Highly-Efficient and Reusable Private Function Evaluation with Linear Complexity

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Abstract. Private function evaluation aims to securely compute a function $f(x_1, \ldots, x_n)$ without leaking any information other than what is revealed by the output, where f is a private input of one of the parties (say Party_1) and x_i is a private input of the i-th party Party_i. In this work, we propose a novel and secure *two-party private function evaluation* (2PFE) scheme based on the DDH assumption. Our scheme introduces a reusability feature that significantly improves the state-of-the-art. Accordingly, our scheme has two variants, one is utilized in the initial execution of the function f, and the other is utilized in its subsequent evaluations. To the best of our knowledge, this is the first and most efficient 2PFE scheme that enjoys a reusablity feature. Our protocols achieve linear communication and computation complexities and a constant number of rounds which is at most three.

Keywords: Private function evaluation, Secure 2-party computation, Communication complexity, Cryptographic protocol.

1 Introduction

Development of computing devices with the proliferation of the Internet has prompted enormous opportunities for cooperative computation. These computations could occur between trusted or partially trusted partners, or even between competitors. Secure multiparty computation (MPC) protocols allow two or more parties to collaborate and compute a public functionality using their private inputs without the need for a trusted third-party. However, the generic solutions for MPC are not adequate for some particular cases where the function itself is also sensitive and required to be kept private. Private function evaluation (PFE) is a special case of MPC, where the function to be computed is known by only one party. PFE is useful in several real-life applications where an algorithm or a function itself needs to remain secret for reasons such as protecting intellectual property or security classification level. Recently, designing efficient PFE protocols have been a challenging and attractive task for cryptography researchers [PSS09, KM11, MS13, Sad15, BBKL19, Bin19, KS16, GKS17].

Efficient and practical PFE schemes are becoming increasingly important as many real-world applications require protection of their valuable assets. For example, many software companies targeting the global market are extremely concerned about illegal reproduction of their software products. Software obfuscation methods usually prevent reverse engineering, but still allow direct copying of programs. Another solution could be providing the software-as-a-service in the cloud to eliminate the risk of exposure. However, this solution also causes another issue, i.e., threatening the privacy of customer data, since computations need to take place at the hands of software vendors.

Fully homomorphic encryption (FHE) [Gen09, HS18, BV11, BV14, BDGM19] provides a natural solution to PFE where a party encrypts his/her input using his/her public key $E_{pk}(x)$ the other evaluates some f homomorphically and returns the output $y = E_{pk}(f(x))$, thus the first party can obtain the result by decrypting y only. Ring-LWE based FHE constructions are the most favourable ones [LPR13a, LPR13b]. One important aspect of this approach is that its communication complexity is proportional to the input and output sizes, but does not depend on the size of f [BDGM19]. However, FHE schemes suffer a polynomial blow up factor (in the security parameter) to the ciphertext size, so that affecting the overall communication cost. Therefore, they are still far from being practical [LN14b, LN14a, OSPG18, GH19, NDR$^+$19].

Another approach targeting those problems falls into the category of PFE. Compared to FHE, PFE is currently much closer to practical use. Moreover, in many occasions such a PFE scheme is quite beneficial, including the ones where a service provider may opt keeping the functionality and/or its specific implementation confidential, and the ones where the disclosure of the function itself means revelation of sensitive information, or causes a security weakness.

The current research goal for secure computation protocols (including PFE) is efficient and practical solutions with low round, communication, and computation complexities. Among these three measures, as also pointed out by Beaver, Micali, and Rogaway, the number of rounds is the most valuable resource [BMR90]. The other important research goal in this area is the minimization of communication complexity. Since hardware trends show that computation power progresses more rapidly compared to communication channels, the main bottleneck for many applications will be the bandwidth usage.

In this paper, we consider the two-party PFE (2PFE) setting where the first party (say Party_1) has a function input f (compiled into a boolean circuit C_f) and optionally a private input bit string x_1, whereas the other party (say Party_2) has an input bit string x_2. The parties aim to evaluate f on x_1 and x_2 so that at least one of them would obtain the resulting $f(x_1, x_2)$ without any of them deducing any information about the other one's private input beyond what $f(x_1, x_2)$ itself reveals.

1.1 Related Work

First proposed by Andrew Yao [Yao82, Yao86], secure two-party computation (2PC) comprises the techniques for joint evaluation of a function by two parties on their respective secret inputs. In recent years, there has been a promising progress over the original Yao's protocol [BMR90, NPS99, KS08a, PSSW09, KMR14, ZRE15, KKS16]. As a consequence of these improvements, secure computation techniques now have promising results.

2PFE differs from the standard 2PC in that the latter involves both parties evaluating a publicly known function on their private inputs, whereas in the former, the function itself is also a private input. 2PFE concept is first appeared in [AFK87, AF90]. So far, there are basically two main approaches that PFE solutions are built upon.

The first one is based on a universal circuit which takes a boolean circuit C with circuit size less than g and an input $(x_1, \ldots, x_n)$, and outputs $C(x_1, \ldots, x_n)$. The idea is that if the regular secure computation techniques can be applied on a universal circuit, then a PFE scheme can be obtained. Consequently, the efforts targeting the efficiency of *universal circuit based PFEs* have generally been towards reducing the size of universal circuits, and the cost of their secure computation [KS08b, SS09, KS16, GKS17].

The second approach is avoiding the use of universal circuits and designing special purpose PFE protocols. Following this line of work, several PFE schemes have been proposed, e.g., as [PSS09, KM11, MS13, Sad15, BBKL19]. A remarkable work embracing this approach is *singly homomorphic encryption* based 2PFE scheme of Katz and Malka (KM11) applied on boolean circuits [KM11]. In KM11 [KM11], the authors utilize a singly homomorphic scheme (e.g., ElGamal [EG85] or Paillier [Pai99]) for the generation of the two random tokens[5] on each wire, later utilized in the 2PC stage. They first propose a basic version of their protocol in [KM11, Sect. 3.1] (which we call KM11-1st) and for the efficiency concerns they propose a more efficient variant in [KM11, Sect. 3.2] (which we call KM11-2nd). Both schemes have only three rounds, and provide $O(g)$ asymptotic complexity in terms of communication and computation, where g denotes the circuit size. The latter one reduces the communication and offline computation complexity.

In [MS13], Mohassel and Sadeghian proposed 2PFE schemes, for boolean circuits and arithmetic circuits. Considering boolean circuits, they propose two types of protocols: one is based on *oblivious evaluation of*

[5] Throughout this paper, the term "token" stands for a random bit string generated for a wire of the boolean circuit, and has hidden semantics of either 0 or 1.

switching networks (OSN)[6] (which we call MS13-OSN) and the other one is based on singly homomorphic encryption (which we call MS13-HE). Even though MS13-OSN is efficient for small sized circuits, it is still inefficient for large circuits due to its $O(g \log(g))$ communication and computation complexities. It fails to outperform asymptotically linear communication and computation complexities of [KM11]. On the other hand, MS13-HE provides linear communication and computation complexities and slightly outperforms KM11-2nd. We remark that to the best of our knowledge, a reusability feature cannot be adapted[7] to protocols proposed in [KM11] and [MS13].

In [MS13], the authors also propose a protocol for arithmetic circuits based on partial (singly) homomorphic encryption. This protocol has equal number of rounds to its gates (see [MS13, p. 570]), whereas the other PFE protocols for boolean circuits have constant number of rounds. For large circuits the number of rounds will be a bottleneck[8]. [MS13] also proposes a multi-party PFE variant based on OSN that remains the most efficient one to date. Their proposals are essentially secure in the semi-honest model, and has later been extended to the malicious model by [MSS14].

Recently BBKL18 [BBKL19] improves the OSN based 2PFE protocol of [MS13]. They show how to utilize the elegant *half gates* technique [ZRE15] to their 2PFE scheme. The optimization of BBKL18 improves MS13-OSN by reducing the overall communication cost about 40%.

1.2 Our Contributions

In this work, we propose a highly efficient 2PFE scheme for boolean circuits secure in the semi-honest model. Our scheme enjoys the cost reduction due to the reusability of tokens that will be used in the 2PC stage. This eliminates some of the computations and exchanged messages in the subsequent executions for the same function. Therefore, one of the strongest aspects of our proposed protocol is the remarkable cost reduction if the same function is evaluated more than once (possibly on varying inputs). We highlight that such a cost reduction is not applicable to the protocols of KM11 [KM11] and MS13 [MS13] since they require running the whole protocol from scratch for each execution. In this respect, we present two protocols of our scheme: (1) a protocol for initial executions (InExe), (2) a resumption protocol for subsequent executions (ReExe). The former protocol is utilized in the first evaluation of the function, while the latter one is utilized in the second or later subsequent evaluations of the same function. We note that the latter protocol is more efficient than the former one due to the fact that it benefits from the reusable tokens generated already in InExe protocol. The latter case is likely to be encountered more frequently in practice, compared to the cases where a function is evaluated only once.

Our proposed protocols significantly enhances the state-of-the-art in terms of communication cost. Compared to MS13-OSN [MS13], BBKL18 [BBKL19], and GKS17 [GKS17] protocols, we achieve an asymptotic cost reduction. Namely, while the asymptotic communication costs of those protocols are equal to $O(g \log(g))$, our scheme provides $O(g)$ communication complexity where g is the number of gates. To illustrate this asymptotic difference, for a thousand-gate circuit, our cost reduction is about 94% over MS13-OSN, about 88% over BBKL18, and about 68% over GKS17. For a billion-gate circuit, our cost reduction is about 98% over MS13-OSN, about 96% over BBKL18, and about 89% over GKS17. The protocols of MS13-HE, KM11-1st, KM11-2nd and ours have linear asymptotic complexity. Thanks to the reusability feature, the advantage of our scheme becomes more pronounced when the number of PFE execution is more than one. Namely, for two executions our cost reduction is about 54% over KM11-1st, 30% over KM11-2nd, and 20% over MS13-HE.

