Delayed-Input Non-Malleable Zero Knowledge and Multi-Party Coin Tossing in Four Rounds

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Abstract

In this work we start from the following two results in the state-of-the art:

1. 4-round non-malleable zero knowledge (NMZK): Goyal et al. in FOCS 2014 showed the first 4-round one-one NMZK argument from one-way functions (OWFs). Their construction requires the prover to know the instance and the witness already at the 2nd round.
2. 4-round multi-party coin tossing (MPCT): Garg et al. in Eurocrypt 2016 showed the first 4-round protocol for MPCT. Their result crucially relies on 3-round 3-robust parallel non-malleable commitments. So far there is no candidate construction for such a commitment scheme under standard polynomial-time hardness assumptions.

We improve the state-of-the art on NMZK and MPCT by presenting the following two results:

1. a *delayed-input* 4-round one-*many* NMZK argument Π_{NMZK} from OWFs; moreover Π_{NMZK} is also a *delayed-input* many-many *synchronous* NMZK argument.
2. a 4-round MPCT protocol Π_{MPCT} from one-to-one OWFs; Π_{MPCT} uses Π_{NMZK} as subprotocol and exploits the special properties (e.g., delayed input, many-many synchronous) of Π_{NMZK}.

Π_{MPCT} makes use of a special proof of knowledge that offers additional security guarantees when played in parallel with other protocols. The new technique behind such a proof of knowledge is an additional contribution of this work and is of independent interest.

1 Introduction

Non-malleable zero-knowledge (NMZK) and secure multi-party computation (MPC) are fundamental primitives in Cryptography. In this work we will study these two primitives and for the case of MPC we will focus on the coin-tossing functionality that is among the most studied functionalities.

NMZK. The first construction of NMZK was given by Dolev at at. in [DDN91]. Later on, Barak in [Bar02] showed the first constant-round construction. An improved construction was then given by Pass and Rosen in [PR05, PR08]. The work of Goyal et al. [GRRV14] obtained the first round-optimal construction requiring only 4 rounds and one-way functions (OWFs). Their construction requires the instance and the witness to be known already when the prover plays his first round. Their definition is the standard one-one definition where the adversary opens two sessions, one with a prover and one with a verifier.

The fact that the instance and the witness need to be known already at the second round is an important limitation when NMZK is used as subprotocol to prove statements about another subprotocol played in parallel. Moreover the one-one security is an important limitation when NMZK is used in a multi-party scenario where several of such argument systems are played in parallel.

The above two limitations clearly raise the following natural and interesting open questions:

Open Question 1: is there a 4-round delayed-input NMZK argument system?

Open Question 2: is there a 4-round many-many synchronous NMZK argument system?

Multi-party coin-flipping (MPCT). In [KOS03], Katz et al. obtained a constant-round secure MPC protocol using sub-exponential hardness assumptions. This result was then improved by Pass in [Pas04] that showed how to get bounded-concurrent secure MPC for any functionality with standard assumptions. Further results of Goyal [Goy11] and Goyal et al. [GLOV12] relied on better assumptions but with a round complexity still far from optimal.

A very recent work of Garg et al. [GMPP16b] makes a long jump ahead towards fully understanding the round complexity of secure MPCT. They show that the existence of a 3-round 3-robust parallel non-malleable commitment scheme implies a 4-round protocol for secure MPCT for polynomially many coins with black-box simulation. Some candidate instantiations of such special commitment scheme are the one of Pass et al. [PPV08] based on non-falsifiable assumptions, or the one of Ciampi et al. [COSV16] based on sub-exponentially strong one-to-one one-way functions (see [GMPP16a, Pol16] for more details). The achieved round complexity (i.e., 4 rounds) is proven optimal in [GMPP16b] when simulation is black box and the number of bits in the output of the functionality is superlogarithmic.

A very recent result of Ananth et al. [ACJ17] constructs a 4-round MPC protocol for any functionality assuming DDH w.r.t. superpolynomial-time adversaries. The above state-of-the art leaves open the following question.

Open Question 3: is there a 4-round secure MPCT protocol under standard assumptions?

1.1 Our Contribution

In this paper we solve the above 3 open problems. More precisely we present the following results:

1. a *delayed-input* 4-round one-*many* NMZK argument Π_{NMZK} from OWFs, therefore solving Open Question 1; moreover Π_{NMZK} is also a *delayed-input* many-many *synchronous* NMZK argument, therefore solving Open Question 2;
2. a 4-round MPCT protocol Π_{MPCT} from one-to-one OWFs, therefore solving Open Question 3[1].

The two constructions are not uncorrelated. Indeed Π_{MPCT} uses Π_{NMZK} as subprotocol and exploits the special properties (e.g., delayed input, many-many synchronous) of Π_{NMZK}. Moreover, Π_{MPCT} makes use of a special proof of knowledge that offers additional security guarantees when played in parallel with other protocols. Designing such a proof of knowledge is an additional contribution of this work and is of independent interest.

Interestingly, several years after the 4-round zero knowledge argument system from OWFs of [BJY97], the same optimal round complexity and optimal complexity assumptions have been shown sufficient in this work for delayed-input NMZK and in [COP+14] for resettably sound zero knowledge.

More details on our two new constructions follow below.

1.2 MPCT from NMZK

A first main idea that allows us to bypass the strong requirements of the construction of [GMPP16b] is that we avoid robust/non-malleable commitments and instead focus on non-malleable zero knowledge. Since we

[1] An unpublished prior work of Goyal et al. [GKP+17] achieves a similar result on MPCT using completely different techniques.

want a 4-round MPCT protocol, we need to rely on 4-round NMZK. The only known construction is the one of [GRRV14]. Unfortunately their NMZK argument system seems to be problematic to use in our design of a 4-round MPCT protocol. There are two main reasons. The first reason is that the construction of [GRRV14] uses the technique of secure computation in the head and therefore requires the instance already in the second round. This is often a problem when the NMZK argument is played in parallel with other subprotocols as in our construction. Indeed these additional subprotocols end in the 3rd or 4th round and typically[2] need to be strengthened by a zero-knowledge proof of correctness. The second reason is that in the setting of 4-round MPCT the adversary can play as a many-many synchronous man-in-the-middle (MiM), while the construction of [GRRV14] is proved one-one non-malleable only.

We therefore improve the state-of-the-art on NMZK constructing a delayed-input NMZK argument system. Our construction only needs one-way functions and is secure even when a) there are polynomially many verifiers (i.e., it is a one-many NMZK argument), and b) there are polynomially many provers and they are played in parallel. We will crucially use both the delayed-input property and security with multiple parallelized provers and verifiers in our secure MPCT construction. Our NMZK argument is also crucially used in [COSV17c].

1.3 Overview of Our Delayed-Input Parallel NMZK Argument from OWFs

Issues in natural constructions. A natural construction of a NMZK argument consists of having: 1) a subprotocol (usually a witness-indistinguishable proof of knowledge) useful to extract a trapdoor from the verifier; 2) a non-malleable commitment of the witness for the statement to be proved; 3) a witness-indistinguishable proof of knowledge (WIPoK) to prove that either the committed message is a witness or the trapdoor is known. Combining the above 3 tools in parallel is necessary to obtain a 4-round construction.

The simulator for such a scheme would complete the transcript for the WIPoK by extracting the trapdoor form the verifier and committing to 0 in the non-malleable commitment.

Unfortunately it is not clear how to prove the security of this scheme when all subprotocols are squeezed into 4 rounds. The problem arises from the interactivity nature of the primitives involved, that are: a 3-round non-malleable commitment and two instantiations of a 3-round WIPoK. More precisely the non-malleable commitment is executed in parallel with a 3-round WIPoK Π. When in the security proof the trapdoor is used as witness of Π, the MiM could do the same and also commit to the message 0 in the non-malleable commitment. To detect this behaviour, in order to break the WI of Π, the reduction should extract the message committed in the non-malleable commitment by rewinding it. This implies that also the WIPoK involved in the reduction must be rewound (we recall that these two subprotocols are executed in parallel). It is important to observe that if we allow the MiM to commit to the message 0 then the simulator has no way to extract a witness from the MiM (extraction is required by the definition of NMZK).

A different approach. To overcome this problem we follow the approach proposed in [COSV17b] relying on non-interactive primitives instead of 3-rounds WIPoK. In this way, in every security reduction to such primitives, it will be always possible to extract the message committed in the non-malleable commitment without interfering with the reduction. Therefore, similarly to [COSV17b], we construct this WIPoK relying on: statistically binding commitment, instance-dependent trapdoor commitments (IDTCom) and special honest-verifier zero knowledge (HVZK).

More in details, let $(\pi_1, \pi_2, \pi_3, \pi_4)$ be the transcript of a 4-round Special HVZK delayed-input[3] proof of knowledge (PoK) for the $\mathcal{NP}$-language L. We require the prover to send an IDTCom $\mathtt{tcom}_0$ of π_2 that is opened only in the last round, when π_4 is sent. The instance used for the IDTCom is the pair $(\mathtt{com}, 0)$

[2] Indeed, even the construction of [GMPP16b] that makes use of a special non-malleable commitments requires also a delayed-input zero-knowledge argument.

[3] By *delayed-input* we mean that the witness and the instance are needed only to play the last round.

where $\texttt{com}$ is a statistically binding commitment. This means that the commitment $\texttt{tcom}_0$ (computed using IDTCom) can be opened to any value if $\texttt{com}$ is a valid commitment of the message 0 and the decommitment information of $\texttt{com}$ is known[4] ($\texttt{tcom}_0$ is binding otherwise).

The actual transcript for such protocol therefore can be denoted by $(\pi_1, (\texttt{tcom}_0, \texttt{com}), \pi_3, (\pi_2, \texttt{tdec}_0, \pi_4))$ where $\texttt{com}$ is a valid commitment of the message 0 but the IDTCom is honestly computed (without using the decommitment information of $\texttt{com}$ in order to open a message different from π_2).

