maskVerif: automated analysis of software and hardware higher-order masked implementations

Gilles Barthe¹, Sonia Belaïd², Pierre-Alain Fouque³, and Benjamin Grégoire⁴

¹ IMDEA Software Institute
gilles.barthe@imdea.org
² CryptoExperts
sonia.belaïd@cryptoexperts.com
³ Rennes Univ
pierre-alain.fouque@univ-rennes1.fr
⁴ Inria Sophia Antipolis
benjamin.gregoire@sophia.inria.fr

Abstract. Masking is a popular countermeasure for protecting both hardware and software implementations against differential power analysis. A main strength of software masking is that its security guarantees can be captured rigorously through formal models, such as the Threshold Probing Model (Ishai, Sahai, Wagner, CRYPTO 2003) and proved automatically using formal verification tools based on Fourier coefficients (Eldib, Wang and Schaumont, ACM TOSSEM 2014) or information flow analyses (Barthe et al., EUROCRYPT 2015). In 2018, Bloem et al. (EUROCRYPT 2018) and Faust et al. (CHES 2018) introduce two security models, respectively called Threshold Probing Model with Glitches, and Robust Threshold Probing Model, to formalize the security of masked hardware implementations. Moreover, Bloem et al. provide an automated tool for proving security in the Threshold Probing Model with Glitches. However, their method is based on the computations of Fourier coefficients, and thus computationally expensive and limited to low orders.

In this paper, we develop new information flow-based techniques for proving security of hardware implementations in the models of (Bloem et al., EUROCRYPT 2018) and (Faust et al., CHES 2018). Our techniques improve over the algorithms of (Barthe et al., EUROCRYPT 2015) in terms of coverage and efficiency, allowing for verification at higher orders and minimizing the possibility of false negatives. We implement our methods in an extension of the maskVerif tool (Barthe et al., EUROCRYPT 2015) that supports efficient verification of both software and hardware implementations. Our tool is able to analyze examples from the literature, including (Bloem et al., EUROCRYPT 2018), much faster and at higher orders, as well as implementations that follow Generalized Masking Scheme (Reparaz et al., CRYPTO 2015) and Domain Oriented Masking (Gross, Mangard, Korak, CHES 2017).

Keywords: Glitches, Masking, Automated verification

1 Introduction

Side-channel attacks provide an effective vector to retrieve key material and more generally secret information from cryptographic implementations. Informally, side-channel attacks exploit physical information, such as timing or noise, that can be observed from the execution of said implementations, and carry statistical analysis to retrieve the desired information from these observations. There exist many successful forms of side-channel attacks. For implementations on embedded devices, one of the most devastating family of attacks is differential power analysis (DPA) [26]. Protecting against such attacks is therefore a major theoretical and practical concern, and has been the subject of a long line of research.

The most deployed countermeasure so far is masking. Masking uses secret-sharing techniques and rests on the observation that combining \( t + 1 \) data from their noisy leakage was proven to be exponentially hard in \( t \) [10, 32]. Given a masking order \( t \), which models the adversary’s capabilities, a masking transformation aims to replace every sensitive variable \( x \) of the implementation by a set of \( t + 1 \) variables \( (x_i)_{0 \leq i \leq t} \), called shares, such that \( t \) of these shares are generated uniformly at
random and the last one is computed such that \( x_0 \cdots \cdots \cdots x_t = x \) for some law \( \star \). The main challenge behind masking transformations is to transform computations that were initially performed on inputs \( x \) into computations that are performed on \( (x_i)_{0 \leq i \leq t} \), while very carefully avoiding to compute intermediate results that depend on all the shares of secret inputs. While this goal is reasonably easy to achieve for the case \( t = 1 \), it is more difficult to achieve for higher values of \( t \). In particular, the complexity of linear functions is multiplied by \( t + 1 \) and the complexity of non-linear functions is often quadratic in \( t \) under a masking transformation.

Motivated by the complexity of designing masking transformations, and the desire to reason formally about their correctness, Ishai, Sahai, and Wagner introduced the \( t \)-threshold probing model \[25\]. Under this model, an implementation is secure if and only if any set of \( t \) intermediate variables is independent from the secret. In 2014, an elegant work by Duc, Dziembowski, and Faust \[15\] managed to demonstrate the practicability of this model in the reality of embedded devices. It has served as a source of motivation for a series of formal methods and automated tools for proving security, or for generating secure masked implementations in the threshold probing model \[30, 17, 2, 3, 11, 39\]. An interesting by-product of this work, specifically \[3\] is a stronger notion of security, called strong non-interference, that addresses the long-standing problem of compositional reasoning for masked implementations.

While these models make sense in a software scenario where we assume that an observation reflects the leakage of a single variable, many masking schemes are insecure when executed on hardware devices. The discrepancy between these models and hardware implementations was first analyzed by Mangard, Popp and Gammel \[27\]. Their work identifies so-called a special kind of transitions, known as hardware glitches, as the main origin of the discrepancy. Informally, glitches happen whenever information does not propagate simultaneously within the different wires of a combinational logic. Glitches introduce unexpected dependencies between several variables within the same combinational logic, and induce additional leakage that can be used to break implementations. Following \[27\], several glitches attacks have thus been successfully applied to \( t \)-probing secure masking schemes \[28, 29, 35, 34\]. In parallel, many techniques have been developed against glitch attacks. In seminal work, Nikova \textit{et al.} \[31, 8\] introduce a hardware countermeasure, called Threshold Implementations, a.k.a. TI, which consists in ensuring the three following properties: (i) correctness, the sum of the output shares is equal to the result of applying the function without masking, (ii) incompleteness, each output share shall be computed from at most \( t \) input shares, or at least one input share must not be used in the computation of each output share (it guarantees that each output share computation will not leak information on sensitive variables), (iii) uniformity, the output sharing must be uniformly distributed if the input sharing is. A special uniformity technique has been proposed by Daemen in \[14\]. Finally a generalization of TI, called Consolidating Masking Scheme (CMS) \[35\] by Reparaz \textit{et al.}, and a new one Domain Masking Scheme (DMS) \[22\] to scale high order masking has been proposed by Groß \textit{et al.} since one of the drawbacks of TI is the randomness required at the end of each non-linear stage.

Recently, Bloem, Groß, Iusupov, Könighofer, Mangard and Winter \[9\] and Faust, Grosso, Merino Del Pozo, Pagbialonga and Standaert \[19\] independently address the problem of proving the security of masked implementations in presence of glitches. Both papers define a new security model, henceforth referred to as the threshold probing model with glitches and the robust threshold probing model, which combine the benefits of the threshold probing model with an abstract but accurate modeling of glitches. In addition, Bloem \textit{et al.} propose an automated method based on (an estimation of) Fourier coefficients for proving that an implementation is secure in their model. They also use their verification method on a representative set of examples, including the S-Boxes of AES, Keccak and FIDES. Due to the computational cost of their approach, the tool primarily applies to the first-order setting.

**Contributions**

The main outstanding contribution of this paper is an efficient, and fully automated tool for proving (different notions of) security of masked implementations in several software and hardware models from the literature.

On the modelling side, we introduce a simple but expressive intermediate representation, called MaskIR, in which each instruction is explicitly annotated by its leakage, which is itself modeled as
an expression in an extended language. The intermediate representation naturally captures several models from the literature, including the threshold probing model, the bounded moment model, and the threshold probing model with glitches. Moreover, it can be used to establish comparisons between the different models.

On the algorithmic side, we present a new algorithm for proving that a tuple is independent from secrets. The new algorithm is inspired from Gaussian elimination but is able to accommodate non-linear systems of equations, and significantly improves over the core algorithm from [2] by: (i) being sound and complete (no attack missed and no false negative) for linear systems of equations; (ii) minimizing false negatives for the non-linear case.