[6] The OSN mechanism is introduced in [MS13] to achieve a solution for the oblivious extended permutation (OEP) problem. OEP allows the oblivious transition of each masked gate output to the input(s) of the next connected gate(s). We refer the readers to the references in [MS13, Bin19, BBKL19] for detailed explanation on OSN.

[7] This is due to the fact that the blinding operations in these protocols are one-time pads (XOR or cyclic addition), therefore, reusing the blinded values inevitably leaks information about the truth values of intermediate wires. On the other hand, our mechanism relies on DDH so that the blinding values would remain unknown to the respective parties.

[8] We can intuitively say that as the latency between parties increases, so does the cost of each additional communication round (we refer to [SZ13] that backs up this discussion). A similar analysis on trade-offs between boolean and arithmetic circuit based protocols has also been addressed in [CKMZ14, p. 527].

For ten executions our cost reduction is about 63% over KM11-1st, 44% over KM11-2nd, and 37% over MS13-HE.

The number of rounds of our InExe protocol is 3 and the number of rounds of our ReExe protocol is equal to 1, or 2, or 3 depending on the input string length of Party_1 (i.e., owner of f)[9]. This also reflects the improvement of ReExe protocol over the existing 2PFE protocols in terms of round complexity (see Table 1).

We also deal with the case that Party_1 runs the 2PFE protocol for the same private function with various Party_2s separately. This is a common scenario where Party_1 may run a business with many customers for her algorithm/software. Trivially, our ReExe protocol can be utilized between the same two parties in the second and subsequent evaluations after the first evaluation. Instead of running the initial execution protocol with each Party_2, we propose a more efficient mechanism for the generation of the reusable tokens by employing a threshold based system.

1.3 Organization

In Section 2, we give a preliminary background that is used throughout the paper. Section 3 presents the descriptions of our InExe and ReExe protocols, and a method for the case where Party_1 would like to execute 2PFE with various Party_2s separately. Section 4 provides a performance analysis of our protocols, and compare them with the existing state-of-the-art 2PFE protocols. In Section 5, we give the security proofs of our protocols in the semi-honest model. Finally, Section 6 concludes our paper.

2 Preliminaries

This section provides some background information on the DDH assumption, describes some notations used in this paper and briefly summarize the concept of the 2PFE framework.

2.1 Decisional Diffie-Hellman Assumption

The Decisional Diffie-Hellman (DDH) assumption for $\mathbb{G}$ provides that the following two ensembles are computationally indistinguishable

$$\{(P_1, P_2, a \cdot P_1, a \cdot P_2) : P_i \in \mathbb{G}, a \in_R \mathbb{Z}_q^*\} \approx_c$$

$$\{(P_1, P_2, a_1 \cdot P_1, a_2 \cdot P_2) : P_i \in \mathbb{G}, a_1, a_2 \in_R \mathbb{Z}_q^*\}.$$

where $X \approx_c Y$ denotes that the sets X and Y are computationally indistinguishable, $\mathbb{G}$ is a group (represented in additive notation) of prime order q (of size 2λ where λ is a security parameter), and $P_1, P_2 \in_R \mathbb{G}$.

The security of our protocols is based on the following lemma of Naor and Reingold [NR04] providing a natural generalization of the DDH assumption for $m > 2$ generators.

Lemma 1 ([NR04]). *Under the DDH assumption on $\mathbb{G}$, for any positive integer m,*

$$\{(P_1, \ldots, P_m, a \cdot P_1, \ldots, a \cdot P_m) : P_i \in \mathbb{G}, a \in_R \mathbb{Z}_q^*\} \approx_c$$

$$\{(P_1, \ldots, P_m, a_1 \cdot P_1, \ldots, a_m \cdot P_m) : P_i \in \mathbb{G}, a_1, \ldots, a_m \in_R \mathbb{Z}_q^*\}.$$

There exist certain elliptic curve groups where the DDH assumption holds. We will not go through the details of these primitives and refer the reader to [Bon98, HMV03]. The main advantage of the elliptic curve DDH assumption over the discrete logarithm based DDH assumption is that the discrete logarithm DDH problem requires sub-exponential time [LV01] while the current best algorithms known for solving the elliptic curve DDH problem requires exponential time resulting the same security with smaller key sizes. Therefore,

[9] More concretely, if Party_1 has $x_1 = \perp$, then the number of rounds is equal to 1. If Party_1 has a non-empty input x_1 in such that the OT extension is not applicable for its garbled input, then the number of rounds is equal to 2. Otherwise, the number of rounds is equal to 3.

the elliptic curve based systems are more practical than the classical discrete logarithm systems since smaller parameters may be chosen to ensure the same level of security. In fact, attacks on elliptic-curve groups require running time that is a square root of the group size [10].

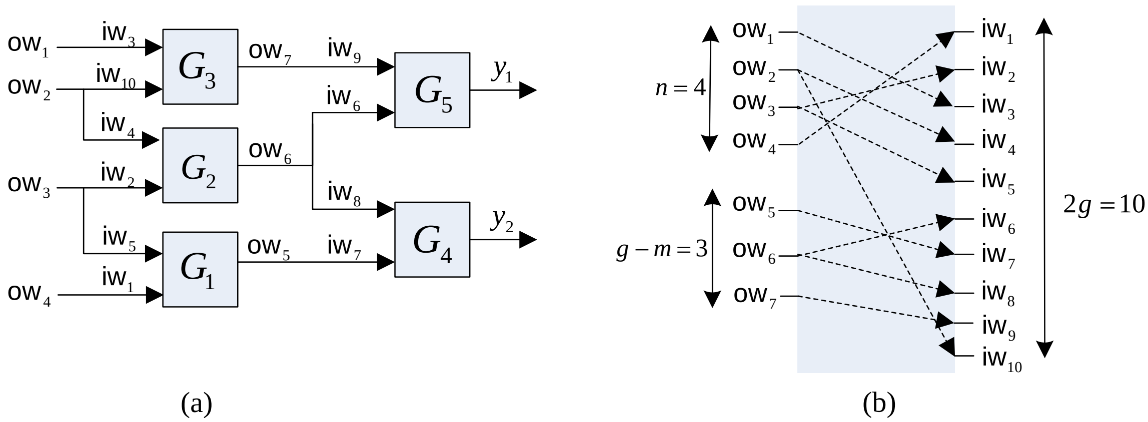


Fig. 1. (a) A circuit representation $\mathcal{C}_f$ of a function f. (b) The mapping π_f of f.

2.2 Notations and Concept of 2PFE Framework

In a two-party private function evaluation (2PFE) scheme, Party_1 has a function input f (compiled into a boolean circuit C_f) and optionally a private input bit string x_1, whereas Party_2 has an input bit string x_2. The parties aim to evaluate f on x_1 and x_2 so that at least one of them would obtain the resulting $f(x_1, x_2)$. The recent 2PFE schemes [KM11, MS13] conform to a generic 2PFE framework (formalized by [MS13]) that basically reduces the 2PFE problem to hiding both parties' input strings and topology of the circuit. The framework is not concerned with hiding the gates since it allows only one type of gate in the circuit structure.

In a nutshell the 2PFE framework is as follows. Before starting the 2PFE protocol, Party_1 compiles the function into a boolean circuit C_f consisting of only one type of gates (e.g., NAND gates). During the protocol execution, Party_1 and Party_2 first engage in a mapping evaluation protocol where Party_2 obliviously obtains the tokens on gate inputs, and then they mutually run a 2PC protocol where Party_2 garbles each gate separately using those tokens, and Party_1 evaluates the garbled circuit. As a result, Party_1 obtains the garbled tokens that map to the corresponding outputs of the function (i.e., $y = f(x_1, x_2)$).

Let g, n, and m denote the number of gates (circuit size), the number of inputs, and the number of outputs of C_f, respectively. Let $\mathsf{OW} = (\mathsf{ow}_1, \ldots, \mathsf{ow}_{n+g-m})$ denote the set of outgoing wires that is the union of the input wires of the circuit and the output wires of its non-output gates. Note that the total number of elements in OW is $M = n + g - m$. Similarly, let $\mathsf{IW} = (\mathsf{iw}_1, \ldots, \mathsf{iw}_{2g})$ denote the set of incoming wires that is the union of the input wires of each gate in the circuit. Note also that the total number of elements in IW is $N = 2g$. Throughout this paper, M and N denote the numbers of outgoing and incoming wires, respectively. Let π_f be a mapping such that $j \leftarrow \pi_f(i)$ if and only if $\mathsf{ow}_i \in \mathsf{OW}$ and $\mathsf{iw}_j \in \mathsf{IW}$ correspond to the same wire in the circuit C_f (see Figure 1).

We define the public information of the circuit C_f as $\mathsf{PubInfo}_{C_f}$ which is composed of: (1) the number of each party's input bits, (2) the number of output bits, (3) the total number of incoming wires N and that of outgoing wires M, (4) the incoming and outgoing/output wire indices that belong to each gate, (5)

[10] For example, for the 112-bit symmetric key security level, a 2048-bit large prime number is required for a prime sub-group of the multiplicative group modulo a prime, whereas only a 224-bit prime p is sufficient for a NIST-elliptic curve over $\mathbb{F}_p$ [Gir16].

the outgoing wire indices corresponding to each party's input bits. Note that, it is a common assumption among PFE schemes [KM11, MS13, BBKL19] that both parties have pre-agreement on the number of gates (g), the number of input wires (n), the number of output wires (m), the number of input bits of Party_1 (q). Both parties generate $\mathsf{PubInfo}_{C_f}$ at the beginning of the protocol execution (without an additional round of communication). Namely, each party runs the following deterministic procedure to obtain $\mathsf{PubInfo}_{C_f}$ on public input (g, n, m, q):

- Set $N := 2g$, $M := n + g - m$.
- For $i = 1, \dots, g$, set iw_{2i-1} and iw_{2i} as the incoming wires of the gate G_i.
- For $i = 1, \dots, g - m$, set ow_i as the outgoing wire of the gate G_i.
- For $i = 1, \dots, q$, set ow_{g-m+i} as the outgoing wire corresponding to Party_1's i-th input bit.
- For $i = 1, \dots, n - q$, set $\mathsf{ow}_{g-m+q+i}$ as the outgoing wire corresponding to Party_2's i-th input bit.
- For $i = 1, \dots, m$, set the output wire y_i as the output of G_{g-m+i}.
- Return $\mathsf{PubInfo}_{C_f} := (M, N, \mathsf{OW}, \mathsf{IW}, y)$.

