching. Specifically, verifying a batch of n transactions can be reduced to a single multi-exponentiation of size $2mn + n\log(m) + O(n)$.

Notation	Description
$\hat{\mathbf{Y}} = \hat{\mathbf{Y}}(u) := \mathbf{R} \circ \mathbf{C}_{\mathcal{R}}^{ou}$	Vector of compressed public keys and coins with randomness $u \in \mathbb{Z}_q$
$\hat{\mathbf{e}} = \hat{\mathbf{e}}(v) := \bar{v}^{ \mathcal{S} } \mathbf{E}$	Vector of compressed unit vectors with randomness $v \in \mathbb{Z}_q$
$\hat{T} = \hat{T}(u, v) := \bar{\mathbf{T}}^{u^2 \bar{v}^{ \mathcal{S} }}$	Compressed tag with randomness $u, v \in \mathbb{Z}_q$
$\xi = \xi(u, v) := -\langle \bar{v}^{ \mathcal{S} }, u \cdot \bar{\mathbf{a}}^{\mathcal{S}} + u^2 \cdot \bar{\mathbf{x}}^{o-1} \rangle$	Compressed secrets with randomness $u, v \in \mathbb{Z}_q$.
$\eta = \eta(u, v) := -\langle \bar{v}^{ \mathcal{S} }, \bar{\mathbf{x}} + u \cdot \bar{\mathbf{r}}^{\mathcal{S}} \rangle$	Note that $(\xi, \eta, \hat{\mathbf{e}})$ satisfies $I = G^\xi H^\eta \hat{\mathbf{T}} \hat{\mathbf{Y}}^{\hat{\mathbf{e}}}$.
$\bar{\mathbf{c}}_L, \bar{\mathbf{c}}_R$	Encoding of witness by honest prover dependent on u and v , see Figure 9.
$m = 3 + \mathcal{R} + \mathcal{R} \mathcal{S} + \beta \mathcal{T} + 3 \mathcal{S} $	Length of $\bar{\mathbf{c}}_L$ and $\bar{\mathbf{c}}_R$
$(\bar{\mathbf{v}}_0, \dots, \bar{\mathbf{v}}_8, \bar{\mathbf{u}}_4) = (\bar{\mathbf{v}}_0, \dots, \bar{\mathbf{v}}_8, \bar{\mathbf{u}}_4)(u, v, y)$	Constraint vectors parameterized by the randomness $u, v, y \in \mathbb{Z}_q$, see Figure 10.
$(\bar{\alpha}, \bar{\beta}, \bar{\delta}, \bar{\theta}, \bar{\zeta}, \bar{\mu}, \bar{\nu}, \bar{\omega}) = (\bar{\alpha}, \bar{\beta}, \bar{\delta}, \bar{\theta}, \bar{\zeta}, \bar{\mu}, \bar{\nu}, \bar{\omega})(u, v, y, z)$	Compressed constraint vectors parameterized by the randomness $u, v, y, z \in \mathbb{Z}_q$, see Figure 10 and Figure 11.
$\text{EQ} = \text{EQ}[\bar{\mathbf{a}}^{\mathcal{T}}, u, v, y]$	System of equations parameterized by the amounts $\bar{\mathbf{a}}^{\mathcal{T}}$ and randomness $u, v, y \in \mathbb{Z}_q$, see Figure 12

Figure 8: Notation for signatures of knowledge construction.

$$\begin{aligned} \bar{\mathbf{c}}_L &:= (\xi \parallel \eta \parallel 1 \parallel \hat{\mathbf{e}} \parallel \text{vec}(\mathbf{E}) \parallel \text{vec}(\mathbf{B}) \parallel \bar{\mathbf{a}}^{\mathcal{S}} \parallel \bar{\mathbf{r}}^{\mathcal{S}} \parallel \bar{\mathbf{x}}) \\ \bar{\mathbf{c}}_R &:= (\bar{0}^{3+|\mathcal{R}|} \parallel \text{vec}(\mathbf{E}) - \bar{1}^{|\mathcal{R}| |\mathcal{S}|} \parallel \text{vec}(\mathbf{B}) - \bar{1}^{|\mathcal{T}|} \parallel \bar{0}^{2|\mathcal{S}|} \parallel \bar{\mathbf{x}}^{o-1}) \end{aligned}$$

Figure 9: Honest encoding of witness.

$$\begin{bmatrix} \bar{\mathbf{v}}_0 \\ \bar{\mathbf{v}}_1 \\ \bar{\mathbf{v}}_2 \\ \bar{\mathbf{v}}_3 \\ \bar{\mathbf{v}}_4 \\ \bar{\mathbf{v}}_5 \\ \bar{\mathbf{v}}_6 \\ \bar{\mathbf{v}}_7 \\ \bar{\mathbf{v}}_8 \\ \bar{\mathbf{u}}_4 \end{bmatrix} := \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \bar{y}^{|\mathcal{R}| |\mathcal{S}| + \beta |\mathcal{T}|} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \bar{y}^{|\mathcal{S}|} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \bar{y}^{|\mathcal{T}|} \otimes \bar{2}^\beta & \cdot & \cdot & \cdot \\ \cdot & \cdot & y^{|\mathcal{S}|} & \cdot & \bar{y}^{|\mathcal{S}|} \otimes \bar{1}^{|\mathcal{R}|} & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot & u \cdot \bar{v}^{|\mathcal{S}|} & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & u \cdot \bar{v}^{|\mathcal{S}|} & \bar{v}^{|\mathcal{S}|} \\ \cdot & \cdot & \cdot & -\bar{y}^{|\mathcal{R}|} & \bar{v}^{|\mathcal{S}|} \otimes \bar{y}^{|\mathcal{R}|} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \bar{1}^{|\mathcal{T}|} \otimes \bar{2}^\beta & -\bar{1}^{|\mathcal{S}|} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \bar{y}^{|\mathcal{R}| |\mathcal{S}| + \beta |\mathcal{T}|} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & u^2 \cdot \bar{v}^{|\mathcal{S}|} \end{bmatrix}$$

Figure 10: Definitions of constraint vectors. (Dots mean zeros.)

6.2 Log-size Transactions

While Omniring produces spend proofs of logarithmic size, the spender needs to communicate the set of destination accounts $\{\text{acc}_i^{\mathcal{T}}, \text{info}_i\}_{i=1}^{|\mathcal{T}|}$, the set of tags $\{\text{tag}_i\}_{i=1}^{|\mathcal{S}|}$, and a set of ring accounts $\{\text{acc}_i^{\mathcal{R}}\}_{i=1}^{|\mathcal{R}|}$ within the transaction to allow the cryptocurrency network to verify the transaction. Since $|\mathcal{T}|$ and $|\mathcal{S}|$ are typically small (we typically have $|\mathcal{T}| = 2$ and $|\mathcal{S}| < 5$), they can be safely neglected. However, the ring size $|\mathcal{R}|$ is a problem if a high level of privacy is desired. The obvious solution to include $|\mathcal{R}|$ ring members in the transaction needs $O(|\mathcal{R}|)$ space which quickly becomes impractical. However, using the *recovery sampling* technique by Chator and Green [19], which is built for this exact purpose, the description of the set of ring members can be as short as $O(\log |\mathcal{R}|)$, yielding a transaction size (not only proof sizes) logarithmic in m .

6.3 Performance Comparison

We compare the performance of Omniring with the RingCT scheme currently employed in Monero, *i.e.*, the scheme by Noether *et al.* [47] with a minor modification [2], together with Bulletproofs [16] range proofs (all of the range proofs in a transaction aggregated into a single Bulletproof). For conciseness we simply use Monero

$$\begin{array}{l}
\vec{\theta} := \sum_{i=0}^1 z^i \cdot \vec{v}_i \quad \vec{\zeta} := \sum_{i=2}^7 z^i \cdot \vec{v}_i \quad \vec{\mu} := \sum_{i=2}^8 z^i \cdot \vec{v}_i \\
\vec{v} := z^8 \cdot \vec{v}_8 \quad \vec{\omega} := z^4 \cdot \vec{u}_4 \\
\vec{\alpha} := \vec{\theta}^{\circ-1} \circ (\vec{\omega} - \vec{v}) \quad \vec{\beta} := \vec{\theta}^{\circ-1} \circ \vec{\mu} \\
\delta := z \cdot \langle \vec{1}^{|\mathcal{S}|}, \vec{y}^{|\mathcal{S}|} \rangle + z^3 \cdot \langle \vec{1}^{|\mathcal{S}|+1}, \vec{y}^{|\mathcal{S}|+1} \rangle + \langle \vec{\alpha}, \vec{\mu} \rangle + \langle \vec{1}^m, \vec{v} \rangle
\end{array}
\quad \text{EQ}(\vec{\gamma}_L, \vec{\gamma}_R) = 0 \iff \left\{ \begin{array}{l}
\langle \vec{\gamma}_L, \vec{\gamma}_R \circ \vec{v}_0 \rangle = 0 \quad (5) \\
\langle \vec{\gamma}_L, \vec{\gamma}_R \circ \vec{v}_1 \rangle = \langle \vec{1}^{|\mathcal{S}|}, \vec{y}^{|\mathcal{S}|} \rangle \quad (6) \\
\langle \vec{\gamma}_L, \vec{v}_2 \rangle = \langle \vec{a}^{\mathcal{T}}, \vec{y}^{|\mathcal{T}|} \rangle \quad (7) \\
\langle \vec{\gamma}_L, \vec{v}_3 \rangle = \langle \vec{1}^{|\mathcal{S}|+1}, \vec{y}^{|\mathcal{S}|+1} \rangle \quad (8) \\
\langle \vec{\gamma}_L, \vec{v}_4 \rangle + \langle \vec{\gamma}_R, \vec{u}_4 \rangle = 0 \quad (9) \\
\langle \vec{\gamma}_L, \vec{v}_5 \rangle = 0 \quad (10) \\
\langle \vec{\gamma}_L, \vec{v}_6 \rangle = 0 \quad (11) \\
\langle \vec{\gamma}_L, \vec{v}_7 \rangle = 0 \quad (12) \\
\langle \vec{\gamma}_L - \vec{\gamma}_R - \vec{1}^m, \vec{v}_8 \rangle = 0 \quad (13)
\end{array} \right.$$

Figure 11: Definitions of constraint vectors (cont.).

Figure 12: A system of equations guaranteeing the integrity of the encoding of witness.

to refer to this scheme. We consider the typical case of $|\mathcal{T}|=2$,¹ and the amount range $\beta=64$ used in Monero. For a fair comparison we also consider only transactions with one source account ($|\mathcal{S}|=1$), to exclude the advantages that our model of RingCT provides for $|\mathcal{S}|>1$ (see Section 1.1.1).

Proof Size In Figure 14 we compare the proof size of known RingCT schemes. We assume (non-pairing) elliptic curves as in Monero and therefore do not differentiate between group elements and scalars because they have roughly the same size. The proof size of Omniring is

$$2\lceil \log_2(3 + |\mathcal{R}| + |\mathcal{R}||\mathcal{S}| + \beta|\mathcal{T}| + 3|\mathcal{S}|) \rceil + 9$$

while that of Monero is

$$(|\mathcal{R}|+2)(|\mathcal{S}|+1) + \log_2(|\mathcal{T}|\beta) + 9.$$

RingCT 3.0 proposed in a concurrent work [28] can be instantiated in the same setting, and has proof size

$$|\mathcal{S}|(2\lceil \log_2|\mathcal{R}| \rceil + 17) + 2\lceil \log_2(\beta|\mathcal{T}|) \rceil + 2.$$

The proof size of RingCT 2.0 [60] is $O(|\mathcal{S}| + \log(\beta|\mathcal{T}|))$ elements where the hidden constant is in the hundreds. The concrete count is incomparable to other schemes and is omitted, as the scheme is based on pairing groups and has a trusted setup.

Figure 13a shows the number of elements in the proof against the size of the ring. Note that even when $|\mathcal{R}|$ is as small as 11, which is the ring size currently enforced in Monero, the proof size of Omniring is already significantly smaller than that of Monero, and for larger $|\mathcal{R}|$, the difference in proof size grows further. Finally, we remark that although our comparison only considers $|\mathcal{S}|=1$ and $|\mathcal{T}|=2$, for general $|\mathcal{S}|$ and $|\mathcal{T}|$, the gap in proof size would only be larger as the proof size of Monero scales linearly with $|\mathcal{S}|$ and $|\mathcal{T}|$ while Omniring's only scales logarithmically.

Running Time To compare the running time, we make use of the fact that our spend algorithm is very similar in structure to a Bulletproof range proof, which has been implemented in Monero; the only difference significant for performance is the size of the vectors in the inner product proof. By modifying the Monero benchmark suite to run Bulletproofs with larger vectors, we could obtain precise estimates for the running time of Omniring, and compare them with running times for the RingCT scheme used in Monero. Our estimates are suitable for a comparison with

¹A typical transaction uses one destination account to pay to the receiver and one *change account* to pay the remaining funds from the source accounts back to a new account of the sender. This model, introduced by Bitcoin, is common in cryptocurrencies and it is actually fundamental to RingCT because the spend proof only reveals that the sum of the source amounts equals the sum of the output amounts. Partial spends of a source account would require accounting for the exact amount that has been spent.

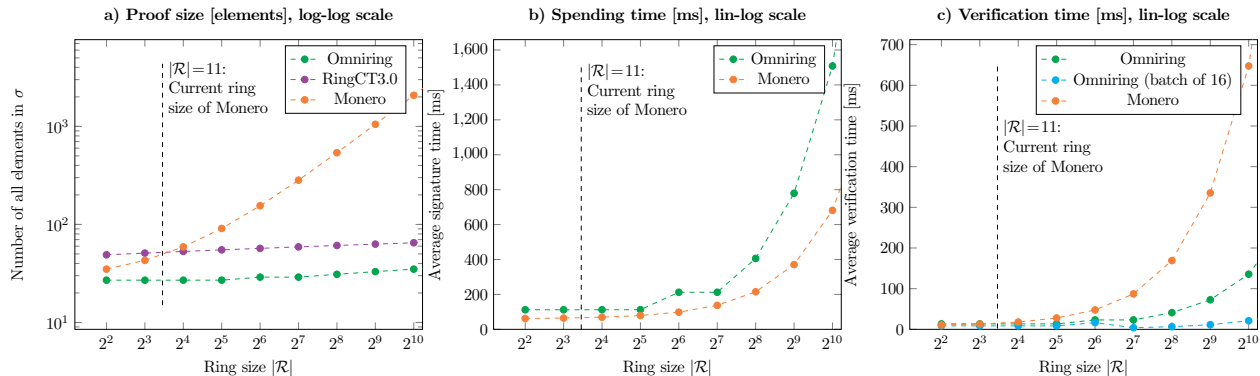


Figure 13: Performance Comparison

Scheme	Spend proof size (in elements)	Pairing	Trusted setup
Monero [2, 47] with BP rangeproofs	$O(\mathcal{R} \mathcal{S} + \log(\beta \mathcal{T}))$	No	No
RingCT 2.0 [60] with BP rangeproofs	$O(\mathcal{S} + \log(\beta \mathcal{T}))$	Yes	Yes
RingCT 3.0 [28] with BP rangeproofs	$O(\mathcal{S} \log \mathcal{R} + \log(\beta \mathcal{T}))$	No	No
Omniring	$O(\log(\mathcal{R} \mathcal{S} + \beta \mathcal{T}))$	No	No

Figure 14: Comparison of RingCT schemes.

Monero because they rely on the same C++ implementation of Bulletproofs, i.e., on the same elliptic curve using the same optimizations. Our modified code is available in an anonymized repository [49]. All experiments were run on a single core of an Intel Core i7-7600U (Kaby Lake) CPU with TurboBoost disabled to get more consistent results.

Figure 13b and Figure 13c show the estimated time needed for generating and verifying a proof respectively against the size of the ring, with the latter being particularly important as each proof generated by a single spender has to be verified by virtually all other users. The time needed for generating proofs in Omniring is about twice of that in Monero. Omniring, however, has considerably faster verification than Monero does for higher $|\mathcal{R}|$. For instance, at $|\mathcal{R}|=128$, verifying an Omniring transaction is 4 times faster than verifying a Monero transaction. This gap is further amplified if we employ batch verification techniques in Omniring; we included a graph for a sample batch size of 16 to demonstrate this. Batch verification is particularly useful in cryptocurrencies to reduce the verification time of nodes catching up with the blockchain, e.g., they can verify all transactions in a block in one batch.

A Related Work

A.1 Comparison with RingCT “1.0” and 2.0

We compare our formalization with the one in [60] and our construction with the one in [47] and “RingCT 2.0” proposed in [60].

Formalizing RingCT The formal model in [60] does not formalize the central property of stealth addresses nor receiver anonymity. While both our model and that of [60] define balance and spender anonymity, our definitions assume stronger adversaries. In the case of balance, we require the property to hold even if all accounts in the transaction are maliciously generated. In contrast, [60] considers only honestly generated accounts. While [60] only considers spender anonymity in the case where all source accounts are not corrupt, we allow some of the source accounts to be corrupt and still require the non-corrupt accounts to be anonymous. Moreover, related to the support of stealth addresses, we allow the source accounts to be the target accounts in previous transactions created by the adversary. This naturally makes it non-trivial to define spender anonymity in contrast to previous works that only relied on a ring signature style anonymity definition. Therefore our model offers stronger security guarantees.

Constructing RingCT The construction of Noether *et al.* [47] produces proofs of size $O(|\mathcal{S}| \cdot |\mathcal{R}|, |\mathcal{T}|, \beta)$. This inefficiency incentivizes users of Monero to employ smaller ring sizes thus effectively reducing the anonymity set, which leads to de-anonymization of the spending accounts as shown by Möser *et al.* [45] and Kumar *et al.* [32]. A core component in this construction is a “range proof” protocol which allows a spender to prove that an amount being transferred lies within a pre-defined range. A recent improvement is made by the Monero team, who replaces such range proof with Bulletproof range proof [16]. This reduces the linear dependency on β to a logarithmic one and the linear dependency on $|\mathcal{S}| \cdot |\mathcal{R}|$ becomes the bottleneck of the system. In contrast we extend the Bulletproof framework (Section 1.3) and exploit its efficiency not just for range proofs but also for the spender to prove his ownership of coins and their spendability.

Sun *et al.* [60] proposed an accumulator-based construction which features signature size independent of $|\mathcal{T}|$ and $|\mathcal{R}|$. However, their scheme relies on a trusted setup, and a pairing-friendly elliptic curve over which operations are computationally more expensive than non-pairing-friendly ones. While setup-free accumulators are known based on unknown order groups, they require considerably larger parameters to be secure. Also, the construction of [60] does not support stealth addresses.

A.2 Comparison with RingCT 3.0

A very recent concurrent work proposes RingCT 3.0 [28] which uses a syntax similar to ours and improves upon RingCT 2.0 mainly by supporting stealth addresses and getting rid of the trusted setup. Their results differ in two main aspects from our findings: First, we argue that their security model is insufficient and does not cover certain real-world attacks. Second, our construction is more efficient as it supports a unified ring and directly incorporates range proofs. In the following paragraph, we discuss the differences in detail.

Regarding their security model, RingCT 3.0 suggests a more restricted definition of balance that forces the adversary to generate its transactions using oracles provided by the experiment. This is necessary to learn the amounts corresponding to the adversarial transaction by witnessing the oracle queries. We believe that this notion is too restrictive because it does not cover adversaries that simply forge proofs of false statements and which do not use the oracle at all. Our approach is different and more general allowing arbitrary adversaries and we only require the existence of an extractor which extracts the amounts by running code of the adversary.

The second weakness in their formal security model is their definition of anonymity, which is split into *anonymity against receivers* and *anonymity against ring insiders*. First, the two properties together do not seem to imply the combined property, *i.e.*, anonymity against a coalition of receivers and ring members. Second, their definition only holds with respect to honestly generated source accounts. We believe that this notion is too weak because it rules out natural real-world attacks where a curious user, who has transferred some money to a one-time account of the victim, tries to determine if the victim’s account is used in a given transaction. Third, their definition only covers spender but not receiver anonymity.spender.

Aside from the definitional issues, the construction seems also less efficient compared to our “unified ring” construction because RingCT 3.0 requires separate rings for separate source accounts like all other previous RingCT schemes. Moreover, range proofs are not directly included in their construction. Instead, for real-world applications, one would need to compose their construction with a separate range proof system. While this is acceptable from a theoretical point of view, it incurs unnecessary computational and communication overheads which impact concrete efficiency.

A.3 Other Privacy-Preserving Currencies

We briefly compare other privacy-preserving currencies with RingCT. The second largest currency that has certain privacy-preserving properties is DASH. Since DASH uses trusted mixers to achieve some form of anonymity, it is incomparable to RingCT. The third largest competitor is ZCash, which is an implementation of the zerocash protocol by Ben-Sasson *et al.* [10]. In contrast to RingCT, the underlying protocol uses a SNARKs that requires a trusted setup. While setup-free SNARKs are known, they do seem to be applicable due to their inefficient nature.