Consider now an experiment where $\texttt{com}$ is still a commitment of the message 0, but π_2 can be opened arbitrarily using the trapdoor procedure. If the output of this new experiment deviates from the previous one, we will have a reduction to the trapdoorness of the IDTCom. The reduction is not problematic since the challenger of the trapdoorness is non-interactive and just sends a pair (commitment, decommitment) that is either computed using the regular procedure or through the use of the trapdoor. Next, in another experiment we can replace the prover of the adaptive-input PoK with the special HVZK simulator that will compute π_2 and π_4 after having as input π_1 and π_3. Again, the output of this experiment will not deviate from the previous one otherwise we can show an adversary for the Special HVZK property. The reduction again is not problematic since the challenger of Special HVZK is non-interactive.

We observe that if the instance used to compute the IDTCom is false (i.e., the instance used is $(\texttt{com}, 0)$ but $\texttt{com}$ is not a commitment of 0) then the protocol described so far is a 3-round adaptive-input PoK for the language L.

We consider now two instantiations of the protocol described above: Π_0 and Π_1. The instance used to run the IDTCom used in Π_0 is $(\texttt{com}, 0)$ and the instance used to execute Π_1 is $(\texttt{com}, 1)$. Basically, the instances used by Π_0 and Π_1 share the same value $\texttt{com}$. We observe that $\texttt{com}$ is a statistically binding commitment, therefore it could contain only one message out of 0 and 1. This means that the execution of both Π_0 and Π_1 represents an adaptive-input PoK for the language $L_{\mathsf{OR}} = L_0$ OR L_1. Indeed given that $\texttt{com}$ is statistically binding, at least one out of the two IDTCom is binding too. Therefore rewinding our protocol using different challenges π_3^0 and π_3^1 yields the extraction of a least one witness. Let's call this protocol (i.e, Π_0 and Π_1 along with a shared $\texttt{com}$) $\Pi_{\mathcal{WI}}$. We observe that $\Pi_{\mathcal{WI}}$ is still interactive like Π_0 and Π_1. However, if executed in parallel with an interactive protocol, the rewinds made to this protocol do not interfere with the security of $\Pi_{\mathcal{WI}}$. More precisely, it is possible to do all the reductions described above even though the third and the second rounds are rewound. Intuitively, this is true because all the primitive involved in the reductions are non-interactive, and so it is possible to rely on their security even in the case that some rewinds occur.

A NMZK argument system: NMZK. We run $\Pi_{\mathcal{WI}}$ in parallel with a 4-round public-coin honest-extractable one-one non-malleable commitment scheme Π_{nm}. The prover now runs $\Pi_{\mathcal{WI}}$ in order to prove either the validity of some $\mathcal{NP}$-statement, or that the non-malleable commitment computed using Π_{nm} contains a trapdoor. The simulator for NMZK works by extracting the trapdoor, then committing to it using the non-malleable commitment, and finally using the knowledge of both the trapdoor and the opening information used to compute the non-malleable commitment, as a witness for $\Pi_{\mathcal{WI}}$. The subprotocol for the trapdoor extraction follows also [COSV17b]. More precisely, the trapdoor is represented by the knowledge of two signatures under a verification key sent by the verifier in the 1st round. In order to allow the extraction of the trapdoor, we consider a verifier that sends a signature for a message chosen in the 2nd round by the prover.

On the limited non-malleability of the commitment that is needed in NMZK. For our construction we use a 4-round public-coin one-one honest-extractable synchronous non-malleable commitment Π_{nm}. The *public-coin* requirement is due to the security reduction since we will have to simulate the last round of

[4]The decommitment information of $\texttt{com}$ represents the trapdoor of the IDTCom.

the receiver without knowing the randomness used to compute the previous round. Of course the public-coin property allows such a simulation. Just for simplicity we state our theorems requiring the public-coin property even though our approach can make use of a broader class of protocols. Moreover, we require the existence of a public-coin commitment scheme that is non-malleable only w.r.t. a specific limited adversary: one-one (that opens only one left and one right session) and synchronous (that she aligns the messages of the left sessions with the messages of the right sessions).

Defeating the issues. Using such limited non-malleable commitment we are able to prove that our protocol NMZK is one-many NMZK (synchronous many-many NMZK). This is done following the approach provided in [COSV16] where a one-one non-malleable commitment Π_{nm} is used in order to construct a one-many (and then a many-many) non-malleable commitment. Indeed, in our protocol the simulator extracts the trapdoor[5], and commits to it using the non-malleable commitment scheme. We need to prove that the MiM adversary does not do the same. Roughly, we are requiring only a specific, weaker form of non-malleability just in order to prevent such attack. Therefore, the reduction to the non-malleability of the underling commitment scheme isolates one right session and checks if the MiM has committed to the trapdoor or not. The distinguisher for the non-malleable commitment takes as input the committed message an checks if it corresponds to two signatures of two different messages for a given signature key.

The above approach works only with synchronous sessions. Indeed the non-malleable commitment that we use is secure only in the synchronous case. In order to deal with the asynchronous case we rely on the *honest-extractability* of Π_{nm}. An honest-extractable commitment scheme is a commitment scheme that guarantees the extraction only from honest senders. We recall that $\Pi_{\mathcal{WI}}$ is executed in parallel with Π_{nm} in order to ensure that the MiM either knows a witness for an $\mathcal{NP}$-statement $x \in L$ or the trapdoor has been *correctly* committed using Π_{nm}. For our propose we only need to ensure that the MiM never commits to the trapdoor and uses it as a witness to execute $\Pi_{\mathcal{WI}}$ in the right sessions. In this way we can rely on the adaptive-input PoK property of $\Pi_{\mathcal{WI}}$ in order to extract a witness for x even when the simulator is executed. We observe that if by contradiction the MiM (with non-negligible probability) is not using the witness for x to execute $\Pi_{\mathcal{WI}}$, we have that he honestly computes a commitment to the signatures using Π_{nm}. Therefore, we can extract the message committed by the MiM in order to break the hiding of Π_{nm}[6].

From one-many NMZK to parallel many-many NMZK. Fortunately, our scheme is also many-many NMZK when the sessions are parallelized. Indeed, the simulator can extract (simultaneously) the trapdoor from the right sessions, playing as described previously. The only substantial difference is that we need to use a many-one non-malleable commitment with all the properties listed above. Following the approach proposed in the security proof of Proposition 1 provided in [LPV08], it is possible to claim that a synchronous (one-one) non-malleable commitment is also synchronous many-one non-malleable.

We end this section by observing that a non-malleable commitment scheme that enjoys all the properties that we require is provided in [GPR16][7] and relies on OWFs. Also the IDTCom, the Special HVZK adaptive-input PoK and the statistically binding commitment can be constructed assuming OWFs only.

A special WIPoK Π_{OR}. In order to construct our MPC coin-tossing protocol Π_{MPCT}, we also propose a generic approach for a special WIPoK Π_{OR} that can be nicely composed with other protocols in parallel in

[5]The trapdoor for our protocol is represented by two signatures for a verification key chosen by the verifier.

[6]A rewind made in an asynchronous session does not interfere with (i.e., does not rewind) the challenger of the hiding of Π_{nm}. This part of the proof actually deals with more interesting subtleties. We refer the reader to the formal proof for more details.

[7]In order to ensure non-malleability against a non-synchronous adversary the construction of Goyal et al. modifies Π and uses assumptions w.r.t. a quasi-polynomial-time adversary. For our propose we just need to use the basic protocol Π of [GPR16] that relies on assumptions w.r.t. polynomial-time adversaries.

the same spirit of $\Pi_{\mathcal{WI}}$. The main difference with $\Pi_{\mathcal{WI}}$ is that our special WIPoK Π_{OR} can be used only when the theorem to be proved is known in the beginning of the protocol (which is sufficient for our MPC application).

In a nutshell, Π_{OR} takes two instantiations of the three-move Special HVZK PoK (like as in Blum's protocol [Blu86a]) and composes them via the OR composition proposed in [CDS94] thus obtaining a WIPoK. Using this WIPoK a reduction can be successfully completed even when there are rewinds due to another protocols played in parallel.

In more details, we combine together two executions of Blum's protocol by using the trick for composing two three-move Special HVZK PoKs Σ_0, Σ_1 to construct a three-move Special HVZK PoK for the $\mathcal{NP}$-language L_0 OR L_1 [CDS94]. Let (x_0, x_1) be the compound statement to be proved, with $x_0 \in L_0$ and $x_1 \in L_1$, and let w_b be the witness for x_b. The protocol Π_{OR} proposed in [CDS94] executes Σ_0 and Σ_1 (respectively for L_0 and L_1) in parallel, but after receiving the challenge c from the verifier, the prover can use as challenges for Σ_0 and Σ_1 every pair (c_0, c_1) s.t. $c_0 \oplus c_1 = c$. Therefore the prover could choose in advance one of the challenge to be used (e.g., c_{1-b}), and compute the other one by setting $c_b = c \oplus c_{1-b}$. In this way the transcript for Σ_{1-b} can be computed using the Special HVZK simulator while the transcript for Σ_b is computed using the witness w_b. Thus the prover has the "freedom" of picking one out of two challenges before seeing c, but still being able to complete the executions of both Σ_0 and Σ_1 for every c. We will show that this "freedom" is sufficient to switch from the use of w_0 to the use of w_1 (in order to prove WI) even when it is required to answer to additional (and different) challenges $c^1, \ldots, c^{\mathsf{poly}(\lambda)}$ (i.e., when some rewinds occur). Indeed it is possible to switch the witness used (from w_0 to w_1) in two steps relying first on the Special HVZK of Σ_1, and then on the Special HVZK of Σ_0. More precisely we consider the hybrid experiment H^{w_0} as the experiment where in Π_{OR} the witness w_0 is used (analogously we define H^{w_1}). We now consider H^{w_0,w_1} that differs from H^{w_0} because both the witnesses w_0 and w_1 are used. We prove that H^{w_0} and H^{w_0,w_1} are indistinguishable due to the Special HVZK of Σ_1 even though Π_{OR} is rewound polynomially many times. The reduction works as follows. A challenge c_1 is chosen before the protocol Π_{OR} starts and the Special HVZK challenger is invoked thus obtaining (a_1, z_1). The transcript for Σ_0 is computed by the reduction using the witness w_0 in order to answer to the challenge $c_0^i = c^i \oplus c_1$ for $i = 1, \ldots, \mathsf{poly}(\lambda)$. We recall the we are in a setting where Π_{OR} could be rewound, and therefore the reduction needs to answer to multiple challenges. We observe that the reduction to the Special HVZK is not disturbed by these rewinds because c_1 can be kept fixed. The same arguments can be used to prove that H^{w_0,w_1} is computationally indistinguishable from H^{w_1}. We also notice that Π_{OR} remains a PoK [CDS94].