Next, Party_1 generates π_f (i.e., the connection between incoming and outgoing wire indices) using the following randomized procedure on input $(C_f, \mathsf{OW}, \mathsf{IW})$.

- Randomly permute the indices $1, \dots, g - m$, and assign it to an ordered set A.
- For $i = 1, \dots, g - m$, assign $G_{A[i]}$ to the i-th non-output gate in topological order.
- For $i = 1, \dots, m$, assign G_{g-m+i} to i-th output gate.
- Extract π_f from C_f according to the connections between ows and iws.
- Return π_f.

We next define *Reusable Mapping Template* in which the efficiency of our scheme mostly due to the reusability of this template.[11]

Definition 1 (Reusable Mapping Template). *Let $\pi_f^{-1}(j)$ be the inverse mapping of π_f that denotes the index of the outgoing wire connected to iw_j. A Reusable Mapping Template is a set $\mathsf{ReuseTemp}_f := (\mathcal{P}, \mathcal{Q})$ such that $\mathcal{P} := (P_1, \dots, P_M)$ where P_i is a generator of the group picked for ow_i by Party_2 and $\mathcal{Q} := (Q_1, \dots, Q_N)$ where $Q_j := t_j \cdot P_{\pi_f^{-1}(j)}$ is a group element generated for iw_j by Party_1 for $t_j \in_R \mathbb{Z}_q^*$, $i = 1, \dots, M$, and $j = 1, \dots, N$.*

3 Our PFE Scheme

In this section, we first present our protocol for initial executions InExe which is optimized by offline/online decomposition (Figure 2). We next introduce our efficient resumption protocol for subsequent executions ReExe (Figure 3). We then propose an efficient method for executions with multiple Party_2s.

3.1 The description of our InExe protocol

InExe scheme is optimized by carrying out some of the computations in the off-line stage. In general, such precomputation techniques enhance real-time performance at the cost of extra preliminary computations and storage consumption. Besides, in today's technological perspectives, memory consumption is rarely considered to be a serious drawback since storage units are abundant in many recent devices. In what follows, we give the full protocol steps of our optimized initial execution InExe protocol. Also, Figure 2 depicts the protocol steps of our InExe protocol. The computations that can be carried out in the precomputation phase include the generation of $\mathcal{P}$, and the computation of the sets $\mathcal{W}^0$ and $\mathcal{W}^1$ by Party_2.

In accordance with the generic 2PFE framework the description of our InExe is as follows.

[11] Although, KM11 [KM11] also involves homomorphic encryption for token generation, it requires all protocol steps to be repeated in each subsequent executions.

$\boldsymbol{Party_1}$ input: $x_1 \in \{0,1\}^*, C_f, \pi_f$	**Pre-shared info:** $\mathbb{G}, q, P, \mathsf{PubInfo}_{C_f}$	$\boldsymbol{Party_2}$ input: $x_2 \in \{0,1\}^*$
	Offline Pre-computation	Pick: $\alpha_0, \alpha_1 \in_R \mathbb{Z}_q^*$. Generate: $\mathcal{P} := (P_1, \ldots, P_M)$, $\mathcal{W}^0 := (W_i^0 \twoheadleftarrow \alpha_0 \cdot P_i)$, $\mathcal{W}^1 := (W_i^1 \twoheadleftarrow \alpha_1 \cdot P_i)$, $i = 1, \ldots, M$.
	$\xleftarrow{\mathcal{P}}$	
Generate: $T := (t_1, \ldots, t_N : t_j \in_R \mathbb{Z}_q^*)$, $\mathcal{Q} = (Q_j \leftarrow t_j \cdot P_{\pi_f^{-1}(j)})$, $j = 1, \ldots, N$. Keep: $\mathsf{ReuseTemp}_f := (\mathcal{P}, \mathcal{Q})$.	$\xrightarrow{\mathcal{Q}}$	Generate: $\mathcal{V}^0 := (V_j^0 \twoheadleftarrow \alpha_0 \cdot Q_j)$, $\mathcal{V}^1 := (V_j^1 \twoheadleftarrow \alpha_1 \cdot Q_j)$, $j = 1, \ldots, N$. Keep: $\mathsf{ReuseTemp}_f := (\mathcal{P}, \mathcal{Q})$. Pick: $Y^0 := (y_i^0 \twoheadleftarrow_R \{0,1\}^\ell)$, $Y^1 := (y_i^1 \twoheadleftarrow_R \{0,1\}^\ell)$, $i = 1, \ldots, m$.
Act as Evaluator	**Run 2PC Protocol**	**Act as Garbler** Generate garbled circ. F by using $(\mathcal{W}^0, \mathcal{W}^1, \mathcal{V}^0, \mathcal{V}^1, Y^0, Y^1, \mathsf{PubInfo}_{C_f})$.
	$\xleftarrow{F, X_2}$	
Using F, X_1, X_2, T, π_f obtain $Y = f(x_1, x_2)$.		

Fig. 2. Sketch of our InExe 2PFE Protocol. $\mathsf{ReuseTemp}_f$ and T are stored (if needed) for the later PFE runs by (ReExe) protocol. Note that in case Party_1 has inputs (x_1) then OT protocol is required (to send the corresponding garbled X_1) which can be trivially combined with the protocol rounds (i.e., the first round of OT protocol is sent accompanied by $\mathcal{P}$ message and second one by $\mathcal{Q}$ and third one by F, X_2) for minimization of the total number of rounds.

Inputs Prior to the protocol execution, both parties should have a pre-agreement on a cyclic group $\mathbb{G}$ of large prime order q (where $|q| = 2\lambda$) with a generator P and the $\mathsf{PubInfo}_{C_f}$ on inputs (g, n, m, q). Each party has the following inputs: (1) Party_1 holds a boolean circuit C_f consisting of only one type of gates (e.g., NAND gates) and the corresponding mapping π_f, and (possibly but not necessarily) his input x_1 (2) Party_2 holds his inputs x_2.

Offline pre-computation phase Party_2 generates the set $\mathcal{P}$ of M random generators. It also picks $\alpha_0, \alpha_1 \in_R \mathbb{Z}_q^*$, and prepares the group element sets $\mathcal{W}^0 := (W_1^0, \ldots, W_M^0 : W_i^0 \twoheadleftarrow \alpha_0 \cdot P_i,\ i = 1, \ldots, M)$ for FALSEs and $\mathcal{W}^1 := (W_1^1, \ldots, W_M^1 : W_i^1 \twoheadleftarrow \alpha_1 \cdot P_i,\ i = 1, \ldots, M)$ for TRUEs, where P_i is the i-th element in $\mathcal{P}$ and each W_i^b is a token for $\mathsf{ow}_i \in \mathsf{OW}$, $b \in \{0,1\}$. Party_2 stores $\mathcal{P}$, $\mathcal{W}^0$, $\mathcal{W}^1$, α_0, and α_1.

Online phase Online phase consists of three rounds as follows.
Round 1: Party_2 sends $\mathcal{P}$ to Party_1.
Round 2: Party_1 generates the blinding set $T := (t_1, \ldots, t_N : t_j \in_R \mathbb{Z}_q^*,\ j = 1, \ldots, N)$, computes the set $\mathcal{Q} = (Q_1, \ldots, Q_N : Q_j \leftarrow t_j \cdot P_{\pi_f^{-1}(j)},\ j = 1, \ldots, N)$, where $\pi_f^{-1}(j)$ denotes the index of the outgoing wire connected to iw_j. Party_1 sends $\mathcal{Q}$ to Party_2. Now, both parties have the knowledge of the set $\mathsf{ReuseTemp}_f := (\mathcal{P}, \mathcal{Q})$.

For the later PFE runs with the same function (if needed), Party_1 stores $\mathsf{ReuseTemp}_f$ (see Figure 3 for the protocol of subsequent executions (ReExe)).

Party_2 prepares the group element sets corresponding to $\mathsf{iw}_j \in \mathsf{IW}$. The set $\mathcal{V}^0$ is for FALSE, $\mathcal{V}^1$ is for TRUE semantic values.

$$\mathcal{V}^0 := (V_1^0, \dots, V_N^0 : V_j^0 \leftarrow \alpha_0 \cdot Q_j,\ j = 1, \dots, N),$$

$$\mathcal{V}^1 := (V_1^1, \dots, V_N^1 : V_j^1 \leftarrow \alpha_1 \cdot Q_j,\ j = 1, \dots, N).$$

Party_2 next picks the following two randomly chosen ordered sets for output wires of the circuit

$$Y^0 := (y_1^0, \dots, y_m^0 : y_i^0 \leftarrow_R \{0,1\}^\ell,\ i = 1, \dots, m),$$

$$Y^1 := (y_1^1, \dots, y_m^1 : y_i^1 \leftarrow_R \{0,1\}^\ell,\ i = 1, \dots, m),$$

where ℓ is the bit length of a group element (i.e., $\ell = \lceil \log_2(q) \rceil$). For the later PFE runs with the same function (if needed), Party_2 stores $\mathsf{ReuseTemp}_f$.