ZeroCoin and Zerocash ZeroCoin [43] and Zerocash [10] are schemes for cryptocurrencies aiming to provide anonymity and the privacy of amounts based on zero-knowledge proofs. Ring signatures in CryptoNote v2.0 [62]

(the underlying scheme of Monero) serve the same purpose as the non-interactive zero-knowledge proofs in Zerocoin. In fact, the zero-knowledge proofs can be seen as a form of ring signatures. Zerocoin was developed as an extension to Bitcoin, and Zerocash is designed as an independent currency and has been implemented in Zcash [63]. Both Zerocoin and Zerocash use a trusted setup, and Zerocash uses zero-knowledge succinct non-interactive arguments of knowledge (ZK-SNARKs) to prove the integrity of computations. Due to the trusted setup, the system per se cannot be considered as completely decentralized. By using cryptographic accumulators which give constant-size membership proofs, Zerocoin and Zerocash can handle very large anonymity sets; the anonymity set is the set of all coins ever created. In contrast, Monero scales only to moderate anonymity sets but does not require a trusted setup; the same is true for another instantiation of Zerocoin proposed by Groth and Kohlweiss [26].

A.4 Brief History of RingCT and Monero

To ensure anonymity in cryptocurrencies while still preventing double-spending, van Saberhagen mentioned the use of linkable ring signatures in a cryptocurrency called CryptoNote v2.0 [62], which ensures that messages signed by the same sender are linkable, independently of the rings or messages. The construction is a slight modification of the scheme by Liu, Wei, and Wong [34], however the security analysis is not detailed and is carried out with respect to informal definitions.

Back [5] observed how to improve CryptoNote v2.0 relying on ideas from [1]. Noether *et al.* [47] generalized Back’s scheme in the name of ring confidential transactions (RingCT) to allow for batch spending with improved confidentiality (by using Confidential Transactions (CT)) and anonymity guarantees (Stealth Addresses). While the original proposal by Maxwell [36] leaves out many cryptographic details, which could just be found in the source code, the concept of CT for confidentiality of amounts has been explained by Gibson [25] and partially formalized by Poelstra *et al.* [52]. However, the analysis of Noether *et al.* [47] is informal. (Some versions of) CryptoNote v2.0 / RingCT was implemented in (some versions of) Monero, one of the major (privacy-preserving) cryptocurrencies which is deployed.

Sun *et al.* [60] gave another attempt to formalize RingCT and provide a construction with constant size signature. However, as we have discussed in Appendix A, their formalization is far from satisfactory. Moreover, their construction achieves (asymptotic) efficiency improvements using accumulators that require a trusted setup, which is especially undesirable for the setting of Monero.

Stealth Addresses To our best knowledge, the concept of “stealth addresses” first appeared in CryptoNote v2.0 [62] in the name “public user keys” without formalization. Meiklejohn and Mercer [38, 39] attempt to formalize stealth addresses. They require that the one-time public keys are identically distributed as randomly chosen ones, in the view of external parties. This requirement is necessary but not sufficient because the spender who derives the one-time public key may know extra information about the key which can be used to link signatures (hence breaking spender anonymity).

Bulletproofs Bulletproofs [16] is a recently proposed general purpose proof system improved from [13] for arithmetic circuit satisfiability. In particular, they allow to prove that a committed value lies within a given range, a relation to be proved when spending in RingCT. As discussed in [12], the tools used to construct general purpose proof systems (*e.g.*, [13]) seem to be more computationally expensive than the ones for specific languages (*e.g.*, low-degree polynomial relations in [12] and this work).

Non-slanderability $\not\Rightarrow$ Linkability In Sun *et al.* [60], it is claimed that non-slanderability implies linkability. Since their claim is informal, it is unclear whether the implication is claimed just for RingCT systems or also for linkable signatures. We show that either case is not true by giving an intuition for constructing counter examples. Consider a RingCT or a linkable ring signature scheme where the tag is a commitment of the signer secret key. A signature consists of a proof where, among other relations, the tag is a commitment of a secret key, and the public key corresponding to the secret key is a member of the ring. It is clear that such a construction can be made unforgeable and anonymous (and balanced in the case of RingCT) when instantiated with appropriate proof system and commitment scheme. In particular, suppose the proof system is perfectly zero-knowledge and

is a PoK, and the commitment scheme is perfectly hiding. In this case, we observe that the scheme can be made non-slanderable yet non linkable, falsifying the claim in Sun *et al.* [60] that non-slanderability implies linkability.

Obviously, the scheme is not linkable since the tag is completely independent of the secret key. Intuitively, the scheme is non-slanderable, since given a tag (a commitment of some secret key sk) along with a proof about it, sk is information-theoretically hidden from the adversary due to the perfectly zero-knowledge and perfectly hiding property. If the adversary manages to produce a proof that the tag is a commitment of a secret key sk' , with overwhelming probability we would have $\text{sk} \neq \text{sk}'$. Since the proof system is a proof of knowledge, we can extract sk' and thus break the binding property of the commitment scheme. We remark that the construction of Sun *et al.* [60] is nevertheless linkable since a tag is uniquely determined by an account.

B Preliminaries

Let $\lambda \in \mathbb{N}$ denote the security parameter. We denote by $\text{poly}(\lambda)$ and $\text{negl}(\lambda)$ the sets of polynomials and negligible functions in λ , respectively. PPT means probabilistic polynomial time. Given a set S , $x \leftarrow_{\$} S$ means sampling an element x from S uniformly at random. For an algorithm A with input x and output z , we write $z \leftarrow A(x)$. For $n \in \mathbb{N}$ the set $[n]$ is defined as $[n] := \{1, \dots, n\}$. Unless specified, all sets are ordered. All algorithms may output \perp upon failure.

B.1 Vectors and Matrices

Vectors (of integers and group elements) are always written as row vectors unless specified. The actual orientation of a vector in a matrix-vector product is implicit and is not specified unless there is an ambiguity. Both ‘,’ and ‘||’ are used as delimiters, *e.g.*, $\vec{v} = (v_1, \dots, v_n) \in \mathbb{Z}^n$ and $\vec{u} = (u_1 || \dots || u_n) \in \mathbb{Z}^n$. Inner products are denoted by $\langle \vec{u}, \vec{v} \rangle = \sum_{i=1}^n u_i v_i$. Let $\vec{g} \in \mathbb{G}^n$ be a vector of group elements, and $x \in \mathbb{Z}$. We define the following operations between vectors of group elements and (vectors of) integers:

- Hadamard Powers: $\vec{g}^{\circ x} := (g_1^x, \dots, g_n^x)$
- Hadamard Products: $\vec{g}^{\vec{v}} := (g_1^{v_1}, \dots, g_n^{v_n})$

We also define the operations between (vectors of) integers:

- Hadamard Products: $\vec{u} \circ \vec{v} := (u_1 v_1, \dots, u_n v_n)$
- Kronecker Products: $\vec{u} \otimes \vec{v} := (u_1 \vec{v}, \dots, u_n \vec{v})$
- Consecutive Powers: $\vec{x}^n = (1, x, \dots, x^{n-1})$

Given a matrix $E \in \mathbb{Z}_q^{m \times n}$, its (row) vectorization is defined as $\text{vec}(\mathbf{E}) := (\vec{e}_1, \dots, \vec{e}_m)$, where \vec{e}_i is the i -th row of E . Conversely, we write $\mathbf{E} = \text{vec}^{-1}(\vec{e}_1, \dots, \vec{e}_m)$.

B.2 Hardness Assumptions over Cyclic Groups

Let $\mathcal{G} = (\mathbb{G}, q, G)$ be the description of a cyclic group \mathbb{G} of prime order q with generator G . In the following, we recall the (generalized) discrete logarithm assumption and the strong decisional Diffie-Hellman inversion (SDDHI) assumption. While the generalized discrete logarithm assumption trivially holds in the generic group model [57], the SDDHI assumption and the GapDH assumption can be proven to hold in the generic group model [17, 56].

Definition B.1 (Discrete Logarithm (DL) Assumption). *We say that the discrete logarithm assumption holds over \mathcal{G} if for every PPT adversary \mathcal{A}*

$$\Pr [\text{DL}_{\mathcal{A}}(\mathcal{G}) = 1] \leq \text{negl}(\lambda),$$

where the game $\text{DL}_{\mathcal{A}}(\mathcal{G})$ is defined in Figure 15.

$\text{DL}_{\mathcal{A}}(\mathcal{G})$ <hr style="width: 80%; margin: 0 auto;"/> $x \leftarrow \mathfrak{s}\mathbb{Z}_q$ $x' \leftarrow \mathcal{A}(\mathbb{G}, q, G, G^{x'})$ $b := (G^x = G^{x'})$ $\mathbf{return } b$

Figure 15: Security Game for Discrete Logarithm

Definition B.2 (Generalized Discrete Logarithm (GDL) Assumption). *We say that the generalized discrete logarithm assumption holds over \mathcal{G} if for all $\ell \in \text{poly}(\lambda)$, every PPT adversary \mathcal{A}*

$$\Pr [\ell\text{-GDL}_{\mathcal{A}}(\mathcal{G})] \leq \text{negl}(\lambda),$$

where the game $\ell\text{-GDL}_{\mathcal{A}}(\mathcal{G})$ is defined in Figure 16.

$\ell\text{-GDL}_{\mathcal{A}}(\mathcal{G})$ <hr style="width: 80%; margin: 0 auto;"/> $x \leftarrow \mathfrak{s}\mathbb{Z}_q$ $x' \leftarrow \mathcal{A}(\mathbb{G}, q, G, G^{x'}, G^{x'^2}, \dots, G^{x'^{\ell}})$ $b := (G^x = G^{x'})$ $\mathbf{return } b$

Figure 16: Security Game for GDL

Definition B.3 (Strong Decisional Diffie-Hellman Inversion (SDDHI) [17]). *We say that the SDDHI assumption according to holds over \mathcal{G} if for all $\ell \in \text{poly}(\lambda)$, every PPT adversary \mathcal{A}*

$$\left| \Pr [\text{SDDHI}_{\mathcal{A}}^0(\mathcal{G}) = 1] - \Pr [\text{SDDHI}_{\mathcal{A}}^1(\mathcal{G}) = 1] \right| \leq \text{negl}(\lambda),$$

where the game $\text{SDDHI}_{\mathcal{A}}(\mathcal{G})$ is defined in Figure 17.

$\text{SDDHI}_{\mathcal{A}}^b(\mathcal{G})$ <hr style="width: 80%; margin: 0 auto;"/> $Z := \emptyset$ $x \leftarrow \mathfrak{s}\mathbb{Z}_q^*$ $s \leftarrow \mathcal{A}_0^{\mathcal{O}^x}(\mathbb{G}, q, G, G^x)$ $b \leftarrow \mathfrak{s}\{0, 1\}$ $y_0 = G^{\frac{1}{x+s}}$ $y_1 \leftarrow \mathfrak{s}\mathbb{G}$ $b' \leftarrow \mathcal{A}_1^{\mathcal{O}^x}(y_b)$ $\mathbf{if } s \in Z \mathbf{ then return } 0$ $\mathbf{return } b'$	$\mathcal{O}_x(z)$ <hr style="width: 80%; margin: 0 auto;"/> $Z := Z \cup \{z\}$ $\mathbf{return } G^{\frac{1}{x+z}}$
---	---

Figure 17: Security Game for SDDHI

Definition B.4 (Decisional Diffie-Hellman (DDH)). *We say that the DDH assumption according to holds over \mathcal{G} if for all $\ell \in \text{poly}(\lambda)$, every PPT adversary \mathcal{A}*

$$\left| \Pr [\text{DDH}_{\mathcal{A}}^0(\mathcal{G})=1] - \Pr [\text{DDH}_{\mathcal{A}}^1(\mathcal{G})=1] \right| \leq \text{negl}(\lambda),$$

where the game $\text{DDH}_{\mathcal{A}}(\mathcal{G})$ is defined in Figure 18.

$\text{DDH}_{\mathcal{A}}^b(\mathcal{G})$

$x, y, z \leftarrow \mathcal{S}\mathbb{Z}_q^*$

$b \leftarrow \mathcal{S}\{0,1\}$

$Z_0 = G^{xy}$

$Z_1 = G^z$

$b' \leftarrow \mathcal{A}(\mathbb{G}, q, G, G^x, G^y, Z_b)$

$b_0 := (b = b')$

return b_0

Figure 18: Security Game for DDH

We now define an interactive variant of the discrete logarithm (representation) assumption, and show that it is equivalent to the standard discrete logarithm assumption.

Definition B.5 (Interactive Discrete Logarithm (iDL) Assumption). *We say that the interactive discrete logarithm assumption holds over \mathcal{G} if for all $\ell \in \text{poly}(\lambda)$, every PPT adversary \mathcal{A}*

$$\Pr [\ell\text{-iDL}_{\mathcal{A}}(\mathcal{G})] \leq \text{negl}(\lambda),$$

where the game $\ell\text{-iDL}_{\mathcal{A}}(\mathcal{G})$ is defined in Figure 19.

$\ell\text{-iDL}_{\mathcal{A}}(\mathcal{G})$

$(\vec{\mathbf{g}}, \text{st}) \leftarrow \mathcal{A}(\mathbb{G}, q, G)$

$\vec{\mathbf{h}} \leftarrow \mathcal{S}\mathbb{G}^\ell$

$(\vec{\mathbf{a}}, \vec{\mathbf{b}}) \leftarrow \mathcal{A}(\text{st}, \vec{\mathbf{h}})$

$b_0 := (I = \vec{\mathbf{g}}^{\vec{\mathbf{a}}} \vec{\mathbf{h}}^{\vec{\mathbf{b}}})$

$b_1 := (\vec{\mathbf{b}} \neq \vec{0}^\ell)$

return $b_0 \wedge b_1$

Figure 19: Security Game for iDL

Theorem B.6. *The interactive discrete logarithm assumption holds over \mathcal{G} if and only if the discrete logarithm assumption holds over \mathcal{G} .*

Proof. The forward direction is trivial. For the backward direction, suppose there exists a PPT adversary \mathcal{A} which solves the interactive discrete logarithm problem for some $\ell \in \mathbb{N}$. We construct a PPT algorithm \mathcal{B} for the discrete logarithm problem as follows.

\mathcal{B} receives the discrete logarithm instance G^x for some unknown x . It runs \mathcal{A} up to the point where \mathcal{A} outputs a vector of group elements $\vec{\mathbf{g}}$. Let $m = |\vec{\mathbf{g}}| \in \text{poly}(\lambda)$ be the length of $\vec{\mathbf{g}}$. \mathcal{B} forks the execution of \mathcal{A} into $m+1$ parallel instances. Let $i^* \leftarrow \mathcal{S}[\ell]$ and $j^* \leftarrow \mathcal{S}[m+1]$. For the j -th instance, \mathcal{B} does the following:

If $j \neq j^*$, \mathcal{B} samples $\vec{x}_j \leftarrow \mathbb{Z}_q^\ell$. If $j = j^*$, \mathcal{B} chooses a random index $i^* \leftarrow [\ell]$ and writes symbolically $x_{i^*,j^*} := x$. For $i \in [\ell] \setminus \{ i^* \}$, it samples $x_{i,j^*} \leftarrow \mathbb{Z}_q$. In either case, it sends $\vec{h}_j := G^{\vec{x}_j}$ to \mathcal{A} , who responds with (\vec{a}_j, \vec{b}_j) with $I = \vec{g}^{\vec{a}_j} \vec{h}_j^{\vec{b}_j}$ and $\vec{b}_j \neq \vec{0}^\ell$. We can write

$$G^{-\langle \vec{b}_j, \vec{x}_j \rangle} = \vec{g}^{\vec{a}_j}.$$

Note that $|\vec{a}_j| = m$ for all $j \in [m+1]$ and therefore the set $\{ \vec{a}_1, \dots, \vec{a}_{m+1} \}$ must be linearly dependent, *i.e.*, there exists c_1, \dots, c_{m+1} not all zero such that

$$c_1 \cdot \vec{a}_1 + \dots + c_{m+1} \cdot \vec{a}_{m+1} = \vec{0}^m$$

Since j^* is chosen randomly, with probability at least $1/(m+1)$, we have $c_{j^*} \neq 0$. We can therefore write

$$\vec{a}_{j^*} = \sum_{j \in [m+1] \setminus \{j^*\}} c'_j \cdot \vec{a}_j$$

for some $c'_1, \dots, c'_{j^*-1}, c'_{j^*+1}, \dots, c'_{m+1}$. This implies

$$\begin{aligned} \vec{g}^{\vec{a}_{j^*}} &= \prod_{j \in [m+1] \setminus \{j^*\}} (\vec{g}^{\vec{a}_j})^{c'_j} \\ G^{\langle \vec{b}_{j^*}, \vec{x}_{j^*} \rangle} &= \prod_{j \in [m+1] \setminus \{j^*\}} \left(G^{\langle \vec{b}_j, \vec{x}_j \rangle} \right)^{c'_j} \\ G^{b_{i^*,j^*} x} &= G^{\sum_{j \in [m+1] \setminus \{j^*\}} c'_j \cdot \langle \vec{b}_j, \vec{x}_j \rangle - \sum_{i \in [\ell] \setminus \{ i^* \}} b_{i,j^*} x_{i,j^*}} \end{aligned}$$

Since i^* is chosen randomly, with probability at least $1/\ell$, we have $b_{i^*,j} \neq 0$. Hence we have $x = (b_{i^*,j})^{-1} \cdot \left(\sum_{j \in [m+1] \setminus \{j^*\}} c'_j \cdot \langle \vec{b}_j, \vec{x}_j \rangle - \sum_{i \in [\ell] \setminus \{ i^* \}} b_{i,j^*} x_{i,j^*} \right)$ which is a solution to the discrete logarithm problem instance. \square

Definition B.7 (Gap Diffie-Hellman (GapDH) [48]). *We say that the GapDH assumption according to holds over \mathcal{G} if for all $\ell \in \text{poly}(\lambda)$, every PPT adversary \mathcal{A}*

$$\Pr [\text{GapDH}_{\mathcal{A}}(\mathcal{G}) = 1] \leq \text{negl}(\lambda),$$

where the game $\text{GapDH}_{\mathcal{A}}(\mathcal{G})$ is defined in Figure 20

$\text{GapDH}_{\mathcal{A}}(\mathcal{G})$	$\mathcal{O}_{\text{DDH}}(A, B, C)$
$x \leftarrow \mathbb{Z}_q^*; y \leftarrow \mathbb{Z}_q^*$	$a := \log_G(A); b := \log_G(B)$
$H \leftarrow \mathcal{A}^{\text{DDH}}(\mathbb{G}, q, G, G^x, G^y)$	$c := \log_G(C)$
return $(H = G^{xy})$	return $(G^{ab} = G^c)$

Figure 20: Security Game for GapDH

B.3 Arguments of Knowledge

Definition B.8 (Arguments of Knowledge). *A triple $(\text{Setup}, \mathcal{P}, \mathcal{V})$ is called an argument of knowledge for a relation \mathcal{R} if it satisfies the following two definitions.*

On input 1^λ the setup algorithm Setup produces a common reference string crs . When interacting the prover \mathcal{P} and verifier \mathcal{V} produce a transcript $tr = \langle \mathcal{P}(\cdot), \mathcal{V}(\cdot) \rangle$ where $\langle \cdot \rangle$ denotes the actual protocol between \mathcal{P} and \mathcal{V} .

Definition B.9 (Perfect completeness). $(\text{Setup}, \mathcal{P}, \mathcal{V})$ has perfect completeness if for all non-uniform polynomial time adversaries \mathcal{A}

$$\Pr \left[\left(\text{crs}, \text{stmt}, \text{wit} \right) \notin \mathcal{R} \vee \left(\mathcal{P}(\text{crs}, \text{stmt}, \text{wit}), \mathcal{V}(\text{crs}, \text{stmt}) = 1 \right) \mid \left(\text{stmt}, \text{wit} \right) \leftarrow \mathcal{A}(\text{crs}) \right] = 1$$

Definition B.10 (Computational Witness-Extended Emulation [33]). $(\text{Setup}, \mathcal{P}, \mathcal{V})$ has witness-extended emulation if for all deterministic polynomial time \mathcal{P}^* there exists an expected polynomial time emulator \mathcal{E} such that for all pairs of interactive adversaries $\mathcal{A}_1, \mathcal{A}_2$ there exists a negligible function $\text{negl}(\lambda)$ such that

$$\left| \Pr \left[\begin{array}{c} \text{crs} \leftarrow \text{Setup}(1^\lambda), \\ (\text{stmt}, \text{wit}) \leftarrow \mathcal{A}_2(\text{crs}), \\ tr \leftarrow \langle \mathcal{P}^*(\text{crs}, \text{stmt}, \text{wit}), \\ \mathcal{V}(\text{crs}, \text{stmt}) \rangle \end{array} \mid \mathcal{A}_1(tr) = 1 \right] - \Pr \left[\begin{array}{c} \text{crs} \leftarrow \text{Setup}(1^\lambda), \\ (\text{stmt}, \text{wit}) \leftarrow \mathcal{A}_2(\text{crs}), \\ (tr \text{ is accepting}) \Rightarrow (\text{crs}, \text{stmt}, \text{wit}) \in \mathcal{R} \end{array} \mid (tr, \text{wit}) \leftarrow \mathcal{E}^{\mathcal{O}}(\text{crs}, \text{stmt}) \right] \right| \leq \text{negl}(\lambda)$$

where the oracle is given by $\mathcal{O} = \langle \mathcal{P}^*(\text{crs}, \text{stmt}, \text{wit}), \mathcal{V}(\text{crs}, \text{stmt}) \rangle$, and permits rewinding to a specific point and resuming with fresh randomness for the verifier from this point onwards.