On the implementation side, we provide a new implementation of maskVerif that supports our intermediate representation MaskIR. In addition, the new implementation proposes several new functionalities and several major efficiency improvements. In particular, it uses graph-based methods to improve the efficiency of the algorithms from [2], and supports our new algorithm for proving independence from secrets. The resulting tool is the first one able to verify t-(strong) non-interference of a circuit in presence of glitches, and also the only one able to verify all the three properties in hardware and software, namely probing security, non-interference, and strong non-interference at higher orders. Table 1 compares the coverage of our tool with existing tools.

On the experimental side, we verify security of hardware masked implementations. The results demonstrate a significant improvement over state-of-the-art. For instance, the first-order hardware implementation of the AES s-box provided by Gross et al. [22] is proven to be secure with presence of glitches within less than a second, whereas Bloem et al. [9] performs the same verification within five to ten hours using parallelization.

On the methodological side, our experiments establish that verifying an implementation in the threshold probing model with glitches is often faster than verifying the same implementation in the threshold probing model (without glitches). This might be somewhat surprising, given that security in the threshold probing model with glitches implies security in the threshold probing model (without glitches). However, this difference is a consequence of maskVerif verification algorithm that iteratively analyzes large observation sets (typically of larger size than the verification order). In presence of glitches, the adversary has more power and thus the observation sets are much bigger. On the other hand, there are much less observation sets to verify. This suggests an interesting methodology to verify software implementations, where the user forces (through adding annotations at selected points) the verification algorithm to consider larger sets. Automating this methodology in maskVerif is an interesting direction for further work.

Cautionary note. Formal verification complements, rather than supplants, existing approaches to perform an empirical assessment of the leakage, using on statistical tests or concrete attacks. While practical, empirical approaches are not designed to guarantee the absence of attacks on a different device or when the adversary is given more leakage traces. On the other hand, formal verification necessarily relies on models, which must be validated empirically.

2 Leakage Models and Existing Verification Approaches

We start with an informal description of the problem and of the main challenges to verify masked implementations. We then proceed with a brief and informal explanation of some of the most important models and security notions. Finally, we conclude with a critical review of prior work on formal verification, providing in each case a brief account of the methods used and of their limitations.

2.1 Problem statement

We consider probabilistic implementations that operate on inputs, perform intermediate computations, and return outputs. Moreover, we suppose given a set of secrets, typically computed as a deterministic function of the inputs.
The definition of security depends on three notions: the execution model, the leakage model, and the adversary model. Execution models capture the operational behavior (a.k.a. semantics) of programs. Program behavior is defined compositionally: one defines the behavior of atomic computations in isolation, and then one defines the behavior of program from the behavior of its sub-components. Note that execution models do not capture leakage.

Leakage models define how much information is leaked by each atomic computation. In the simplest software models, leakage typically only depends on the values manipulated by the current computation. However, other models, including hardware models, allow leakage to depend on the results of previous sub-computations. Thus, in contrast to program semantics, program leakage needs not be fully compositional, in the sense that it is not always possible to define leakage of atomic computations in isolation.

Adversary models specify how much information an adversary can gain from an execution of a complete implementation. In some models, the adversary obtains the joint distribution of all atomic leakages. In most models, however, this is not the case. Instead, it is common to assume that adversaries can only observe a maximal number \( t \) of atomic leakages, called the order of an adversary. Given a (possibly adversarially chosen) set \( O \) of atomic leakages of size \( \leq t \), we define the leakage of an implementation w.r.t. \( O \) to be the joint distribution of atomic leakages for all elements of \( O \). Throughout the next sections, we let \( L_O(x) \) denote the leakage obtained by an adversary which observes for an observation set \( O \) of his choice the execution of the implementation on input \( x \).

Given execution, leakage and adversary models, one can now define the notion of secure implementation. Informally, an implementation is secure if for all possible choices of the adversary, leakage does not reveal any information about secrets. Prior work captures this intuition through two distinctive flavours of security definitions, using notions of “small” and “equivalence” (we make the definition of the latter more precise in Section 2.2):

- in the simulation-based paradigm, one proves that for every set of observations, leakage can be computed from a “small” subset on inputs. More precisely, one exhibits a simulator that takes as input a “small” subset of inputs, and computes the leakage. One then argues that “small” subsets of inputs are (in general uniformly distributed and always) independent from the secret, and concludes that the set of observations reveals nothing to the adversary.
- in the information flow paradigm, one proves that for every set of observations, there exists a “small” subset of inputs, such that leakage is equal for every two runs that coincide on this “small” subset of inputs. As in the previous case, one then argues that “small” subsets of inputs are (in general uniformly distributed and always) independent from the secret, and conclude similarly.

The simulation-based paradigm is familiar in cryptography. However, it involves an existential quantification over simulators. In contrast, the information flow paradigm is familiar in programming languages and formal verification, and only involves universal quantification (over pairs of related inputs). Fortunately, it is often possible to prove equivalence between simulation-based and information flow-based definitions. Still, verification of masked implementations is a challenge for two reasons:

- combinatorial explosion: The number of adversarial choices of observation sets grows exponentially with the order \( t \) and the size of the implementation [2, 11, 9].
- non-compositionality: The sequential composition of two secure implementations is not always secure [36, 13, 33, 3].

As a consequence, manual verification of masked implementations at higher orders is at best error-prone and more generally unrealistic. This has been a main source of motivation for developing automated verification tools, and for proposing new, compositional, notions of security.

### 2.2 Security notions

We recall three (still informal) security notions from the literature. We start from the weaker definition, called threshold probing security, and then present an intermediate definition, called
non-interference in the literature, and conclude with the strongest notion, called strong non-
interference in the literature.

For the clarity of exposition, we consider the case of programs with two inputs. However, all
definitions extend without any difficulty to programs with an arbitrary number of inputs. Concretely,
we let the inputs be \( x = (x_1, \ldots, x_t) \) and \( x' = (x'_1, \ldots, x'_{t+1}) \), and assume that the
secrets that should not be leaked by computation are \( s = x_1 + \ldots + x_{t+1} \) and \( s' = x'_1 + \ldots + x'_{t+1} \).

We also define \( \mu_s \) as the uniform distribution over all tuples of inputs \( (x_1, \ldots, x_{t+1}) \) such that
\( s = x_1 + \ldots + x_{t+1} \). Then, \( \mu_{s'} \) is defined similarly.

We shall also need the following definition. Let \( I \subseteq \{1, \ldots, t+1\} \). We say that two tuples
\( v = (v_1, \ldots, v_{t+1}) \) and \( v' = (v'_1, \ldots, v'_{t+1}) \) are \( I \)-equivalent, written \( v \simeq_I v' \), if \( v_i = v'_i \) for every
\( i \in I \). We also let \( v_I \) denote the subvector containing only indices from \( I \).