Round 3: Now, both parties then engage in a 2PC protocol where Party_2 and Party_1 play the garbler and evaluator roles, respectively. Party_2 garbles the whole circuit by using $\mathcal{W}^0$, $\mathcal{W}^1$, $\mathcal{V}^0$, $\mathcal{V}^1$, Y^0, Y^1, and $\mathsf{PubInfo}_{C_f}$. Note that in contrast to the usual garbling in [KM11, MS13], in our garbling phase, Party_2 has group elements instead of random tokens. To use group elements as keys, we now define an instantiation of a dual-key cipher (DKC) notion of [BHR12] using a pseudorandom function as

$$\mathsf{Enc}_{P_1,P_2}(\mathsf{m}) := [H(P_1, P_2, \mathsf{gateID})]_\ell \oplus \mathsf{m}$$

where P_1 and P_2 are two group elements used as keys, m is the ℓ-bit plaintext, gateID is the index number of the gate, $H : \mathbb{G} \times \mathbb{G} \times \{0,1\}^* \to \{0,1\}^{\ell+\tau}$ is a hash-function (which we model as a random oracle), τ is an integer such that $\tau > 2\log_2(4g)$ for preventing collisions in the τ rightmost bits of hashes, and $[H(X)]_\ell$ denotes the truncated hash value (of the message X) which is cropped to the ℓ leftmost bits of $H(X)$ for some X. Also, we denote $[H(X)]_\tau$ for the truncated hash value (of the message X) which is cropped to the τ rightmost bits of $H(X)$ for some X. The former truncated hash value is used for encryption, while the latter is utilized for the point and permute optimization of Beaver et al. [BMR90]. Note that the encryption scheme Enc is based on the encryption scheme in [LPS08] and differs from it only by utilization of group elements as keys.

Let G_a be a non-output NAND gate for some $a \in \{1, \dots, g\}$. Let also $\mathsf{iw}_i, \mathsf{iw}_j$ be the incoming wires and ow_z be the outgoing wire of G_a where $i, j \in \{1, \dots, M\}$ and $z \in \{1, \dots, N\}$. To garble G_a, Party_2 prepares the following four ciphertexts

$$\mathsf{ct}_a^1 := \mathsf{Enc}_{V_i^0, V_j^0}(\overline{W_z^1}),\ \mathsf{ct}_a^2 := \mathsf{Enc}_{V_i^0, V_j^1}(\overline{W_z^1}),$$

$$\mathsf{ct}_a^3 := \mathsf{Enc}_{V_i^1, V_j^0}(\overline{W_z^1}),\ \mathsf{ct}_a^4 := \mathsf{Enc}_{V_i^1, V_j^1}(\overline{W_z^0})$$

where $\overline{W_z^1}$ and $\overline{W_z^1}$ are the ℓ-bit string representations of the group elements. Similarly, let G_b be an output NAND gate for some $b \in \{1, \dots, g\}$. Let also $\mathsf{iw}_i, \mathsf{iw}_j$ be the incoming wires and z be the output wire index of G_b where $i, j \in \{1, \dots, M\}$ and $z \in \{1, \dots, m\}$. To garble G_b, Party_2 prepares the following four ciphertexts

$$\mathsf{ct}_b^1 := \mathsf{Enc}_{V_i^0, V_j^0}(y_z^1),\ \mathsf{ct}_b^2 := \mathsf{Enc}_{V_i^0, V_j^1}(y_z^1),$$

$$\mathsf{ct}_b^3 := \mathsf{Enc}_{V_i^1, V_j^0}(y_z^1),\ \mathsf{ct}_b^4 := \mathsf{Enc}_{V_i^1, V_j^1}(y_z^0).$$

For the point and permute optimization [BMR90], for each gate G_a in the circuit, Party_2 picks random indices $I_a^1, I_a^2 \in \{1, \dots, \tau\}$ such that

$$\{(\mathsf{X}[I_a^1], \mathsf{X}[I_a^2]), (\mathsf{Y}[I_a^1], \mathsf{Y}[I_a^2]), (\mathsf{Z}[I_a^1], \mathsf{Z}[I_a^2]), (\mathsf{T}[I_a^1], \mathsf{T}[I_a^2])\} =$$

$$\{(0,0), (0,1), (1,0), (1,1)\} \text{ where}$$

$\mathsf{X} = [H(V_i^0, V_j^0, \mathsf{gateID})]_\tau$, $\mathsf{Y} = [H(V_i^0, V_j^1, \mathsf{gateID})]_\tau$, $\mathsf{Z} = [H(V_i^1, V_j^0, \mathsf{gateID})]_\tau$, $\mathsf{T} = [H(V_i^1, V_j^1, \mathsf{gateID})]_\tau$, and $\mathsf{S}[I_a^i]$ denotes the I_a^i-th bit of the bit string S. We denote each garbled gate GG_a, which is then composed

***Party*$_1$**		***Party*$_2$**
input: $x_{1,k} \in \{0,1\}^*, C_f, \pi_f, T$	**Pre-shared info:** $\mathbb{G}, q, P, \mathsf{PubInfo}_{C_f}, \mathsf{ReuseTemp}_f$	input: $x_{2,k} \in \{0,1\}^*$
	Offline Pre-computation	Pick: $\alpha_{0,k}, \alpha_{1,k} \in_R \mathbb{Z}_q^*$. Generate: $\mathcal{W}_k^0 := (W_{i,k}^0 \twoheadleftarrow \alpha_{0,k} \cdot P_i)$, $\mathcal{W}_k^1 := (W_{i,k}^1 \twoheadleftarrow \alpha_{1,k} \cdot P_i)$, $i = 1, \ldots, M$. $\mathcal{V}_k^0 := (V_{j,k}^0 \twoheadleftarrow \alpha_{0,k} \cdot Q_j)$, $\mathcal{V}_k^1 := (V_{j,k}^1 \twoheadleftarrow \alpha_{1,k} \cdot Q_j)$, $j = 1, \ldots, N$. Pick: $Y_k^0 := (y_{i,k}^0 \twoheadleftarrow_R \{0,1\}^\ell)$, $Y_k^1 := (y_{i,k}^1 \twoheadleftarrow_R \{0,1\}^\ell)$, $i = 1, \ldots, m$. Generate garbled circ. F_k by using $(\mathcal{W}_k^0, \mathcal{W}_k^1, \mathcal{V}_k^0, \mathcal{V}_k^1, Y_k^0, Y_k^1, \mathsf{PubInfo}_{C_f})$.
Act as Evaluator	**Run 2PC Protocol**	**Act as Garbler**
	$\xleftarrow{F_k, X_{2,k}}$	
Using $F_k, X_{1,k}, X_{2,k}, T, \pi_f$ obtain $Y = f(x_{1,k}, x_{2,k})$.	$\xleftrightarrow{OT\ (if\ needed)}$	

Fig. 3. Sketch of our ReExe protocol for the k-th execution ($k > 1$). The number of rounds is equal to 1, or 2, or 3 depending on the input string length of Party_1.

of four ℓ-bit ciphertexts, ct_a^1, ct_a^2, ct_a^3, and ct_a^4, and an index pair (I_a^1, I_a^2). Note that the set of ciphertexts in the GG_a are ordered according to I_a^1-th and I_a^2-th bits of their corresponding X, Y, Z, and T values. For example, let $\mathsf{X} = 011001\ldots1$, $\mathsf{Y} = 101111\ldots0$, $\mathsf{Z} = 110001\ldots0$, and $\mathsf{T} = 010111\ldots1$. If $(I_a^1, I_a^2) = (1,5)$ then $(\mathsf{X}[1], \mathsf{X}[5]) = (0,0)$, $(\mathsf{Y}[1], \mathsf{Y}[5]) = (1,1)$, $(\mathsf{Z}[1], \mathsf{Z}[5]) = (1,0)$, $(\mathsf{T}[1], \mathsf{T}[5]) = (0,1)$, and therefore, we have $GG_a = (\mathsf{ct}_a^1, \mathsf{ct}_a^4, \mathsf{ct}_a^3, \mathsf{ct}_a^2, (I_a^1, I_a^2))$. A trivial method for finding such a pair (I_a^1, I_a^2) could be as follows. First, Party_2 can find I_a^1 such that $\{\mathsf{X}[I_a^1], \mathsf{Y}[I_a^1], \mathsf{Z}[I_a^1], \mathsf{T}[I_a^1]\} = \{0,0,1,1\}$ with probability of $6/16$ in each trial. Then, I_a^2 could also be found with probability of $4/16$ in each trial. Therefore, the expected number of trials to find a pair of (I_a^1, I_a^2) is 7. Party_2 garbles all the gates in the circuit in the above-mentioned way, and obtains the garbled circuit F. Party_2 then sends F and its garbled input X_2 (i.e., the W_i group elements for outgoing wires corresponding to x_2) to Party_1. As usual, Party_1 gets its own garbled input X_1 (i.e., the W_i group elements for outgoing wires corresponding to x_1) from Party_2 using oblivious transfers (OT) (or one invocation of the OT extension schemes [IKNP03, KK13, ALSZ13]). Note that this does not increase the round complexity of our overall protocol, since the exchange messages needed for OT rounds can be accompanied to the protocol rounds (i.e., the first round of OT is sent with $\mathcal{P}$ message and second one with $\mathcal{Q}$ and third one with F, X_2).

Using F, the garbled input $X = (X_1, X_2)$, T, and π_f, Party_1 evaluates the whole garbled circuit in topological order. If an outgoing wire ow_d is mapped to an incoming wire iw_e, then the group element V_e of the e-th incoming wire is computed by the multiplication of the group element W_d of the d-th outgoing wire and the blinding value t_e (i.e., if $\pi_f(d) = e$, then $V_e = t_e \cdot W_d$). Each garbled gate GG_a can be evaluated when both group elements (V_i, V_j) on its incoming wires $(\mathsf{iw}_i, \mathsf{iw}_j)$ are computed. To evaluate each GG_a, Party_1 first computes $H(V_i, V_j, \mathsf{gateID})$, and then XORs the ciphertext in the GG_a pointed by I_a^1-th and I_a^2-th bits of the $H(V_i, V_j, \mathsf{gateID})_\tau$. At the end, Party_1 obtains the token set $Y = (y_1, \ldots, y_m)$ for the output bits of the function $y = f(x_1, x_2)$.