Definition B.11 (Public coin). An argument of knowledge $(\text{Setup}, \mathcal{P}, \mathcal{V})$ is called public coin if all messages sent from the verifier to the prover are chosen uniformly at random and independently of the prover's messages, i.e., the challenges correspond to the verifier's randomness ρ .

Definition B.12 (Perfect Special Honest-Verifier Zero-Knowledge). A public coin argument of knowledge $(\text{Setup}, \mathcal{P}, \mathcal{V})$ is a perfect special honest verifier zero knowledge (SHVZK) argument of knowledge for \mathcal{R} if there exists a probabilistic polynomial time simulator \mathcal{S} such that for all pairs of interactive adversaries $\mathcal{A}_1, \mathcal{A}_2$

$$\Pr \left[\begin{array}{c} (\text{crs}, \text{stmt}, \text{wit}) \in \mathcal{R} \wedge \\ \mathcal{A}_1(tr) = 1 \end{array} \mid \begin{array}{c} \text{crs} \leftarrow \text{Setup}(1^\lambda), \\ (\text{stmt}, \text{wit}, \rho) \leftarrow \mathcal{A}_2(\text{crs}), \\ tr \leftarrow \langle \mathcal{P}(\text{crs}, \text{stmt}, \text{wit}), \mathcal{V}(\text{crs}, \text{stmt}; \rho) \rangle \end{array} \right] = \Pr \left[\begin{array}{c} (\text{crs}, \text{stmt}, \text{wit}) \in \mathcal{R} \wedge \\ \mathcal{A}_1(tr) = 1 \end{array} \mid \begin{array}{c} \text{crs} \leftarrow \text{Setup}(1^\lambda), \\ (\text{stmt}, \text{wit}, \rho) \leftarrow \mathcal{A}_2(\text{crs}), \\ tr \leftarrow \mathcal{S}(\text{stmt}, \rho) \end{array} \right]$$

B.4 Signatures of Knowledge

The term signatures of knowledge was widely used in the literature before it was formalized by [18]. We present a simplified definition which captures schemes in the random oracle model.

Definition B.13 (Signatures of Knowledge). Let \mathcal{R} be an NP relation for the language $\mathcal{L}_R = \{\text{stmt} \mid \exists \text{wit} : \mathcal{R}(\text{crs}, \text{stmt}, \text{wit}) = 1\}$ for a statement x and witness w . Let H be a random oracle. A signature of knowledge for \mathcal{L} and the message m in presence of H is a tuple of algorithms $\Sigma = (\text{Setup}, \text{SoKSign}^{\mathsf{H}}, \text{SoKVerify}^{\mathsf{H}}, \mathcal{S})$ defined below. Note that SoKSign and SoKVerify have oracle access to the random oracle H .

$\text{pp} \leftarrow \text{Setup}(1^\lambda, \mathcal{L})$ on input the security parameter 1^λ and the description of the language \mathcal{L} outputs the public parameters pp .

$\sigma \leftarrow \text{SoKSign}^{\mathsf{H}}(\text{pp}, \text{stmt}, \text{wit}, m)$ on input the public parameters pp , a statement $\text{stmt} \in \mathcal{L}$, the corresponding witness wit , and a message $m \in \mathcal{M}$, outputs a signature σ .

$b \leftarrow \text{SoKVerify}^{\mathsf{H}}(\text{pp}, \text{stmt}, \sigma, m)$ on input the public parameters pp , a statement $\text{stmt} \in \mathcal{L}$, a signature σ , and a message $m \in \mathcal{M}$, outputs a bit b deciding whether σ is a valid signature on m .

$\sigma \leftarrow \mathcal{S}(\text{pp}, \text{stmt}, m)$ on input the public parameters pp , a statement $\text{stmt} \in \mathcal{L}$, and a message $m \in \mathcal{M}$, outputs a signature σ .

Definition B.14 (Perfect Completeness). For all $\lambda \in \mathbb{N}$, $\text{pp} \in \text{Setup}(1^\lambda, \mathcal{L})$, (x, w) such that $R(\text{stmt}, \text{wit}) = 1$, $m \in \mathcal{M}$, $\sigma \in \text{SoKSign}^{\mathsf{H}}(\text{pp}, \text{stmt}, \text{wit}, m)$, we have $\text{SoKVerify}^{\mathsf{H}}(\text{pp}, \text{stmt}, \sigma, m) = 1$.

Definition B.15 (Perfect Simulatability). *It holds that that*

$$\left\{ \begin{array}{l} (\text{pp}, \sigma): \\ \text{pp} \leftarrow \text{Setup}(1^\lambda, \mathcal{L}) \\ \sigma \leftarrow \text{SoKSign}^H(\text{pp}, \text{stmt}, \text{wit}, m) \end{array} \right\} = \left\{ \begin{array}{l} (\text{pp}, \sigma): \\ \text{pp} \leftarrow \text{Setup}(1^\lambda, \mathcal{L}) \\ \sigma \leftarrow \mathcal{S}(\text{pp}, \text{stmt}, m) \end{array} \right\}$$

Definition B.16 (Extractability). *For all PPT adversaries \mathcal{A} , there exists a PPT extractor $\mathcal{E}_{\mathcal{A}}$ such that for all $\text{stmt} \in \{0,1\}^*$, if*

$$\Pr \left[\begin{array}{l} \text{pp} \leftarrow \text{Setup}(1^\lambda, \mathcal{L}) \\ b=1: (\sigma, m) \leftarrow \mathcal{A}^H(\text{pp}) \\ b \leftarrow \text{SoKVerify}^H(\text{pp}, \text{stmt}, \sigma, m) \end{array} \right] > \text{negl}(\lambda)$$

then

$$\Pr \left[R(x, w) = 1: \begin{array}{l} \text{pp} \leftarrow \text{Setup}(1^\lambda, \mathcal{L}) \\ w \leftarrow \mathcal{E}_{\mathcal{A}}(\text{pp}, \text{stmt}) \end{array} \right] > 1 - \text{negl}(\lambda)$$

It is straightforward to show that a perfectly complete, perfectly special honest verifier zero-knowledge, public-coin logarithmic-round argument of knowledge scheme for a language \mathcal{L} with extended-witness emulation can be transformed into a perfectly complete, extractable, perfectly simulatable signature of knowledge scheme for \mathcal{L} and the message space $\mathcal{M} = \{0,1\}^*$ using the Fiat-Shamir heuristics [24].

B.5 Labeled Public-key Encryption Scheme

The notion of the labeled public-key encryption scheme is formally considered by Shoup [58]. Compared with the standard public-key encryption, the encryption and decryption algorithms of a labeled encryption scheme take an additional labeled as input. A label can be considered as a binary string with a length polynomially bounded by the security parameter.

Definition B.17 (Labeled Public-key Encryption). *A labeled public-key encryption scheme is a tuple of algorithms $\text{PKE} = (\text{Setup}, \text{KGen}, \text{Enc}, \text{Dec})$ defined below.*

$\text{pp} \leftarrow \text{Setup}(1^\lambda)$ is a probabilistic algorithm which takes as input the security parameter 1^λ , and outputs a public parameter pp .

$(\text{pk}, \text{sk}) \leftarrow \text{KGen}(\text{pp})$ is a probabilistic algorithm which takes as input the public parameter pp , and outputs a public key pk and a secret key sk . We assume the existence of an algorithm $\text{SKVerify}(\text{pk}, \text{sk})$ which checks if pk is a valid public key corresponding to sk .

$c \leftarrow \text{Enc}(\text{pk}, \tau, m)$ is a probabilistic algorithm which inputs the public key pk , a labeled τ , and a message m , and outputs a ciphertext c .

$m \leftarrow \text{Dec}(\text{sk}, \tau, c)$ is a deterministic algorithm which takes as input the secret key sk , a labeled τ , and a ciphertext c , and outputs a message m (or \perp upon failure).

Correctness of PKE requires that for all message-label pairs (m, τ) , all $\text{pp} \in \text{Setup}(1^\lambda)$, and all $(\text{pk}, \text{sk}) \in \text{KGen}(\text{pp})$, $\text{Dec}(\text{sk}, \tau, \text{Enc}(\text{pk}, \tau, m))$ always returns m .

We follow the CCA security requirement of the labeled encryption considered in [58]. This notion is stronger than weak CCA in [29, 35] by allowing the adversary to make any query to the decryption oracle provided that the queried label and the queried ciphertext are not the challenging ones simultaneously.

Definition B.18 (IND-CCA). *PKE is indistinguishable under chosen-ciphertext attack (IND-CCA) if for every PPT adversary \mathcal{A} there exists a negligible function $\text{negl}(\lambda)$ such that*

$$\left| \Pr[\text{IND-CCA}_{\text{PKE}, \mathcal{A}}^0(1^\lambda) = 1] - \Pr[\text{IND-CCA}_{\text{PKE}, \mathcal{A}}^1(1^\lambda) = 1] \right| \leq \text{negl}(\lambda)$$

where the game $\text{IND-CCA}_{\text{PKE}, \mathcal{A}}^b(1^\lambda)$ is defined in Figure 21.

$\text{IND-CCA}_{\text{PKE},\mathcal{A}}^b$	$\text{DecO}(\tau,c)$
$\text{pp} \leftarrow \text{PKE.Setup}(1^\lambda)$	if $(\tau,c) = (\tau^*,c^*)$
$(\text{pk},\text{sk}) \leftarrow \text{PKE.KGen}(\text{pp})$	return \perp
$(\text{st},m_0,m_1,\tau^*) \leftarrow \mathcal{A}^{\text{DecO}}(\text{pk})$	else
$c^* \leftarrow \text{PKE.Enc}(\text{pk},\tau^*,m_b)$	return $\text{PKE.Dec}(\text{sk},\tau,c)$
$b' \leftarrow \mathcal{A}^{\text{DecO}}(\text{st},c^*)$	endif
return b'	

Figure 21: Security Game for IND-CCA of PKE

$\text{IK-CCA}_{\text{PKE},\mathcal{A}}^b$	$\text{DecO}(i,\tau,c)$
$\text{pp} \leftarrow \text{PKE.Setup}(1^\lambda)$	if $(\tau,c) = (\tau^*,c^*)$
$(\{\text{pk}_i,\text{sk}_i\}_{i=0}^1) \leftarrow \text{PKE.KGen}(\text{pp})$	return \perp
$(\text{st},m^*,\tau^*) \leftarrow \mathcal{A}^{\text{DecO}}(\text{pk}_0,\text{pk}_1)$	else
$c^* \leftarrow \text{PKE.Enc}(\text{pk}_b,\tau^*,m^*)$	return $\text{PKE.Dec}(\text{sk}_i,\tau,c)$
$b' \leftarrow \mathcal{A}^{\text{DecO}}(\text{st},c^*)$	endif
return b'	

Figure 22: Security Game for IK-CCA of PKE

Key-privacy of the public-key encryption is first introduced by Bellare *et al.* [8], which requires no PPT adversary viewing a chosen message encrypted under one of two public keys can guess which public key is used. It implies that the receiver of the ciphertext (*i.e.*, the owner of the public key) is anonymous from the point of view of the adversary. Key-privacy property is essential for the scenario where identities are needed to be protected, *e.g.*, anonymous communications and cryptocurrencies. We give the definition of key privacy for a labeled public-key encryption scheme as below.

Definition B.19 (IK-CCA). PKE is key-private under chosen ciphertext attack (IK-CCA) if for every PPT adversary \mathcal{A} there exists a negligible function $\text{negl}(\lambda)$ such that

$$\left| \Pr[\text{IK-CCA}_{\text{PKE},\mathcal{A}}^0(1^\lambda) = 1] - \Pr[\text{IK-CCA}_{\text{PKE},\mathcal{A}}^1(1^\lambda) = 1] \right| \leq \text{negl}(\lambda)$$

where the game $\text{IK-CCA}_{\text{PKE},\mathcal{A}}^b(1^\lambda)$ is defined in Figure 22.

B.6 Homomorphic Commitment Scheme

A commitment scheme allows the sender to commit to a value and later reveal that value to a receiver by showing the value together with the opening. The receiver is able to verify that this value was indeed contained in the commitment. A commitment scheme should be hiding that the commitment does not tell anything about the committed value, and binding that the commitment can only be opened to the value which it was committed to.

Definition B.20 (Commitment). A commitment scheme consists of two probabilistic algorithms (Setup, Com) defined below.

$\text{crs} \leftarrow \text{Setup}(1^\lambda)$ is a probabilistic algorithm which takes as input the security parameter 1^λ , and outputs a common reference string crs , which specifies a message space \mathcal{M} , a randomness space χ and a commitment space \mathcal{C} .

$C \leftarrow \text{Com}_{\text{crs}}(m;r)$ is a probabilistic algorithm which takes as input the common reference string crs , a message $m \in \mathcal{M}$, and a randomness $r \in \chi$, and outputs a commitment $C \in \mathcal{C}$.

$\text{Hiding}_{\text{HC},\mathcal{A}}^b(1^\lambda)$ <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $\text{crs} \leftarrow \text{Setup}(1^\lambda)$ $(m_0, m_1) \leftarrow \mathcal{A}(\text{crs})$ $C \leftarrow \text{Com}_{\text{crs}}(m_b)$ $b' \leftarrow \mathcal{A}(C)$ $\mathbf{return } b'$
--

Figure 23: Hiding Experiment for Commitments

$\text{Binding}_{\text{HC},\mathcal{A}}(1^\lambda)$ <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $\text{crs} \leftarrow \text{Setup}(1^\lambda)$ $(m_0, r_0, m_1, r_1) \leftarrow \mathcal{A}(\text{crs})$ $b_0 := ((m_0, r_0) \neq (m_1, r_1))$ $b_1 := (\text{Com}_{\text{crs}}(m_0; r_0) = \text{Com}_{\text{crs}}(m_1; r_1))$ $\mathbf{return } b_0 \wedge b_1$

Figure 24: Security Game for Binding of Commitments

Definition B.21 (Perfect Hiding). A commitment scheme $\text{HC} = (\text{Setup}, \text{Com})$ is perfectly hiding if for every PPT adversary \mathcal{A} there exists a negligible function $\text{negl}(\lambda)$ such that

$$\Pr[\text{Hiding}_{\text{HC},\mathcal{A}}^0(1^\lambda) = 1] = \Pr[\text{Hiding}_{\text{HC},\mathcal{A}}^1(1^\lambda) = 1]$$

where the game $\text{Hiding}_{\text{HC},\mathcal{A}}^b(1^\lambda)$ is defined in Figure 23.

Definition B.22 (Computationally Binding). A commitment scheme $(\text{Setup}, \text{Com})$ is computationally binding if for every PPT adversary \mathcal{A} there exists a negligible function $\text{negl}(\lambda)$ such that

$$\Pr[\text{Binding}_{\text{HC},\mathcal{A}}(1^\lambda) = 1] \leq \text{negl}(\lambda)$$

where the game $\text{Binding}_{\text{HC},\mathcal{A}}(1^\lambda)$ is defined in Figure 24.

Suppose that $(\mathcal{M}, +)$, $(\chi, +)$, and (\mathcal{C}, \cdot) are groups. Then HC is said to be homomorphic if the product of two commitments is a commitment to the sum of the two committed values.

Definition B.23 (Homomorphic Commitment Scheme). A commitment scheme is homomorphic if for all $\text{crs} \in \text{Setup}(1^\lambda)$, all $m, m' \in \mathcal{M}$ and all $r, r' \in \chi$

$$\text{Com}_{\text{crs}}(m + m', r + r') = \text{Com}_{\text{crs}}(m, r) \cdot \text{Com}_{\text{crs}}(m', r').$$

C Tracking and Viewing

C.1 Definitions

We extend the model to cover transaction tracking and viewing, which are features supported by some previous schemes.

Definition C.1 (Extended RingCT). An extended RingCT is a RingCT scheme with a slightly extended syntax, where an original account acc is split into an extended form, consisting of an account acc , some tracking information $\text{info}_{\text{Track}}$, and some viewing information $\text{info}_{\text{View}}$.

C.1.1 Trackability

CryptoNote [62], the predecessor of RingCT, introduces a feature called tracking. It allows a user to voluntarily delegate a tracking key to a trusted third party², so that the latter can track incoming transactions on behalf of the user. This is particularly useful for a computationally constrained user as tracking incoming transactions requires monitoring all new messages posted on the public ledger.

Note that the trackability is not meant to be a security feature. Indeed, a user can easily avoid being tracked by not delegating the tracking key or simply creating another master public key. Tracking also relies on the well-formedness of the account acc in a transaction. As a spend proof does not necessarily guarantee the well-formedness of the entire account acc while still being considered as valid, a “cheating” spender can easily help the receiver to avoid being tracked³.

Definition C.2 (Trackability). *An extended RingCT scheme is said to be trackable if the following holds:*

1. *There exists additionally a tuple of PPT algorithms (TKGen,TKVerify,Track) defined as follows:*

$\text{tsk} \leftarrow \text{TKGen}(\text{msk})$: *The tracking key generation algorithm inputs a master secret key msk , and outputs a tracking key tsk .*

$b \leftarrow \text{TKVerify}(\text{mpk}, \text{tsk})$: *The tracking key verification algorithm inputs a master public key mpk and a tracking key tsk . It outputs a bit b indicating if tsk is a valid tracking key corresponding to the master public key mpk .*

$b \leftarrow \text{Track}(\text{tsk}, \text{acc}, \text{info}_{\text{Track}})$: *The track algorithm inputs a tracking key tsk , an account acc , and some tracking information. It outputs a bit b indicating if acc is an account generated from the master public key mpk corresponding to tsk .*

2. *For all $\lambda, \alpha, \beta \in \mathbb{N}$, all $\text{pp} \in \text{Setup}(1^\lambda, 1^\alpha, 1^\beta)$, all $(\text{mpk}, \text{msk}) \in \text{SAKGen}(\text{pp})$, all $\text{tsk} \in \text{TKGen}(\text{msk})$, it holds that $\text{TKVerify}(\text{mpk}, \text{tsk}) = 1$.*
3. *For all $\lambda, \alpha, \beta \in \mathbb{N}$, all $\text{pp} \in \text{Setup}(1^\lambda, 1^\alpha, 1^\beta)$, all (mpk, tsk) such that $\text{TKVerify}(\text{mpk}, \text{tsk}) = 1$, all $a \in \{0, \dots, 2^\beta - 1\}$, all $(\text{ck}, \text{acc}, \text{info}_{\text{Track}}, \text{info}_{\text{View}}) \in \text{OTAccGen}(\text{mpk}, a)$, it holds that $\text{Track}(\text{tsk}, \text{acc}, \text{info}_{\text{Track}}) = 1$.*

C.1.2 Viewability

RingCT extends the tracking capability such that the designated third party can also learn the amount to be received in an incoming transaction. Viewability is useful in scenarios where a user wishes to have incoming transactions to its address audited. For instance, a charity fund may want a third party or even the general public to audit the amount of donations that it receives, given that the donors are willing to disclose it. To allow more fine-grained tracking permissions, we call this new feature viewability. Similar to trackability, viewability is not meant to be a security feature.

Definition C.3 (Viewability). *An extended RingCT scheme is said to be viewable if the following holds:*

1. *There exists additionally a tuple of PPT algorithms (VKGen, VKVerify, View) defined as follows:*

$\text{vsk} \leftarrow \text{VKGen}(\text{msk})$: *The viewing key generation algorithm inputs a master secret key msk , and outputs a viewing key vsk .*

$b \leftarrow \text{VKVerify}(\text{mpk}, \text{vsk})$: *The viewing key verification algorithm inputs a master public key mpk and a viewing key vsk . It outputs a bit b indicating if vsk is a valid viewing key corresponding to the master public key mpk .*

$a \leftarrow \text{View}(\text{vsk}, \text{acc}, \text{info}_{\text{View}})$: *The view algorithm inputs a viewing key vsk , an account acc , and some viewing information $\text{info}_{\text{View}}$. It outputs an amount a stored in the account acc .*

2. *For all $\lambda, \alpha, \beta \in \mathbb{N}$, all $\text{pp} \in \text{Setup}(1^\lambda, 1^\alpha, 1^\beta)$, all $(\text{mpk}, \text{msk}) \in \text{SAKGen}(\text{pp})$, all $\text{vsk} \in \text{VKGen}(\text{msk})$, it holds that $\text{VKVerify}(\text{mpk}, \text{vsk}) = 1$.*
3. *For all $\lambda, \alpha, \beta \in \mathbb{N}$, all $\text{pp} \in \text{Setup}(1^\lambda, 1^\alpha, 1^\beta)$, all (mpk, vsk) such that $\text{VKVerify}(\text{mpk}, \text{vsk}) = 1$, all $a \in \{0, \dots, 2^\beta - 1\}$, all $(\text{ck}, \text{acc}, \text{info}_{\text{Track}}, \text{info}_{\text{View}}) \in \text{OTAccGen}(\text{mpk}, a)$, it holds that $\text{View}(\text{vsk}, \text{acc}, \text{info}_{\text{View}}) = a$.*

²Different users can delegate to different third parties.