**Threshold probing security.** The first notion is threshold probing security, which can be under-
stood informally as a notion of non-interference under uniform inputs. We say that an imple-
mentation is \( t \)-threshold probing secure or \( t \)-non-interfering under uniform inputs, if for every \( (s, s') \) and \( (u, u') \), and every observation set \( O \) such that \( |O| \leq t \),
\[
L_O(\mu_s, \mu_{s'}) = L_O(\mu_u, \mu_{u'})
\]

**Non-interference.** We say that an implementation is \( t \)-non-interfering, written \( t\text{-Ni} \), if for every observation set \( O \) such that \( |O| \leq t \), there exists two sets \( I \) and \( I' \) such that \( |I|, |I'| \leq t \) and for every pair of inputs \( (x, x') \) and \( (y, y') \),
\[
x \simeq_I y \land x' \simeq_{I'} y' \implies L_O(x, x') = L_O(y, y').
\]
This is equivalent to requiring for every set of observations \( O \) the existence of two sets \( I \) and \( I' \) with \( |I|, |I'| \leq t \) and a simulator \( S \) that takes as inputs \( x_I \) and \( x'_{I'} \) such that
\[
L_O(x, x') = S(x_I, x'_{I'}). \]

**Strong non-interference.** The final notion is strong non-interference, and is used for composi-
tional reasoning. Strong non-interference distinguishes between internal and output observations.
For every observation set \( O \), we let \( \| O \| \) denote the size of its subset of internal observations. Then,
we say that an implementation is \( t \)-strong non-interfering, written \( S\text{NI} \), if for every observation set \( O \) such that \( |O| \leq t \), there exists two sets \( I \) and \( I' \), such that \( |I|, |I'| \leq \| O \| \) and for every pair of inputs \( (x, x') \) and \( (y, y') \),
\[
x \simeq_I y \land x' \simeq_{I'} y' \implies L_O(x, x') = L_O(y, y').
\]
As for non-interference, strong non-interference admits an equivalent but simulation-based defini-
tion of security. Obviously, it can be verified that

\[ \text{strong non-interference} \implies \text{non-interference} \implies \text{threshold probing security} \]

More interestingly, the different definitions serve different purposes and have complementary
strengths:

- Non-interference (\( \text{NI} \)) is generally easier to prove than threshold probing security.
  Intuitively, this is because determining the dependencies between a set of intermediate vari-
able and some input shares is simpler than reasoning about distributions. In addition, \( \text{NI} \) is a
  simpler definition for reasoning compositionally as observations can be propagated through
  the circuit (i.e., simulation with some input shares become observations on output variables)
  to reason on the global security. Non-interference is used as a baseline definition in almost all
  works on formal verification.

- While the \( \text{NI} \) property is a convenient target for verification, it cannot be used on its own for
  analyzing the security of complete implementations. Therefore, Barthe et al. [3] introduce the
  \( \text{SNI} \) property which supports compositional reasoning and can be used to justify the security
  of complete implementations.
2.3 Leakage models

In this paragraph, we review some important leakage models from the literature that are of particular interest for this paper. Noisy leakage model, bounded moment model, and threshold probing model with transitions are additionally recalled in Appendix A.

**Threshold probing model.** This model was introduced by Ishai, Sahai, and Wagner [25]. In this model, the adversary chooses a set of intermediate variables of his choice; the requirement is that the size of the set is bounded by some natural number \( t \), called the order.

Leakage is then defined as the joint distribution of the intermediate values. A circuit is said to be \( t \)-probing secure or \( t \)-private if and only if, for all adversarial choices of sets \( X \) of intermediate variables (subject to the cardinality constraint \( |X| \leq t \)), the leakage, modeled as a joint distribution, does not reveal any information about the secret.

The conceptual simplicity of the threshold probing model makes it particularly convenient for reasoning formally about the security of masked implementations. It is therefore not surprising that a large majority of prior works, in particular those focused on masking of software implementations, is based on the threshold probing model. Moreover, the threshold probing model has been a main target for formal verification, as detailed in the next subsection.

**Robust Threshold Probing Model.** Even when they are secure in the threshold probing model, hardware implementations can still be vulnerable to glitches attacks. As a consequence, recent works introduce an extension of the threshold probing model to reason on hardware security. As part as a more generic hardware leakage model, Faust et al. [20] consider an extension of the authorized set of probes in the threshold probing model so that putting one probe on a gadget reveals to the adversary the whole set of this gadget’s inputs (assuming the gadget is performed within the same combinatorial logic). Their model rests on two observations: first, glitches may break locality of leakage, in the sense that computations may leak values that depend on prior computations; this is similar to the model with transitions, except that non-locality arises at a significantly higher scale. Second, glitches may yield unexpected computations. Their model renders these two observations concrete by letting adversaries learn, under provisos that reflect physical constraints imposed by hardware, all the inputs of a combinatorial set at the cost of a single observation.

**Robust/Threshold Probing Model/with Glitches.** Independently, Bloem, Groß, Iusupov, Könighofer, Mangard and Winter [9] introduce an extension of the threshold probing model, henceforth called, the threshold probing model with glitches. Contrary to Faust et al. ’s model which provides the adversary the whole set of inputs for an observation within a combinatorial logic set, Bloem et al’s model let adversaries select, within similar hardware constraints, modified implementations on which the observations will be made. Crucially, their definition of security in presence of glitches states that a program \( P \) is secure at order \( t \) in the threshold probing model with glitches iff a program \( P' \) built from \( P \) (along the lines discussed above) is also secure at order \( t \) in the threshold probing model.

The two models are equivalent since authorizing observations on any function of a combinatorial set’s inputs is equivalent to authorizing observations on all its inputs. In the rest of this paper, we will refer to these models as the threshold probing model with glitches.

**Relationships between models** We have:

- threshold probing model w/ glitches ⇔ robust threshold probing model
- threshold probing model w/ glitches ⇒ threshold probing model

2.4 Verification Tools

In this section, we review the state-of-the-art for the verification of masked implementations.
Verification of software-based implementations. Moss, Oswald, Page and Tunstall [30] were the first to consider the use of automated methods to build or verify masked implementations. Specifically, they propose and implement a type-based masking compiler that track which variables are masked by random values and iteratively modifies an unprotected program until all secrets are masked. This strategy suffices to ensure security against first-order differential power analysis, and works well on many examples.

While type-based verification is generally efficient and scalable, it is also often overly conservative, i.e. it rejects secure programs. Logic-based verification often strikes interesting trade-offs between efficiency and expressiveness. This possibility was first explored in the context of masked implementations by Bayrak, Regazzoni, Novo and Ienne [5]. Concretely, they propose a SMT-based method for analyzing the security of masked implementations against first-order differential power analysis. In contrast to the type system of [30], which targets proving a stronger property of programs, their method directly targets proving statistical independence between secrets and leakage. Their approach is limited to first-order masking but was extended to higher orders by Eldib, Wang and Schaumont [17]. Their approach is based on a logical characterization of security, akin to non-interference (NI), and is based on model counting. Unfortunately, model counting incurs an exponential blow-up in the security order, and becomes infeasible even for relatively small orders. Eldib, Wang and Schaumont circumvent the issue by developing sophisticated (and somewhat unintuitive) methods for incremental verification. Although such methods help, the scope of application of their methods remains limited. Recently, Zhang, Gao, Song and Wang [39] show how abstraction-refinement techniques provide significant improvement in terms of precision and scalability. They implement their technique in a tool, called SCInfer, that alternates between fast and moderately precise approaches (partly inspired from [2], which we describe below) and computationally expensive but precise approaches. Their tool delivers practical results for the benchmarks taken from [17].

Independently, Barthe, Belaïd, Dupressoir, Fouque, Grégoire and Strub [2] propose a different approach for proving security in the threshold probing model. Their approach establishes and leverages a tight connection between the security of masked implementations and probabilistic non-interference, for which they propose efficient verification methods. Specifically, they show how a relational program logic previously used for mechanizing proofs of provable security can be specialized into an efficient procedure for proving probabilistic non-interference, and develop techniques that overcome the combinatorial explosion of observation sets for high orders. Informally, the main idea of their algorithm is to carefully select sets of \( t \) or more intermediate variables and to repeatedly apply optimistic sampling on the tuple of expressions that represent the results of these intermediate variables until they do not depend on the secret. The concrete outcome of their work is the maskVerif framework, which achieves practicality at reasonably high orders. For instance, they report using maskVerif to automatically and formally verify the probing security of the original Ishai-Sahai-Wagner multiplication [25] at order 5. A tweaked version of maskVerif also offers the possibility to verify the security of higher-order implementations in the transition-based model.