3.2 Optimization with reusability feature: Our ReExe protocol

One of the novelties of our scheme over the state-of-the-art is that our scheme results in a significant cost reduction when the same private function is evaluated more than once between the same or varying evaluating parties. This feature is quite beneficial in relevant real-life scenarios where individuals (or enterprises) can mutually and continuously have long-term business relationship instead of a single deal. Note that such a cost reduction is not available in the protocols of KM11 [KM11] and MS13 [MS13], since they require that token generation and 2PC procedures be repeated in all executions. However, our scheme involves *Reusable Mapping Tamplate* ($\mathsf{ReuseTemp}_f := (\mathcal{P}, \mathcal{Q})$) that is reusable for the generation of tokens on incoming and outgoing wires. The reusability of $\mathsf{ReuseTemp}_f$ incurs upto 42% reduction in communication overhead over [MS13], i.e., asymptotically approaching 42% as protocol is re-executed for the same function.

Figure 3 depicts the sketch of our optimized ReExe protocol. In ReExe protocol most of the calculations are performed in the offline pre-computation phase. For the k-th evaluation, Party_2 picks $\alpha_{0,k}, \alpha_{1,k} \in_R \mathbb{Z}_q^*$ values then prepares the sets $\mathcal{W}_k^0, \mathcal{W}_k^1, \mathcal{V}_k^0, \mathcal{V}_k^1, Y_k^0$ and Y_k^1. Then using $\mathcal{W}_k^0, \mathcal{W}_k^1, \mathcal{V}_k^0, \mathcal{V}_k^1, Y_k^0, Y_k^1$ and $\mathsf{PubInfo}_{C_f}$, Party_2 prepares the garbled circuit F as in the InExe protocol. The online phase then includes only the 2PC stage that also runs the same way as in Section 3.1. During the evaluation procedure of the 2PC stage, Party_1 always use the same T in all protocol runs.

3.3 Executing with Various Party_2s

In the previous section, we have addressed the case where the same two parties would like to evaluate the same function multiple times. In this section, we deal with the case that Party_1 would like to run the 2PFE protocol for the same private function with various Party_2s separately. This is a relevant scenario where Party_1 may run a business with many customers for her algorithm/software. Suppose that a cryptological research institution invents a practical algorithm for breaking RSA. Since such an algorithm would clearly attract a substantial demand, the institution may prefer hiding the details of the algorithm selling only its use. On the other hand, in many cases the clients would not like to share the keys (i.e., private inputs) with the institution. This is one of the several scenarios that a 2PFE protocol for the same private function with various Party_2s is suitable for.

First of all, we recall that the execution of our second protocol in Figure 3 requires the preknowledge of $\mathsf{ReuseTemp}_f := (\mathcal{P}, \mathcal{Q})$ by Party_2 and the set T by Party_1. Trivially, once $\mathsf{ReuseTemp}_f$ and T are produced during InExe with any Party_2 as in our first protocol in Figure 2, then they can be stored, and our second protocol can be made use of in the subsequent executions with the same Party_2. We are here interested in a more efficient mechanism running with various Party_2s by eliminating the costs of our first protocol for generating the preknowledge. The goal of this mechanism is to generate the generator set $\mathcal{P}$ in such a way that Party_1 does not know the relation between any two of its elements. T and $\mathcal{Q}$ can be subsequently computed, once the generator set $\mathcal{P}$ is given to Party_1. In order to do so, we utilize a distributed system[12] based on a t-out-of-n threshold mechanism (fault tolerant against arbitrary behaviour of up to t malicious and colluding authorities) which takes $(\mathbb{G}, q, P, M)$ as input and outputs $\mathcal{P}$. For further reducing communication and round costs in the first execution, $\mathcal{P}$ might be generated via a random oracle by both parties in an offline phase. This will result in reduction of one round and M tokens of InExe.

In the offline stage of our new mechanism, the generator set $\mathcal{P}$ is generated by the distributed authorities, and given to Party_1. Next, Party_1 computes the sets T and $\mathsf{ReuseTemp}_f$. It then publishes $\mathsf{PubInfo}_{C_f}$ and $\mathsf{ReuseTemp}_f$ so that any prospective k-th party $\mathsf{Party}_{2,k}$ can utilize them in a 2PFE protocol run. This offline stage is dealt with only once, and its outputs (i.e., T and $\mathsf{ReuseTemp}_f$) are used in the subsequent executions. Note that the flow of re-executions for all $\mathsf{Party}_{2,k}$s is exactly the same as our ReExe protocol. We would like to stress that the costs of any execution in our new mechanism with a distributed system does not differ from the ReExe protocol.

4 Complexity Analysis

In this section, we first present the costs of our InExe and ReExe protocols in terms of communication, online computation, and round complexities. We then compare these protocols with the existing boolean circuit

[12] One can also suggest a single semi-trusted authority for generation of the generator set $\mathcal{P}$. However, the knowledge of the relations among the elements of $\mathcal{P}$ by a single party may violate the privacy of inputs, and therefore, it is better to distribute the trust among multiple authorities.

Table 1. Comparison of the existing 2PFE schemes in terms of overall communication (in bits) and online computation costs (in terms of symmetric-key operations), offline computation costs (in terms of symmetric-key operations), and the number of rounds.

	Communication	**Online Comp.**	**Offline Comp.**	**Rounds**
KM11-1st [KM11]	$(4M+10N)\lambda$	N Asym. + $2.5N$ Sym.	$4(M+N)$ Asym.	3
KM11-2nd [KM11]	$(2M+7N)\lambda$	N Asym. + $2.5N$ Sym.	$2(M+N)$ Asym.	3
MS13-OSN [MS13]	$(10N\log_2 N+4N+5)\lambda$	$(6N\log_2 N+2.5N+3)$ Sym.	$O(\lambda)$ Sym.	6
MS13-HE [MS13]	$(2M+6N)\lambda$	N Asym. + $2.5N$ Sym.	$2(M+N)$ Asym.	3
GKS17 [GKS17]	$(2N\log_2 N)\lambda$	$0.7N\log_2 N$ Sym.	$2N\log_2 N$ Sym.	3
BBKL18 [BBKL19]	$(6N\log_2 N+0.5N+3)\lambda$	$(6N\log_2 N+N+3)$ Sym.	$O(\lambda)$ Sym.	6
Our InExe	$(2M+6N)\lambda$	$4N$ Asym. + $2.5N$ Sym.	$(3M-1)$ Asym.	3
Our ReExe	$4N\lambda$	N Asym. + $0.5N$ Sym.	$2(M+N)$ Asym. + 2 Sym.	1 / 2 / 3

based 2PFE schemes. M, N and λ denote the number of outgoing wires (i.e., equal to $n+g-m$), the number of incoming wires (i.e., $N=2g$) and the security parameter, respectively.

4.1 Complexity of Our Scheme

Communication cost Considering our InExe protocol, the overall communication overhead is $(2M+6N)\lambda$ bits, composed of (i) the set $\mathcal{P}$ (M of 2λ-bit strings) is sent by Party_2 in Round 1, (ii) the set $\mathcal{Q}$ (N of 2λ-bit strings) is sent by Party_1 in Round 2, (iii) the garbled circuit ($2N$ of 2λ-bit strings) is sent by Party_2 in Round 3, where M is the number of outgoing wires and N is the number of incoming wires ($N=2g$). Considering our ReExe protocol, the use of $\mathsf{ReuseTemp}_f$ eliminates the transmission of $(2M+2N)\lambda$ bits (required for token generation). Therefore, in total only $4N\lambda$ bits (required for the garbled circuit) are transmitted.

Table 2. Comparison of the existing 2PFE schemes in terms of overall communication costs for various circuit sizes. Here we take $N=2M$ and $\lambda=128$.

	Number of Gates				
	$\mathbf{2^{10}}$	$\mathbf{2^{15}}$	$\mathbf{2^{20}}$	$\mathbf{2^{25}}$	$\mathbf{2^{30}}$
KM11-1st [KM11]	0.38 MB	12.00 MB	0.38 GB	12.00 GB	384.00 GB
KM11-2nd [KM11]	0.25 MB	8.00 MB	0.25 GB	8.00 GB	256.00 GB
MS13-OSN [MS13]	3.56 MB	164.00 MB	6.69 GB	264.00 GB	10,048.00 GB
MS13-HE [MS13]	0.22 MB	7.00 MB	0.22 GB	7.00 GB	224.00 GB
GKS17 [GKS17]	0.68 MB	32.00 MB	1.31 GB	52.00 GB	1,984.00 GB
BBKL18 [BBKL19]	1.89 MB	90.50 MB	3.77 GB	151.00 GB	5,776.00 GB
Our InExe	0.22 MB	7.00 MB	0.22 GB	7.00 GB	224.00 GB
Our ReExe	0.13 MB	4.00 MB	0.13 GB	4.00 GB	128.00 GB

Computation cost In terms of online computation complexity, InExe protocol requires $4N$ elliptic curve point multiplications, composed of (i) N operations by Party_1 in Round 2, (ii) $2N$ operations by Party_2 in Round 3, (iii) N operations by Party_1 during the evaluation of the garbled circuit. There is also a relatively small cost of $2.5N$ symmetric-key operations during the 2PC stage (composed of $2N$ operations by Party_2 for garbling and $0.5N$ operations by Party_1 for evaluating). ReExe protocol reduces the online computation costs to N elliptic curve point multiplications and $0.5N$ symmetric-key operations (carried out only by Party_1).

Note that Beaver's OT pre-computation technique [Bea95] can be used for decomposing OT's for Party_1's input bits into online/offline stages. This eliminates online public-key operations of OT by carrying out them offline.