³This could happen in a scenario where the sender is donating some money to a political entity and the entity wishes to hide it from the IRS.

Extending syntax of accounts	
// All accounts $\text{acc} = (\text{pk}, \text{co}, \tilde{\text{ek}}, \tilde{\text{ck}})$ are now split into $\text{acc} = (\text{pk}, \text{co})$, $\text{info}_{\text{Track}} = \tilde{\text{ek}}$, and $\text{info}_{\text{View}} = \tilde{\text{ck}}$.	
<div style="border-bottom: 1px solid black; margin-bottom: 5px;">TKGen(msk)</div> parse msk as (tsk, vsk, x) return tsk	<div style="border-bottom: 1px solid black; margin-bottom: 5px;">VKGen(msk)</div> parse msk as (tsk, vsk, x) return vsk
<div style="border-bottom: 1px solid black; margin-bottom: 5px;">TKVerify(mpk, tsk)</div> parse mpk as (tpk, vpk, X) return SKVerify(tpk, tsk)	<div style="border-bottom: 1px solid black; margin-bottom: 5px;">VKVerify(mpk, vsk)</div> parse mpk as (tpk, vpk, X) return SKVerify(vpk, vsk)
<div style="border-bottom: 1px solid black; margin-bottom: 5px;">Track(tsk, acc, info_{Track})</div> return $b := \text{Dec}(\text{tsk}, \text{acc}, \text{info}_{\text{Track}}) \neq \perp$	<div style="border-bottom: 1px solid black; margin-bottom: 5px;">View(vsk, acc, info_{View})</div> $(a, r) \leftarrow \text{Dec}(\text{vsk}, \text{acc}, \text{info}_{\text{View}})$ if $\text{co} = \text{Com}(a; r)$ then return a else return \perp

Figure 25: RingCT Construction (Extensions).

C.1.3 On Trackability and Viewability against Malicious Parties

The above definitions for traceability and viewability assume honest spenders and receivers, which is sufficient for the intended purposes as discussed above. For curiosity, we briefly discuss trackability and viewability against malicious spenders and receivers, and even trackers and viewers.

From a definitional point of view, defining such notions is not an issue. Indeed, similar notions are well-known in related primitives, such as the traceability and non-frameability of group signatures [7]. It is straightforward to adopt these definitions to RingCT.

Any schemes satisfying these strengthened notions however are likely much less efficient than those that do not. Intuitively, in a scheme which is trackable against malicious receivers, a user should not be able to spend from an account unless the account can be tracked by the tracker. Consider the following construction template. The user generates a tracking key and proves that the key is bound to its master public key. To spend, the user must prove that the source account is associated to some master public key. This convinces the tracker that the user is only able to spend from accounts that can be tracked. However, notice that the spender has to prove that the master public key associated to the source account is a member of the set of all master public keys. This is much more expensive than proving that the source account is a member of a set of ring accounts, and is against the design philosophy of RingCT.

Lastly, even if the RingCT scheme provides trackability and viewability against malicious parties, the users can still easily avoid being tracked or viewed by simply creating another stealth address.

C.2 Extension to Construction

We extend our construction in Section 4 to support tracking and viewing. The descriptions of the algorithms are in Figure 25.

Extending syntax of accounts All accounts (output of OTAccGen, input to Spend, Receive, and Vf) $\text{acc} = (\text{pk}, \text{co}, \tilde{\text{ek}}, \tilde{\text{ck}})$ are now split into $\text{acc} = (\text{pk}, \text{co})$, $\text{info}_{\text{Track}} = \tilde{\text{ek}}$, and $\text{info}_{\text{View}} = \tilde{\text{ck}}$.

Tracking key generation Given the master secret key msk the tracking key generation algorithm parses msk as (tsk, vsk, x) and returns tsk.

<p>InitOracles()</p> <hr/> <p>// In addition to the original InitOracles()... // Initialize Sets Tracked := Viewed := TKRevealed := VKRevealed := \emptyset</p>	<p>VKGen$\mathcal{O}(k)$</p> <hr/> <p>// Reveal viewing key of an honest user. vsk \leftarrow VKGen(MSK[k]) VKRevealed := VKRevealed \cup {k} return vsk</p>
<p>TKGen$\mathcal{O}(k)$</p> <hr/> <p>// Reveal tracking key of an honest user. tsk \leftarrow TKGen(MSK[k]) TKRevealed := TKRevealed \cup {k} return tsk</p>	<p>View$\mathcal{O}(k, \text{acc}, \text{info}_{\text{View}})$</p> <hr/> <p>// Instruct (auditor of) user k view amount of // account acc. vsk := VKGen(MSK[k]) a \leftarrow View(vsk, acc) Viewed := Viewed \cup {(k, acc, info_{View})} return a</p>
<p>Track$\mathcal{O}(k, \text{acc}, \text{info}_{\text{Track}})$</p> <hr/> <p>// Instruct (auditor of) user k to track acc. tsk := TKGen(MSK[k]) b \leftarrow Track(tsk, acc) Tracked := Tracked \cup {(k, acc, info_{Track})} return b</p>	

Figure 26: Oracles for track and view

Track Given a tracking key tsk, an account acc, and some tracking information info_{Track}, the tracking algorithm checks if $\text{Dec}(\text{tsk}, \text{acc}, \text{info}_{\text{Track}}) \neq \perp$.

Viewing key generation Given the master secret key msk the viewing key generation algorithm parses msk as (tsk, vsk, x) and returns vsk.

View Given a viewing key vsk, an account acc, and some viewing information info_{view}, the viewing algorithm checks if $\text{Dec}(\text{vsk}, \text{acc}, \text{info}_{\text{View}}) = (a, r) \neq \perp$ and returns a if successful.

To prove security in the presence of these additional algorithms we define the oracles Figure 26.

D Security Proofs for Ω

We present here the full security analysis of our RingCT construction Ω in Section 4. In addition to the oracles given in Figure 1 we extend the security games for privacy and non-slanderability by providing the adversary with the additional oracles described in Figure 26 to model the implications of Tracking and Viewing on security.

D.1 Proof of Theorem 4.2 (Balance)

Proof. First we show that CheckTag is computationally binding. Suppose not, let \mathcal{A} be a PPT adversary who outputs an account $\text{acc} = (\text{pk}, \text{co}, \text{ek}, \text{ck})$ and two distinct inputs (sk, tag) and $(\text{sk}', \text{tag}')$ such that both satisfy the predicate CheckTag. That is, $\text{tag} = \text{TagEval}(\text{sk})$, $\text{tag}' = \text{TagEval}(\text{sk}')$, and $\text{pk} = \text{Com}(0, \text{sk}) = \text{Com}(0, \text{sk}')$. From the last relation, we must have $\text{sk} = \text{sk}'$ or otherwise we can construct an adversary against the binding property of HC. It then follows that $\text{tag} = \text{tag}'$ since TagEval is deterministic. We therefore have $(\text{sk}, \text{tag}) = (\text{sk}', \text{tag}')$ which is a contradiction.

Next we show that CheckAmount is computationally binding. Suppose not, let \mathcal{A} be a PPT adversary who outputs an account $\text{acc} = (\text{pk}, \text{co}, \text{ek}, \text{ck})$ and two distinct inputs (ck, a) and (ck', a') such that both satisfy the predicate CheckAmount. That is, $\text{co} = \text{Com}(a; \text{ck}) = \text{Com}(a'; \text{ck}')$. This directly contradicts with the binding property of HC.

We then construct an extractor \mathcal{E} given any PPT adversary \mathcal{A} which outputs (tx, σ) such that $\text{Vf}(\text{tx}, \sigma) = 1$ with non-negligible probability. By extractability of the SoK, there exists an efficient extraction algorithm $\text{SoK}.\mathcal{E}_{\mathcal{A}}$ which

extracts a witness wit for the statement $\text{stmt} = \text{stmt}(\text{tx})$ with overwhelming probability. Parse stmt and wit as

$$\begin{aligned}\text{stmt} &= \left(\{ \text{acc}_i^{\mathcal{R}} \}_{i=1}^{|\mathcal{R}|}, \{ \text{tag}_i \}_{i=1}^{|\mathcal{S}|}, \{ \text{acc}_i^{\mathcal{T}} \}_{i=1}^{|\mathcal{T}|} \right) \\ \text{wit} &= \left(\{ (j_i, x_i, a_i^{\mathcal{S}}, r_i^{\mathcal{S}}) \}_{i=1}^{|\mathcal{S}|}, \{ (a_i^{\mathcal{T}}, r_i^{\mathcal{T}}) \}_{i=1}^{|\mathcal{T}|} \right)\end{aligned}$$

We have

$$\begin{aligned}\forall i \in [|\mathcal{S}|], & \begin{cases} \text{pk}_{j_i}^{\mathcal{R}} = \text{Com}(0; x_i) \\ \text{co}_{j_i}^{\mathcal{R}} = \text{Com}(a_i^{\mathcal{S}}; r_i^{\mathcal{S}}) \\ \text{tag}_i = \text{TagEval}(x_i) \end{cases} \\ \forall i \in [|\mathcal{T}|], & \begin{cases} \text{co}_i^{\mathcal{T}} = \text{Com}(a_i^{\mathcal{T}}; r_i^{\mathcal{T}}) \\ a_i^{\mathcal{T}} \in \{0, \dots, 2^\beta - 1\} \end{cases} \\ & \sum_{i \in [|\mathcal{S}|]} a_i^{\mathcal{S}} = \sum_{i \in [|\mathcal{T}|]} a_i^{\mathcal{T}}\end{aligned}$$

This implies the following:

- $\forall i \in [|\mathcal{S}|], \text{CheckTag}(\text{acc}_{j_i}^{\mathcal{R}}, \text{sk}_i, \text{tag}_i) = 1$
- $\forall i \in [|\mathcal{S}|], \text{CheckAmount}(\text{acc}_{j_i}^{\mathcal{R}}, \text{ck}_i^{\mathcal{S}}, a_i^{\mathcal{S}}) = 1$
- $\forall i \in [|\mathcal{T}|], \text{CheckAmount}(\text{acc}_i^{\mathcal{T}}, \text{ck}_i^{\mathcal{T}}, a_i^{\mathcal{T}}) = 1$

Furthermore, since $a_i^{\mathcal{T}} \in \{0, \dots, 2^\beta - 1\}$ for all $i \in [|\mathcal{T}|]$, $|\mathcal{T}| \leq 2^\alpha$, and $\{0, \dots, 2^{\alpha+\beta} - 1\} \subseteq \mathcal{M}$, $\sum_{i \in [|\mathcal{T}|]} a_i^{\mathcal{T}}$ has the same value when interpreted as an element in \mathcal{M} and as an element in \mathbb{Z} . On the other hand, for each $i \in [|\mathcal{S}|]$, the value of $a_i^{\mathcal{S}}$ when interpreted as an element in \mathbb{Z} must be greater than that when interpreted as an element in \mathcal{M} . Therefore $\sum_{i \in [|\mathcal{S}|]} a_i^{\mathcal{S}} = \sum_{i \in [|\mathcal{T}|]} a_i^{\mathcal{T}}$ (in \mathcal{M}) implies $\sum_{i \in [|\mathcal{S}|]} a_i^{\mathcal{S}} \geq \sum_{i \in [|\mathcal{T}|]} a_i^{\mathcal{T}}$ (in \mathbb{Z}).

Now, for any PPT adversary \mathcal{A} , let $\mathcal{E} = \text{SoK}.\mathcal{E}_{\mathcal{A}}$. By the analysis above, it must hold that $\Pr[\text{Balance}_{\Omega, \mathcal{A}, \mathcal{E}_{\mathcal{A}}}(1^\lambda, 1^\alpha, 1^\beta) = 1] \leq \text{negl}(\lambda)$. □

D.2 Proof of Theorem 4.3 (Privacy)

We extend the privacy game from Figure 3 to include viewing and tracing.

Proof. We give a brief intuition of the proof.

Intuition. We prove by hybrid arguments. To give some intuition as to how we progress through the hybrids: We start off in the real experiment for $\text{Privacy}_{\Omega, \mathcal{A}}^0$ where the challenge bit b is set to 0. This means that the challenge spend proof given to the adversary is associated with the keys from the sets \mathcal{S}_0 and \mathcal{T}_0 (remember, two sets of source accounts \mathcal{S}_0 and \mathcal{S}_1 with $|\mathcal{S}_0| = |\mathcal{S}_1|$ and target accounts \mathcal{T}_0 and \mathcal{T}_1 with $|\mathcal{T}_0| = |\mathcal{T}_1|$ were specified by the adversary). We then transition through four hybrids where, in the end the spend proof is independent of the sets \mathcal{S}_0 and \mathcal{T}_0 , while the tags are generated corresponding to the keys in \mathcal{S}_1 .

We now begin switching the target accounts from being associated to the set \mathcal{T}_0 to the set \mathcal{T}_1 . We can do this without worrying about the spending keys because the spending keys are already delinked from the tags and are independent from the target accounts. After a sequence of hybrids we will completely switch to having the target accounts as being associated to set \mathcal{T}_1 . Now we switch the spend proof from being simulated to being honestly generated from $(\mathcal{S}_1, \mathcal{T}_1)$. With this we finally end up with a hybrid that is identical to the real experiment for $\text{Privacy}_{\Omega, \mathcal{A}}^1$ where the challenge bit b is set to 1. Proving the indistinguishability of the successive hybrids results in proving indistinguishability in the theorem. We define the hybrid experiments as

```

Privacy $_{\Omega, \mathcal{A}}^b(1^\lambda, 1^\alpha, 1^\beta)$ 
pp  $\leftarrow$  Setup( $1^\lambda, 1^\alpha, 1^\beta$ ), InitOracles()
 $\mathcal{O} := \{\text{SAKGen}\mathcal{O}, \text{TKGen}\mathcal{O}, \text{VKGen}\mathcal{O}, \text{Spend}\mathcal{O}, \text{Receive}\mathcal{O}, \text{Track}\mathcal{O}, \text{View}\mathcal{O}\}$ 
( $I, J, \mathcal{R}, \mathcal{S}, \mathcal{T}, \mu$ )  $\leftarrow$   $\mathcal{A}^{\mathcal{O}}$ (pp)
 $\mathcal{S}_0 := \mathcal{S}_1 := \mathcal{S}, \mathcal{T}_0 := \mathcal{T}_1 := \mathcal{T}$ 
// Preparing honest spenders as instructed by adversary.
parse  $I$  as  $\left\{ (s_i, \{j_{t,i}, k_{t,i}^S, \ell_{t,i}\}_{t=0}^1) \right\}_{i=1}^{|I|}$ 
for  $i \in [|I|]$  do
  for  $t \in \{0,1\}$  do
    ( $\text{ck}_{t,i}^S, \text{sk}_{t,i}^S, a_{t,i}^S, \text{tag}_{t,i}, \text{acc}_{t,i}^S, \text{info}_{\text{Track}, t, i}^S, \text{info}_{\text{View}, t, i}^S$ ) := Wallet( $k_{t,i}^S$ )[ $\ell_{t,i}$ ]
     $\mathcal{R}[j_{t,i}] := \text{acc}_{t,i}^S$ 
     $\mathcal{S}_t[s_i] := (j_{t,i}, \text{ck}_{t,i}^S, \text{sk}_{t,i}^S, a_{t,i}^S, \text{tag}_{t,i})$ 
  endfor
  if  $\text{tag}_{0,i} \neq \text{tag}_{1,i} \wedge \{\text{tag}_{0,i}, \text{tag}_{1,i}\} \cap \text{Spent} \neq \emptyset$  then return 0
endfor
// Preparing honest receivers as instructed by adversary.
parse  $J$  as  $\left\{ (d_j, \{k_{t,j}^T, a_{t,j}^T\}_{t=0}^1) \right\}_{j=1}^{|J|}$ 
for  $j \in [|J|]$  do
  for  $t \in \{0,1\}$  do
    ( $\text{ck}_{t,j}^T, \text{acc}_{t,j}^T, \text{info}_{\text{Track}, t, j}^T, \text{info}_{\text{View}, t, j}^T$ ) := OTAccGen(MPK( $k_{t,j}^T$ ),  $a_{t,j}^T$ )
     $\mathcal{T}_t[d_j] := (\text{ck}_{t,j}^T, a_{t,j}^T, \text{acc}_{t,j}^T, \text{info}_{\text{Track}, t, j}^T, \text{info}_{\text{View}, t, j}^T)$ 
  endfor
endfor
for  $t \in \{0,1\}$  do
   $\text{tx}_t := \text{tx}(\mathcal{R}, \mathcal{S}_t, \mathcal{T}_t, \mu)$ 
   $\sigma_t \leftarrow \text{Spend}(\mathcal{R}, \mathcal{S}_t, \mathcal{T}_t, \mu)$ 
  if  $\forall f(\text{tx}_t, \sigma_t) = 0$  then return 0
endfor
 $b_0 \leftarrow \mathcal{A}^{\mathcal{O}}$ ( $\text{tx}_b, \sigma_b$ )
 $b_1 := (\text{TKRevealed} \cup \text{VKRevealed}) \cap \{k_{t,j}^T : t \in \{0,1\}, j \in [|J|]\} = \emptyset$ 
 $b_2 := (\text{Received} \cap \{(k_{t,j}^T, \text{acc}_{b,j}^T, \text{info}_{\text{Track}, b, j}^T, \text{info}_{\text{View}, b, j}^T) : t \in \{0,1\}, j \in [|J|]\}) = \emptyset$ 
 $b_3 := (\text{Tracked} \cap \{(k_{t,j}^T, \text{acc}_{b,j}^T, \text{info}_{\text{Track}, b, j}^T) : t \in \{0,1\}, j \in [|J|]\}) = \emptyset$ 
 $b_4 := (\text{Viewed} \cap \{(k_{t,j}^T, \text{acc}_{b,j}^T, \text{info}_{\text{View}, b, j}^T) : t \in \{0,1\}, j \in [|J|]\}) = \emptyset$ 
return  $b_0 \wedge b_1 \wedge b_2 \wedge b_3 \wedge b_4$ 

```

Figure 27: Privacy Experiment (with Tracking and Viewing)

follows:

Definition of main hybrid experiments.

Hyb_1 is identical to the privacy experiment $\text{Privacy}_{\Omega, \mathcal{A}}^0$.

Hyb_2 differs from Hyb_1 in the way the signature σ is generated whenever the Spend algorithm is executed (both in the spend oracle and the challenge selection). Instead of using SoKSig, the challenger computes σ using the simulator \mathcal{S} that is guaranteed to exist by the simulatability of SoK.

Note that in Hyb_2 the only information about b available to the adversary is the transaction

$$\text{tx}_0 = \left(\left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_{0,i} \right\}_{i=1}^{|\mathcal{S}|}, \left\{ \text{acc}_{0,i}^{\mathcal{T}}, \text{info}_{\text{Track},0,i}^{\mathcal{T}}, \text{info}_{\text{View},0,i}^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|}, \mu \right).$$

The proof σ_0 does not give any extra information as it is computed from $\text{stmt}(\text{tx}_0)$. In the following hybrids, we gradually switch

$$\left\{ \text{tag}_{0,i} \right\}_{i=1}^{|\mathcal{S}|} \text{ and } \left\{ \text{acc}_{0,i}^{\mathcal{T}}, \text{info}_{\text{Track},0,i}^{\mathcal{T}}, \text{info}_{\text{View},0,i}^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|}$$

to

$$\left\{ \text{tag}_{1,i} \right\}_{i=1}^{|\mathcal{S}|} \text{ and } \left\{ \text{acc}_{1,i}^{\mathcal{T}}, \text{info}_{\text{Track},1,i}^{\mathcal{T}}, \text{info}_{\text{View},1,i}^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|}$$

respectively.

Hyb_3 differs from Hyb_2 in the way the proof is simulated. The proof is now simulated from

$$\left(\left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_{1,i} \right\}_{i=1}^{|\mathcal{S}|}, \left\{ \text{acc}_{0,i}^{\mathcal{T}}, \text{info}_{\text{Track},0,i}^{\mathcal{T}}, \text{info}_{\text{View},0,i}^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|}, \mu \right)$$

instead of from

$$\left(\left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_{0,i} \right\}_{i=1}^{|\mathcal{S}|}, \left\{ \text{acc}_{0,i}^{\mathcal{T}}, \text{info}_{\text{Track},0,i}^{\mathcal{T}}, \text{info}_{\text{View},0,i}^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|}, \mu \right).$$

Hyb_4 differs from Hyb_3 in the way the proof is simulated. The proof is now simulated from

$$\left(\left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_{1,i} \right\}_{i=1}^{|\mathcal{S}|}, \left\{ \text{acc}_{1,i}^{\mathcal{T}}, \text{info}_{\text{Track},1,i}^{\mathcal{T}}, \text{info}_{\text{View},1,i}^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|}, \mu \right)$$

instead of from

$$\left(\left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_{1,i} \right\}_{i=1}^{|\mathcal{S}|}, \left\{ \text{acc}_{0,i}^{\mathcal{T}}, \text{info}_{\text{Track},0,i}^{\mathcal{T}}, \text{info}_{\text{View},0,i}^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|}, \mu \right).$$

Hyb_5 differs from Hyb_4 in the way the signature σ is generated whenever the `Spend` algorithm is executed (both in the spend oracle and the challenge selection). Instead of using the simulator \mathcal{S} of `SoK`, the challenger computes σ using `SoKSig`. Note that this hybrid is identical to the privacy experiment $\text{Privacy}_{\Omega, \mathcal{A}}^b$ with $b=1$.