1.4 4-Round Secure Multi-Party Coin Tossing

Our MPCT protocol will critically make use of our delayed-input synchronous many-many NMZK from OWFs, and of a special WIPoK Π_{OR}. Similarly to [GMPP16b] our protocol consists of each party committing to a random string r, that is then sent in the clear in the last round. Moreover there will be a simulatable proof of correctness of the above commitment w.r.t. r, that is given to all parties independently. The output consists of the $\bigoplus$ of all opened strings. We now discuss in more details the construction of a the special WIPoK Π_{OR} and the messages exchanged by a pair of parties P_1 and P_2 in our multi-party coin tossing protocol Π_{MPCT}. The generalization to n players is straight-forward and discussed in Section 4.1.

Informal description of the protocol. P_1, using a perfectly binding computationally hiding commitment scheme, commits in the first round to a random string r_1 two times thus obtaining $\mathtt{com}_0, \mathtt{com}_1$. Moreover P_1 runs Π_{OR} in order to prove knowledge of either the message committed in $\mathtt{com}_0$ or the message committed in $\mathtt{com}_1$. In the last (fourth) round P_1 sends r_1. In parallel, an execution of a NMZK ensures that both $\mathtt{com}_0$ and $\mathtt{com}_1$ contain the same message r_1 (that is sent in the fourth round)[8]. When P_1 receives the last round

[8]Notice here how crucial is the delayed-input property of the synchronous many-many NMZK.

that contains r_2, P_1 computes and outputs $r_1 \oplus r_2$. P_2 symmetrically executes the same steps using as input r_2.

The simulator for Π_{MPCT} runs the simulator of NMZK and extracts the input $r^\star$ from the malicious party using the PoK extractor of Π_{OR}. At this point the simulator invokes the functionality thus obtaining r and plays in the last round $r_s = r \oplus r^\star$. Note that the values that the simulator commits in $\mathtt{com}_0$ and $\mathtt{com}_1$ are unrelated to r_s and this is possible because the NMZK is simulated. The extraction of the input from the adversary made by the simulator needs more attention. Indeed the security of NMZK will ensure that, even though the simulator cheats (he commits to a random string in both $\mathtt{com}_0$ and $\mathtt{com}_1$) the adversary can not do the same. Therefore the only way he can complete an execution of Π_{MPCT} consists of committing two times to $r^\star$ in the first round, and of sending the same value in the fourth round. This means that the value extracted (in the third round) from the PoK extractor of Π_{OR} is the input of the malicious party.

Our security proof consists of showing the indistinguishability of a sequence hybrid experiments where the firs hybrid corresponds to the real world experiment, and the last hybrid corresponds to the simulated experiment. The second hybrid experiment differs from the real game because the simulator of NMZK is used (instead of the honest prover). The simulator, in order to extract the trapdoor from the adversary, rewinds from the third to the second round, thus rewinding also Π_{OR}. Indeed the adversary, for every different second round of the NMZK could sent a different second round for Π_{OR}. This becomes a problem when we consider the hybrid experiment H_i where the witness for Π_{OR} changes. Due to the rewinds made by the simulator of the NMZK it is not clear how to rely on the security of the WI property of Π_{OR} (the challenger of WI would be rewound). This is the reason why we need to consider an intermediate hybrid experiment H^{w_0,w_1} where both witnesses of Π_{OR} can be used. Then we can prove the indistinguishability between H^{w_0,w_1} and H_i still relying on the Special HVZK of the sub-protocol used in Π_{OR} (which is Blum's protocol).

2 Definitions and Tools

Preliminaries. We denote the security parameter by λ and use "$||$" as concatenation operator (i.e., if a and b are two strings then by $a||b$ we denote the concatenation of a and b). For a finite set Q, $x \leftarrow Q$ sampling of x from Q with uniform distribution. We use the abbreviation PPT that stays for probabilistic polynomial time. We use $\mathsf{poly}(\cdot)$ to indicate a generic polynomial function. A *polynomial-time relation* Rel (or *polynomial relation*, in short) is a subset of $\{0,1\}^* \times \{0,1\}^*$ such that membership of (x, w) in Rel can be decided in time polynomial in $|x|$. For $(x, w) \in \mathsf{Rel}$, we call x the *instance* and w a *witness* for x. For a polynomial-time relation Rel, we define the $\mathcal{NP}$-language L_{Rel} as $L_{\mathsf{Rel}} = \{x | \exists\, w : (x, w) \in \mathsf{Rel}\}$. Analogously, unless otherwise specified, for an $\mathcal{NP}$-language L we denote by $\mathsf{Rel_L}$ the corresponding polynomial-time relation (that is, $\mathsf{Rel_L}$ is such that $L = L_{\mathsf{Rel_L}}$). We also use $\hat{L}$ to denotes the language that includes L and all well formed instances that are not in L. Let A and B be two interactive probabilistic algorithms. We denote by $\langle A(\alpha), B(\beta)\rangle(\gamma)$ the distribution of B's output after running on private input β with A using private input α, both running on common input γ. A *transcript* of $\langle A(\alpha), B(\beta)\rangle(\gamma)$ consists of the messages exchanged during an execution where A receives a private input α, B receives a private input β and both A and B receive a common input γ. Moreover, we will refer to the *view* of A (resp. B) as the messages it received during the execution of $\langle A(\alpha), B(\beta)\rangle(\gamma)$, along with its randomness and its input. We denote by A_r an algorithm A that receives as randomness r.

2.1 (Delayed-Input) Proof/Argument Systems

Definition 1 (Proof/argument system). *A pair of* PPT *interactive algorithms* $\Pi = (\mathcal{P}, \mathcal{V})$ *constitute a* proof system *(resp., an* argument system*) for an* $\mathcal{NP}$*-language* L*, if the following conditions hold:*

Completeness: *For every* $x \in L$ *and* w *such that* $(x, w) \in \mathsf{Rel_L}$*, it holds that:*

$$\text{Prob}\left[\ \langle \mathcal{P}(w), \mathcal{V}\rangle(x) = 1\ \right] = 1.$$

Soundness: *For every interactive (resp.,* PPT *interactive) algorithm* $\mathcal{P}^\star$*, there exists a negligible function* ν *such that for every* $x \notin L$ *and every* z*:*

$$\text{Prob}\left[\ \langle \mathcal{P}^\star(z), \mathcal{V}\rangle(x) = 1\ \right] < \nu(|x|).$$

A proof/argument system $\Pi = (\mathcal{P}, \mathcal{V})$ for an $\mathcal{NP}$-language L, enjoys *delayed-input* completeness if $\mathcal{P}$ needs x and w only to compute the last round and $\mathcal{V}$ needs x only to compute the output. Before that, $\mathcal{P}$ and $\mathcal{V}$ run having as input only the size of x. The notion of delayed-input completeness was defined in [CPS+16a]. For a protocol that enjoys delayed-input completeness we consider also the notion of adaptive-input arguments/proof system. That is, the soundness holds against a stronger adversary $\mathcal{P}^*$ that can choose the statement to be proved in the last round of the interaction with $\mathcal{V}$. Analogously we also consider the notion of adaptive-input arguments/proof of knowledge (see App. A for more details).

We say that an interactive protocol $\Pi = (\mathcal{P}, \mathcal{V})$ is *public coin* if, at every round, $\mathcal{V}$ simply tosses a predetermined number of coins (random challenge) and sends the outcome to the $\mathcal{P}$. Moreover we say that the transcript τ of an execution $b = \langle \mathcal{P}(z), \mathcal{V}\rangle(x)$ is *accepting* if $b = 1$.

3 4-Round Delayed-Input NMZK from OWFs

3.1 Notation, Non-Malleability Definitions and Tools

Delayed-Input non-malleable zero knowledge. Following [LP11a] we use a definition that gives to the adversary the power of adaptive-input selection. More precisely, in [LP11a] the adversary selects the instance and a Turing machine outputs the witness in exponential time. Here we slightly deviate (similarly to [SCO+01]) by 1) requiring the adversary to output also the witness and 2) allowing the adversary to make this choice at the last round. This choice is due to our application where delayed-input non-malleable zero knowledge is used. Indeed we will show that this definition is enough for our propose.

Let $\Pi = (\mathcal{P}, \mathcal{V})$ be a delayed-input interactive argument system for a $\mathcal{NP}$-language L with witness relation $\mathsf{Rel_L}$. Consider a PPT MiM adversary $\mathcal{A}$ that is simultaneously participating in one left session and $\mathsf{poly}(\lambda)$ right sessions. Before the execution starts, $\mathcal{P}, \mathcal{V}$ and $\mathcal{A}$ receive as a common input the security parameter in unary 1^λ. Additionally $\mathcal{A}$ receives as auxiliary input $z \in \{0,1\}^\star$[9]. In the left session $\mathcal{A}$ verifies the validity of a statement x (chosen adaptively in the last round of Π) by interacting with $\mathcal{P}$ using identity $\mathtt{id}$ of his choice. In the right sessions $\mathcal{A}$ proves the validity of the statements $\tilde{x}_1 \ldots, \tilde{x}_{\mathsf{poly}(\lambda)}$ (chosen adaptively in the last round of Π) to the honest verifiers $\mathcal{V}_1, \ldots, \mathcal{V}_{\mathsf{poly}(\lambda)}$, using identities $\tilde{\mathtt{id}}_1, \ldots, \tilde{\mathtt{id}}_{\mathsf{poly}(\lambda)}$ of his choice.