A follow-up work by the same authors [3] addresses the problem of compositional reasoning by introducing the notion of strong non-interference (SNI) discussed in the previous paragraph, and adapts maskVerif to check SNI. The adaptation achieves similar coverage as the original tool, i.e. it achieves practicality at reasonably high-orders. In addition, [3] proposes an information flow type system with cardinality constraints, which forms the basis of a compiler, called maskComp. This compiler transforms an unprotected implementation into an implementation that is protected at any desired order—the order is passed as an argument. Somewhat similar to the masking compiler of [30], maskComp uses typing information to control and to minimize the insertion of mask refreshing gadgets. In the same line of work, Belaïd, Goudarzi, and Rivain recently propose tightPROVE [6] which exactly and directly verifies the threshold probing security of a circuit based on standard operations at any order.

Also recently, Coron [11] presents an alternative tool, called checkMasks. checkMasks achieves similar functionalities as maskVerif, but exploits a more extensive set of transformations for operating on tuples of expressions. This is useful to achieve better verification times on selected examples.
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<th>Tools</th>
<th>probing</th>
<th>NI</th>
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<td>maskVerif [2, 3]</td>
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<tr>
<td>checkMasks [11]</td>
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<td>Bloem et al. [9]</td>
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<td>new maskVerif</td>
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Verification of hardware-based implementations. Bloem, Groß, Iusupov, Könighofer, Mangard and Winter [9] present a formal technique for proving security of implementations in the threshold probing model with glitches. Their method is based on Xiao-Massey lemma, which provides a necessary and sufficient condition for a boolean function to be statistically independent from a subset of its variables. Informally, the lemma states that a boolean function \( f \) is statistically independent of a set of variables \( X \) if and only if the so-called Fourier coefficients of every non-empty subset of \( X \) is null. However, since the computation of Fourier coefficients is computationally expensive, they use instead an approximation method, whose correctness is established in their paper. By encoding their approximation in logical form, they are able to instantiate their approach using SAT-based solvers. Their tool is able to verify implementations of S-Boxes of AES, Keccak and FIDES. However, the cost of the verification is significant.

To conclude this description of current properties and techniques, Table 1 recall the four state-of-the-art automated tools verifying concrete higher-order masked implementations at fixed orders. While all current tools verify at most three settings, our new version of maskVerif covers all settings, is more efficient and is able to verify implementations at higher orders than previous tools.

3 Framework

This section describes the MaskIR intermediate representation to describe software and hardware implementations, and demonstrates its use to represent different leakage models. We also prove general results for transferring security proofs from one model to another, and for optimizing security proofs.

3.1 Rationale

The basic idea between MaskIR is two-fold: on the one hand, existing leakage models artificially enforce restrictions on security notions. For instance, our definition of input equivalence enforces that every element in the partition of inputs has size \( t + 1 \), whereas it would suffice that every element in the partition contains at least \( t + 1 \) elements. While such coincidences can be important for some specific proof techniques, they are never exploited in the approach followed by maskVerif.

On the other hand, formal verification is only possible if leakage is made explicit. A common approach to model the non-functional behavior of programs, of which leakage is an instance, is to use ghost code. In our case, ghost code is added at each program point to model its leakage. In principle, one could execute the program, together with its ghost annotations, to compute leakage for chosen sets of inputs. However, we only use ghost code for verification purposes.

Finally, MaskIR also provides a convenient framework to compare and establish relations between different leakage models, in the spirit of [4], which proves that security in the threshold probing model entails security in the bounded moment model. We use this ability of MaskIR to prove equivalence between the notion of security from [9], in which the leakage functions are adversarially controlled, and a simpler notion of security. In a similar way, MaskIR can be used to justify optimizations of the verification algorithm. In particular, we introduce a notion of covering set of observation sets, and prove that the absence of leakage for all observation sets in the covering set suffices to ensure security.
3.2 Programming language and leakage model

The syntax of programs is shown in Figure 1. Programs are modelled as sequences of deterministic and probabilistic assignments. Leakage is modelled explicitly by tagging instructions with an expression ℓ that defines its leakage. More formally, deterministic assignments are of the form $x \leftarrow e \mid ℓ$, where $x \in X$ is a deterministic variable, $e \in E$ is a program expression, and $ℓ \in L$ is a leakage expression. Probabilistic assignments are of the form $r \leftarrow \mu \mid ℓ$, where $r \in R$ is a probabilistic variable, $\mu$ is a distribution expression, and $ℓ \in L$ is a leakage expression. The sets $E$, $L$, and $R$ are defined inductively from variables and operators. In general, leakage expressions may use a richer set of operators than program expressions. In particular, leakage expressions often represent tuples of values, although tuples may not be supported in the core language. Moreover, operators used in leakage expressions may be probabilistic, for instance to return noise values.

The operational semantics of MaskIR programs produces for each program point a value, and a leakage, both of which may be probabilistic. It is then direct to define the leakage of an observation set, which we model as a set of program points. As before, we write $L_O(x)$ for the leakage gained by an adversary observing at program points in $O$ of his choice the execution of the implementation under consideration on input $x$. We note that without loss of generality, we can assume that programs are written in single static assignment (SSA) form, i.e. every variable $x$ or $r$ appears at most once on the left-hand side of an assignment. However, putting a program in SSA form induces a loss of information that is relevant for some models, e.g. when transitions are involved. In this case, we assume given a predecessor function on variables.

Expressions

\[
e ::= x \mid r \mid f(e_1, \ldots, e_n) \\
ℓ ::= x \mid r \mid g(ℓ_1, \ldots, ℓ_n) \mid ⟨ℓ_1, \ldots, ℓ_n⟩
\]

where $x$ ranges over variables and $r$ ranges over deterministic variables, $f$ ranges over deterministic operators from a set $F$, $g$ ranges over deterministic and probabilistic operators from a set $G$ (in general $F \subseteq G$) and $(\ldots)$ is syntax for tuples.

Statements

\[
s ::= x \leftarrow e \mid ℓ \text{ deterministic assignment} \\
r \leftarrow \mu \mid ℓ \text{ probabilistic assignment} \\
s; s \quad \text{sequential composition} \\
\text{return } e \quad \text{return expression}
\]

Example 1. We briefly indicate how different leakage models can be encoded in MaskIR. The case of glitches is discussed in the next paragraph.

- threshold probing model: we set $ℓ = x$ for a deterministic assignment $x \leftarrow e \mid ℓ$, and $ℓ = r$ for a probabilistic assignment $r \leftarrow \mu \mid ℓ$;
- threshold probing model with transitions: in this model, it is assumed that the consecutive storage of two variables in the same register may leak both values during the second assignment. In that case, the leakage $ℓ$ of the second assignment is thus defined as a pair of values corresponding to the successively stored variables. More concretely, we assume that programs are written in SSA form and we assume given a predecessor function on variables. We then set $ℓ = (x, y)$ for a deterministic assignment $x \leftarrow e \mid ℓ$, where $y$ is the predecessor of $x$;
- noisy leakage model: for each deterministic assignment (resp. probabilistic assignment), $ℓ$ is expressed as the sum of $x$ (resp. $r$) and a (generally Gaussian) noise;
- bounded moment model: this model features parallel assignments of the form $(x_1, \ldots, x_n) \leftarrow (e_1, \ldots, e_n)$. Without loss of generality we can assume that programs are written in SSA form so $x_1, \ldots, x_n$ do not occur in $e_1, \ldots, e_n$. One can then translate the parallel assignment above into a sequence of assignments

\[
x_1 \leftarrow e_1 \mid ℓ; \ldots; x_{n-1} \leftarrow e_{n-1} \mid ℓ; x_n \leftarrow e_n \mid ℓ
\]
where $\ell$ is a vector that contains all the mixed moments $\ell^{(o_1, \ldots, o_n)}$ at orders $t_1, t_2, \ldots, t_n$ such that $\sum_{1 \leq i \leq n} m_{o_i} \leq t_o$, and

$$\ell^{(o_1, \ldots, o_n)} = E(x_1^{o_1} \times x_2^{o_2} \times \ldots \times x_n^{o_n}).$$

### 3.3 Modeling glitches

In order to handle verification of hardware masked implementations, we rely on the threshold probing model with glitches introduced in two different ways by Faust et al. [20] and by Bloem at al. in [9]. Basically, we consider a hardware implementation as an oriented graph of vertices (i.e., operation gates) and edges (i.e., variables) organized with so-called combinatorial logic sets separated by registers. Combinatorial logic sets are sub-graphs that gather operations (vertices) on variables which aim to compute a same output which is to be stored in a register. In hardware, storing a variable in a register creates a synchronization point which stops the propagation of glitches. In the threshold probing model with glitches we rely on, the adversary is thus allowed to make at most $t$ observations on wires, each one resulting in the whole set of inputs that are manipulated so far within the corresponding combinatorial logic set.