Proving indistinguishability of hybrids.

$\text{Hyb}_1 \equiv \text{Hyb}_2$ The equivalence of Hyb_2 and Hyb_1 follows directly from the perfect simulatability of the signature of knowledge `SoK`.

$\text{Hyb}_2 \approx_c \text{Hyb}_3$ To show the indistinguishability between Hyb_2 and Hyb_3 we build a series of $|\mathcal{S}|+1$ sub-hybrids with $\text{Hyb}_2 = \text{Hyb}_{2,0}$ and $\text{Hyb}_3 = \text{Hyb}_{2,|\mathcal{S}|}$. In $\text{Hyb}_{2,\ell}$, The proof is now simulated from

$$\left(\left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_{1,i} \right\}_{i=1}^{\ell} \cup \left\{ \text{tag}_{0,i} \right\}_{i=\ell+1}^{|\mathcal{S}|}, \left\{ \text{acc}_{0,i}^{\mathcal{T}}, \text{info}_{\text{Track},0,i}^{\mathcal{T}}, \text{info}_{\text{View},0,i}^{\mathcal{T}} \right\}_{i=1}^{|\mathcal{T}|}, \mu \right).$$

It remains to show that $\text{Hyb}_{2,\ell-1} \approx_c \text{Hyb}_{2,\ell}$. Note that at every point in the experiment `TagEval` is called

with a randomly chosen input of the form $x + s$, where s is an output of the random oracle. Therefore we can use the related-input pseudorandomness of the tag function as defined in Definition 4.1 to argue that $\text{tag}_{0,\ell}$ and $\text{tag}_{1,\ell}$ are both indistinguishable from a random tag in ψ , and hence indistinguishable from each other.

$\text{Hyb}_3 \approx_c \text{Hyb}_4$ To show the indistinguishability between Hyb_3 and Hyb_4 we build a series of $|\mathcal{T}| + 1$ sub-hybrids with $\text{Hyb}_3 = \text{Hyb}_{3,0}$ and $\text{Hyb}_4 = \text{Hyb}_{3,|\mathcal{T}|}$. In $\text{Hyb}_{3,\ell}$, The proof is now simulated from

$$\left(\begin{array}{l} \{ \text{acc}_i^{\mathcal{R}} \}_{i=1}^{|\mathcal{R}|}, \{ \text{tag}_{1,i} \}_{i=1}^{|\mathcal{S}|}, \\ \{ \text{acc}_{1,i}^{\mathcal{T}}, \text{info}_{\text{Track},1,i}^{\mathcal{T}}, \text{info}_{\text{View},1,i}^{\mathcal{T}} \}_{i=1}^{\ell} \cup \\ \{ \text{acc}_{0,i}^{\mathcal{T}}, \text{info}_{\text{Track},0,i}^{\mathcal{T}}, \text{info}_{\text{View},0,i}^{\mathcal{T}} \}_{i=\ell+1}^{|\mathcal{T}|}, \mu \end{array} \right).$$

It remains to show that $\text{Hyb}_{3,\ell-1} \approx_c \text{Hyb}_{3,\ell}$. For this we define 4 sub-hybrids with $\text{Hyb}_{3,\ell-1} = \text{Hyb}_{3,\ell-1,0}$ and $\text{Hyb}_{3,\ell} = \text{Hyb}_{3,\ell-1,3}$. In $\text{Hyb}_{3,\ell-1,1}$, the proof is simulated from

$$(\text{acc}_{1,i}^{\mathcal{T}}, \text{info}_{\text{Track},0,i}^{\mathcal{T}}, \text{info}_{\text{View},0,i}^{\mathcal{T}}).$$

In $\text{Hyb}_{3,\ell-1,2}$, the proof is simulated from

$$(\text{acc}_{1,i}^{\mathcal{T}}, \text{info}_{\text{Track},1,i}^{\mathcal{T}}, \text{info}_{\text{View},0,i}^{\mathcal{T}}).$$

In $\text{Hyb}_{3,\ell-1,3}$, the proof is simulated from

$$(\text{acc}_{1,i}^{\mathcal{T}}, \text{info}_{\text{Track},1,i}^{\mathcal{T}}, \text{info}_{\text{View},1,i}^{\mathcal{T}}).$$

Recall that $\text{acc}_{0,\ell}^{\mathcal{T}}$ and $\text{acc}_{1,\ell}^{\mathcal{T}}$ are of the form

$$\begin{aligned} \text{acc}_{0,\ell}^{\mathcal{T}} &= (\text{pk}_{0,\ell}^{\mathcal{T}}, \text{co}_{0,\ell}^{\mathcal{T}}) \\ \text{acc}_{1,\ell}^{\mathcal{T}} &= (\text{pk}_{1,\ell}^{\mathcal{T}}, \text{co}_{1,\ell}^{\mathcal{T}}) \end{aligned}$$

Since HC is computationally hiding, clearly $\text{Hyb}_{3,\ell-1,0} \approx_c \text{Hyb}_{3,\ell-1,1}$.

Next we argue that $\text{Hyb}_{3,\ell-1,1} \approx_c \text{Hyb}_{3,\ell-1,2}$. Note that in $\text{Hyb}_{3,\ell-1,1}$, $\text{info}_{\text{Track},0,i}^{\mathcal{T}}$ is of the form

$$\text{info}_{\text{Track},0,i}^{\mathcal{T}} \leftarrow \text{PKE.Enc}(\text{tpk}_0, (\text{pk}_{1,\ell}^{\mathcal{T}}, \text{co}_{1,\ell}^{\mathcal{T}}), \text{ek}_0)$$

for some tpk_0 and ek_0 . In $\text{Hyb}_{3,\ell-1,2}$, $\text{info}_{\text{Track},1,i}^{\mathcal{T}}$ is of the form

$$\text{info}_{\text{Track},1,i}^{\mathcal{T}} \leftarrow \text{PKE.Enc}(\text{tpk}_1, (\text{pk}_{1,\ell}^{\mathcal{T}}, \text{co}_{1,\ell}^{\mathcal{T}}), \text{ek}_1)$$

for some tpk_1 and ek_1 . Note that by the definition of the privacy experiment, the adversary cannot succeed if it requests for the keys tsk_0 or tsk_1 , or queries the tracking oracle on $((\text{pk}_{1,\ell}^{\mathcal{T}}, \text{co}_{1,\ell}^{\mathcal{T}}), \text{info}_{\text{Track},0,i}^{\mathcal{T}})$ or $((\text{pk}_{1,\ell}^{\mathcal{T}}, \text{co}_{1,\ell}^{\mathcal{T}}), \text{info}_{\text{Track},1,i}^{\mathcal{T}})$. We can thus use the IK-CCA and IND-CCA security of PKE to show that $\text{Hyb}_{3,\ell-1,1} \approx_c \text{Hyb}_{3,\ell-1,2}$.

The argument for $\text{Hyb}_{3,\ell-1,2} \approx_c \text{Hyb}_{3,\ell-1,3}$ is similar to that for $\text{Hyb}_{3,\ell-1,1} \approx_c \text{Hyb}_{3,\ell-1,2}$, except that now the adversary might be able to learn the amount hidden in $\text{co}_{1,\ell}^{\mathcal{T}}$ using the ViewO oracle. To do so, the adversary could query the ViewO oracle on $((\text{pk}, \text{co}_{1,\ell}^{\mathcal{T}}), c)$ for some pk and c . If the oracle outputs some $a \neq \perp$, then the adversary could learn the amount hidden in $\text{co}_{1,\ell}^{\mathcal{T}}$. We argue that this would only happen with negligible probability.

Note that since HC is perfectly hiding, $\text{co}_{1,\ell}^{\mathcal{T}}$ contains no information about the amount that is committed. Therefore the probability that the ViewO oracle outputs the ‘‘correct’’ amount is negligible. Suppose the oracle outputs a different amount than what is committed, then the oracle would have obtained a different opening to $\text{co}_{1,\ell}^{\mathcal{T}}$ which breaks the binding property of HC. We can therefore conclude that the ViewO oracle outputs \perp on such inputs with overwhelming probability.

$\text{Hyb}_4 \equiv \text{Hyb}_5$ The equivalence of Hyb_4 and Hyb_5 follows directly from the perfect simulatability of the signature of knowledge SoK.

□

D.3 Proof of Theorem 4.4 (Non-Slanderability)

We extend the non-slanderability game from Figure 4 to include viewing and tracing.

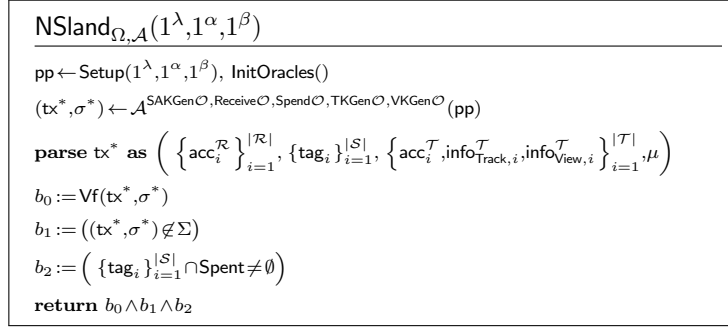


Figure 28: Non-slanderability Experiment (with Tracking and Viewing)

Proof. We prove by hybrid arguments. Consider a PPT adversary \mathcal{A} who participates in the experiment $\text{NSland}_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta)$. Without loss of generality, we assume that \mathcal{A} makes at most q_H , q_R , and q_S queries to the H oracle, receive oracle, and spend oracle respectively. We define a hybrid experiment $\text{NSland}'_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta)$ as follows:

$\text{NSland}'_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta)$ differs from $\text{NSland}_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta)$ in the way the $\text{Spend}\mathcal{O}$ behaves. In particular, in $\text{NSland}'_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta)$, after running the line $\text{tx} := \text{tx}(\mathcal{R},\mathcal{S},\mathcal{T},\mu)$ and $\text{stmt} := \text{stmt}(\text{tx})$, the challenger runs the simulator \mathcal{S} (which is guaranteed to exist by the simulatability property of SoK) to obtain σ .

We argue that the two hybrid experiments are functionally identical, *i.e.*, $\text{NSland}_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta) \equiv \text{NSland}'_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta)$, which follows directly from the simulatability property of SoK.

Now, we show that if there exists a PPT adversary \mathcal{A} such that

$$\Pr \left[\text{NSland}'_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta) = 1 \right] > \frac{1}{p(\lambda)}$$

for some polynomial p , then we can construct another PPT adversary \mathcal{B} that can find a pre-impair of Tag with non-negligible probability thus breaking the OneWay property of Tag .

The algorithm \mathcal{B} simulates the $\text{NSland}'_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta)$ experiment for \mathcal{A} by generating $\text{pp} \leftarrow \Omega.\text{Setup}(1^\lambda)$ and simulating the oracles in the following way:

H oracle: When the adversary queries with a master public key mpk and an ephemeral key ek as inputs for the j -th time, \mathcal{B} queries $\text{Tag}\mathcal{O}_x()$ and receives $(s_j, \text{TagEval}(x+s_j))$. \mathcal{B} programs $\text{H}(\text{mpk},\text{ek}) := s_j$ and returns s_j . Since we assume that \mathcal{A} only queries H at most q_H times, we have that $j \in [q_H]$.

Receive oracle: When \mathcal{A} queries the receive oracle with an account $\text{acc} = (\text{pk}, \text{co}, \tilde{\text{ek}}, \tilde{\text{ck}})$ as inputs, \mathcal{B} first performs the integrity check honestly. Then \mathcal{B} is able to set $\text{tag} := \text{TagEval}(x+s)$, as $s = s_j$ for some $j \in [q_H]$ due to how H is simulated.

Spending oracle: \mathcal{B} can simulate the responses to the $\text{Spend}\mathcal{O}$ queries honestly as in $\text{NSland}'_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta)$, since SoK is simulatable.

All key generating oracles are simulated honestly as they do not depend on the tag.

\mathcal{A} outputs a slander (tx^*, σ^*) . Parse tx^* as

$$\left(\left\{ \text{acc}_i^{\mathcal{R}} \right\}_{i=1}^{|\mathcal{R}|}, \left\{ \text{tag}_i \right\}_{i=1}^{|\mathcal{S}|}, \left\{ (\text{acc}_i^{\mathcal{T}}, \text{info}_i) \right\}_{i=1}^{|\mathcal{T}|}, \mu \right)$$

Since $\text{NSland}'_{\Omega,\mathcal{A}}(1^\lambda,1^\alpha,1^\beta) = 1$, the following conditions hold:

- $\text{SoKVf}(\text{stmt}^*, \sigma^*, \text{tx}^*) = 1$ and

- $\exists \text{tag}_{i^*} \in \left(\{\text{tag}_i\}_{i=1}^{|\mathcal{S}|} \cap \text{Spent} \neq \emptyset \right)$

where $\text{stmt}^* = \text{stmt}(\text{tx}^*)$.

\mathcal{B} exploits the PPT extractor \mathcal{E} of SoK to extract a witness wit^* for the statement stmt^* .

Parse wit^* as $\left(\{(j_i, x_i, a_i^{\mathcal{S}}, r_i^{\mathcal{S}})\}_{i=1}^{|\mathcal{S}|}, \{(a_i^{\mathcal{T}}, r_i^{\mathcal{T}})\}_{i=1}^{|\mathcal{T}|} \right)$.

Since $\text{tag}_{i^*} \in \text{Spent}$, it must be the case that in a previous receive oracle query on some account acc' and help information info' , the algorithm computes the tag tag' where $\text{tag}_{i^*} = \text{tag}'$. Note that tag' was set as $\text{tag}' = \text{TagEval}(x + s^*)$ where $s^* = H(\text{mpk}, \text{ek}')$ for some ek' . \mathcal{B} then simply outputs x_{i^*} as a pre-image of Tag .

Due to the extractability property of SoK, $R(\text{stmt}^*, \text{wit}^*) = 1$ with non-negligible probability. So x_{i^*} is a pre-image of Tag with non-negligible probability, which violates the security of Tag from Definition 4.1. \square

E Security Proofs for Argument of Knowledge Construction

We present the formal security proofs of the argument of knowledge construction in Section 5.

E.1 Proof of Theorem 5.1 (Zero-Knowledge)

Proof. By inspection, \mathcal{V} is public-coin and Π consists of 8 rounds. Π is also trivially perfectly complete. Next, we show that Π is perfect special honest-verifier zero-knowledge by constructing an efficient simulator \mathcal{S} .

Let $\text{stmt} = (\vec{\mathbf{R}}, \vec{\mathbf{C}}_{\mathcal{R}}, \vec{\mathbf{T}}, \vec{\mathbf{C}}_{\mathcal{T}})$ and the verifier public coin $(h, \vec{\mathbf{p}}, \vec{\mathbf{g}}', \vec{\mathbf{h}}, u, v, w, x, y, z)$ be provided by \mathcal{A} . Recall that \mathcal{S} computes $\hat{\mathbf{Y}}, \hat{\mathbf{T}}$, and $\vec{\mathbf{g}}_w$ as in Π . That is,

$$\begin{aligned} \hat{\mathbf{Y}} &= \vec{\mathbf{R}} \circ \vec{\mathbf{C}}_{\mathcal{R}}^{\circ u} \\ \hat{\mathbf{T}} &= \vec{\mathbf{T}}^{u^2 v^{|\mathcal{S}|}} \\ \vec{\mathbf{g}}_w &= ((G \| H \| \hat{\mathbf{T}} \| \hat{\mathbf{Y}})^{\circ w} \circ \vec{\mathbf{p}} \| \vec{\mathbf{g}}') \end{aligned}$$

\mathcal{S} then samples $A, T_2 \leftarrow \mathfrak{s}\mathbb{G}$, $\tau, r \leftarrow \mathfrak{s}\mathbb{Z}_q$, and $\vec{\ell}, \vec{r} \leftarrow \mathfrak{s}\mathbb{Z}_q^m$. It computes $t = \langle \vec{\ell}, \vec{r} \rangle$. It then computes S and T_1 as follows:

$$\begin{aligned} S &= (h^{-r} A \vec{\mathbf{g}}_w^{\vec{\alpha} - \vec{\ell}} \vec{\mathbf{h}}^{\vec{\beta} - \vec{\theta} \circ \vec{r}})^{-1/x} \\ T_1 &= (G^{\delta(y,z) - t} H^{-\tau} \vec{\mathbf{C}}_{\mathcal{T}}^{z^2 \cdot \vec{y}^{|\mathcal{T}|}} T_2^{x^2})^{-1/x} \end{aligned}$$

Finally, \mathcal{S} outputs $(A, S, T_1, T_2, \tau, r, \vec{\ell}, \vec{r}, t)$.

Since all elements produced by \mathcal{S} and those produced by Π are either independently randomly distributed or fully determined by the verification equations, they are identically distributed. \square

E.2 Proof of Theorem 5.2 (Soundness)

To prove that Π is sound, or more precisely has extended-witness emulation, we first state some useful lemmas (Lemma 1 and Lemma 2). For that we establish the following notation.

For $i \in \{L, R\}$, let $\vec{\gamma}_i = (\gamma_{i,1}, \gamma_{i,2}, \gamma_{i,3}, \vec{\gamma}_{i,4}, \dots, \vec{\gamma}_{i,9}) \in \mathbb{Z}_q^m$ be variable vectors of the same format as $\vec{\mathbf{c}}_i$, *i.e.*, $\vec{\gamma}_{i,1}, \vec{\gamma}_{i,2}, \vec{\gamma}_{i,3} \in \mathbb{Z}$, $\vec{\gamma}_{i,4} \in \mathbb{Z}_q^{|\mathcal{R}|}$, $\vec{\gamma}_{i,5} \in \mathbb{Z}_q^{|\mathcal{R}||\mathcal{S}|}$, $\vec{\gamma}_{i,6} \in \mathbb{Z}_q^{|\mathcal{T}|}$, $\vec{\gamma}_{i,7}, \vec{\gamma}_{i,8}, \vec{\gamma}_{i,9} \in \mathbb{Z}_q^{|\mathcal{S}|}$, and $\vec{\gamma}_{i,5} = \text{vec}(\Gamma_i)$ and $\vec{\gamma}_{i,6} = \text{vec}(\Delta_i)$ for some matrices $\Gamma_i \in \mathbb{Z}_q^{|\mathcal{S}| \times |\mathcal{R}|}$ and $\Delta_i \in \mathbb{Z}_q^{|\mathcal{T}| \times \beta}$.

We define a system of constraints $\text{CS} = \text{CS}[\vec{\mathbf{a}}^{\mathcal{T}}, u, v]$ parameterized by $\vec{\mathbf{a}}^{\mathcal{T}}, u, v$ as follows:

$$\text{CS}(\vec{\gamma}_L, \vec{\gamma}_R) = 0 \iff$$

$$\left\{ \begin{array}{ll} (\vec{\gamma}_{L,5}, \vec{\gamma}_{L,6}) \circ (\vec{\gamma}_{R,5}, \vec{\gamma}_{R,6}) = \vec{0}^{|\mathcal{R}||\mathcal{S}|+\beta|\mathcal{T}|} & (14) \\ \vec{\gamma}_{R,9} = \vec{\gamma}_{L,9}^{\circ-1} & (15) \\ \Delta_L \vec{2}^\beta = \vec{\mathbf{a}}^\mathcal{T} & (16) \\ \Gamma_L \vec{1}^{|\mathcal{R}|} = \vec{1}^{|\mathcal{S}|} & (17) \\ \gamma_{L,1} = -\langle \vec{v}^{|\mathcal{S}|}, u \cdot \vec{\gamma}_{L,7} + u^2 \cdot \vec{\gamma}_{R,9} \rangle & (18) \\ \gamma_{L,2} = -\langle \vec{v}^{|\mathcal{S}|}, \vec{\gamma}_{L,9} + u \cdot \vec{\gamma}_{L,8} \rangle & (19) \\ \vec{\gamma}_{L,4} = \vec{v}^{|\mathcal{S}|} \Gamma_L & (20) \\ \langle \vec{1}^{|\mathcal{S}|}, \vec{\gamma}_{L,7} \rangle = \vec{1}^{|\mathcal{T}|} \Delta_L \vec{2}^\beta & (21) \\ (\vec{\gamma}_{R,5}, \vec{\gamma}_{R,6}) = (\vec{\gamma}_{L,5}, \vec{\gamma}_{L,6}) - \vec{1}^{|\mathcal{R}||\mathcal{S}|+\beta|\mathcal{T}|} & (22) \\ \gamma_{L,3} = 1 & (23) \end{array} \right.$$

Lemma 1. Fix $q > 2^\lambda$, $\vec{\mathbf{a}}^\mathcal{T} \in \mathbb{Z}_q^{|\mathcal{T}|}$, $u, v \in \mathbb{Z}_q$. Suppose there exists $\vec{\gamma}_L, \vec{\gamma}_R \in \mathbb{Z}_q^m$ such that, for $|\mathcal{R}||\mathcal{S}| + \beta|\mathcal{T}|$ different values of y we have $\text{EQ}(\vec{\gamma}_L, \vec{\gamma}_R) = 0$, then $\text{CS}(\vec{\gamma}_L, \vec{\gamma}_R) = 0$.