More precisely in the left session $\mathcal{A}$, before the last round of Π is executed, adaptively selects the statement x to be proved and the witness w, s.t. $(x, w) \in \mathsf{Rel_L}$, and sends them to $\mathcal{P}$[10].

Let $\mathsf{View}^{\mathcal{A}}(1^\lambda, z)$ denote a random variable that describes the view of $\mathcal{A}$ in the above experiment.

Definition 2 (Delayed-input NMZK)**.** *A delayed-input argument system* $\Pi = (\mathcal{P}, \mathcal{V})$ *for an* $\mathcal{NP}$*-language* L *with witness relation* $\mathsf{Rel_L}$ *is delayed-input non-malleable zero knowledge (NMZK) if for any MiM adversary* $\mathcal{A}$ *that participates in one left session and* $\mathsf{poly}(\lambda)$ *right sessions, there exists a expected* PPT *machine* $S(1^\lambda, z)$ *such that:*

1. *The probability ensembles* $\{S^1(1^\lambda, z)\}_{\lambda\in\mathbb{N}, z\in\{0,1\}^\star}$ *and* $\{\mathsf{View}^{\mathcal{A}}(1^\lambda, z)\}_{\lambda\in\mathbb{N}, z\in\{0,1\}^\star}$ *are computationally indistinguishable over* λ*, where* $S^1(1^\lambda, z)$ *denotes the first output of* $S(1^\lambda, z)$*.*
2. *Let* $(\mathsf{View}, w_1, \ldots, w_{\mathsf{poly}(\lambda)})$ *denote the output of* $S(1^\lambda, z)$*, for some* $z \in \{0,1\}^\star$*. Let* $\tilde{x}_1, \ldots, \tilde{x}_{\mathsf{poly}(\lambda)}$ *be the right-session statements appearing in* View *and let* $\mathtt{id}$ *and* $\tilde{\mathtt{id}}_1, \ldots, \tilde{\mathtt{id}}_{\mathsf{poly}(\lambda)}$ *be respectively the*

[9] We denote (here and in the rest of the paper) by $\tilde{\delta}$ a value associated with the right session where δ is the corresponding value in the left session.

[10] The witness w sent by $\mathcal{A}$ will be just ignored by the simulator.

identities used in the left and right sessions appearing in View. *Then for every* $i \in \{1, \dots, \mathsf{poly}(\lambda)\}$, *if the* i*-th right session is accepting and* $\mathtt{id} \neq \tilde{\mathtt{id}}_i$, *then* $\tilde{w}_i$ *is s.t.* $(\tilde{x}_i, \tilde{w}_i) \in \mathsf{Rel_L}$.

The above definition of NMZK allows the adversary to select statements adaptively in the last round both in left and in the right sessions. Therefore any argument system that is NMZK according to the above definition enjoys also adaptive-input argument of knowledge. In our paper we also consider the notion of *many-many synchronous delayed-input NMZK*, that is equal to the notion of delayed-input NMZK except that polynomially many left and right sessions are played in parallel.

In the rest of the paper, following [GRRV14], we assume that identities are known before the protocol begins, though strictly speaking this is not necessary, as the identities do not appear in the protocol until after the first prover message. The MiM can choose his identity adversarially as long as it differs from the identities used by honest senders. As already observed in previous work, when the identity is selected by the sender the id-based definitions guarantee non-malleability as long as the MiM does not behave like a proxy (an unavoidable attack). Indeed the sender can pick as id the public key of a signature scheme signing the transcript. The MiM will have to use a different id or to break the signature scheme.

3.2 Non-Malleable Commitments

A commitment scheme involves two players: sender and receiver. Informally, it consists of two phases, a commitment phase and a decommitment phase. In the commitment phase the sender, with a secret input m, interacts with the receiver. In the end of this interaction we say that a *commitment* of the message m has been computed. Moreover the receiver still does not know what m is (i.e. m is hidden) and at the same time the sender can subsequently (i.e., during the decommitment phase) open this commitment only to m (see Def. 13 for a formal definition of commitment scheme).

In order to define a non-malleable commitment we follow [LPV08, LPV09]. Let $\Pi = (\mathsf{Sen}, \mathsf{Rec})$ be a statistically binding commitment scheme. And let λ be the security parameter. Consider a MiM adversary $\mathcal{A}$ that, on auxiliary input z participates in a left and a right session. In the left sessions the MiM adversary $\mathcal{A}$ interacts with Sen receiving commitment to value m using an identity id of its choice. In the right session $\mathcal{A}$ interacts with Rec attempting to commit to a related value $\tilde{m}$ again using identity of its choice $\tilde{\mathtt{id}}$. If the right commitment is invalid, or undefined, its value is set to $\perp$. Furthermore, if $\tilde{\mathtt{id}} = \mathtt{id}$ then $\tilde{m}$ is also set to $\perp$ (i.e., a commitment where the adversary uses the same identity of the honest senders is considered invalid). Let $\mathsf{mim}_{\Pi}^{\mathcal{A},m}(z)$ denote a random variable that describes the values $\tilde{m}$ and the view of $\mathcal{A}$ in the above experiment.

Definition 3 (Non-malleable commitment scheme [LPV08, LPV09])**.** *A commitment scheme is* non-malleable with respect to commitment *if, for every* PPT *MiM adversary* $\mathcal{A}$, *for every* $m_0 \in \{0,1\}^{\mathsf{poly}(\lambda)}$ *and* $m_1 \in \{0,1\}^{\mathsf{poly}(\lambda)}$ *the following holds*

$$\{\mathsf{mim}_{\Pi}^{\mathcal{A},m_0}(z)\}_{z\in\{0,1\}^\star} \approx \{\mathsf{mim}_{\Pi}^{\mathcal{A},m_1}(z)\}_{z\in\{0,1\}^\star}.$$

We say that a commitment is valid or well formed if it admits a decommitment to a message $m \neq \perp$.

For our propose we use a 4-round synchronous honest-extractable non-malleable commitment. That is, a commitment scheme that enjoys 1) non-malleability only against synchronous adversaries, 2) is extractable w.r.t. honest sender (honest-extractable) and 3) is public-coin. The non-malleable commitment Π provided in Figure 2 of [GPR16] enjoys non-malleability against synchronous adversary (as proved in Theorem 1 of [GPR16]), is public coin and can be instantiated in 4 rounds relying on OWFs (the protocol can be squeezed to 3 rounds using one-to-one OWFs).

Also, as stated in Section 5 of [GPR16], given a commitment computed by the sender of Π one can rewind the sender in order to obtain a new accepting transcript with the same first round (resp., first two rounds if we consider the instantiation that relies on OWFs) in order to extract a message m. Moreover, if the sender is honest, then it is possible to claim that m is the actual message committed by the sender. We remark that we do not require any form of extractability against malicious senders.

3.2.1 2-Round Instance-Dependent Trapdoor Commitments.

Here we define a special commitment scheme based on an $\mathcal{NP}$-language L where the sender and receiver also receive as input an instance x. While correctness and computational hiding hold for any x, we require that statistical binding holds for $x \notin L$ and moreover knowledge of a witness for $x \in L$ allows to equivocate. Finally, we require that a commitment along with two valid openings to different messages allows to compute the witness for $x \in L$. We recall that $\hat{L}$ denotes the language that includes L and all well formed instances that are not in L.

Definition 4. *Let 1^λ be the security parameter, L be an $\mathcal{NP}$-language and $\mathsf{Rel_L}$ be the corresponding $\mathcal{NP}$-relation. A triple of PPT algorithms $\mathsf{TC} = (\mathsf{Sen}, \mathsf{Rec}, \mathsf{TFake})$ is a 2-Round Instance-Dependent Trapdoor Commitment scheme if the following properties hold.*

Correctness. *In the 1st round, Rec on input 1^λ and $x \in \hat{L}$ outputs ρ. In the 2nd round Sen on input the message m, 1^λ, ρ and $x \in L$ outputs $(\mathtt{com}, \mathtt{dec})$. We will refer to the pair $(\rho, \mathtt{com})$ as the commitment of m. Moreover we will refer to the execution of the above two rounds including the exchange of the corresponding two messages as the commitment phase. Then Rec on input m, x, $\mathtt{com}$, $\mathtt{dec}$ and the private coins used to generate ρ in the commitment phase outputs 1. We will refer to the execution of this last round including the exchange of $\mathtt{dec}$ as the decommitment phase. Notice that an adversarial sender $\mathsf{Sen}^\star$ could deviate from the behavior of Sen when computing and sending $\mathtt{com}$ and $\mathtt{dec}$ for an instance $x \in \hat{L}$. As a consequence Rec could output 0 in the decommitment phase. We will say that $\mathtt{dec}$ is a valid decommitment of $(\rho, \mathtt{com})$ to m for an instance $x \in \hat{L}$, if Rec outputs 1.*

Hiding. *Given a PPT adversary $\mathcal{A}$, consider the following hiding experiment $\mathsf{ExpHiding}^b_{\mathcal{A},\mathsf{TC}}(\lambda, x)$ for $b = 0, 1$ and $x \in \hat{L}_R$:*