Hardware leakage with glitches can easily be described in MaskIR programming language. Basically, for each deterministic assignment $\ell$ is set to a tuple formed by the current $x$ and all the variables involved in the computation of $e$ which belongs to the same combinatorial logic set. For probabilistic assignments, $\ell$ is simply set to $r$.

We illustrate the threshold probing model with glitches on a concrete example with the hardware first-order implementation of the DOM AND provided by Groß, Mangard, and Korak [22] and displayed as a graph in Figure 2. It takes as inputs secrets $a$ and $b$ that are respectively additively split into shares $a.0, a.1$, and $b.0, b.1$. Gates designed by FF represent registers, and variable $r$ is a uniform random variable. In the threshold probing model (i.e., without glitches), leakage for each wire is illustrated on Figure 2. Note that leakage at the output of the registers (FF) is the same as on its input. In the threshold probing model with glitches, leakage for each wire is displayed on Figure 3.

The threshold probing security with glitches can be extended to define (S)NI security with glitches as well. Basically, as in the threshold probing security model with glitches, each observation is replaced by the set of intermediate variables that are previously manipulated in the same combinatorial logic set since the previous synchronization point (register). The two (informal) security definitions directly follow.

**Definition 1 (NI with glitches).** An implementation is $t$-NI with glitches iff any set of at most $t$ observations can be perfectly simulated with at most $t$ shares of each input when each
observation is replaced by the set of intermediate variables that are previously manipulated in the same combinatorial logic set.

For SNI security notion, when outputs are not stored in register, then observations on the output are also replaced by the set of intermediate variables that are involved in the current computation since the last register.

Definition 2 (SNI with glitches). An implementation is $t$-SNI with glitches iff any set of at most $t_1$ observations whose $t_1$ on internal variables and $t_2$ on output variables can be perfectly simulated with at most $t_1$ shares of each input when each observation is replaced by the set of intermediate variables that are previously manipulated in the same combinatorial logic set.

3.4 Comparing models

Thus far, we have shown how MaskIR provides a unifying framework for modelling leakage. We now prove that it can also be used for comparing models. To achieve this goal, it is first necessary to define general notions of security that subsume the notions to be compared. This can be done at little expense. First, see that without loss of generality, we can assume that each leakage expression corresponds to one observation from the adversary, since expressions can be split into smaller ones to achieve this effect. Then, assume as before that inputs are split into shares; however, we do not require that the number of shares is related to the order $t$ against which the security analysis will be performed. Moreover, we assume given a norm function that maps every observation set $O$ to a natural number $\|O\|$, called its norm. Then we say that an implementation verifies general non-interference at order $t$ (and w.r.t. $\|\cdot\|$) iff for every observation set $O$ such that $|O|\leq t$, there exists two sets $I$ and $I'$, such that $|I|, |I'| \leq \|O\|$ and for every pair of inputs $(x, x')$ and $(y, y')$,

$$x \simeq_I y \land x' \simeq_{I'} y' \implies L_O(x, x') = L_O(y, y').$$

Note that this definition can still be generalized in multiple dimensions. However, it suffices for recovering prior definitions and for our purposes.

With this unified definition, MaskIR provides a convenient formalism to compare models. In this paragraph, we provide a set of rules for increasing leakage in an implementation. By repeatedly applying these rules, one can reduce security in one model to security in another model.

Definition 3. The leakage amplification relation $P \Rightarrow P'$ is the smallest reflexive transitive relation closed under the following rules:

- simplification: this is a local rule which allows to replace a leakage expression by its components:

  $$x \leftarrow e \mid f(\ell_1, \ldots, \ell_n) \Rightarrow x \leftarrow e \mid (\ell_1, \ldots, \ell_n)$$
extension: this is a local rule which allows to add a leakage expression:

\[ x \leftarrow e | \langle \ell_1, \ldots, \ell_n \rangle \Rightarrow x \leftarrow e | \langle \ell_1, \ldots, \ell_{n+n'} \rangle \]

permutation: this is an administrative local rule which is used to capture the fact that the order of leakage expressions is irrelevant; below \( \sigma \) is a permutation over \( \{1, \ldots, n\} \):

\[ x \leftarrow e | \langle \ell_1, \ldots, \ell_n \rangle \Rightarrow x \leftarrow e | \langle \ell_{\sigma(1)}, \ldots, \ell_{\sigma(n)} \rangle \]

cancellation: this is a global rule which allows to eliminate a leakage expression that is contained in a leakage expression of another instruction:

\[ x \leftarrow e | \langle \ell_1, \ldots, \ell_n \rangle \Rightarrow x \leftarrow e | e \]

provided there exists another instruction of the form \( x' \leftarrow e' | \langle \ell_1, \ldots, \ell_{n+n'} \rangle \) in the program.

Note that similar rules exist for random assignments.

The correctness of leakage amplification is captured by the following statement, which is proved by induction on the derivation of \( P \Rightarrow P' \).

**Proposition 1.** If \( P \Rightarrow P' \) and \( P' \) is non-interfering w.r.t. \( \| \cdot \| \) then \( P \) is non-interfering w.r.t. \( \| \cdot \| \).

Proposition 1 can be used to relate models.

We note that leakage amplification also opens interesting possibility for hybrid verification between models. This is typically interesting in situations involving two models: a stronger, but easier to verify, model, and a weaker, but more precise model. Embedding theorems can only be used for whole programs. In contrast, leakage amplification offers the possibility to reason in the stronger model for the part of the program where the embedding preserves security, and remains in the weaker model for the part of the program where the embedding fails. Practical applications of such hybrid techniques are an interesting direction for future work.

### 4 Algorithmic verification

In this section, we present new verification and attack validation algorithms that are used in maskVerif.

#### 4.1 Verification

**maskVerif** combines two main algorithms: a verification algorithm determines whether a tuple of expressions jointly depends on secrets, and an exploration algorithm which (adaptively) goes through all the possible sets of intermediate variables to analyze. Verification succeeds if the verification algorithm proves absence of leakage on all inputs given by the exploration algorithm, until there remains no further set to explore.

**Verification of single observation set.** The verification algorithm determines whether an arbitrary set of observations \( V = (v_1, \ldots, v_n) \) jointly depends on a secret \( k \) or not. We first compute from the program a tuple of expressions, which we also denote \( (v_1, \ldots, v_n) \) by abuse of notation. The original algorithm is given in Figure 4. Let us take a small example to illustrate the behaviour of this algorithm. Let us consider the following set of expressions \( V = (x_1, x_0 + r_1 + x_2, r_2) \) where \( r_1, r_2, x_0, \) and \( x_1 \) are random variables, and \( x_2 = s + x_0 + x_1 \) are the three shares that compose a secret \( s \). The verification operates as follows:

1. **Step 1:** \( s \) is involved in the computation of \( x_2 \) which is itself involved in the computation of \( v_2 \), thus we go to **Step 2**.
2. **Step 2:** in the second expression \( v_2 = x_0 + r_1 + x_2 \), there exists a random variable \( r_1 \) such that the entire expression forming \( v_2 \) is bijective in \( r_1 \). Thus, we replace \( v_2 \) by \( r_1 \) and set \( V \) is now equal to \( (x_1, r_1, r_2) \). We then go back to **Step 1**.