Proof. Since $\text{EQ}(\vec{\gamma}_L, \vec{\gamma}_R) = 0$ for $|\mathcal{R}||\mathcal{S}| + \beta|\mathcal{T}|$ different values of y , then the following polynomials (in y) of degree at most $|\mathcal{R}||\mathcal{S}| + \beta|\mathcal{T}| - 1$ each has $|\mathcal{R}||\mathcal{S}| + \beta|\mathcal{T}|$ different roots and therefore must be all equal to the zero polynomial. These polynomials are:

$$\begin{aligned} \langle (\vec{\gamma}_{L,5}, \vec{\gamma}_{L,6}) \circ (\vec{\gamma}_{R,5}, \vec{\gamma}_{R,6}), \vec{y}^{|\mathcal{R}||\mathcal{S}|+\beta|\mathcal{T}|} \rangle &= 0 && \text{by Eq (5)} \\ \langle (\vec{\gamma}_{L,9} \circ \vec{\gamma}_{R,9}) - \vec{1}^{|\mathcal{S}|}, \vec{y}^{|\mathcal{S}|} \rangle &= 0 && \text{by Eq (6)} \\ \langle \vec{y}^{|\mathcal{T}|}, (\Delta_L \vec{2}^\beta - \vec{\mathbf{a}}^\mathcal{T}) \rangle &= 0 && \text{by Eq (7)} \\ (\gamma_{L,3} - 1)y^{|\mathcal{S}|} + \langle \vec{\Gamma}_L \vec{1}^{|\mathcal{R}|} - \vec{1}^{|\mathcal{S}|}, \vec{y}^{|\mathcal{S}|} \rangle &= 0 && \text{by Eq (8)} \\ \gamma_{L,1} + \langle \vec{\gamma}_{L,7}, u \cdot \vec{v}^{|\mathcal{S}|} \rangle + \langle \vec{\gamma}_{R,9}, u^2 \cdot \vec{v}^{|\mathcal{S}|} \rangle &= 0 && \text{by Eq (9)} \\ \gamma_{L,2} + \langle \vec{\gamma}_{L,8}, u \cdot \vec{v}^{|\mathcal{S}|} \rangle + \langle \vec{\gamma}_{L,9}, \vec{v}^{|\mathcal{S}|} \rangle &= 0 && \text{by Eq (34)} \\ \langle \vec{v}^{|\mathcal{S}|} \Gamma_L - \vec{\gamma}_{L,4}, \vec{y}^{|\mathcal{R}|} \rangle &= 0 && \text{by Eq (11)} \\ \vec{1}^{|\mathcal{T}|} \Delta_L \vec{2}^\beta - \langle \vec{1}^{|\mathcal{S}|}, \vec{\gamma}_{L,7} \rangle &= 0 && \text{by Eq (12)} \\ \langle (\vec{\gamma}_{L,5}, \vec{\gamma}_{L,6}) - (\vec{\gamma}_{R,5}, \vec{\gamma}_{R,6}) - \vec{1}^{|\mathcal{R}||\mathcal{S}|+\beta|\mathcal{T}|}, \vec{y}^{|\mathcal{R}||\mathcal{S}|+\beta|\mathcal{T}|} \rangle &= 0 && \text{by Eq (13)} \end{aligned}$$

By comparing coefficients, we conclude that $\text{CS}(\vec{\gamma}_L, \vec{\gamma}_R) = 0$ □

The following is obtained by observing the system CS.

Lemma 2. If $\text{CS}(\vec{\gamma}_L, \vec{\gamma}_R) = 0$, then:

- Each row of Γ_L is a unit vector of length $|\mathcal{R}|$.
- The i -row of Δ_L is the length- β binary representation of $a_i^\mathcal{T}$.
- $\langle \vec{1}^{|\mathcal{S}|}, \vec{\gamma}_{L,7} \rangle = \sum_{i \in [|\mathcal{T}|]} a_i^\mathcal{T}$.

Using the above lemmas, we now prove that Π has witness-extended emulation.

Proof. (Theorem 5.2) Assuming the discrete logarithm assumption holds over \mathcal{G} , by Theorem B.6, the interactive discrete logarithm assumption (as defined in Definition B.5) also holds over \mathcal{G} . The following is a straightforward corollary of Theorem B.6.

Corollary 1. If there exists a PPT adversary \mathcal{A} which does the following:

1. On input (\mathbb{G}, q, G, H) , choose vector of group elements $(\hat{T} \parallel \hat{\mathbf{Y}})$ and an integer w .
2. Receive a uniformly random vector of group elements $(h \parallel \vec{\mathbf{p}} \parallel \vec{\mathbf{g}}' \parallel \vec{\mathbf{h}})$ of appropriate dimensions.
3. Produce a non-zero integer vector $(r \parallel \vec{\mathbf{a}} \parallel \vec{\mathbf{b}})$ such that $I = h^r \vec{\mathbf{g}}_w^{\vec{\mathbf{a}}} \vec{\mathbf{h}}^{\vec{\mathbf{b}}}$, where $\vec{\mathbf{g}}_w = ((G \parallel H \parallel \hat{T} \parallel \hat{\mathbf{Y}})^w \circ \vec{\mathbf{p}} \parallel \vec{\mathbf{g}}')$.

then there exists a PPT algorithm solving the discrete logarithm problem over \mathcal{G} .

Proof. Suppose \mathcal{A} exists, we construct an adversary \mathcal{B} against the interactive discrete logarithm assumption. \mathcal{B} receives (\mathbb{G}, q, G) and samples a random group element H . It passes (\mathbb{G}, q, G, H) to \mathcal{A} , and receives from the latter a vector of group elements $(\hat{T} \parallel \hat{\mathbf{Y}})$ and an integer w . \mathcal{B} then sends to its challenger the vector $(G \parallel H \parallel \hat{T} \parallel \hat{\mathbf{Y}})$, and receives from the latter a uniformly random vector of group elements $(h \parallel \vec{\mathbf{p}} \parallel \vec{\mathbf{g}}' \parallel \vec{\mathbf{h}})$. \mathcal{B} simply forwards $(h \parallel \vec{\mathbf{p}} \parallel \vec{\mathbf{g}}' \parallel \vec{\mathbf{h}})$ to \mathcal{A} , and receives a non-zero integer vector $(r \parallel \vec{\mathbf{a}} \parallel \vec{\mathbf{b}})$ such that $I = h^r \vec{\mathbf{g}}_w^{\vec{\mathbf{a}}} \vec{\mathbf{h}}^{\vec{\mathbf{b}}}$.

Let $\vec{\mathbf{a}} = (\vec{\mathbf{a}}_1, \vec{\mathbf{a}}_2)$ be of the appropriate dimensions. We can thus write

$$I = (G \parallel H \parallel \hat{T} \parallel \hat{\mathbf{Y}})^{w \cdot \vec{\mathbf{a}}_1} (h \parallel \vec{\mathbf{p}} \parallel \vec{\mathbf{g}}' \parallel \vec{\mathbf{h}})^{(r \parallel \vec{\mathbf{a}} \parallel \vec{\mathbf{b}})}$$

Since $(r \parallel \vec{\mathbf{a}} \parallel \vec{\mathbf{b}})$ is non-zero, $(w \cdot \vec{\mathbf{a}}_1 \parallel r \parallel \vec{\mathbf{a}} \parallel \vec{\mathbf{b}})$ is a valid solution to the interactive discrete logarithm problem instance. \mathcal{B} therefore outputs $(w \cdot \vec{\mathbf{a}}_1 \parallel r \parallel \vec{\mathbf{a}} \parallel \vec{\mathbf{b}})$. \square

With the above corollary, we can proceed to construct an extractor. Let $\mathbf{pp} \leftarrow \text{Setup}(1^\lambda, \mathcal{L})$ and $(\text{stmt}, \text{wit}) \leftarrow \mathcal{A}_2(\mathbf{pp})$. We construct an extractor \mathcal{E} which, on input \mathbf{pp} and stmt , outputs a transcript and, if the transcript is accepting, a witness wit' to the statement stmt . It is trivial for \mathcal{E} to produce a transcript which is indistinguishable to that produced by $\langle \mathcal{P}^*(\mathbf{pp}, \text{stmt}, \text{wit}), \mathcal{V}(\mathbf{pp}, \text{stmt}) \rangle$ for any prover \mathcal{P}^* as \mathcal{E} is given an oracle $\mathcal{O} = \langle \mathcal{P}^*(\mathbf{pp}, \text{stmt}, \text{wit}), \mathcal{V}(\mathbf{pp}, \text{stmt}) \rangle$. We thus focus on describing how \mathcal{E} can extract a witness wit' in the case where the transcript is accepting.

\mathcal{E} runs \mathcal{P}^* on 1 uniformly random chosen (u, v) , 2 different values of w , $|\mathcal{R}| |\mathcal{S}| + \beta |\mathcal{T}|$ different values of y , 9 different values of z , 3 different values of x . This results in $54(|\mathcal{R}| |\mathcal{S}| + \beta |\mathcal{T}|)$ transcripts. Fix a particular choice of (w, y, z) , \mathcal{E} obtains three transcripts of the form $(A, S, T_1, T_2, \tau_{x_i}, r_{x_i}, \vec{t}_{x_i}, \vec{r}_{x_i}, t_{x_i})$ for $i = 1, 2, 3$. Below, we show how the extractor can extract the discrete logarithm representations of A , S , T_1 , and T_2 .

Extracting A Choose $\kappa_{i=1,2} \in \mathbb{Z}_q$ such that $\sum_{i=1,2} \kappa_i = 1$ and $\sum_{i=1,2} \kappa_i x_i = 0$. This leads to the extraction of A by Equation (3) as

$$\begin{aligned} A &= h^{\sum_{i=1}^2 \kappa_i r_{x_i}} \vec{\mathbf{g}}_w^{\sum_{i=1}^2 \kappa_i \vec{t}_{x_i} - \vec{\alpha}} \vec{\mathbf{h}}^{\sum_{i=1}^2 \kappa_i \vec{\theta}^{\circ-1} \circ \vec{r}_{x_i} - \vec{\beta}} \\ &:= h^{r'_A} \vec{\mathbf{g}}_w^{c'_L} \vec{\mathbf{h}}^{c'_R} \end{aligned}$$

Note that r'_A , c'_L , and c'_R depend on w . To obtain a discrete logarithm representation which is independent of w , \mathcal{E} repeats the above for the other choice of w , which we denote by w' . With this additional transcript the extractor can extract A as $h^{r'_A} \vec{\mathbf{g}}_w^{c'_L} \vec{\mathbf{h}}^{c'_R}$. Note that we now have two (possibly different) representations of A . Write $\vec{c}'_L = (\vec{c}'_{L,1} \parallel \vec{c}'_{L,2})$ and $\vec{c}''_L = (\vec{c}''_{L,1} \parallel \vec{c}''_{L,2})$ with appropriate dimensions. We have

$$\begin{aligned} h^{r'_A} \vec{\mathbf{g}}_w^{c'_L} \vec{\mathbf{h}}^{c'_R} &= h^{r'_A} \vec{\mathbf{g}}_w^{c''_L} \vec{\mathbf{h}}^{c''_R} \\ I &= h^{r'_A - r''_A} (G \parallel H \parallel \hat{T} \parallel \hat{\mathbf{Y}})^{w \cdot \vec{c}'_{L,1} - w' \cdot \vec{c}''_{L,1}} \\ &\quad \cdot (\vec{\mathbf{p}} \parallel \vec{\mathbf{g}}')^{\vec{c}'_L - \vec{c}''_L} \vec{\mathbf{h}}^{c'_R - c''_R} \end{aligned}$$

We can assume $r'_A = r''_A$, $\vec{c}'_L = \vec{c}''_L$ and $\vec{c}'_R = \vec{c}''_R$. Suppose not, since $(\vec{\mathbf{p}} \parallel \vec{\mathbf{g}}' \parallel \vec{\mathbf{h}})$ is chosen uniformly after fixing $(G \parallel H \parallel \hat{T} \parallel \hat{\mathbf{Y}})$, we would have an efficient algorithm against interactive discrete logarithm assumption. Note that $w \neq w'$. Let $\vec{c}'_{L,1} = (\xi' \parallel \eta' \parallel \psi' \parallel \hat{\mathbf{e}}')$. We can obtain the relations

$$\begin{aligned} I &= (G \parallel H \parallel \hat{T} \parallel \hat{\mathbf{Y}})^{(w-w') \cdot \vec{c}'_{L,1}} \\ I &= (G \parallel H \parallel \hat{T} \parallel \hat{\mathbf{Y}})^{\vec{c}'_{L,1}} \\ I &= G^{\xi'} H^{\eta'} \hat{T}^{\psi'} \hat{\mathbf{Y}}^{\hat{\mathbf{e}}'} \end{aligned} \tag{24}$$

Extracting S Similar to the extraction of A , \mathcal{E} can extract S from Equation (3) by sampling some $\kappa_{i=1,2}$ with $\sum_{i=1,2}\kappa_i=0$ and $\sum_{i=1,2}\kappa_i x_i=1$. This leads to

$$\begin{aligned} S &= h^{\sum_{i=1}^2 \kappa_i \tau_{x_i}} \bar{\mathbf{g}}_w^{\sum_{i=1}^2 \kappa_i \bar{l}_{x_i}} \bar{\mathbf{h}}^{\sum_{i=1}^2 \kappa_i \bar{\theta}^{\circ-1} \circ \bar{r}_{x_i}} \\ &:= h^{t'_S} \bar{\mathbf{g}}_w^{s'_L} \bar{\mathbf{h}}^{s'_R} \end{aligned}$$

Note that for fixed w , the above expressions of A and S hold for all choices of (x, y, z) , for otherwise we would obtain a non-trivial discrete logarithm representation base $(h \| \bar{\mathbf{g}}_w \| \bar{\mathbf{h}})$, which violates the discrete logarithm assumption due to Corollary 1.

Putting the expressions of A and S back into Equation (3), it follows that for all challenges (x, y, z)

$$\begin{aligned} \bar{l}'_x &= \bar{\mathbf{c}}'_L + \bar{\theta}^{\circ-1} \circ (\bar{\omega} - \bar{\nu}) + \bar{\mathbf{s}}'_L \cdot x \\ \bar{r}'_x &= \bar{\theta} \circ (\bar{\mathbf{c}}'_R + \bar{\mathbf{s}}'_R \cdot x) + \bar{\mu} \end{aligned}$$

or otherwise we would have obtained a non-trivial representation of the identity element base $(h \| \bar{\mathbf{g}}_w \| \bar{\mathbf{h}})$, and thus violated the discrete logarithm assumption due to Corollary 1.

Extracting T_1 and T_2 To extract T_1 , the extractor chooses $\kappa_{i=1,2,3} \leftarrow \mathbb{Z}_q$ with $\sum_{i=1}^3 \kappa_i = 0$, $\sum_{i=1}^3 \kappa_i x_i = 1$ and $\sum_{i=1}^3 \kappa_i x_i^2 = 0$. Together with Equation (4) we have

$$\begin{aligned} T_1 &= G^{\sum_{i=1}^3 \kappa_i t_{x_i}} H^{\sum_{i=1}^3 \kappa_i \tau_{x_i}} \\ &=: G^{t'_1} H^{\tau'_1} \end{aligned}$$

Similarly, by choosing $\kappa'_{i=1,2,3} \leftarrow \mathbb{Z}_q$ with $\sum_{i=1}^3 \kappa'_i = 0$, $\sum_{i=1}^3 \kappa'_i x_i = 0$ and $\sum_{i=1}^3 \kappa'_i x_i^2 = 1$ and Equation (4) we have

$$\begin{aligned} T_2 &= G^{\sum_{i=1}^3 \kappa'_i t_{x_i}} H^{\sum_{i=1}^3 \kappa'_i \tau_{x_i}} \\ &=: G^{t'_2} H^{\tau'_2} \end{aligned}$$

Note that the above expressions of T_1 and T_2 hold for all challenge x , or otherwise we would have obtained a non-trivial discrete logarithm representation of the identity element base $(G \| H)$ which directly violates the discrete logarithm assumption.

Extracting $\bar{\mathbf{C}}_{\mathcal{T}}$. Fix a certain choice of (y, z) . By putting the representations of T_1 and T_2 back to Equation (4), the extractor can find some (a, r) with $G^a H^r = \bar{\mathbf{C}}_{\mathcal{T}}^{\bar{y}^{|\mathcal{T}|}}$. Repeating this for $|\mathcal{T}|$ different y , and using the technique similar to that for extracting A , S , T_1 , and T_2 , \mathcal{E} can extract $(a_i^{\mathcal{T}'}, r_i^{\mathcal{T}'})$ such that $\text{co}_i^{\mathcal{T}} = G^{a_i^{\mathcal{T}'}} H^{r_i^{\mathcal{T}'}}$ for all $i \in [|\mathcal{T}|]$. In the following, we write $\bar{\mathbf{a}}^{\mathcal{T}'} := (a_1^{\mathcal{T}'} \| \dots \| a_{|\mathcal{T}|}^{\mathcal{T}'})$ and $\bar{\mathbf{r}}^{\mathcal{T}'} := (r_1^{\mathcal{T}'} \| \dots \| r_{|\mathcal{T}|}^{\mathcal{T}'})$.

Note that the above expressions of $\text{co}_i^{\mathcal{T}}$ hold for all challenges (x, y, z) , for otherwise we would obtain a non-trivial discrete logarithm representation base $(G \| H)$, which directly violates the discrete logarithm assumption.

Outputting Witness Write $\bar{\mathbf{c}}'_L$ as

$$\bar{\mathbf{c}}'_L = (\xi' \| \eta' \| \psi' \| \epsilon' \| \text{vec}(\mathbf{E}') \| \text{vec}(\mathbf{B}')) \| \bar{\mathbf{a}}^{S'} \| \bar{\mathbf{r}}^{S'} \| \bar{\mathbf{x}}'.$$

Together with the vectors $\bar{\mathbf{a}}^{\mathcal{T}'}$ and $\bar{\mathbf{r}}^{\mathcal{T}'}$ extracted above, \mathcal{E} outputs the witness

$$\text{wit}' = \left(\mathbf{E}', \bar{\mathbf{x}}', \bar{\mathbf{a}}^{S'}, \bar{\mathbf{r}}^{S'}, \mathbf{B}', \bar{\mathbf{a}}^{\mathcal{T}'}, \bar{\mathbf{r}}^{\mathcal{T}'} \right).$$

Showing Well-Formedness It remains to show that $\vec{\mathbf{c}}'_L$ (and $\vec{\mathbf{c}}'_R$) are well-formed, and hence wit' is a valid witness to stmt . Putting the expression of $\vec{\mathbf{C}}_{\mathcal{T}}$ back to Equation (4), we have

$$t'_x = z^2 \cdot \langle \vec{\mathbf{a}}^{\mathcal{T}'}, \vec{\mathbf{y}}^{|\mathcal{T}'|} \rangle + \delta + t'_1 x + t'_2 x^2$$

for all challenges (x, y, z) , or we would have a discrete logarithm relation between G and H . Let

$$\begin{aligned} t'_0 &:= z^2 \cdot \langle \vec{\mathbf{a}}^{\mathcal{T}'}, \vec{\mathbf{y}}^{|\mathcal{T}'|} \rangle + \delta \\ l'(X) &:= \vec{\mathbf{c}}'_L + \vec{\alpha} + \vec{\mathbf{s}}'_L \cdot X \\ r'(X) &:= \vec{\theta} \circ (\vec{\mathbf{c}}'_R + \vec{\mathbf{s}}'_R \cdot X) + \vec{\mu} \\ t'(X) &:= \langle l'(X), r'(X) \rangle \end{aligned}$$

We have that for each choice of (y, z) the following quadratic polynomial

$$\sum_{i=0}^2 t'_i X^i - t'(X)$$

has at least three distinct roots and therefore has to be the zero polynomial. In particular, for all (y, z) it holds that $t'_0 = t'(0)$. Examining both sides of the equation, we have:

$$\begin{aligned} t'_0 &= z^2 \cdot \langle \vec{\mathbf{a}}^{\mathcal{T}'}, \vec{\mathbf{y}}^{|\mathcal{T}'|} \rangle + \delta \\ &= z \cdot \langle \vec{\mathbf{1}}^{|\mathcal{S}|}, \vec{\mathbf{y}}^{|\mathcal{S}|} \rangle + z^2 \cdot \langle \vec{\mathbf{a}}^{\mathcal{T}'}, \vec{\mathbf{y}}^{|\mathcal{T}'|} \rangle + \\ &\quad z^3 \cdot \langle \vec{\mathbf{1}}^{|\mathcal{S}|+1}, \vec{\mathbf{y}}^{|\mathcal{S}|+1} \rangle + \\ &\quad \langle \vec{\alpha}, \vec{\mu} \rangle + \langle \vec{\mathbf{1}}^m, \vec{\nu} \rangle \end{aligned}$$

$$\begin{aligned} t'(0) &= \langle \vec{\mathbf{c}}'_L, \vec{\theta} \circ \vec{\mathbf{c}}'_R \rangle + \langle \vec{\mathbf{c}}'_L, \vec{\mu} \rangle + \\ &\quad \langle \vec{\mathbf{c}}'_R, \vec{\omega} - \vec{\nu} \rangle + \langle \vec{\alpha}, \vec{\mu} \rangle \\ &= \langle \vec{\mathbf{c}}'_L, \vec{\theta} \circ \vec{\mathbf{c}}'_R \rangle + (\langle \vec{\mathbf{c}}'_L, \vec{\zeta} \rangle + \langle \vec{\mathbf{c}}'_R, \vec{\omega} \rangle) + \\ &\quad \langle \vec{\mathbf{c}}'_L - \vec{\mathbf{c}}'_R, \vec{\nu} \rangle + \langle \vec{\alpha}, \vec{\mu} \rangle \end{aligned}$$