- *On input 1^λ and x, $\mathcal{A}$ outputs a message m, along with ρ.*
- *The challenger on input x, m, ρ, b works as follows: if $b = 0$ then it runs Sen on input m, x and ρ, obtaining a pair $(\mathtt{com}, \mathtt{dec})$, otherwise it runs TFake on input x and ρ, obtaining a pair $(\mathtt{com}, \mathtt{aux})$. The challenger outputs $\mathtt{com}$.*
- *$\mathcal{A}$ on input $\mathtt{com}$ outputs a bit b' and this is the output of the experiment.*

We say that hiding *holds if for any PPT adversary $\mathcal{A}$ there exist a negligible function ν, s.t.:*

$$\left| \text{Prob}\left[\ \mathsf{ExpHiding}^0_{\mathcal{A},\mathsf{TC}}(\lambda, x) = 1\ \right] - \text{Prob}\left[\ \mathsf{ExpHiding}^1_{\mathcal{A},\mathsf{TC}}(\lambda, x) = 1\ \right] \right| < \nu(\lambda).$$

Special Binding. *There exists a PPT algorithm that on input a commitment $(\rho, \mathtt{com})$, the private coins used by Rec to compute ρ, and two valid decommitments $(\mathtt{dec}, \mathtt{dec}')$ of $(\rho, \mathtt{com})$ to two different messages m and m', outputs w s.t. $(x, w) \in \mathsf{Rel_L}$ with overwhelming probability.*

Instance-Dependent Binding. *For every malicious unbounded sender $\mathsf{Sen}^\star$ there exists a negligible function ν s.t. for a commitment $(\rho, \mathtt{com})$ $\mathsf{Sen}^\star$, with probability at most $\nu(\lambda)$, outputs two decommitments (m_0, d_0) and (m_1, d_1) with $m_0 \neq m_1$ s.t. Rec on input the private coins used to compute ρ and $x \notin L$ accepts both decommitments.*

Trapdoorness. *For any PPT adversary $\mathcal{A}$ there exist a negligible function ν, s.t. for all $x \in L$ it holds that:*

$$\left| \text{Prob}\left[\ \mathsf{ExpCom}_{\mathcal{A},\mathsf{TC}}(\lambda, x) = 1\ \right] - \text{Prob}\left[\ \mathsf{ExpTrapdoor}_{\mathcal{A},\mathsf{TC}}(\lambda, x) = 1\ \right] \right| < \nu(\lambda)$$

where $\mathsf{ExpCom}_{\mathcal{A},\mathsf{TC}}(\lambda, x)$ and $\mathsf{ExpTrapdoor}_{\mathcal{A},\mathsf{TC}}(\lambda, x)$ are defined below[11].

[11] We assume wlog that $\mathcal{A}$ is stateful.

$\mathsf{ExpCom}_{\mathcal{A},\mathsf{TC}}(\lambda, x)$:
-*On input* 1^λ *and* x, $\mathcal{A}$ *outputs* (ρ, m).
-Sen *on input* 1^λ, x, m *and* ρ, *outputs* $(\mathtt{com}, \mathtt{dec})$.
-$\mathcal{A}$ *on input* $(\mathtt{com}, \mathtt{dec})$ *outputs a bit* b *and this is the output of the experiment.*

$\mathsf{ExpTrapdoor}_{\mathcal{A},\mathsf{TC}}(\lambda, x)$:
-*On input* 1^λ *and* x, $\mathcal{A}$ *outputs* (ρ, m).
-TFake *on input* 1^λ, x *and* ρ, *outputs* $(\mathtt{com}, \mathtt{aux})$.
-TFake *on input* $\mathtt{tk}$ *s.t.* $(x, \mathtt{tk}) \in \mathsf{Rel_L}$, x, ρ, $\mathtt{com}$, $\mathtt{aux}$ *and* m *outputs* $\mathtt{dec}$.
-$\mathcal{A}$ *on input* $(\mathtt{com}, \mathtt{dec})$ *outputs a bit* b *and this is the output of the experiment.*

3.3 Overview of Our Protocol

For our construction of a 4-round delayed-input non-malleable zero knowledge $\mathsf{NMZK} = (\mathcal{P}_{\mathsf{NMZK}}, \mathcal{V}_{\mathsf{NMZK}})$ we use the following tools.

1. A signature scheme $\Sigma = (\mathsf{Gen}, \mathsf{Sign}, \mathsf{Ver})$;
2. A 2-round statistically binding, computationally hiding commitment scheme $\mathsf{PBCOM} = (\mathsf{Com}, \mathsf{Dec})$.
3. A 4-round public-coin synchronous honest-extractable one-one non-malleable commitment scheme $\mathsf{NM} = (\mathcal{S}, \mathcal{R})$ that fixes the opening information in the second round.
4. A 2-round IDTC scheme $\mathsf{TC}_0 = (\mathsf{Sen}_0, \mathsf{Rec}_0, \mathsf{TFake}_0)$ for the $\mathcal{NP}$-language $L_0 = \{(\mathtt{com}, 0) : \exists\ \mathtt{dec}$ s.t. Dec accepts $\mathtt{dec}$ as a decommitment of $\mathtt{com}$ to $0\}$.
5. A 2-round IDTC scheme $\mathsf{TC}_1 = (\mathsf{Sen}_1, \mathsf{Rec}_1, \mathsf{TFake}_1)$ for the $\mathcal{NP}$-language $L_1 = \{(\mathtt{com}, 1) : \exists\ \mathtt{dec}$ s.t. Dec accepts $\mathtt{dec}$ as a decommitment of $\mathtt{com}$ to $1\}$.
6. A 4-round delayed-input public-coin Special HVZK $\mathsf{LS}_L = (\mathcal{P}_L, \mathcal{V}_L)$ for the $\mathcal{NP}$-language L that is adaptive-input PoK for the corresponding relation $\mathsf{Rel_L}$.
7. A 4-round delayed-input public-coin Special HVZK $\mathsf{LS}_{\mathsf{nm}} = (\mathcal{P}_{\mathsf{nm}}, \mathcal{V}_{\mathsf{nm}})$ for the following $\mathcal{NP}$-language

$$L_{\mathsf{nm}} = \{(\mathsf{vk}, \tau = (\mathtt{id}, \mathsf{nm}_1, \mathsf{nm}_2, \mathsf{nm}_3, \mathsf{nm}_4), s_1 : \exists(\mathsf{dec}_{\mathsf{nm}}, s_0, \sigma_1, \mathtt{msg}_1, \sigma_2, \mathtt{msg}_2) \text{ s.t.}$$
$$\mathsf{Ver}(\mathsf{vk}, \mathtt{msg}_1, \sigma_1) = 1 \text{ AND } \mathsf{Ver}(\mathsf{vk}, \mathtt{msg}_2, \sigma_2) = 1 \text{ AND } \mathtt{msg}_1 \neq \mathtt{msg}_2 \text{ AND}$$
$$\mathcal{R} \text{ accepts } (\mathtt{id}, s_0, \mathsf{dec}_{\mathsf{nm}}) \text{ as a valid decommitment of } \tau \text{ AND } s_0 \oplus s_1 = \sigma_1 || \sigma_2\}$$

that is adaptive-input PoK for the corresponding relation $\mathsf{Rel}_{\mathsf{L_{nm}}}$.
Informally by running $\mathsf{LS}_{\mathsf{nm}}$ one can prove that s_0 is committed using a non-malleable commitment and $s_0 \oplus s_1 = \sigma_1 || \sigma_2$. Moreover σ_1 and σ_2 are two signatures for two different messages w.r.t. the verification key vk.

3.3.1 Our protocol: NMZK.

We now give an high-level description of our delayed-input non-malleable ZK protocol depicted in Fig. 1. For a formal description see Fig. 2.

In the **first round** $\mathcal{V}_{\mathsf{NMZK}}$ computes a signature-verification key pair $(\mathsf{sk}, \mathsf{vk})$ and sends vk to $\mathcal{P}_{\mathsf{NMZK}}$. $\mathcal{V}_{\mathsf{NMZK}}$ computes and sends a (public coin) first rounds ls^1_L of LS_L and the first round $\mathsf{ls}^1_{\mathsf{nm}}$ of $\mathsf{LS}_{\mathsf{nm}}$. $\mathcal{V}_{\mathsf{NMZK}}$ computes and sends a first rounds nm_1 of NM. Finally, $\mathcal{V}_{\mathsf{NMZK}}$ computes and sends fresh first round of Naor's commitment for the remain tools listed above involved in the construction. To not overburden the notation we omit these additional messages.

In the **second round** $\mathcal{P}_{\mathsf{NMZK}}$ runs Com on input the bit 1 thus obtaining $\mathtt{com}, \mathtt{dec}$ (where $\mathtt{com}$ represents the statistically binding commitment of the bit 1). $\mathcal{P}_{\mathsf{NMZK}}$ sends $\mathtt{com}$, computes ls^2_L by running $\mathcal{P}_L$ on input ls^1_L and commits to it using the 2-round IDTC scheme TC_0. More precisely, $\mathcal{P}_{\mathsf{NMZK}}$ runs Sen_0 on input the instance $(\mathtt{com}, 0)$ and the message to be committed ls^2_L in order to compute the pair $(\mathtt{tcom}_0, \mathtt{tdec}_0)$ and sends $\mathtt{tcom}_0$. We observe that the commitment $\mathtt{tcom}_0$ is binding because of the Instance-Dependent Binding property enjoyed by the IDTC scheme (the instance used to compute $\mathtt{tcom}_0$ is false, indeed $\mathtt{com}$

is a commitment of the bit 1). $\mathcal{P}_{\mathsf{NMZK}}$ now runs the trapdoor procedure of the IDTC scheme TC_1. More precisely, $\mathcal{P}_{\mathsf{NMZK}}$ runs TFake_1 on input the instance $(\mathtt{com}, 1)$ to compute the pair $(\mathtt{tcom}_1, \mathtt{aux})$ and then sends $\mathtt{tcom}_1$ to $\mathcal{V}_{\mathsf{NMZK}}$. In this case $\mathtt{tcom}_1$ can be equivocated to any message using the trapdoor (the opening information of $\mathtt{com}$) due to the trapdoorness of the IDTC scheme. $\mathcal{P}_{\mathsf{NMZK}}$ now starts the procedure to commit to a random message s_0 using the non-malleable commitment NM. So, he runs $\mathcal{S}$ on input s_0, the identity $\mathtt{id}$ and nm_1 thus obtaining $(\mathsf{nm}_2, \mathsf{dec}_{\mathsf{nm}})$ and then sends nm_2.[12] In addition, $\mathcal{P}_{\mathsf{NMZK}}$ sends a random message $\mathtt{msg}$.