Inputs: \( \mathcal{V} = (v_1, \ldots, v_n) \), flag \( b = 0 \)

**Step 1**: if \( k \) is involved in the computation of at least one expression in \( \mathcal{V} \), then go to **Step 2**. Otherwise return `True`.
**Step 2**: while there exists a random variable \( r \) involved exactly once in the computation of a unique expression \( v_i \) of \( \mathcal{V} \), then the biggest expression \( e \) in \( v_i \) which is bijective in \( r \) is replaced by \( r \). If at least such a transformation occurred, go to **Step 1**. Otherwise go to **Step 3**.
**Step 3**: if \( b \neq 0 \), then return `False`. Otherwise, mathematically simplify the expressions in \( \mathcal{V} \). Then, set \( b \) to one and go back to **Step 1**.

**Fig. 4.** Original verification algorithm

Inputs: \( \mathcal{V} = (v_1, \ldots, v_n) \), flag \( b = 0 \), set \( \mathcal{R} = \emptyset \)

**Step 1**: if \( k \) is involved in the computation of at least one expression in \( \mathcal{V} \), then go to **Step 2**. Otherwise return `True`.
**Step 2**: while there exists a random variable \( r \notin \mathcal{R} \) involved in the computation of an expression \( v_i \) of \( \mathcal{V} \), then find a sub-expression \( e \) in \( v_i \) such that \( r \mapsto e + r \) is bijective and substitute \( r \) by \( e + r \) in all expressions. Extend \( \mathcal{R} \) with \( \{r\} \). If at least such a transformation occurred, go to **Step 1**. Otherwise go to **Step 3**.
**Step 3**: if \( b \neq 0 \), then return `False`. Otherwise, mathematically simplify the expressions in \( \mathcal{V} \). Then, set \( b \) to one and go back to **Step 1**.

**Fig. 5.** New verification algorithm

3. **Step 1**: there is no more expression in \( \mathcal{V} \) which depends on the secret \( s \), thus we return `True`.

This algorithm is correct, in the sense that it will only return `True` if the tuple is independent of secrets, but may return false negatives. The problem arises in **Step 2**, where we require that there exists a random variable \( r \) involved exactly once in the computation of a unique variable \( v_i \) of \( \mathcal{V} \). This requirement fails for examples such as \((r_1 + r_2, r_2 + r_3, r_1 + r_3)\) in \( \mathbb{F}_2 \). This is innocuous in our context, since **Step 1** would take care of this case. However, one would intuitively expect the above tuple to simplify to \((r_1, r_2, r_1 + r_2)\), by applying the bijections \( r_1 \mapsto r_1 + r_2 \) and \( r_2 \mapsto r_2 + r_3 \). This informally corresponds to the well-known method of Gaussian elimination. Indeed, it turns out that a variant of Gaussian elimination (not requiring expressions to be linear) is the key to deal with threshold security and glitches. Informally, the method proceeds by selecting a sub-expression of the form \( r + e \) and performing the substitution \( r \mapsto r + e \). The critical step is then to select which substitutions to perform and to guarantee that the method terminates. We do so by defining the multiplicative depth of a random variable and by performing rewrites in increasing order of multiplicative depth. For instance, in the expression \( r + (r' + e') \times e' \), we assign multiplicative depth 0 to \( r \) and 1 to \( r' \). Moreover, we require that each variable \( r \) is substituted at most once. The new algorithm is summarized in Figure 5.

Our novel algorithm subsumes the original algorithm in the general case. For completeness, we also give in Figure 6 examples of tuples generated from verification of AES S-box that cannot be handled with the algorithm from [2]. Interestingly, the second example consists of a single expression and involves multiplications, whereas the first example consists of a tuple of linear expressions.

**Formal properties.** Our new algorithm always terminates, and provably does not miss attacks (soundness). Moreover, it provably does not return false negatives when all expressions considered are linear (completeness).

**Exploration.** The exploration algorithm ensures that the main algorithm will analyze all the possible sets of at most \( t \) intermediate variables in the implementation to guarantee (at least) \( t \)-probing security. While this step is still exponential in the number of intermediate variables, `maskVerif` provides an efficient way to go through these sets. Instead of exploring all the sets containing exactly \( t \) variables, i.e., \( \binom{m}{t} \) sets for \( m \) variables, the idea in `maskVerif` is to recursively verify large sets. The verification of larger sets turns out to be practically better than the...
verification of all the intermediate sets of size $t$. Concretely, the total number of sets that are actually verified is generally drastically lower than $\binom{m}{t}$, with the cost of verifying a large set being sensibly equal to the cost of verifying a set of $t$-observations, and the verification is consequently much faster.

In the case of hardware implementations, there is also a natural order on the observation sets. Consider the example from Figure 3. In this setting, observations are sets of expressions, and thus an observation may be included in another. Looking at the bottom of the picture, we have

$$\{a.0\} \subseteq \{a.0, b.1\} \subseteq \{a.0, b.1, r\}$$

Without loss of generality, we can always assume that an adversary picks observations that yield the maximal set of expressions with respect to $\subseteq$. This yields a significant decrease in the number of sets to consider. Concretely, only four observations are considered for the example from Figure 3:

$$(b, 1, a, 0, r)$$
$$(b, 0, a, 1, r)$$
$$(b, 1, a, 0) + r, b, 1, a, 1)$$
$$(b, 1, a, 0) + r, b, 0, a, 0)$$

### 4.2 Validating attacks

Originally, maskVerif either provided a formal proof of security or a set of potentially flawed tuples that users were requested to inspect manually to determine whether it corresponded to a real attack. In particular, maskVerif did not provide a method to validate attacks, allowing for the possibility of false negatives.

Our new implementation brings two improvements to the implementation. On the one hand, the new verification algorithm reduces significantly the number of false negatives. In particular, the new algorithm is able to fully analyze the AES S-Box at order 3 in the software model, whereas the old algorithm fails for several thousand tuples (admittedly, out of more than $10^9$ tuples).

In addition, maskVerif now provides a simple but effective mechanism for validating attacks in the threshold probing model. Concretely, upon detection of a potentially flawed tuple, the new implementation computes the joint distribution of every potentially flawed tuple that it detects, so as to verify exactly whether this tuple is an attack in the threshold probing model or not. This step is exact, therefore all false negatives are removed. This method can also be used for verifying SNI, in case the violating tuple only consists of output observations. Indeed, in this case,
the definition of SNI forces that the tuple does not depend on any input share, and so the same method applies. More complete attack validation methods for SNI and attack validation methods for NI are left for future work.

5 Tool overview

This section describes the process of verifying masked implementations using maskVerif. Figure 7 describes pictorially the workflow: the user selects a verification order t, a security property (threshold probing security, NI or SNI), and a model (hardware or software). Additionally, the user must provide the implementation to be verified in an input language called MaskPC, that is common to software and hardware implementations. maskVerif then analyzes the program and either returns a set of flawed tuples (that are formally validated when possible) or a proof of security.

Note that out of the six modes supported by maskVerif, four are new. Indeed, the original paper of Barthe et al. [2] only verifies threshold probing security for software implementations, and Barthe et al. [3, 4] further verify NI and SNI for software implementations. Moreover as recalled in Table 1, no other tool supports NI and SNI verification with glitches.