Number of rounds Our InExe protocol has 3 rounds. The number of rounds of our ReExe protocol is equal to 1, or 2, or 3 depending on the input string length of Party_1. Namely, if Party_1 has $x_1 = \perp$, then the number of rounds is equal to 1 (i.e., no rounds needed for OT). If Party_1's input bits are not many, it is more efficient to use separate OTs for Party1's input tokens in parallel instead of an OT extension scheme. There exists OT schemes with 2 rounds (e.g., [Bea95] and [NP01]). Hence, this choice results in a PFE scheme with overall 2 rounds. If Party_1's input bits are many, then using an OT extension scheme is more efficient. Note that Ishai based OT extension schemes are composed of $O(\lambda)$ parallel OTs (again can be realized by Naor and Pinkas's OT [NP01]) and an additional round. Similarly, this choice results in a PFE scheme with overall 3 rounds.

4.2 Comparison

We now compare our 2PFE protocols with the state-of-the-art constant-round 2PFE protocols. In our scheme, we utilize an EC cyclic group where the DDH assumption holds for state-of-the-art efficiency. For [KM11], we take into account both protocols: (1) their "$\mathcal{C}$-PFE protocol" (see [KM11, Sect. 3.1], which we call KM11-1st) and (2) their "A More Efficient Variant" (see [KM11, Sect. 3.2], which we call KM11-2nd). For a fair comparison, we assume that the point and permute optimization [BMR90] is directly applied to the MS13 and KM11 protocols during the 2PC phase[13]. Regarding the HE based schemes, for a fair comparison, we assume that EC-ElGamal is used. Also, considering KM11 and our protocols, we assume that each element of $\mathbb{G}$ has a length $\ell = 2\lambda$ bits for a λ-bit security.

Table 1 compares the existing 2PFE schemes in terms of overall communication cost, online/offline computation costs, and the number of rounds. We also provide Table 2 that depicts a comparison in terms of overall communication costs for various circuit sizes. In general, MS13-OSN, GKS17, BBKL18 performs $O(NlogN)$, whereas MS13-HE, KM11 and our protocols achieves linear complexity[14]. Note that although the complexity of MS13-HE is same as our InExe protocol, for the later executions our ReExe protocol enjoys a significant cost reduction due to the reusability feature, which is not possible for MS13-HE and KM11 protocols. For all circuit sizes, the communication costs of ReExe protocol are significantly lower than that of existing 2PFE protocols.

The advantage of our scheme becomes more pronounced when the number of executions is more than one. To demonstrate this, we define the normalized cost efficiency (NCE) function that takes a protocol (Prot_i), a circuit C_f and the number of executions (k), then outputs an efficiency ratio wrt our scheme. The normalized cost efficiency is calculated via dividing the cumulative communication cost of our protocol by that of Prot_i.

$$NCE(\mathsf{Prot}_i, C_f, k) = \frac{f_c(\mathsf{InExe}, C_f) + (k-1)f_c(\mathsf{ReExe}, C_f)}{kf_c(\mathsf{Prot}_i, C_f)},$$

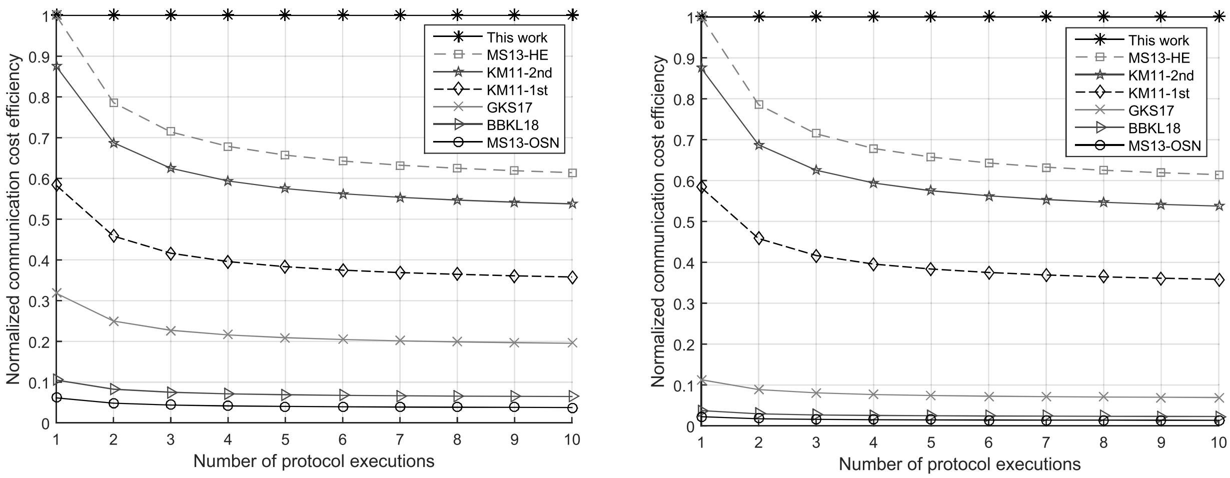
where f_c is the cost function that outputs the communication cost value for given protocol and C_f.

Figure 4 depicts the normalized cost efficiency comparison of the protocols for circuits with 2^{10} and 2^{30} gates. Also, without loss of generality, we take $N = 2M$. Considering MS13-HE, although it performs the same efficiency in the initial execution, after the second execution, its efficiency is about 0.8 (meaning that our protocol saves about 20% bandwidth as compared to MS13-HE), and after ten executions it is about 0.63 (i.e., we achieve 37% saving). Moreover, for two executions our cost reduction is about 54% over KM11-1st, 30% over KM11-2nd. For ten executions our cost reduction is about 63% over KM11-1st, 44% over KM11-2nd. For large number of runs (e.g. a hundred runs) our savings over KM11-1st, KM11-2nd and MS13-HE are 66%, 50% and 43%, respectively.

[13] In [MS13] and [KM11], for the 2PC phases, the authors do not suggest any optimization. However, a point and permute optimization is available for both schemes.

[14] Note that $M \leq N$, therefore $O(M + N) = O(N)$

Figure 4 (a) and (b) shows how the normalized cost efficiency changes wrt circuit size. For the protocols that has linear complexity, the normalized cost efficiency does not change as the number of gates increases. However, for the protocols with $O(NlogN)$ complexity, their normalized efficiency dramatically decreases. For instance, after two executions, our cost reduction is about 74% and 91% over GKS17; about 91% and 97% over BBKL18; and about 95% and 98% over MS13-OSN for a thousand and a billion-gate circuits, respectively.



(a) Number of gates = 2^{10} (b) Number of gates = 2^{30}

Fig. 4. Comparision of cummulative communication cost via normalized bandwidth efficiency vs. number of PFE executions. (a) gives a comparison using a circuit 2^{10} gates where (b) gives a comparison using a circuit 2^{30} gates.

Regarding the computation costs, Table 1 gives both asymmetric (denoted by Asym.) and symmetric (denoted by Sym.) cryptographic operations, separately. On the other hand, comparing the complexity of asymmetric operations with symmetric operations is subject to discussion. To compare with each other, one can define a computation cost ratio as the cost of a primitive asymmetric operation divided by the cost of a primitive symmetric operation for the same security level. Note that the value of this ratio depends upon several factors such as the software implementations, the underlying symmetric-key encryption scheme, the availability of short-cut algorithms, the type of the chosen elliptic curve, the hardware infrastructure, and the type of utilized processors. For example, according to [EIV18], in a setting where curve25519 and SHA256 are picked as the EC and the hash function, respectively, and the operations take place on an Intel Xeon Processor E3-1220 v6 (amd64, 4x3GHz), this ratio is roughly 130.

Among all protocols, our ReExe protocol performs the best result in terms of round complexity. Namely, the number of rounds in ReExe is equal to 1 if Party_1 has $x_1 = \perp$, or 2 if Party_1 has a non-empty input x_1 in such that the OT extension is not applicable for its garbled input, or 3 otherwise. Note that the arithmetic circuit based protocol of [MS13] provides $O(g)$ round complexity (see [MS13, p. 570])).

5 Security of Our Protocols

In this section, we give simulation-based security proofs of our InExe protocol in Figure 2, ReExe protocol in Figure 3, and our mechanism with various Party_2s in Sect. 3.3 in accordance with the security proof of [KM11].

Theorem 1. *If the following three conditions hold then the 2PFE protocol proposed in Figure 2 is secure against semi-honest adversaries: (1) the DDH assumption is hard in the cyclic group $\mathbb{G}$, (2) the hash-function*

$H : \mathbb{G} \times \mathbb{G} \times \{0,1\}^* \rightarrow \{0,1\}^{\ell+\tau}$ *involved in the instantiation of DKC scheme is modeled as a random oracle, (3) the OT scheme securely realizes $\mathcal{F}_{OT}$ functionality in the OT-hybrid model against semi-honest adversaries.*

Proof. First, consider the case that Party_1 is corrupted. For any probabilistic polynomial time adversary $\mathcal{A}_1$, controlling Party_1 in the real world, we construct a simulator $\mathcal{S}_1$ that simulates $\mathcal{A}_1$'s view in the ideal world. $\mathcal{S}_1$ runs $\mathcal{A}_1$ on Party_1's inputs, f and x_1, the function output token set $Y = (y_1, \ldots, y_m)$, the pre-shared group parameters, and $\mathsf{PubInfo}_{C_f}$ as follows.