In the **third round** of the protocol $\mathcal{V}_{\mathsf{NMZK}}$, upon receiving $\mathtt{msg}$, computes and sends a signature σ of $\mathtt{msg}$ by running $\mathsf{Sign}(\mathsf{sk}, \mathtt{msg})$. Also, $\mathcal{V}_{\mathsf{NMZK}}$ computes and sends all the (public coin) second rounds ls^3_L, $\mathsf{ls}^3_{\mathsf{nm}}$, nm_3 for the respective protocols LS_L, $\mathsf{LS}_{\mathsf{nm}}$ and NM.

In the **fourth round** $\mathcal{P}_{\mathsf{NMZK}}$ checks whether or not σ is a valid signature for $\mathtt{msg}$ w.r.t. the verification key vk. In the negative case $\mathcal{P}_{\mathsf{NMZK}}$ aborts, otherwise he continues with the following steps. Upon receiving the instance x to be proven and the witness w s.t. $(x, w) \in \mathsf{Rel_L}$, $\mathcal{P}_{\mathsf{NMZK}}$ completes the transcript for LS_L thus obtaining ls^4_L by running the prover procedure $\mathcal{P}_L$ on input x, w and ls^3_L. At this point $\mathcal{P}_{\mathsf{NMZK}}$ can send the opening $(\mathtt{tdec}_0, \mathsf{ls}^2_L)$ for the commitment $\mathtt{tcom}_0$ computed in the second round, and completes an accepting transcript for LS_L for the instance x by sending ls^4_L.

$\mathcal{P}_{\mathsf{NMZK}}$ completes the commitment of s_0 by running $\mathcal{S}$ on input nm_3 thus obtaining nm_4 and sends nm_4. $\mathcal{P}_{\mathsf{NMZK}}$ picks a random string s_1, sets $x_{\mathsf{nm}} = (\mathsf{vk}, \mathtt{id}, (\mathsf{nm}_1, \mathsf{nm}_2, \mathsf{nm}_3, \mathsf{nm}_4), s_1)$ and runs the Special HVZK simulator of $\mathsf{LS}_{\mathsf{nm}}$ on input $(x_{\mathsf{nm}}, \mathsf{ls}^1_{\mathsf{nm}}, \mathsf{ls}^3_{\mathsf{nm}})$ thus obtaining $(\mathsf{ls}^2_{\mathsf{nm}}, \mathsf{ls}^4_{\mathsf{nm}})$. Now $\mathcal{P}_{\mathsf{NMZK}}$ opens the trapdoor commitment $\mathtt{tcom}_1$ to $\mathsf{ls}^2_{\mathsf{nm}}$. More precisely he runs TFake_1 on input $\mathtt{tk} = (\mathtt{dec})$, $\mathtt{tcom}_1$, $\mathtt{aux}$ and $\mathsf{ls}^2_{\mathsf{nm}}$ in order to compute $\mathtt{tdec}_1$ and sends $((\mathtt{tdec}_1, \mathsf{ls}^2_{\mathsf{nm}}), \mathsf{ls}^4_{\mathsf{nm}}, x_{\mathsf{nm}})$ to $\mathcal{V}_{\mathsf{NMZK}}$. We recall that $\mathtt{tcom}_1$ has been computed in *trapdoor mode* in the second round of the protocol using the instance $(\mathtt{com}, 1)$, where $\mathtt{com}$ is a valid commitment of the bit 1.

The verifier $\mathcal{V}_{\mathsf{NMZK}}$ **accepts** x iff the following conditions are satisfied.

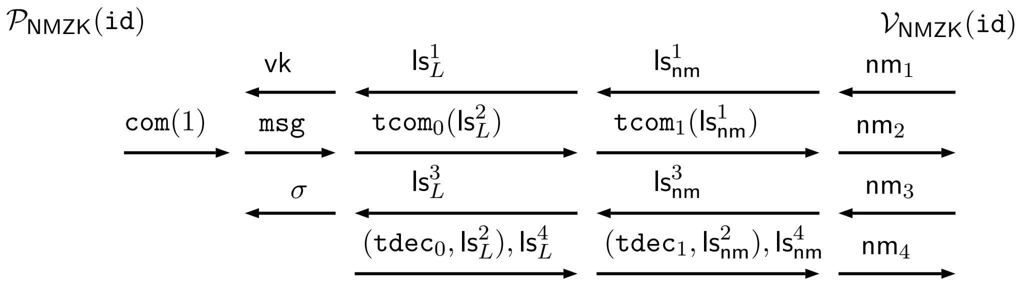
1. Rec_0 on input $(\mathtt{com}, 0), \mathtt{tcom}_0, (\mathtt{tdec}_0, \mathsf{ls}^2_L)$ accepts $(\mathsf{ls}^2_L, \mathtt{tdec}_0)$ as a decommitment of $\mathtt{tcom}_0$ (we recall that $(\mathtt{com}, 0)$ is the instance used to run the algorithms of the IDTC scheme TC_0).
2. Rec_1 on input $(\mathtt{com}, 1), \mathtt{tcom}_1, (\mathtt{tdec}_1, \mathsf{ls}^2_{\mathsf{nm}})$ accepts $(\mathsf{ls}^2_{\mathsf{nm}}, \mathtt{tdec}_1)$ as a decommitment of $\mathtt{tcom}_1$ (we recall that $(\mathtt{com}, 1)$ is the instance used to run the algorithms of the IDTC scheme TC_1).
3. $(\mathsf{ls}^1_L, \mathsf{ls}^2_L, \mathsf{ls}^3_L, \mathsf{ls}^4_L)$ is accepting for $\mathcal{V}_L$ with respect to the instance x.
4. $(\mathsf{ls}^1_{\mathsf{nm}}, \mathsf{ls}^2_{\mathsf{nm}}, \mathsf{ls}^3_{\mathsf{nm}}, \mathsf{ls}^4_{\mathsf{nm}})$ is accepting for $\mathcal{V}_{\mathsf{nm}}$ with respect to the instance x_{nm}.

The simulator extractor. Informally, the simulator $\mathsf{Sim}_{\mathsf{NMZK}}$ of our protocol interacts with the adversary $\mathcal{A}_{\mathsf{NMZK}}$ by emulating both the prover in the left session and polynomially many verifiers in the right sessions. In the left sessions $\mathsf{Sim}_{\mathsf{NMZK}}$ interacts with $\mathcal{A}_{\mathsf{NMZK}}$ as the honest verifiers do. While, in the right session for an instance $x \in L$ chosen adaptively by $\mathcal{A}_{\mathsf{NMZK}}$, $\mathsf{Sim}_{\mathsf{NMZK}}$ equivocates the commitment computed using TC_0, and runs the SHVZK simulator of LS_L to completes the transcript for LS_L w.r.t. the instance x. In order to use the trapdoor procedure of TC_0 the simulator has to commit to 0 in $\mathtt{com}$. In this way the commitment computed using TC_1 becomes statistically binding, therefore the transcript for $\mathsf{LS}_{\mathsf{nm}}$ needs to be computed using the honest prover procedure. We recall that the transcript for $\mathsf{LS}_{\mathsf{nm}}$ proves (in the sense of knowledge of a witness) that the message committed in NM contains a value s_0[13] such $s_0 \oplus s_1$ corresponds to two signatures (σ_1, σ_2)[14] of two different messages w.r.t. the verification key vk (s_1 is sent in clear in the fourth round). In order to compute these s_0 and s_1, $\mathsf{Sim}_{\mathsf{NMZK}}$ extracts two signatures for two different messages rewinding $\mathcal{A}_{\mathsf{NMZK}}$ from the third to the second round. We use this trick following [COSV16] in order to avoid any additional requirement. Indeed if the sender of NM is delayed-input (i.e. the message to be committed can be decided in the last round), then $\mathsf{Sim}_{\mathsf{NMZK}}$ can simply compute the first round of NM,

[12] We recall that the decommitment information of the non-malleable commitment scheme can be computed together with the second (the first in this informal description) round of the protocol.

[13] For ease of exposition we will simply say that $\mathcal{A}_{\mathsf{NMZK}}$ commits to two signatures using NM.

[14] W.l.o.g. we assume that the signatures σ_1, σ_2 include the signed messages.



Upon receiving x, w
s.t. $(x, w) \in \mathsf{Rel_L}$

- $\mathtt{com}$ is the commitment of the bit 1 computed using PBCOM.
- vk is a a verification key of a signature scheme and σ is a valid signature of the message $\mathtt{msg}$.
- $\tau_L = (\mathsf{ls}_L^1, \mathsf{ls}_L^2, \mathsf{ls}_L^3, \mathsf{ls}_L^4)$ is the transcript output from the honest prover procedure of LS_L proving knowledge of the witness for $x \in L$.
- $\mathtt{tcom}_0$ is the commitment of ls_L^2 computed using the honest sender procedure of the IDTC TC_0. $\mathtt{tdec}_0$ represents the decommitment information of $\mathtt{tcom}_0$.
- $\mathtt{tcom}_1$ is computed using the trapdoor procedure of TC_1. $\mathtt{tdec}_1$ represents the decommitment information (computed using the trapdoor procedure of TC_1) of $\mathtt{tcom}_1$ w.r.t. the message $\mathsf{ls}_{\mathsf{nm}}^2$.
- s_0, s_1 are random strings.
- $\tau = (\mathtt{id}, \mathsf{nm}_1, \mathsf{nm}_2, \mathsf{nm}_3, \mathsf{nm}_4)$ represents the transcript of $\langle \mathcal{S}(s_0), \mathcal{R}\rangle(\mathtt{id})$ that is, a commitment of the message s_0 computed using the synchronous honest-extractable one-one non-malleable commitment scheme NM.
- $\tau_{\mathsf{nm}} = (\mathsf{ls}_{\mathsf{nm}}^1, \mathsf{ls}_{\mathsf{nm}}^2, \mathsf{ls}_{\mathsf{nm}}^3, \mathsf{ls}_{\mathsf{nm}}^4)$ is the transcript output of a Special HVZK simulator of $\mathsf{LS}_{\mathsf{nm}}$ proving knowledge of a decommitment of τ to the message s_0 s.t. $s_0 \oplus s_1 = \sigma_1 || \sigma_2$ where σ_1, σ_2 are two signatures of two different messages w.r.t vk.