The above implies that

$$\begin{aligned} & z \cdot \langle \vec{\mathbf{1}}^{|\mathcal{S}|}, \vec{\mathbf{y}}^{|\mathcal{S}|} \rangle + z^2 \cdot \langle \vec{\mathbf{a}}^{\mathcal{T}'}, \vec{\mathbf{y}}^{|\mathcal{T}'|} \rangle + \\ & z^3 \cdot \langle \vec{\mathbf{1}}^{|\mathcal{S}|+1}, \vec{\mathbf{y}}^{|\mathcal{S}|+1} \rangle \\ &= \langle \vec{\mathbf{c}}'_L, \vec{\theta} \circ \vec{\mathbf{c}}'_R \rangle + (\langle \vec{\mathbf{c}}'_L, \vec{\zeta} \rangle + \langle \vec{\mathbf{c}}'_R, \vec{\omega} \rangle) + \\ & \langle \vec{\mathbf{c}}'_L - \vec{\mathbf{c}}'_R - \vec{\mathbf{1}}^m, \vec{\nu} \rangle \\ &= \sum_{i=0}^1 z^i \langle \vec{\mathbf{c}}'_L, \vec{\mathbf{c}}'_R \circ \vec{\mathbf{v}}_i \rangle + \\ & \sum_{i=2}^7 z^i \langle \vec{\mathbf{c}}'_L, \vec{\mathbf{v}}_i \rangle + z^4 \langle \vec{\mathbf{c}}'_R, \vec{\mathbf{u}}_4 \rangle + \\ & z^8 \langle \vec{\mathbf{c}}'_L - \vec{\mathbf{c}}'_R - \vec{\mathbf{1}}^m, \vec{\mathbf{v}}_8 \rangle. \end{aligned}$$

Since the above holds for 9 different values of z , the system of equations $\text{EQ}[\vec{\mathbf{a}}^{\mathcal{T}'}, u, v, y](\vec{\mathbf{c}}'_L, \vec{\mathbf{c}}'_R)$ as defined in Figure 12 is satisfied for $|\mathcal{R}||\mathcal{S}| + \beta|\mathcal{T}'|$ different values of y . By Lemma 1, we have $\text{CS}[\vec{\mathbf{a}}^{\mathcal{T}'}, u, v](\vec{\mathbf{c}}'_L, \vec{\mathbf{c}}'_R) = 0$. Then from the definition of $\text{CS}[\vec{\mathbf{a}}^{\mathcal{T}'}, u, v]$, and by Lemma 2, the following conditions must hold:

- Each row of \mathbf{E}' is a unit vector of length $|\mathcal{R}|$.
- The ℓ -th row of \mathbf{B}' is the length- β binary representation of $a_\ell^{\mathcal{T}'}$.
- $\sum_{\ell \in [|\mathcal{S}|]} a_\ell^{\mathcal{S}'} = \sum_{\ell \in [|\mathcal{T}'|]} a_\ell^{\mathcal{T}'}$.

Furthermore, let $\text{vec}(\mathbf{E}') = (\vec{\mathbf{e}}'_1, \dots, \vec{\mathbf{e}}'_{|\mathcal{S}|})$, we can write

$$\begin{aligned}\xi' &= -\langle \vec{v}^{|\mathcal{S}|}, u \cdot \vec{\mathbf{a}}^{\mathcal{S}'} + u^2 \cdot \vec{\mathbf{x}}'^{\circ-1} \rangle \\ \eta' &= -\langle \vec{v}^{|\mathcal{S}|}, \vec{\mathbf{x}}' + u \cdot \vec{\mathbf{r}}^{\mathcal{S}'} \rangle \\ \psi' &= 1 \\ \hat{\mathbf{e}}' &= \vec{v}^{|\mathcal{S}|} \mathbf{E}' = \sum_{\ell \in [|\mathcal{S}|]} v^{\ell-1} \cdot \vec{\mathbf{e}}'_\ell.\end{aligned}$$

By Equation (24), we have

$$\begin{aligned}I &= G^{\xi'} H^{\eta'} \hat{T}^{\psi'} \hat{\mathbf{Y}}^{\hat{\mathbf{e}}'} \\ &= G^{-\langle \vec{v}^{|\mathcal{S}|}, u \cdot \vec{\mathbf{a}}^{\mathcal{S}'} + u^2 \cdot \vec{\mathbf{x}}'^{\circ-1} \rangle} H^{-\langle \vec{v}^{|\mathcal{S}|}, \vec{\mathbf{x}}' + u \cdot \vec{\mathbf{r}}^{\mathcal{S}'} \rangle} \prod_{\ell \in [|\mathcal{S}|]} \text{tag}_\ell^{v^{\ell-1} u^2} \\ &\quad (\vec{\mathbf{R}} \circ \vec{\mathbf{C}}_{\mathcal{R}}^{\text{ou}})^{\sum_{\ell \in [|\mathcal{S}|]} v^{\ell-1} \cdot \vec{\mathbf{e}}'_\ell} \\ &= \prod_{\ell \in [|\mathcal{S}|]} G^{-(a_\ell^{\mathcal{S}'}) u v^{\ell-1}} \prod_{\ell \in [|\mathcal{S}|]} G^{\left(\frac{-1}{x'_\ell}\right) u^2 v^{\ell-1}} \\ &\quad \prod_{\ell \in [|\mathcal{S}|]} H^{-(x'_\ell) v^{\ell-1}} \prod_{\ell \in [|\mathcal{S}|]} H^{-(r'_\ell) u v^{\ell-1}} \prod_{\ell \in [|\mathcal{S}|]} \text{tag}_\ell^{u^2 v^{\ell-1}} \\ &\quad \prod_{\ell \in [|\mathcal{S}|]} (\vec{\mathbf{R}}^{\vec{\mathbf{e}}'_\ell})^{v^{\ell-1}} \prod_{\ell \in [|\mathcal{S}|]} (\vec{\mathbf{C}}_{\mathcal{R}}^{\vec{\mathbf{e}}'_\ell})^{u v^{\ell-1}} \\ &= \prod_{\ell \in [|\mathcal{S}|]} (H^{-x'_\ell} \vec{\mathbf{R}}^{\vec{\mathbf{e}}'_\ell})^{v^{\ell-1}} \\ &\quad \prod_{\ell \in [|\mathcal{S}|]} (G^{-a_\ell^{\mathcal{S}'}} H^{-r'_\ell} \vec{\mathbf{C}}_{\mathcal{R}}^{\vec{\mathbf{e}}'_\ell})^{u v^{\ell-1}} \\ &\quad \prod_{\ell \in [|\mathcal{S}|]} (G^{\frac{-1}{x'_\ell}} \text{tag}_\ell)^{u^2 v^{\ell-1}}.\end{aligned}$$

The last equality can be viewed as an evaluation of a degree- $(|\mathcal{S}|+1)$ polynomial (in the exponent) at a random point (u, v) to zero. By the Schwartz–Zippel lemma, the probability that this happens when the polynomial is non-zero is bounded by $\frac{|\mathcal{S}|+1}{q}$ which is negligible given $q > 2^\lambda$. We can therefore assume that the polynomial is always zero. That is, for all $\ell \in [|\mathcal{S}|]$, the following equations hold:

$$\begin{aligned}\vec{\mathbf{R}}^{\vec{\mathbf{e}}'_\ell} &= H^{x'_\ell} \\ \vec{\mathbf{C}}_{\mathcal{R}}^{\vec{\mathbf{e}}'_\ell} &= G^{a_\ell^{\mathcal{S}'}} H^{r'_\ell} \\ \text{tag}_\ell &= G^{1/x'_\ell}\end{aligned}$$

To conclude, wit' extracted by \mathcal{E} is indeed a valid witness corresponding to stmt . □

F Detailed Instantiation

We describe the details of the instantiation mentioned in Section 4.4

F.1 Tag function

We instantiate the tagging scheme $\text{Tag} = (\text{TagSetup}, \text{TagEval})$ as $(\mathbb{G}, q, G) \leftarrow \text{TagSetup}(1^\lambda)$ and $\text{TagEval}(x) := G^{\frac{x}{\#}}$

Theorem F.1. *If the generalized discrete logarithm assumption holds over $\mathcal{G} = (\mathbb{G}, q, G)$, then the tagging scheme Tag instantiated as above is a secure tagging scheme according to Definition 4.1.*

Proof. We prove related-input one-wayness and related-input pseudorandomness separately.

Related-input one-wayness Suppose Tag is not related-input one-way, let \mathcal{A} be a PPT adversary for which

$$\Pr[\text{OneWay}_{\mathcal{A}}(1^\lambda) = 1] > \text{negl}(\lambda),$$

i.e. \mathcal{A} can guess a preimage for $\text{TagEval}(x + s^*)$. Without loss of generality, we assume \mathcal{A} makes at most $l-1$ queries to the oracle $\text{Tag}\mathcal{O}_x$. We construct an adversary \mathcal{B} against ℓ -GDL from Definition B.2 with input $(\mathbb{G}, q, G, \dots, G^{x^\ell})$ as follows:

- Sample $s^*, s_1, \dots, s_{l-1} \leftarrow \mathbb{Z}_q$.
- Denote symbolically the polynomial $p(X) := (X + s^*) \prod_{i=1}^{l-1} (X + s_i) = \sum_{i=1}^l a_i X^i$.
- Set $T := G^{p(X)} = G^{(x+s^*) \prod_{i=1}^{l-1} (x+s_i)}$.
- Set $\text{pp} = (\mathbb{G}, q, T)$ and $\text{tag}^* = T^{\frac{1}{x+s^*}} (= G^{\frac{(x+s^*) \prod_{i=1}^{l-1} (x+s_i)}{(x+s^*)}})$.
- Run \mathcal{A} on $(\text{pp}, s^*, \text{tag}^*)$. Note that since $G, G^x, \dots, G^{x^\ell}$ are not given to \mathcal{A} , pp generated this way has the same distribution as those generated using Setup .
- On the j -th query to $\text{Tag}\mathcal{O}_x$ return $T^{\frac{1}{x+s_j}} (= G^{\frac{(x+s^*) \prod_{i=1}^{l-1} (x+s_i)}{(x+s_j)}}$.
- Eventually, \mathcal{A} returns x' . \mathcal{B} outputs $x = x' - s^*$.

With non-negligible probability, \mathcal{A} is successful in breaking the related-input one-wayness of Tag , *i.e.*, $x' = x + s^*$. Then, with the same non-negligible probability, \mathcal{B} recovers x correctly and solve the ℓ -GDL instance, violating the generalized discrete logarithm assumption.

Related-input pseudorandomness Suppose Tag is not related-input pseudorandom, let \mathcal{A} be a PPT adversary for which

$$\left| \Pr[\text{PR}_{\mathcal{A}}^0(1^\lambda) = 1] - \Pr[\text{PR}_{\mathcal{A}}^1(1^\lambda) = 1] \right| > \text{negl}(\lambda)$$

holds, *i.e.* it can distinguish $\text{Tag}(x + s^*)$ from a random value $y_1 \leftarrow \mathbb{G}$ given s^* .

We construct an adversary \mathcal{B} against SDDHI from Definition B.3, which uses \mathcal{A} as follows:

- On input (\mathbb{G}, q, G, G^x) sample $s^* \leftarrow \mathbb{G}$ and output s^* to get a challenge y_b .
- Set $\text{pp} = (\mathbb{G}, q, G)$ and run \mathcal{A} on (pp, s^*, y_b) .
- Upon receiving a query to $\text{Tag}\mathcal{O}_x$ from \mathcal{A} , sample $s \leftarrow \mathbb{G}$ and return $(s, \mathcal{O}_x(s))$.
- Eventually, \mathcal{A} outputs b' which is also output by \mathcal{B} .

Note that if \mathcal{B} is participating in $\text{SDDHI}_{\mathcal{B}}^0$, then it simulates the environment of $\text{PR}_{\mathcal{A}}^0$ perfectly. Likewise, if \mathcal{B} is participating in $\text{SDDHI}_{\mathcal{B}}^1$, then it simulates the environment of $\text{PR}_{\mathcal{A}}^1$ perfectly. Furthermore, with overwhelming probability, we have $s \neq s^*$. Therefore, for each $b \in \{0,1\}$, the probability that \mathcal{B} outputs 1 in $\text{SDDHI}_{\mathcal{B}}^b$ is negligibly close to that of \mathcal{A} outputting 1 in $\text{PR}_{\mathcal{A}}^b$. Hence,

$$\left| \Pr[\text{SDDHI}_{\mathcal{B}}^0(\mathbb{G}, q, G) = 1] - \Pr[\text{SDDHI}_{\mathcal{B}}^1(\mathbb{G}, q, G) = 1] \right| > \text{negl}(\lambda)$$

which violates the SDDHI assumption. □

F.2 Homomorphic Commitments

The Pedersen commitment [50] can commit to a vector of messages $\mathbf{m} = (m_1, \dots, m_n) \in \mathbb{Z}_q^n$ by picking group elements $G_1, \dots, G_n \leftarrow \mathbb{G}$ and computing $\text{Com}_{\text{crs}}(\mathbf{m}; r) := H^r \prod_{i=1}^n G_i^{m_i}$. The Pedersen commitment is naturally homomorphic. If the DL assumption holds in \mathbb{G} , this commitment scheme is perfectly hiding and computationally binding.

F.3 Labeled Encryption

The Elliptic Curve Integrated Encryption Scheme (ECIES) [56] is a practical hybrid encryption scheme on elliptic curves. We provide below an abstract description of a labeled variant over generic groups. Below, let $H: \{0,1\}^* \rightarrow \{0,1\}^{2\lambda}$ be a hash function modeled as a random oracle, and let SKE and MAC be a symmetric key encryption scheme and a message authentication code scheme respectively both with key space $\{0,1\}^\lambda$.

Setup(1^λ): On input the security parameter 1^λ , it outputs a public parameter pp consisting of a description of a cyclic group \mathbb{G} of order q together with a group elements $G \in \mathbb{G}$.

KGen(pp): On input the public parameter pp , the algorithm samples $x \leftarrow \mathbb{Z}_q$ and computes $H := G^x$. H is the public key pk and x is the secret key sk .

Enc(pk, τ, m): On input the public key pk , a label τ , and a message $m \in \mathbb{G}$, the algorithm samples $r \leftarrow \mathbb{Z}_q$ and computes $\bar{R} := G^r$, $P := H^r$, $(\text{sk}_{\text{SKE}}, \text{sk}_{\text{MAC}}) := H(P, \tau)$, $e \leftarrow \text{SKE.Enc}(\text{sk}_{\text{SKE}}, m)$, and $\sigma \leftarrow \text{MAC.Sig}(\text{sk}_{\text{MAC}}, e)$. The ciphertext is then $c := (R, e, \sigma)$.

Dec(sk, τ, c): On input the secret key sk , a label τ , and a ciphertext c , the algorithm computes $P := R^x$ and $(\text{sk}_{\text{SKE}}, \text{sk}_{\text{MAC}}) := H(P, \tau)$. It checks if $\text{MAC.Vf}(\text{sk}_{\text{MAC}}, e, \sigma) = 1$. If so, it outputs $m := \text{SKE.Dec}(\text{sk}_{\text{SKE}}, e)$. Otherwise, it outputs \perp .

If the GapDH assumption (Definition B.7) holds in \mathbb{G} , SKE is IND-CPA and MAC is a strongly unforgeable MAC, then it is well-known that ECIES is IND-CCA in the random oracle model [22, 56]; this can easily be extended to IK-CCA.

G Adaption to Monero

Our RingCT scheme Omniring presented in the body of the paper is incompatible with Monero due to the difference in the formats of tags. In Omniring, the tag for an account with public key $\text{pk} = H^x$ is given by $\text{tag} = G^{x^{-1}}$. On the other hand, the corresponding tag in Monero would be $\text{tag} = H(\text{pk})^x$ for some hash function H . If Monero were to adopt Omniring while keeping the current set of unspent transaction outputs, a spender who has already spent from an account with $\text{tag} = H(\text{pk})^x$ would be able to spend from that account again using a transaction with $\text{tag} = G^{x^{-1}}$, which would wrongly not be rejected as a double-spend.

To resolve this issue and make Omniring usable in Monero, we describe slight changes to the instantiations of the tagging scheme and the argument system, which ensure that tags have the same format as those currently used in Monero.

G.1 Tagging Scheme

The tagging scheme $\text{Tag}_{\text{in}} = (\text{TagSetup}_{\text{in}}, \text{TagEval}_{\text{in}})$ in Monero is as follows. $\text{TagSetup}_{\text{in}}$ chooses a hash function $H: \mathbb{G} \rightarrow \mathbb{G}$ (modeled as a random oracle) which maps group elements to group elements, and a group element H which is shared with the commitment scheme. On input $x \in \mathbb{Z}_q$, $\text{TagEval}_{\text{in}}$ outputs $H(H^x)^x$.

We show that Tag satisfies (related-input) one-wayness and pseudorandomness.

Theorem G.1. *If the discrete logarithm assumption holds over $\mathcal{G} = (\mathbb{G}, q, G)$, then the tagging scheme Tag_{in} instantiated as in Monero is a secure tagging scheme according to Definition 4.1 in ROM.*

Proof. We prove related-input one-wayness and related-input pseudorandomness separately.

Related-input one-wayness Suppose Tag_{in} is not related-input one-way, let \mathcal{A} be a PPT adversary for which

$$\Pr[\text{OneWay}_{\mathcal{A}}(1^\lambda) = 1] > \text{negl}(\lambda),$$

i.e. \mathcal{A} can guess a preimage for $\text{TagEval}_{\text{in}}(x + s^*)$. Without loss of generality, we assume \mathcal{A} makes at most q_t queries to the oracle $\text{Tag}\mathcal{O}_x$ and q_h queries to the random oracle. We construct an adversary \mathcal{B} against DL from Definition B.1 with input (\mathbb{G}, q, G, G^x) as follows:

- Sample $s^*, a^* \leftarrow \mathbb{Z}_q$.
- Compute $G^{x+s^*} = G^x \cdot G^{s^*}$ and set $H(G^{x+s^*}) = G^{a^*}$.
- Compute the challenge tag $\text{tag}^* = G^{(x+s^*) \cdot a^*} = (G^x)^{a^*} \cdot G^{s^* \cdot a^*}$.
- Set $\text{pp} = (\mathbb{G}, q, G)$ and run \mathcal{A} on $(\text{pp}, s^*, \text{tag}^*)$.
- On the j -th query to $\text{Tag}\mathcal{O}_x$, sample $s_j, a_j \leftarrow \mathbb{Z}_q$ and set $H(G^{x+s_j}) = G^{a_j}$. Then return $(s_j, G^{(x+s_j) \cdot a_j})$.
- On querying the random oracle with input X , check if $X = G^{x+s_j}$ for some s_j where $H(G^{x+s_j})$ was set during the j -th query to the $\text{Tag}\mathcal{O}_x$. If so, return the set value. If not, sample $a \leftarrow \mathbb{Z}_q$ and return G^a as the reply.
- Eventually, \mathcal{A} returns x' . \mathcal{B} outputs $x = x' - s^*$.

With non-negligible probability, \mathcal{A} is successful in breaking the related-input one-wayness of Tag_{in} , *i.e.*, $x' = x + s^*$. Then, with the same non-negligible probability, \mathcal{B} recovers x correctly and solve the DL instance, violating the discrete logarithm assumption.

Related-input pseudorandomness Suppose Tag_{in} is not related-input pseudorandom, let \mathcal{A} be a PPT adversary for which

$$\left| \Pr[\text{PR}_{\mathcal{A}}^0(1^\lambda) = 1] - \Pr[\text{PR}_{\mathcal{A}}^1(1^\lambda) = 1] \right| > \text{negl}(\lambda)$$

holds, *i.e.* it can distinguish $\text{Tag}_{\text{in}}(x + s^*)$ from a random value $y_1 \leftarrow \mathbb{G}$ given s^* .