1. $\mathcal{S}_1$ generates the generator set $\tilde{\mathcal{P}} := (\tilde{P}_1, \ldots, \tilde{P}_M)$. $\mathcal{S}_1$ also prepares the group element sets $\tilde{\mathcal{W}}^0 := (\tilde{W}_1^0, \ldots, \tilde{W}_M^0 : \tilde{W}_i^0 \twoheadleftarrow \tilde{\alpha}_{0,i} \cdot P,\ \tilde{\alpha}_{0,i} \in_R \mathbb{Z}_q^*,\ i = 1, \ldots, M)$ and $\tilde{\mathcal{W}}^1 := (\tilde{W}_1^1, \ldots, \tilde{W}_M^1 : \tilde{W}_i^1 \twoheadleftarrow \tilde{\alpha}_{1,i} \cdot P,\ \tilde{\alpha}_{1,i} \in_R \mathbb{Z}_q^*,\ i = 1, \ldots, M)$. $\mathcal{S}_1$ gives $\tilde{\mathcal{P}}$ to $\mathcal{A}_1$.
2. $\mathcal{S}_1$ receives the blinding set $T := (t_1, \ldots, t_j : t_j \in_R \mathbb{Z}_q^*,\ j = 1, \ldots, N)$ from $\mathcal{A}_1$, and prepares the sets $\tilde{\mathcal{V}}^0 := (\tilde{V}_1^0, \ldots, \tilde{V}_N^0 : \tilde{V}_j^0 \twoheadleftarrow t_j \cdot \tilde{W}^0_{\pi_f^{-1}(j)},\ j = 1, \ldots, N)$ and $\tilde{\mathcal{V}}^1 := (\tilde{V}_1^1, \ldots, \tilde{V}_N^1 : \tilde{V}_j^1 \twoheadleftarrow t_j \cdot \tilde{W}^1_{\pi_f^{-1}(j)},\ j = 1, \ldots, N)$.
3. $\mathcal{S}_1$ prepares the garbled circuit $\tilde{F}$ by garbling each gate as follows. $\mathcal{S}_1$ garbles each non-output NAND gate by encrypting only the group element for FALSE on its outgoing wire with all four possible input token combinations (i.e., for a gate whose incoming wires are iw_i and iw_j, outgoing wire is ow_z, $\mathcal{S}_1$ prepares the following four ciphertexts: $\tilde{\mathsf{ct}}_a^1 = \mathsf{Enc}_{\tilde{V}_i^0, \tilde{V}_j^0}(\overline{\tilde{W}_z^0})$, $\tilde{\mathsf{ct}}_a^2 = \mathsf{Enc}_{\tilde{V}_i^0, \tilde{V}_j^1}(\overline{\tilde{W}_z^0})$, $\tilde{\mathsf{ct}}_a^3 = \mathsf{Enc}_{\tilde{V}_i^1, \tilde{V}_j^0}(\overline{\tilde{W}_z^0})$, $\tilde{\mathsf{ct}}_a^4 = \mathsf{Enc}_{\tilde{V}_i^1, \tilde{V}_j^1}(\overline{\tilde{W}_z^0})$. To garble an output NAND gate whose incoming wires are iw_i and iw_j, and output wire is z, $\mathcal{S}_1$ prepares the four ciphertexts: $\tilde{\mathsf{ct}}_b^1 = \mathsf{Enc}_{\tilde{V}_i^0, V_j^0}(y_z)$, $\tilde{\mathsf{ct}}_b^2 = \mathsf{Enc}_{\tilde{V}_i^0, \tilde{V}_j^1}(y_z)$, $\tilde{\mathsf{ct}}_b^3 = \mathsf{Enc}_{\tilde{V}_i^1, \tilde{V}_j^0}(y_z)$, $\tilde{\mathsf{ct}}_b^4 = \mathsf{Enc}_{\tilde{V}_i^1, \tilde{V}_j^1}(y_z)$. For each garbled gate $\tilde{GG}_a$, $\mathcal{S}_1$ then permutes $\tilde{\mathsf{ct}}_a^2$, $\tilde{\mathsf{ct}}_a^3$, $\tilde{\mathsf{ct}}_a^4$, and picks $\tilde{I}_a^1, \tilde{I}_a^2 \in_R \{1, \ldots, \tau\}$, and places $\tilde{\mathsf{ct}}_a^1$ in the order pointed by $\tilde{I}_a^1$-th and $\tilde{I}_a^2$-th bits of $[H(\tilde{V}_i^0, \tilde{V}_j^0, \mathsf{gateID})]_\tau$ among the other three ciphertexts. Each garbled gate $\tilde{GG}_a$ is then composed of four ℓ-bit ciphertexts and two $\log_2(\tau)$-bit random values $\tilde{I}_a^1$ and $\tilde{I}_a^2$.
4. $\mathcal{S}_1$ gives $\tilde{F}$ to $\mathcal{A}_1$ along with the simulated garbled input consisting of only the group elements for $\mathsf{FALSE}s$ on both parties' input wires $\tilde{X} = (\tilde{X}_1, \tilde{X}_2)$. This completes our simulation.

In what follows, we prove that the information obtained by Party_1 in the real execution $(\mathcal{P}, \mathcal{W}, F)$ is identically distributed to $(\tilde{\mathcal{P}}, \tilde{\mathcal{W}}, \tilde{F})$, where for outgoing wires, Party_1 obtains the group elements $\mathcal{W} = (W_1, \ldots, W_M)$ while $\mathcal{A}_1$ obtaining the group elements $\tilde{\mathcal{W}} = (\tilde{W}_1^0, \ldots, \tilde{W}_M^0)$. We now show the computational indistinguishability of $(\mathcal{P}, \mathcal{W})$ and $(\tilde{\mathcal{P}}, \tilde{\mathcal{W}})$ by utilizing Lemma 1, which ultimately ties the security of our protocol to the DDH assumption. More concretely, we need to show

$$\{(P_1, \ldots, P_M, W_1, \ldots, W_M)\} \approx_c \{(\tilde{P}_1, \ldots, \tilde{P}_M, \tilde{W}_1^0, \ldots, \tilde{W}_M^0)\}$$

$$\{(r_1 \cdot P, \ldots, r_M \cdot P, \alpha_{\mathsf{b}_1} \cdot (r_1 \cdot P), \ldots, \alpha_{\mathsf{b}_M} \cdot (r_M \cdot P))\} \approx_c$$
$$\{(\tilde{r}_1 \cdot P, \ldots, \tilde{r}_M \cdot P, \tilde{\alpha}_{0,1} \cdot P, \ldots, \tilde{\alpha}_{0,M} \cdot P)\}$$

where $\mathsf{b}_i \in \{0,1\}$ is the semantic value on ow_i and $\tilde{P}_i = \tilde{r}_i \cdot P$. For the sake of a simpler representation, we replace $\alpha_{\mathsf{b}_i} r_i$ with r_{M+i}, and $\tilde{\alpha}_{0,i}$ with $\tilde{r}_{M+i}$ for $i = 1, \ldots, M$. Note that $(r_1, \ldots, r_{2M})$ is not identically distributed to $(\tilde{r}_1, \ldots, \tilde{r}_{2M})$, while it is only sufficient to show that

$$\{(r_1 \cdot P, \ldots, r_{2M} \cdot P)\} \approx_c \{(\tilde{r}_1 \cdot P, \ldots, \tilde{r}_{2M} \cdot P)\}.$$

For this purpose, we generate a new set $\mathcal{R} := (R_1, \ldots, R_{2M})$ by picking $2M$ random generators. Hence, we now need to show

$$\{(R_1, \ldots, R_{2M}, r_1 \cdot P, \ldots, r_{2M} \cdot P)\} \approx_c$$
$$\{(R_1, \ldots, R_{2M}, \tilde{r}_1 \cdot P, \ldots, \tilde{r}_{2M} \cdot P)\}$$

Thanks to Lemma 1 and the underlying DDH assumption, we have both

$$\{(R_1,\dots,R_{2M},\gamma\cdot R_1,\dots,\gamma\cdot R_{2M})\}\approx_c$$
$$\{(R_1,\dots,R_{2M},r_1\cdot P,\dots,r_{2M}\cdot P)\}$$

and

$$\{(R_1,\dots,R_{2M},\gamma\cdot R_1,\dots,\gamma\cdot R_{2M})\}\approx_c$$
$$\{(R_1,\dots,R_{2M},\tilde{r}_1\cdot P,\dots,\tilde{r}_{2M}\cdot P)\}$$

where $\gamma\in_R\mathbb{Z}_q^*$. Hence, the following sets are computationally indistinguishable

$$\{(r_1\cdot P,\dots,r_{2M}\cdot P)\}\approx_c\{(\tilde{r}_1\cdot P,\dots,\tilde{r}_{2M}\cdot P)\}$$

which effectively concludes the proof for $\{(\mathcal{P},\mathcal{W})\}\approx_c\{(\tilde{\mathcal{P}},\tilde{\mathcal{W}})\}$. Furthermore, since the same values in T are used among the outgoing wire tokens and incoming wire tokens in both the real and the ideal executions, we have $\{(\mathcal{P},\mathcal{W},\mathcal{V})\}\approx_c\{(\tilde{\mathcal{P}},\tilde{\mathcal{W}},\tilde{\mathcal{V}})\}$ where for each incoming wire $\mathcal{V}=(V_1,\dots,V_N)$ is the set of tokens obtained by Party_1 and $\tilde{\mathcal{V}}=(\tilde{V}_1^0,\dots,\tilde{V}_N^0)$ is the set of tokens obtained by $\mathcal{A}_1$. In contrast to [KM11], it is relatively simple to prove the computational indistinguishability of F and $\tilde{F}$ in our scheme since we use a hash function modeled as random oracle during garbling. Once the distribution of four hash outputs for each gate (in the real and ideal executions) are proven to be computationally indistinguishable random values, outputs of our instantiation of DKC is also proven to be computationally indistinguishable. This results in the computational indistinguishability of each garbled gate GG_a and $\tilde{GG}_a$, and eventually computational indistinguishability of F and $\tilde{F}$. For a gate whose incoming wires are iw_i and iw_j, in the real execution, we have four hash outputs involved in the garbling as follows:

$$H(V_i^0,V_j^0,\mathsf{gateID}),H(V_i^0,V_j^1,\mathsf{gateID}),$$
$$H(V_i^1,V_j^0,\mathsf{gateID}),H(V_i^1,V_j^1,\mathsf{gateID}).$$

Similarly, for each gate, in the ideal execution, we have the following four hash outputs in the garbling as follows:

$$H(\tilde{V}_i^0,\tilde{V}_j^0,\mathsf{gateID}),H(\tilde{V}_i^0,\tilde{V}_j^1,\mathsf{gateID}),$$
$$H(\tilde{V}_i^1,\tilde{V}_j^0,\mathsf{gateID}),H(\tilde{V}_i^1,\tilde{V}_j^1,\mathsf{gateID}).$$

Since in Party_1's view, resulting from the indistinguishability of $\mathcal{V}$ and $\tilde{\mathcal{V}}$, the hash inputs are computationally indistinguishable, and therefore, the hash outputs are computationally indistinguishable random values. This completes the proof for $\{(\mathcal{P},\mathcal{W},F)\}\approx_c\{(\tilde{\mathcal{P}},\tilde{\mathcal{W}},\tilde{F})\}$.