Figure 1: Our 4-round delayed-input NMZK AoK

extract the signatures, and compute the last round of NM committing to the signatures. It is important to observe that even though the non-malleable commitment scheme of [GPR16] fixes the message to be committed in the third round, there is no guarantee that the scheme is secure against an adversary that adaptively chooses the challenge messages in the last round of the non-malleability security game. Therefore, even though the completeness of our scheme would work without using the trick of [COSV16], it would be unclear, in this case, how to prove the security of our scheme. A formal description of $\mathsf{Sim}_{\mathsf{NMZK}}$ can be found in the proof of Theorem 1.

3.4 Construction of Delayed-Input NMZK from OWFs

The formal construction of our delayed-input NMZK AoK $\mathsf{NMZK} = (\mathcal{P}_{\mathsf{NMZK}}, \mathcal{V}_{\mathsf{NMZK}})$ is showed in Fig. 2. The security proof of our protocol follows below.

Theorem 1. *If OWFs exist, then* NMZK *is a delayed-input NMZK AoK for* $\mathcal{NP}$.

Proof. We divide the security proof in two parts, proving that NMZK enjoys delayed-input completeness and non-malleable ZK; the proof of NMZK is divided in two additional lemmas, one for each of the two properties of Def. 2. Before that, we recall that $\mathsf{LS}_{\mathsf{nm}}$ and LS_L can be constructed from OWFs (see App. A) as well as Σ using [Rom90]. The 4-round public-coin synchronous honest-extractable non-malleable commitment scheme NM can be instantiated from OWFs (see Sec. 3.1), as well as PBCOM [Nao91]. We also observe that if PBCOM relies on OWFs, then also TC_0 and TC_1 can be constructed from OWFs (see [COSV17b]).

(Delayed-Input) Completeness. The completeness follows directly from the completeness of $\mathsf{LS}_{\mathsf{nm}}$ and LS_L, the correctness of PBCOM, NM, TC_0, TC_1 and the validity of Σ. We observe that, due to the delayed-input property of LS_L, the statement x and the respective witness w are needed by $\mathcal{P}_{\mathsf{NMZK}}$ only to compute the last round; therefore NMZK enjoys delayed-input completeness as well.

(Delayed-Input) NMZK. Following Definition 2 we start by describing how the simulator $\mathsf{Sim}_{\mathsf{NMZK}}$ for NMZK works. In the left session $\mathsf{Sim}_{\mathsf{NMZK}}$ interacts with the MiM adversary $\mathcal{A}_{\mathsf{NMZK}}$ in the following way. Upon receiving the first round, vk, ls^1_L, $\mathsf{ls}^1_{\mathsf{nm}}$, nm_1, ρ_0, ρ_1, ρ, from $\mathcal{A}_{\mathsf{NMZK}}$, $\mathsf{Sim}_{\mathsf{NMZK}}$ runs Com on input ρ and the message 0 in order to obtain $\mathtt{com}, \mathtt{dec}$. Furthermore, $\mathsf{Sim}_{\mathsf{NMZK}}$ on input $\mathsf{ls}^1_{\mathsf{nm}}$ computes $\mathsf{ls}^2_{\mathsf{nm}}$ by running $\mathcal{P}_{\mathsf{nm}}$ and commits to it using the IDTC scheme TC_1. More precisely, $\mathsf{Sim}_{\mathsf{NMZK}}$ runs Sen_1 on input the instance $((\rho, \mathtt{com}), 0)$, ρ_1 and message to be committed $\mathsf{ls}^2_{\mathsf{nm}}$ thus obtaining the pair $(\mathtt{tcom}_1, \mathtt{tdec}_1)$. Then $\mathsf{Sim}_{\mathsf{NMZK}}$ runs the trapdoor procedure of the IDTC scheme TC_0. That is, $\mathsf{Sim}_{\mathsf{NMZK}}$ runs TFake_0 on input the instance $((\rho, \mathtt{com}), 0)$ and ρ_0 thus obtaining the pair $(\mathtt{tcom}_0, \mathtt{aux})$. $\mathsf{Sim}_{\mathsf{NMZK}}$, in order to commit to a random message s_0 uses the scheme NM, runs $\mathcal{S}$ on input nm_1, the identity $\mathtt{id}$ and s_0 thus obtaining nm_2. $\mathsf{Sim}_{\mathsf{NMZK}}$ sends $\mathtt{tcom}_0, \mathtt{tcom}_1, \mathsf{nm}_2, \mathtt{com}$ and a random message $\mathtt{msg}_1$ to $\mathcal{A}_{\mathsf{NMZK}}$.

Upon receiving the third round (ls^3_L, $\mathsf{ls}^3_{\mathsf{nm}}$, nm_3, σ_1) and the instance x to be proved from $\mathcal{A}_{\mathsf{NMZK}}$, the simulator checks whether or not σ_1 is a valid signature for $\mathtt{msg}_1$ w.r.t. the verification key vk. In the negative case $\mathsf{Sim}_{\mathsf{NMZK}}$ aborts, otherwise he rewinds $\mathcal{A}_{\mathsf{NMZK}}$ from the third to the second round in order to obtain a second signature σ_2 for a different message $\mathtt{msg}_2$. After the extraction of the signatures $\mathsf{Sim}_{\mathsf{NMZK}}$ returns to the execution that he started before the rewinding procedure (that we will denote as the main thread) and computes the fourth round as follows[15].

$\mathsf{Sim}_{\mathsf{NMZK}}$ completes the commitment of s_0 by running $\mathcal{S}$ on input nm_3 thus obtaining $(\mathsf{nm}_4, \mathsf{dec}_{\mathsf{nm}})$ and then sending nm_4. Furthermore, $\mathsf{Sim}_{\mathsf{NMZK}}$ sets s_1 s.t. $s_1 = (\sigma_1 || \sigma_2) \oplus s_0$, $x_{\mathsf{nm}} = (\mathsf{vk}, \mathtt{id}, \mathsf{nm}_1, \mathsf{nm}_2, \mathsf{nm}_3, \mathsf{nm}_4, s_1)$, $w_{\mathsf{nm}} = (\mathsf{dec}_{\mathsf{nm}}, s_0, \sigma_1, \mathtt{msg}_1, \sigma_2, \mathtt{msg}_2)$ and completes the transcript for $\mathsf{LS}_{\mathsf{nm}}$ thus obtaining $\mathsf{ls}^4_{\mathsf{nm}}$ by running the prover procedure $\mathcal{P}_{\mathsf{nm}}$ on input x_{nm}, w_{nm} and $\mathsf{ls}^3_{\mathsf{nm}}$. Then the values $((\mathtt{tdec}_1, \mathsf{ls}^2_{\mathsf{nm}}), \mathsf{ls}^4_{\mathsf{nm}}, x_{\mathsf{nm}})$ are sent to $\mathcal{A}_{\mathsf{NMZK}}$. At this point $\mathsf{Sim}_{\mathsf{NMZK}}$ runs the SHVZK simulator of LS_L on input $(x, \mathsf{ls}^1_L, \mathsf{ls}^3_L)$ thus obtaining $(\mathsf{ls}^2_L, \mathsf{ls}^4_L)$. Then $\mathsf{Sim}_{\mathsf{NMZK}}$ opens, using the trapdoor procedure, the commitment $\mathtt{tcom}_0$ to ls^2_L. More precisely, $\mathsf{Sim}_{\mathsf{NMZK}}$ runs TFake_0 on input $\mathtt{tk} = (\mathtt{dec})$, ρ_0, $\mathtt{tcom}_0$, $\mathtt{aux}$ and ls^2_L in order to compute $\mathtt{tdec}_0$ and sends $((\mathtt{tdec}_0, \mathsf{ls}^2_L), \mathsf{ls}^4_L, x)$ to $\mathcal{A}_{\mathsf{NMZK}}$.

At the end of the execution $\mathsf{Sim}_{\mathsf{NMZK}}$ outputs $\mathcal{A}_{\mathsf{NMZK}}$'s view in the main thread. Furthermore, he uses the extractor of LS_L to extract and output, from the $\mathsf{poly}(\lambda)$ right sessions, the witnesses $\tilde{w}_1, \dots, \tilde{w}_{\mathsf{poly}(\lambda)}$ used by $\mathcal{A}_{\mathsf{NMZK}}$ to compute the transcript of LS_L (the witnesses correspond to statements $\tilde{x}_i$ proved by $\mathcal{A}_{\mathsf{NMZK}}$ in the i-th right session, for $i = 1, \dots, \mathsf{poly}(\lambda)$).

Lemma 1. $\{\mathsf{Sim}_{\mathsf{NMZK}}{}^1(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*} \approx \{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$, *where* $\mathsf{Sim}_{\mathsf{NMZK}}{}^1(1^\lambda, z)$ *denotes the 1st output of* $\mathsf{Sim}_{\mathsf{NMZK}}$.