We construct an adversary \mathcal{B} against DDH from Definition B.4, which uses \mathcal{A} as follows:

- On input $(\mathbb{G}, q, G, G^x, G^y, Z)$, sample $s^* \leftarrow \mathbb{G}$ and set $H(G^{x+s^*}) = H(G^x \cdot G^{s^*}) = G^y$ and $\text{tag}^* = Z \cdot (G^y)^{s^*}$.
- Set $\text{pp} = (\mathbb{G}, q, G)$ and run \mathcal{A} on $(\text{pp}, s^*, \text{tag}^*)$.
- On the j -th query to $\text{Tag}\mathcal{O}_x$, sample $s_j, a_j \leftarrow \mathbb{Z}_q$ and set $H(G^{x+s_j}) = G^{a_j}$. Then return $(s_j, G^{(x+s_j) \cdot a_j})$.
- On querying the random oracle with input X , check if $X = G^{x+s_j}$ for some s_j where $H(G^{x+s_j})$ was set during the j -th query to the $\text{Tag}\mathcal{O}_x$. If so, return the set value. If not, sample $a \leftarrow \mathbb{Z}_q$ and return G^a as the reply.

- Eventually, \mathcal{A} outputs b' which is also output by \mathcal{B} .

Note that if \mathcal{B} is participating in $\text{DDH}_{\mathcal{B}}^0$, then it simulates the environment of $\text{PR}_{\mathcal{A}}^0$ perfectly. Likewise, if \mathcal{B} is participating in $\text{DDH}_{\mathcal{B}}^1$, then it simulates the environment of $\text{PR}_{\mathcal{A}}^1$ perfectly. Therefore, for each $b \in \{0,1\}$, the probability that \mathcal{B} outputs 1 in $\text{DDH}_{\mathcal{B}}^b$ equals the probability of \mathcal{A} outputting 1 in $\text{PR}_{\mathcal{A}}^b$. Hence,

$$\left| \Pr[\text{DDH}_{\mathcal{B}}^0(\mathbb{G},q,G)=1] - \Pr[\text{DDH}_{\mathcal{B}}^1(\mathbb{G},q,G)=1] \right| > \text{negl}(\lambda)$$

which violates the DDH assumption. \square

G.2 Language for Spend Proofs

With the new instantiation of the tagging scheme, the language that needs to be proven by spenders changes slightly. Given a vector of public keys $\vec{\mathbf{R}}$, we define a new vector $\vec{\mathbf{H}}$ of hashes of public keys as

$$\vec{\mathbf{H}} := \left(\text{H}(\text{pk}_1^{\mathcal{R}}), \dots, \text{H}(\text{pk}_{|\mathcal{R}|}^{\mathcal{R}}) \right).$$

The corresponding language is changed to the following:

$$\mathcal{L}_{\text{sp}}[\mathbb{G},q,G,H] := \left\{ \begin{array}{l} \text{stmt} = (\vec{\mathbf{R}}, \vec{\mathbf{C}}_{\mathcal{R}}, \vec{\mathbf{H}}, \vec{\mathbf{T}}, \vec{\mathbf{C}}_{\mathcal{T}}) : \\ \exists \text{wit} = (\vec{\mathbf{E}}, \vec{\mathbf{x}}, \vec{\mathbf{a}}^{\mathcal{S}}, \vec{\mathbf{r}}^{\mathcal{S}}, \vec{\mathbf{B}}, \vec{\mathbf{a}}^{\mathcal{T}}, \vec{\mathbf{r}}^{\mathcal{T}}) \text{ s.t.} \\ \forall i \in [|\mathcal{S}|], \left\{ \begin{array}{l} \vec{\mathbf{e}}_i \text{ is a unit vector of length } |\mathcal{R}| \\ \vec{\mathbf{R}}^{\vec{\mathbf{e}}_i} = H^{x_i} \\ \vec{\mathbf{C}}_{\mathcal{R}}^{\vec{\mathbf{e}}_i} = G^{a_i^{\mathcal{S}}} H^{r_i^{\mathcal{S}}} \\ \vec{\mathbf{H}}^{\vec{\mathbf{e}}_i} = \text{tag}_i^{x_i^{-1}} \end{array} \right. \\ \forall i \in [|\mathcal{T}|], \left\{ \begin{array}{l} \vec{\mathbf{b}}_i \text{ is the binary rep. of } a_i^{\mathcal{T}} \text{ of length } \beta \\ \vec{\mathbf{C}}_{\mathcal{T}}^{\vec{\mathbf{b}}_i} = G^{a_i^{\mathcal{T}}} H^{r_i^{\mathcal{T}}} \end{array} \right. \\ \sum_{i \in [|\mathcal{S}|]} a_i^{\mathcal{S}} = \sum_{i \in [|\mathcal{T}|]} a_i^{\mathcal{T}} \end{array} \right.$$

G.3 Argument System

As in Section 5 we describe a protocol Π_{sp} with linear communication for the updated language \mathcal{L}_{sp} . A protocol with logarithmic communication can be obtained using the same squashing technique presented in Section 5. The notation used in the protocol below is defined above (for $\vec{\mathbf{H}}$), and in Section 4.4, Figures 29 to 31 and Table 1.

Setup_{sp}($1^\lambda, \mathcal{L}_{\text{sp}}$):

Recall that \mathcal{L}_{sp} is specified by a tuple (\mathbb{G}, q, G, H) . Output $\text{crs} = (\mathbb{G}, q, G, H)$.

$\langle \mathcal{P}_{\text{sp}}(\text{crs}, \text{stmt}, \text{wit}), \mathcal{V}_{\text{sp}}(\text{crs}, \text{stmt}) \rangle$:

\mathcal{V}_{sp} :

1. $u, v \leftarrow_{\mathcal{S}} \mathbb{Z}_q$
2. $h \leftarrow_{\mathcal{S}} \mathbb{G}$, $\vec{\mathbf{p}} \leftarrow_{\mathcal{S}} \mathbb{G}^{2+|\mathcal{R}|+|\mathcal{S}|}$, $\vec{\mathbf{g}}' \leftarrow_{\mathcal{S}} \mathbb{G}^{m-|\mathcal{R}|-|\mathcal{S}|-2}$, $\vec{\mathbf{h}} \leftarrow_{\mathcal{S}} \mathbb{G}^m$

$\mathcal{P}_{\text{sp}} \leftarrow \mathcal{V}_{\text{sp}} : u, v, h, \vec{\mathbf{p}}, \vec{\mathbf{g}}', \vec{\mathbf{h}}$

$\mathcal{P}_{\mathfrak{w}}, \mathcal{V}_{\mathfrak{w}}$:

1. $\hat{\mathbf{Y}} := \vec{\mathbf{R}} \circ \vec{\mathbf{C}}_{\mathcal{R}}^{\circ u} \circ \vec{\mathbf{H}}^{\circ u^2}$
2. $\hat{\mathbf{T}} := \vec{\mathbf{T}}^{\circ -u^2 \bar{v}^{|S|}}$
3. For $w \in \mathbb{Z}_q$, denote

$$\vec{\mathbf{g}}_w := ((G \| H \| \hat{\mathbf{Y}} \| \hat{\mathbf{T}})^{\circ w} \circ \vec{\mathbf{p}} \| \vec{\mathbf{g}}') \quad (25)$$

$\mathcal{P}_{\mathfrak{w}}$:

1. $r_A \leftarrow \mathbb{Z}_q$
2. $A := h^{r_A} \vec{\mathbf{g}}_0^{\vec{\mathbf{c}}_L} \vec{\mathbf{h}}^{\vec{\mathbf{c}}_R}$

Note that $\vec{\mathbf{g}}_w^{\vec{\mathbf{c}}_L} = \vec{\mathbf{g}}_{w'}^{\vec{\mathbf{c}}_L}$ for all $w, w' \in \mathbb{Z}_q$ since $I = G^\xi H^\eta \hat{\mathbf{Y}}^{\hat{\mathbf{e}}} \hat{\mathbf{T}}^{\bar{x}^{\circ-1}}$. Thus $A = h^{r_A} \vec{\mathbf{g}}_w^{\vec{\mathbf{c}}_L} \vec{\mathbf{h}}^{\vec{\mathbf{c}}_R}$ for all $w \in \mathbb{Z}_q$.

$\mathcal{P}_{\mathfrak{w}} \rightarrow \mathcal{V}_{\mathfrak{w}} : A$

$\mathcal{V}_{\mathfrak{w}} : w \leftarrow \mathbb{Z}_q$

$\mathcal{P}_{\mathfrak{w}} \leftarrow \mathcal{V}_{\mathfrak{w}} : w$

$\mathcal{P}_{\mathfrak{w}}$:

1. $r_S \leftarrow \mathbb{Z}_q, \vec{\mathbf{s}}_L, \vec{\mathbf{s}}_R \leftarrow \mathbb{Z}_q^m$
2. $S := h^{r_S} \vec{\mathbf{g}}_w^{\vec{\mathbf{s}}_L} \vec{\mathbf{h}}^{\vec{\mathbf{s}}_R}$.

$\mathcal{P}_{\mathfrak{w}} \rightarrow \mathcal{V}_{\mathfrak{w}} : S$

$\mathcal{V}_{\mathfrak{w}} : y, z \leftarrow \mathbb{Z}_q$

$\mathcal{P}_{\mathfrak{w}} \leftarrow \mathcal{V}_{\mathfrak{w}} : y, z$

$\mathcal{P}_{\mathfrak{w}}$:

1. Define the following polynomials (in X):

$$l(X) := \vec{\mathbf{c}}_L + \vec{\alpha} + \vec{\mathbf{s}}_L \cdot X$$

$$r(X) := \vec{\theta} \circ (\vec{\mathbf{c}}_R + \vec{\mathbf{s}}_R \cdot X) + \vec{\mu}$$

$$t(X) := \langle l(X), r(X) \rangle = t_2 X^2 + t_1 X + t_0$$

for some $t_0, t_1, t_2 \in \mathbb{Z}_q$. In particular

$$t_0 = z^2 \cdot \langle \vec{\mathbf{a}}^{\mathcal{T}}, \vec{y}^{|\mathcal{T}|} \rangle + \delta$$

2. $\tau_1, \tau_2 \leftarrow \mathbb{Z}_q$

3. $T_1 := g^{t_1} h^{\tau_1}, T_2 := g^{t_2} h^{\tau_2}$

$\mathcal{P}_{\mathfrak{w}} \rightarrow \mathcal{V}_{\mathfrak{w}} : T_1, T_2$

$\mathcal{V}_{\mathfrak{w}} : x \leftarrow \mathbb{Z}_q$

$\mathcal{P}_{\mathfrak{w}} \leftarrow \mathcal{V}_{\mathfrak{w}} : x$

$\mathcal{P}_{\mathfrak{w}}$:

1. $\tau := z^2 \cdot \langle \vec{\mathbf{r}}^{\mathcal{T}}, \vec{y}^{|\mathcal{T}|} \rangle + \tau_1 x + \tau_2 x^2$
2. $r := r_A + r_S x$

3. $(\vec{l}, \vec{r}, t) := (l(x), r(x), t(x))$

$\mathcal{P}_{\text{uj}} \rightarrow \mathcal{V}_{\text{uj}} : \tau, r, \vec{l}, \vec{r}, t$

\mathcal{V}_{uj} : Check if the following relations hold:

$$t = \langle \vec{l}, \vec{r} \rangle \tag{26}$$

$$h^r \vec{\mathbf{g}}_w^{\vec{l}} \vec{\mathbf{h}}^{\vec{\theta}^{\circ-1} \circ \vec{r}} = AS^x \vec{\mathbf{g}}_w^{\vec{\alpha}} \vec{\mathbf{h}}^{\vec{\beta}} \tag{27}$$

$$G^t H^\tau = G^\delta \vec{\mathbf{C}}_{\mathcal{T}}^{z^2 \cdot \vec{y}^{|\tau|}} T_1^x T_2^{x^2} \tag{28}$$

Theorem G.2. *The verifier \mathcal{V}_{uj} is public-coin. Π_{uj} is constant-round, perfectly complete, and perfect special honest-verifier zero-knowledge.*

Theorem G.3. *Assuming the discrete logarithm assumption holds over \mathcal{G} , Π_{uj} has computational witness-extended emulation.*

The proofs of the above theorems are almost identical to those of Theorems 5.1 and 5.2 and are omitted.

Notation	Description
$\hat{\mathbf{Y}} = \hat{\mathbf{Y}}(u) := \vec{\mathbf{R}} \circ \vec{\mathbf{C}}_{\mathcal{R}}^{\circ u} \circ \vec{\mathbf{H}}^{\circ u^2}$	Vector of compressed public keys and coins with randomness $u \in \mathbb{Z}_q$
$\hat{\mathbf{T}} = \hat{\mathbf{T}}(u, v) := \vec{\mathbf{T}}^{\circ -u^2 \vec{v}^{ \mathcal{S} }}$	Scaled tag with randomness $u, v \in \mathbb{Z}_q$
$\xi = \xi(u, v) := -\langle \vec{v}^{ \mathcal{S} }, u \cdot \vec{\mathbf{a}}^{\mathcal{S}} \rangle$	Compressed secrets with randomness $u, v \in \mathbb{Z}_q$.
$\eta = \eta(u, v) := -\langle \vec{v}^{ \mathcal{S} }, \vec{\mathbf{x}} + u \cdot \vec{\mathbf{r}}^{\mathcal{S}} \rangle$	Note that $(\xi, \eta, \hat{\mathbf{e}}, \vec{\mathbf{x}}^{\circ -1})$ satisfies $I = G^\xi H^\eta \hat{\mathbf{Y}} \hat{\mathbf{e}} \hat{\mathbf{T}}^{\vec{\mathbf{x}}^{\circ -1}}$.
$\hat{\mathbf{e}} = \hat{\mathbf{e}}(v) := \vec{v}^{ \mathcal{S} } \mathbf{E}$	
$\vec{\mathbf{c}}_L, \vec{\mathbf{c}}_R$	Encoding of witness by honest prover dependent on u and v , see Figure 9.
$m = 2 + \mathcal{R} + \mathcal{R} \mathcal{S} + \beta \mathcal{T} + 3 \mathcal{S} $	Length of $\vec{\mathbf{c}}_L$ and $\vec{\mathbf{c}}_R$
$(\vec{\mathbf{v}}_0, \dots, \vec{\mathbf{v}}_8, \vec{\mathbf{u}}_5) = (\vec{\mathbf{v}}_0, \dots, \vec{\mathbf{v}}_8, \vec{\mathbf{u}}_5)(u, v, y)$	Constraint vectors parameterized by the randomness $u, v, y \in \mathbb{Z}_q$, see Figure 10.
$(\vec{\alpha}, \vec{\beta}, \vec{\delta}, \vec{\theta}, \vec{\zeta}, \vec{\mu}, \vec{\nu}, \vec{\omega}) = (\vec{\alpha}, \vec{\beta}, \vec{\delta}, \vec{\theta}, \vec{\zeta}, \vec{\mu}, \vec{\nu}, \vec{\omega})(u, v, y, z)$	Compressed constraint vectors parameterized by the randomness $u, v, y, z \in \mathbb{Z}_q$, see Figure 10 and Figure 11.
$\text{EQ} = \text{EQ}[\vec{\mathbf{a}}^{\mathcal{T}}, u, v, y]$	System of equations parameterized by the amounts $\vec{\mathbf{a}}^{\mathcal{T}}$ and randomness $u, v, y \in \mathbb{Z}_q$, see Figure 12

Table 1: Notation for signatures of knowledge construction (for Monero).

$$\begin{aligned} \vec{\mathbf{c}}_L &:= (\xi \parallel \eta \parallel \hat{\mathbf{e}} \parallel \vec{\mathbf{x}}^{\circ -1} \parallel \text{vec}(\mathbf{E}) \parallel \text{vec}(\mathbf{B}) \parallel \vec{\mathbf{a}}^{\mathcal{S}} \parallel \vec{\mathbf{r}}^{\mathcal{S}}) \\ \vec{\mathbf{c}}_R &:= (\vec{\mathbf{0}}^{2+|\mathcal{R}|+|\mathcal{S}|} \parallel \vec{\mathbf{x}} \parallel \text{vec}(\mathbf{E}) - \vec{\mathbf{1}}^{|\mathcal{R}||\mathcal{S}|} \parallel \text{vec}(\mathbf{B}) - \vec{\mathbf{1}}^{\beta|\mathcal{T}|} \parallel \vec{\mathbf{0}}^{2|\mathcal{S}|}) \end{aligned}$$

Figure 29: Honest encoding of witness (for Monero).

$$\begin{bmatrix} \vec{\mathbf{v}}_0 \\ \vec{\mathbf{v}}_1 \\ \vec{\mathbf{v}}_2 \\ \vec{\mathbf{v}}_3 \\ \vec{\mathbf{v}}_4 \\ \vec{\mathbf{v}}_5 \\ \vec{\mathbf{v}}_6 \\ \vec{\mathbf{v}}_7 \\ \vec{\mathbf{v}}_8 \\ \vec{\mathbf{u}}_5 \end{bmatrix} := \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \vec{y}^{|\mathcal{R}||\mathcal{S}|+\beta|\mathcal{T}|} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \vec{y}^{|\mathcal{S}|} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \vec{y}^{|\mathcal{T}|} \otimes \vec{\mathbf{2}}^\beta & \cdot \\ \cdot & \cdot & \cdot & \cdot & \vec{y}^{|\mathcal{S}|} \otimes \vec{\mathbf{1}}^{|\mathcal{R}|} & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot & u \cdot \vec{v}^{|\mathcal{S}|} \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & u \cdot \vec{v}^{|\mathcal{S}|} \\ \cdot & \cdot & -\vec{y}^{|\mathcal{R}|} & \cdot & \vec{v}^{|\mathcal{S}|} \otimes \vec{y}^{|\mathcal{R}|} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \vec{\mathbf{1}}^{|\mathcal{T}|} \otimes \vec{\mathbf{2}}^\beta & -\vec{\mathbf{1}}^{|\mathcal{S}|} \\ \cdot & \cdot & \cdot & \cdot & \vec{y}^{|\mathcal{R}||\mathcal{S}|+\beta|\mathcal{T}|} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \vec{v}^{|\mathcal{S}|} & \cdot & \cdot & \cdot \end{bmatrix}$$

Figure 30: Definitions of constraint vectors (for Monero). (Dots mean zeros.)

$$\begin{aligned}
\vec{\theta} &:= \sum_{i=0}^1 z^i \cdot \vec{\mathbf{v}}_i & \vec{\zeta} &:= \sum_{i=2}^7 z^i \cdot \vec{\mathbf{v}}_i & \vec{\mu} &:= \sum_{i=2}^8 z^i \cdot \vec{\mathbf{v}}_i \\
\vec{\nu} &:= z^8 \cdot \vec{\mathbf{v}}_8 & \vec{\omega} &:= z^5 \cdot \vec{\mathbf{u}}_5 \\
\vec{\alpha} &:= \vec{\theta}^{\circ-1} \circ (\vec{\omega} - \vec{\nu}) & \vec{\beta} &:= \vec{\theta}^{\circ-1} \circ \vec{\mu} \\
\delta &:= z \cdot \langle \vec{\mathbb{1}}^{|\mathcal{S}|}, \vec{y}^{|\mathcal{S}|} \rangle + z^3 \cdot \langle \vec{\mathbb{1}}^{|\mathcal{S}|}, \vec{y}^{|\mathcal{S}|} \rangle + \langle \vec{\alpha}, \vec{\mu} \rangle + \langle \vec{\mathbb{1}}^m, \vec{\nu} \rangle
\end{aligned}$$

Figure 31: Definitions of constraint vectors (for Monero) (cont.).

$$\text{EQ}(\vec{\gamma}_L, \vec{\gamma}_R) = 0 \iff \left\{ \begin{array}{ll} \langle \vec{\gamma}_L, \vec{\gamma}_R \circ \vec{\mathbf{v}}_0 \rangle & = 0 & (29) \\ \langle \vec{\gamma}_L, \vec{\gamma}_R \circ \vec{\mathbf{v}}_1 \rangle & = \langle \vec{\mathbb{1}}^{|\mathcal{S}|}, \vec{y}^{|\mathcal{S}|} \rangle & (30) \\ \langle \vec{\gamma}_L, \vec{\mathbf{v}}_2 \rangle & = \langle \vec{\mathbf{a}}^{\mathcal{T}}, \vec{y}^{|\mathcal{T}|} \rangle & (31) \\ \langle \vec{\gamma}_L, \vec{\mathbf{v}}_3 \rangle & = \langle \vec{\mathbb{1}}^{|\mathcal{S}|}, \vec{y}^{|\mathcal{S}|} \rangle & (32) \\ \langle \vec{\gamma}_L, \vec{\mathbf{v}}_4 \rangle & = 0 & (33) \\ \langle \vec{\gamma}_L, \vec{\mathbf{v}}_5 \rangle + \langle \vec{\gamma}_R, \vec{\mathbf{u}}_5 \rangle & = 0 & (34) \\ \langle \vec{\gamma}_L, \vec{\mathbf{v}}_6 \rangle & = 0 & (35) \\ \langle \vec{\gamma}_L, \vec{\mathbf{v}}_7 \rangle & = 0 & (36) \\ \langle \vec{\gamma}_L - \vec{\gamma}_R - \vec{\mathbb{1}}^m, \vec{\mathbf{v}}_8 \rangle & = 0 & (37) \end{array} \right.$$

Figure 32: A system of equations guaranteeing the integrity of the encoding of witness (for Monero).

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