We now consider the case that Party_2 is corrupted. For any probabilistic polynomial-time adversary $\mathcal{A}_2$, controlling Party_2 during our first protocol in the real world, we construct a simulator $\mathcal{S}_2$ that simulates $\mathcal{A}_2$'s view in the ideal world. $\mathcal{S}_2$ runs $\mathcal{A}_2$ on Party_2's input, and the pre-shared group parameters, and $\mathsf{PubInfo}_{C_f}$ as follows.

1. $\mathcal{S}_2$ asks $\mathcal{A}_2$ to generate $\tilde{\mathcal{P}}\twoheadleftarrow\textsc{Init}(\mathbb{G},q,P,M)$ and receives $\tilde{\mathcal{P}}$.
2. $\mathcal{S}_2$ then picks $\tilde{t}_j\in_R\mathbb{Z}_q^*$ for $j=1,\dots,N$, and computes $\tilde{Q}_j\twoheadleftarrow\tilde{t}_j\cdot P$ which are now random group elements in $\mathbb{G}$. $\mathcal{S}_2$ assigns $\tilde{\mathcal{Q}}=(\tilde{Q}_1,\dots,\tilde{Q}_N)$, and gives $\tilde{\mathcal{Q}}$ to $\mathcal{A}_2$. This completes our simulation.

In the real execution of our protocol, Party_2 receives only the message $\mathcal{Q}:=(Q_1,\dots,Q_N:Q_j\twoheadleftarrow t_j\cdot P_{\pi_f^{-1}(j)},j=1,\dots,N)$ in Round 2 (apart from the exchanged messages during the OT protocol for Party_1's garbled input). However, the transcripts received by Party_2 during the OT do not leak any information to Party_2 because of the ideal execution of $\mathcal{F}_{OT}$ in the OT-hybrid model. Due to DDH assumption, in Party_2's view, the distributions of $\tilde{\mathcal{Q}}$ and $\mathcal{Q}$ are identical (i.e., $\tilde{\mathcal{Q}}\approx_c\mathcal{Q}$). This concludes the proof for the InExe protocol.

Theorem 2. *If the 2PFE protocol proposed in Figure 2 is secure against semi-honest adversaries (i.e., the three conditions in Theorem 1 are satisfied), then the 2PFE protocol proposed in Figure 3 is also secure against semi-honest adversaries.*

Proof (Sketch). The main difference of the ReExe protocol from the first one is the utilization of $\mathsf{ReuseTemp}_f$. Therefore, the proof will be complete once we show that the utilization of the sets $\mathcal{W}_k^0$, $\mathcal{W}_k^1$, $\mathcal{V}_k^0$, and $\mathcal{W}_k^1$ computed from the same $\mathsf{ReuseTemp}_f$ in the k-th execution gives Party_1 no advantage in deducing Party_2's inputs.

We now show that in Party_1's view, $(\mathcal{W}_k, \mathcal{V}_k, \mathcal{W}_{k+1}, \mathcal{V}_{k+1})$ in two consecutive real executions are computationally indistinguishable from $(\tilde{\mathcal{W}}_1, \tilde{\mathcal{V}}_1, \tilde{\mathcal{W}}_2, \tilde{\mathcal{V}}_2)$ where $\tilde{\mathcal{W}}_1 := (\tilde{W}_{1,1}, \ldots, \tilde{W}_{M,1} : \tilde{W}_{i,1} = \tilde{q}_{i,1} \cdot P,\ \tilde{q}_{i,1} \in_R \mathbb{Z}_q^*,\ i = 1, \ldots, M)$, $\tilde{\mathcal{V}}_1 := (\tilde{V}_{1,1}, \ldots, \tilde{V}_{N,1} : \tilde{V}_{j,1} \twoheadleftarrow t_j \cdot \tilde{W}_{\pi_f^{-1}(j),1},\ j = 1, \ldots, N)$, $\tilde{\mathcal{W}}_2 := (\tilde{W}_{1,2}, \ldots, \tilde{W}_{M,2} : \tilde{W}_{i,2} = \tilde{q}_{i,2} \cdot P,\ \tilde{q}_{i,2} \in_R \mathbb{Z}_q^*,\ i = 1, \ldots, M)$, and $\tilde{\mathcal{V}}_2 := (\tilde{V}_{1,2}, \ldots, \tilde{V}_{N,2} : \tilde{V}_{j,2} \twoheadleftarrow t_j \cdot \tilde{W}_{\pi_f^{-1}(j),2},\ j = 1, \ldots, N)$.

More concretely, we have

$$
\begin{aligned}
&\{((\overline{\overline{1,k}}), \ldots, (\overline{\overline{M,k}}), t_1 \cdot (\overline{\overline{\pi_f^{-1}(1),k}}), \ldots, t_N \cdot (\overline{\overline{\pi_f^{-1}(N),k}}),\\
&(\overline{\overline{1,k+1}}), \ldots, (\overline{\overline{M,k+1}}), t_1 \cdot (\overline{\overline{\pi_f^{-1}(1),k+1}}),\\
&\ldots, t_N \cdot (\overline{\overline{\pi_f^{-1}(N),k+1}}))\}\\
&\approx_c \{(\tilde{q}_{1,1} \cdot P, \ldots, \tilde{q}_{M,1} \cdot P, t_1 \cdot (\tilde{q}_{\pi_f^{-1}(1),1} \cdot P),\\
&\ldots, t_N \cdot (\tilde{q}_{\pi_f^{-1}(N),1} \cdot P), \tilde{q}_{1,2} \cdot P, \ldots, \tilde{q}_{M,2} \cdot P,\\
&t_1 \cdot (\tilde{q}_{\pi_f^{-1}(1),2} \cdot P), \ldots, t_N \cdot (\tilde{q}_{\pi_f^{-1}(N),2} \cdot P))\}
\end{aligned}
$$

where $(\overline{\overline{i,j}})$ is the abbreviation for $\alpha_{\mathsf{b}_{i,j},j} \cdot P_i$, and $\mathsf{b}_{i,k} \in \{0,1\}$ is the semantic bit value of ow_i in the k-th execution. The proof of their indistinguishability relies on the same flow as the proof of Theorem 1, which depends on Lemma 1 and ultimately on the DDH assumption.

Theorem 3. *If the threshold system is secure against malicious adversaries at most $t-1$ of whom are allowed to collude, and the 2PFE protocol proposed in Figure 3 is secure against semi-honest adversaries; then our mechanism with various Party_2s in Sect. 3.3 is also secure against semi-honest adversaries.*

Proof (Sketch). First, the Party_1's view in the 2PFE mechanism is equivalent to the one in the protocol in Figure 3. Observe that the generator set is generated by the distributed system and the tokens (that are used in preparation of the garbled input X_k and the garbled circuit F_k) are computed from $\alpha_{0,k}$ or $\alpha_{1,k}$ in each evaluation as in Figure 3. Therefore, the 2PFE mechanism prevents Party_1 from deducing any information about $\mathsf{Party}_{2,k}$'s input. Second, $\mathsf{Party}_{2,k}$s cannot obtain any information about Party_1's input in none of the executions since the OT outputs are only obtained by Party_1 due the $\mathcal{F}_{OT}$ functionality in the OT-hybrid model. Also, due to Theorem 1, no one can obtain information about π_f from the $\mathsf{ReuseTemp}_f$. Moreover, any $\mathsf{Party}_{2,k}$ has a negligible advantage on distinguishing the exchanged messages in an evaluation between Party_1 and $\mathsf{Party}_{2,l}$ from a random string due to the underlying DDH assumption for $l \neq k$. More concretely, the tokens (that are used in preparation of $\mathsf{Party}_{2,l}$'s garbled input $X_{2,l}$ and the garbled circuit F_l) are computed by multiplying the elements of the $\mathsf{ReuseTemp}_f$ with the private values $\alpha_{0,l}$ or $\alpha_{1,l}$ of $\mathsf{Party}_{2,l}$.

6 Conclusion

We have proposed a secure and highly efficient 2PFE scheme for boolean circuits based on DDH assumption. Our scheme consists of two protocols: (1) a protocol for initial executions (InExe), (2) a resumption protocol (ReExe) for subsequent executions. The latter protocol is more efficient due to the fact that it benefits from the reusable tokens generated already in the former one. The reusability case is more likely to be frequent in practice, compared to the cases where parties execute a function only once. We note that such cost reduction by the reusability feature is not applicable in the existing PFE protocols (i.e., for each PFE execution the parties should run the all the steps from scratch). Our protocols achieve linear communication and computation complexities and a constant number of rounds which is at most three. To the best of our knowledge, this is the first and most efficient 2PFE scheme that enjoys a reusablity feature.

Our scheme asymptotically reduces the communication cost compared to MS13-OSN [MS13], BBKL18 [BBKL19], and GKS17 [GKS17] protocols (i.e., from $O(g\log(g))$ to $O(g)$ where g is the number of gates). For instance, for a billion-gate circuit, our cost reduction is about 98% over MS13-OSN, about 96% over BBKL18, and about 89% over GKS17. Comparing with the protocols that has linear complexity, for ten executions (regardless of number of gates) our cost reduction is about 63% over KM11-1st, 44% over KM11-2nd, and 37% over MS13-HE.

We also propose a solution for the case that Party_1 runs the 2PFE protocol for the same private function with various Party_2s separately. This is a common scenario where Party_1 may run a business with many customers for her algorithm/software. Instead of running InExe protocol with each Party_2, we have proposed a more efficient mechanism for the generation of the reusable tokens by utilizing a threshold based system.

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