![Fig. 7. Overview of software and hardware formal verifications with maskVerif.](image)

5.1 Input Programming Language MaskPC

maskVerif takes as input implementations written in an input language MaskPC. MaskPC is similar to MaskIR, except that it features different forms of assignments that are used to describe succinctly the leakage for each instruction. Concretely, a MaskPC program starts with a header that specifies public variables, secret variables, output variables, and random variables. In particular, these annotations allow to specify which input wires correspond to the sharing of a secret input, as well as for output, and also which input wires are random values. Then, the body of a MaskPC program is a sequence of deterministic and random assignments. However, leakage is not declared explicitly, rather modelled through three kinds of assignment:

\[ x := e \quad (1) \quad x = !e \quad (2) \quad x = \{e\} \quad (3) \]

In the software scenario, all three assignments are treated equivalently and allow the adversary to learn x at the cost of one observation. In the hardware scenario, they have additional properties. Namely, (1) is for simple assignment that generates glitches and propagates then, (2) is for storage in a register that stops glitches propagation. The last, (3), is useful for the encoding probing model, it does not allow the adversary to learn any subexpression of e. This is exactly what is needed for the initial sharing of secret which is assumed to be perfect in the threshold probing model: at
order 2 the sharing of a secret $s \in K$ is given by the three shares $s_0 = r_0$, $s_1 = r_1$, $s_2 = s + r_0 + r_1$, where $r_0$ and $r_1$ are generated uniformly at random from $K$. The computation of the share $s_2$ is assumed to be perfect, i.e. the adversary can not observe the intermediate result $s + r_0$ or she could recover $s$ using another observation on $r_0$. In MaskPC we simply use the notation

$$s_2 = \{s + r_0 + r_1\}.$$

5.2 Verification of Software Implementations

Software implementations are expected to be written in MaskPC programming language. An example of MaskPC program (top) and of its pending MaskIR program (bottom) is given in Figure 8 for a first-order implementation of a multiplication.

Verification proceeds as follows: first, maskVerif generates the corresponding MaskIR program (this is straightforward for software implementations) and performs verification on all observation sets.

Fig. 8. Example of a first-order DOM AND implementation as programmed in MaskPC (top) then MaskIR (bottom) in the software scenario.

5.3 Verification of Hardware Implementations

Hardware implementations are expected to be written in Verilog. In order to handle the verification of hardware implementations (i.e., with glitches) with maskVerif, we first follow the same
steps than in [9]. Namely, we use Yosys synthesis tool [38] to generate .ilang 5 files from Verilog implementations. Then, we manually add the MaskPC header and translate the .ilang file to MaskPC syntax.

Figure 9 presents the MaskPC representation of DOM AND extracted from the Verilog implementation in [9]. The header declares public variables, secret input variables, output variables, random variables, and local variables are first displayed. Then, each line describes a single instruction between at most two variables with two possible assignments. Symbol := refers to a definition, while symbol !=[X] refers to a definition followed by a storage in a register. Appendix B recalls the first steps of this parsing for the Verilog implementation DOM AND as provided in [9]. The resulting file in MaskPC is displayed below. Note that the name of the intermediate variables was changed here to make the reading easier. In particular, variables a (originally \texttt{\XxDI}), and b (originally \texttt{\YxDI}) respectively split into a.\texttt{[0]} and a.\texttt{[1]}, and b.\texttt{[0]} and b.\texttt{[1]} are the secret inputs, and r (\texttt{\ZxDI}) is a uniformly distributed random variable.

\begin{verbatim}
proc \dom_and:
  publics: p1, p2
  inputs : (a, [a.\texttt{[1]}, a.\texttt{[0]}]),
           (b, [b.\texttt{[1]}, b.\texttt{[0]}])
  outputs: [c.\texttt{[1]}, c.\texttt{[0]}]
  randoms: r
  others : tmp9, tmp11, g4, g3, g2, g1, tmp2, tmp4, tmp1, tmp3, tmp8,
           clk3, tmp10, tmp13, tmp6, clk2, tmp12, tmp15, tmp7, clk1,
           tmp5, clk0, t3, tmp14, tmp16, t0, t;

  clk3 := p2                      tmp6 := tmp2 + r
  clk2 := p2                      tmp7 := tmp3 + r
  clk1 := p2                      tmp8 := tmp4
  clk0 := p2                      tmp9 := tmp6
  g4 := p1                        tmp10 := ![tmp6]
  g3 := p1                        tmp11 := tmp7
  g2 := p1                        tmp12 := ![tmp7]
  g1 := p1                        tmp13 := tmp10
  t := ![p1]
  tmp1 := b.\texttt{[0]} * a.\texttt{[0]}
  c.\texttt{[1]} := tmp10 + tmp4
  tmp2 := b.\texttt{[1]} * a.\texttt{[0]}
  tmp15 := tmp12
  tmp3 := b.\texttt{[0]} * a.\texttt{[1]}
  tmp16 := tmp12
  tmp4 := b.\texttt{[1]} * a.\texttt{[1]}
  c.\texttt{[0]} := tmp1 + tmp12
  tmp5 := tmp1

\end{verbatim}

\textbf{Fig. 9.} Input file for DOM AND

Verification proceeds as follows: first, maskVerif generates the corresponding MaskIR program. Basically, the corresponding leakage \(\ell\) for an instruction is the tuple of all intermediate variables that are involved in the current computation from their last storage in a register which is symbolized with assignments of the form =![X]. Second, maskVerif computes the maximal observations with respect to set-theoretical inclusion. Finally, maskVerif is called on all possible maximal observation sets.

6 Experiments

This section reports on experimental evaluation of the new implementation of maskVerif, and discusses some general lessons learned.

\footnote{[9] generates .json files, but we found that .ilang is more human readable and more easy to annotate.}
Examples. Our examples are mainly extracted from the available database provided by the authors of [9]. It gathers four different Verilog implementations of a masked multiplication. Three of them are implemented at the first masking order only, while the last one, referred to as DOM AND and designed in [22], is available up to order \( t = 4 \). For the latter, we also consider modified versions that achieve non-interference and strong non-interference. Larger implementations are also provided, namely three S-boxes. AES S-box as designed in [22] and both versions of FIDES S-box as designed in [7] are implemented at the first order. We also consider a second-order and third-order AES S-box [21]. Moreover, we consider Keccak S-box as designed in [23] is implemented from the first to the third order. To this existing set of examples, we added a few additional ones. First, Keccak S-box is also analyzed at two extra orders, namely \( t = 4 \) and \( t = 5 \). Then, two versions of a different multiplication PARA AND provided in [4] are considered from the first to the fourth order.

Benchmarks. Table 2 summarizes the verification outcome of the examples\(^6\). We use a 2.8 GHz Intel Core i7 with 16 Go of RAM running on macOS High Sierra, while Bloem et al. [9] use a Intel Xeon E5-2699v4 CPU with a clock frequency of 3.6 GHz and 512 GB of RAM running in a 64-bit Linux OS environment.

The table reports on verification for the six modes of maskVerif. Concretely, verifications are performed for the three main security properties, namely SNI, NI, and threshold probing security, and for two scenarios: a hardware scenario (HW), i.e. in presence of glitches, and in a software scenario (SW), i.e. without glitch.

The first column of the table (# obs) indicates the number of possible observations in the targeted implementation. In the software scenario, this number corresponds to the number of intermediate variables. In the hardware scenario with glitches, the number of observations corresponds to optimal observations. Note that it is much lower than in the software scenario since non-optimal observations are ignored. Also note that while this first column displays the number of observations \( n \) that will be further treated, verification at order \( t \) requires the analysis of \( \binom{n}{t} \) tuples. For instance, the verification of Keccak S-box in the software scenario at the fourth-order requires the analysis of \( \binom{450}{4} \approx 2^{31} \) tuples.

The second, third, and fourth column report on the verification times in the 6 modes. We report 0.01s when the result is instantaneous and \( \infty \) when the computations takes more than 10 hours. Note that when an implementation is insecure in a weaker model, then its verification time is equal for the stronger model.

To report the outcome, a cross is displayed when a concrete attack is exhibited. Otherwise, the verification ends up successfully, indicating that the implementation is secure. maskVerif is able to validate most flawed tuples fully automatically. As expected, the validation time is often high compared to the discovery of the flawed tuple. For instance, it takes maskVerif 0.01s to discover a flawed tuple in DOM Keccak S-Box at order 2 for SNI, and 16s to verify the tuple. Also note that for many examples where automated validation fails, manual inspection trivially shows that the flawed tuple is indeed an attack. Typically, such tuples contain expressions that are built only from secrets (no randomness). Implementing further heuristics that improve attack validation is left for future work.

The last column indicates the timings from [9], and thus are specialized to threshold probing security with and without glitches. (Note that their timings are obtained with a more powerful machine). A dash is displayed when the example is not tested. The results shows that maskVerif performs significantly better than the algorithm provided in [9]. For instance, the verification of the hardware first-order masked implementation of AES S-box is at the very least 7826 times much faster with our new version of maskVerif. In particular, note that some of the benchmarks provided for the tool of Bloem et al. only concern the verification of one secret (the ranking correspond to the fastest and the lowest verification of the secrets). They are highlighted with a symbol *. As a consequence, without parallelization (which we do not use in this work), these timings should probably be significantly higher.

Lessons learned. Our first main finding is that, quite expectedly, most of the implementations presented here do not satisfy the SNI notion with glitches. In fact for those that are SNI in the

\(^6\) All the programs and logs are available at https://sites.google.com/view/maskverif/home
software scenario and NI in the hardware scenario it is generally sufficient to store the result into a register before returning it (i.e. remove glitches from the output) to achieve SNI security in the hardware scenario. This is the solution implemented for PARA AND SNI and AND DOM SNI. This observation is already discussed in [19], but is not validated formally with verification tools. In general, it would be interesting given an implementation that is secure in the software scenario to infer an optimal implementation that is secure in the hardware scenario.

Our second main finding is that verification in the threshold probing model with glitches is often faster than verification in the threshold probing model without glitches, for the same implementation. As stated in the introduction, this seemingly counterintuitive phenomenon stems from the verification method used by maskVerif. First, the adversary can get access to more intermediate variables via glitches, which entails that there are less observation sets to verify. Second, one can assume without loss of generality that the adversary only selects maximal observations with respect to set-theoretical inclusion, which further reduces the number of observation sets to consider. The key insight is then that, even if the sets to analyze are much bigger, their verification is not much longer. For instance the fourth-order implementation of Keccak S-box requires 1 minute and 51 seconds for a verification of the non-interference property with glitches and more than 7 minutes when the same property is verified without glitches. This finding opens the possibility for an interesting methodological approach for speeding up the verification of software implementations: rather than analyzing the real implementation, one can annotate the implementation with hardware assignments that give the adversary more power, and analyze the resulting implementation. Automatically picking the assignments that achieve optimal speedup is left for future work.
Table 2. Overview of verification of masked hardware circuits

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7 Conclusion

We have presented a general framework for analyzing the security of masked implementations, and adopted the maskVerif tool to support verification of different models. We believe that our framework and tool can be applied without any difficulty to the transition-based threshold probing model and secure multi-party computation based on additive secret sharing. We also contend that it should be straightforward to extend our work beyond purely qualitative security definitions and to consider quantitative definitions that upper bound how much leakage reveals about secrets—using total variation (a.k.a. statistical) distance [18]. An important direction for future work is to leverage automated verification to full AES implementations. Our natural option would be to extend the maskComp tool [3] to deal with glitches. More speculatively, it would be interesting to extend our framework and verification models to active adversaries, who have the additional ability to tamper computations [24]. We hope that the tools presented here will be useful to the community and help to advance the state of the art in masked computation.
step would be to extend the correspondence between information flow and simulation-based security to the case of active adversaries. An appealing possibility would be to exploit the well-known dual view of information flow security for confidentiality and integrity.

References


A Leakage Models

Noisy leakage model. Another well-known model to reason on the security of masked implementations is the noisy leakage model, introduced in 1999 by Chari et al. [10] and later extended by Prouff and Rivain [32]. The noisy leakage model assumes that the adversary has access to noisy functions of all the intermediate variables of the implementation. This provides a realistic model for side-channel attacks on embedded devices.

In 2014, Duc, Dziembowski, and Faust [15] establish a reduction between the threshold probing model and the noisy leakage model. In a nutshell, they prove that a $t$-probing secure implementation is also secure in the noisy leakage model for a certain level of noise. Beyond its foundational interest, their result is practically very important, since it makes it possible to prove security directly in the $t$-probing model, and to derive practical guarantees. Further work by Dziembowski, Faust and Skórski [16] improves the equivalence by deriving tighter bounds.

Proving security in the noisy leakage model and its variants involves complex calculations. For this reason, the noisy leakage model has not been used directly in formal verification. Of course, it is possible to rely on the equivalence with the threshold probing model, and to carry the verification in the latter.

Bounded moment model. The bounded moment model was recently introduced by Barthe, Dupressoir, Faust, Grégoire, Standaert, and Strub [4]. The bounded moment model reasons about parallel implementations, and thus provides an intermediate ground between the threshold probing model and hardware implementations. Formally, the bounded moment model reasons about programs with parallel assignments. The definition of leakage for such parallel assignments is based on their mixed moments. Barthe et al [4] prove that in order for a parallel implementation is secure at order $t$ in the bounded moment model, it suffices that a serialization of the implementation is secure at order $t$ in the threshold probing model.

Proving security in the bounded moment model requires reasoning about expectations. For this reason, the bounded moment model has also not been used directly in formal verification. As for the noisy leakage model, it is always possible to rely on the equivalence with the threshold probing model, and to carry the verification in the latter.

Threshold probing model with transitions. So far, the described models generally assume that intermediate variables of the targeted implementation may leak at the time they are manipulated in the cryptographic algorithms. Nevertheless, it has been observed in the literature [1, 12] that software devices could also leak on pairs of variables when they are consecutively stored in the same register. By opposition to the leakage of single intermediate variables as considered in the original threshold probing model, this extended notion is generally referred to as the transition-based leakage. When such a leakage is considered, the adversary is allowed to perform $t$ observations such that each observation may capture two variables, namely the targeted variable and the variable that was previously stored in the same register. In that case, the adversary can get up to $2t$ variables to perform its attack, which make the registers allocation critical to avoid higher-order attacks. In summary, the threshold probing model with transitions is a minor variant of the threshold probing model, and as such lies within the scope of formal verification.

B Parsing Verilog implementations

Our method directly starts with a Verilog masked implementation. All along this subsection, we illustrate the different steps which lead to a formal verification with the example of the DOM AND gadget as used in [9], named here dom_and.v and graphically represented on Figure 3.
Once generated, the .ilang file is manually annotated with keywords in order to specify the public variables, the secret input variables, the secret output variables, and the random variables at the beginning of the procedure. For our example the added notations are:

```plaintext
## public \Clk\ Rst
## input \XxDI
## input \YxDI
## output \QxDO
## random \ZxDI
```

\XxDI is in the implementation a vector of wires (of size 2) containing the two shares of the first secret input, \XxDI correspond to second input, \QxDO contains the output shares and \ZxDI is a random input share. The ## annotations correspond to ilang comment, so they can be ignored by ilang tools. In some cases, the input of the circuit is not so naturally split into share. For example, it is possible to define the same gadget taking only one vector of secret input of size 4, say \( Z \) with the following semantic

\[
Z = \{ \XxDI[0], \YxDI[0], \XxDI[1], \YxDI[1] \}
\]

this can be captured by using the following annotations for secret input (or output)

```plaintext
## input : a \Z[0 2]
## input : b \Z[1 3]
```