In order to prove the above lemma we consider the series of hybrid experiments described below. In the proof we denote with $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_i}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ the random variable that describes the view of $\mathcal{A}_{\mathsf{NMZK}}$ in the hybrid $\mathcal{H}_i(1^\lambda, z)$. Let p the probability that in the real execution $\mathcal{A}_{\mathsf{NMZK}}$ completes the left session.

- We start considering the hybrid experiment $\mathcal{H}_0(1^\lambda, z)$ in which in the left session $\mathcal{P}_{\mathsf{NMZK}}$ interacts with $\mathcal{A}_{\mathsf{NMZK}}$ and in the i-th right session $\mathcal{V}_{\mathsf{NMZK}_i}$ interacts with $\mathcal{A}_{\mathsf{NMZK}}$, for $i = 1, \dots, \mathsf{poly}(\lambda)$. Note that $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_0}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*} = \{\mathsf{View}^{\mathcal{A}}_{\mathsf{NMZK}}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$.

[15] Note that it is possible to complete the main thread, due to the delayed-input of $\mathsf{LS}_{\mathsf{nm}}$, and to the fact that we do not need to change the first round of NM (that is, we do not need to change the committed message s_0) in order to have $x_{\mathsf{nm}} \in L_{\mathsf{nm}}$.

- The hybrid experiment $\mathcal{H}_1(1^\lambda, z)$ differs from $\mathcal{H}_0(1^\lambda, z)$ only in the fact that in the left session of $\mathcal{H}_1(1^\lambda, z)$ $\mathcal{A}_{\mathsf{NMZK}}$ is rewound from the third to the second round, in order to extract two signatures σ_1, σ_2 for two distinct messages $(\mathtt{msg}_1, \mathtt{msg}_2)$ w.r.t. a verification key vk. Note that after $1/p$ rewinds the probability of not obtaining a valid new signature is less than $1/2$. Therefore the probability that $\mathcal{A}_{\mathsf{NMZK}}$ does not give a second valid signature for a randomly chosen message after λ/p rewinds is negligible in λ. For the above reason the procedure of extraction of signatures of different messages in $\mathcal{H}_1(1^\lambda, z)$ succeeds except with negligible probability. Observe that the above deviation increases the abort probability of the experiment only by a negligible amount, therefore $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_0}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*} \equiv_s \{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_1}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$.
- The hybrid experiment $\mathcal{H}_2(1^\lambda, z)$ differs from $\mathcal{H}_1(1^\lambda, z)$ only in the message committed using NM. Indeed $\mathcal{P}_{\mathsf{NMZK}}$ commits using NM to two signatures σ_1, σ_2 of two distinct messages $(\mathtt{msg}_1, \mathtt{msg}_2)$ instead of a random message. In more details, $\mathcal{P}_{\mathsf{NMZK}}$ commits to a random string s_0 using NM and in the 4th round sets and sends $s_1 = (\sigma_1 || \sigma_2) \oplus s_0$, instead of sending $s_1 \leftarrow \{0,1\}^\lambda$. Observe that the procedure to extract the signatures succeeds in $\mathcal{H}_2(1^\lambda, z)$ with overwhelming probability, because the first three rounds are played exactly as in $\mathcal{H}_1(1^\lambda, z)$. Now we can claim that $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_2}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ and $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_1}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ are computationally indistinguishable by using the computational hiding property of NM. Suppose by contradiction that there exist an adversary $\mathcal{A}_{\mathsf{NMZK}}$ and a distinguisher $\mathcal{D}_{\mathsf{ZK}}$ such that $\mathcal{D}_{\mathsf{ZK}}$ distinguishes $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_1}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ from $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_2}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$. Then we can construct an adversary $\mathcal{A}_{\mathsf{Hiding}}$ that breaks the computational hiding of NM in the following way. $\mathcal{A}_{\mathsf{Hiding}}$ sends to the challenger of the hiding game $\mathcal{C}_{\mathsf{Hiding}}$ two random messages (m_0, m_1). Then, in the left session $\mathcal{A}_{\mathsf{Hiding}}$ acts as in $\mathcal{H}_2(1^\lambda, z)$ (and $\mathcal{H}_1(1^\lambda, z)$) except for messages of NM for which he acts as proxy between $\mathcal{C}_{\mathsf{Hiding}}$ and $\mathcal{A}_{\mathsf{NMZK}}$. When $\mathcal{A}_{\mathsf{Hiding}}$ computes the last round of the left session he sets and sends $s_1 = \sigma_1 || \sigma_2 \oplus m_0$. In the right sessions $\mathcal{A}_{\mathsf{Hiding}}$ interacts with $\mathcal{A}_{\mathsf{ZK}}$ acting as $\mathcal{V}_{\mathsf{NMZK}}$ does. At the end of the execution $\mathcal{A}_{\mathsf{Hiding}}$ runs $\mathcal{D}_{\mathsf{ZK}}$ and outputs what $\mathcal{D}_{\mathsf{ZK}}$ outputs. It easy to see that if $\mathcal{C}_{\mathsf{Hiding}}$ commits to m_1 then, $\mathcal{A}_{\mathsf{ZK}}$ acts as in $\mathcal{H}_1(1^\lambda, z)$, otherwise he acts as in $\mathcal{H}_2(1^\lambda, z)$. Note that the reduction to the hiding property of NM is possible because the rewinds to extract a second signature do not affect the execution with the challenger of NM that remains straight-line.
- The hybrid experiment $\mathcal{H}_3(1^\lambda, z)$ differs from $\mathcal{H}_2(1^\lambda, z)$ in the way the transcript of $\mathsf{LS}_{\mathsf{nm}}$ is computed. More precisely, the prover $\mathcal{P}_{\mathsf{nm}}$ of $\mathsf{LS}_{\mathsf{nm}}$ is used to compute the messages $\mathsf{ls}^2_{\mathsf{nm}}$ and $\mathsf{ls}^4_{\mathsf{nm}}$ instead of the Special HVZK simulator. Note that due to the delayed-input property of $\mathsf{LS}_{\mathsf{nm}}$ the statement $x_{\mathsf{nm}} = (\mathsf{vk}, \mathsf{nm}_1, \mathsf{nm}_2, \mathsf{nm}_3, \mathsf{nm}_4, s_1)$ and the witness $w_{\mathsf{nm}} = (\mathtt{dec}_{\mathsf{nm}}, s_0, \sigma_1, \mathtt{msg}_1, \sigma_2, \mathtt{msg}_2)$ are needed by $\mathcal{P}_{\mathsf{nm}}$ only to compute $\mathsf{ls}^4_{\mathsf{nm}}$ and are not needed to compute $\mathsf{ls}^2_{\mathsf{nm}}$. Observe that the procedure of extraction of the signatures succeeds in $\mathcal{H}_3(1^\lambda, z)$ with overwhelming probability because before that the 4th round of the left session is played the view of $\mathcal{A}_{\mathsf{NMZK}}$ is identically distributed to the view of $\mathcal{H}_2(1^\lambda, z)$. From the Special HVZK of $\mathsf{LS}_{\mathsf{nm}}$ it follows that $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_2}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ and $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_3}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ are computationally indistinguishable.
- The hybrid $\mathcal{H}_4(1^\lambda, z)$ differs from $\mathcal{H}_3(1^\lambda, z)$ in the way the commitment $(\rho_1, \mathtt{com}_1)$ of TC_1 and the corresponding decommitment information are computed. In more details, the values $\mathtt{tcom}_1, \mathtt{tdec}_1$ are computed by running Sen_1 instead of TFake_1. Observe that the procedure of extraction of the signatures succeeds in $\mathcal{H}_4(1^\lambda, z)$ with overwhelming probability otherwise we can break the hiding of TC_1. From the trapdoorness of TC_1 it follows that $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_4}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ and $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_3}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ are computationally indistinguishable.
- The hybrid $\mathcal{H}_5(1^\lambda, z)$ differs from $\mathcal{H}_4(1^\lambda, z)$ in the message committed using PBCOM. In the left session $\mathcal{P}_{\mathsf{NMZK}}$ commits to 0 using PBCOM, instead of committing to 1. Observe that the procedure of extraction of the signatures succeeds in $\mathcal{H}_5(1^\lambda, z)$ with overwhelming probability otherwise we can break the hiding of PBCOM. From the hiding of PBCOM it also follows that $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_5}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ and $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_4}(1^\lambda, z)\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ are computationally indistinguishable.
- The hybrid $\mathcal{H}_6(1^\lambda, z)$ differs from $\mathcal{H}_5(1^\lambda, z)$ in the way the commitment $(\rho_0, \mathtt{com}_0)$ of TC_0 and the corre-

sponding decommitment information are computed. More precisely, TFake_0 is run to compute a commitment $(\rho_0, \mathtt{tcom}_0)$ and the corresponding decommitment information $\mathtt{tdec}_0$ with respect to the message ls^2_L. Observe that the procedure of extraction of the signatures succeeds in $\mathcal{H}_6(1^\lambda, z)$ with overwhelming probability otherwise we can break the hiding of TC_0. From the trapdoorness of the 2-round instance-dependent trapdoor commitment scheme TC_0 it follows that $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_6}(1^\lambda, z)\}_{\lambda\in\mathbb{N}, z\in\{0,1\}^*}$ and $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_5}(1^\lambda, z)\}_{\lambda\in\mathbb{N}, z\in\{0,1\}^*}$ are computationally indistinguishable.

- The hybrid $\mathcal{H}_7(1^\lambda, z)$ differs from $\mathcal{H}_6(1^\lambda, z)$ in the way the transcript of LS_L is computed. More precisely, the Special HVZK simulator of LS_L is used to compute the messages ls^2_L and ls^4_L instead of the honest prover procedure. Observe that the procedure of extraction of the signatures succeeds in $\mathcal{H}_7(1^\lambda, z)$ with overwhelming probability because before that the 4th round of the left session is played the view of $\mathcal{A}_{\mathsf{NMZK}}$ is identically distributed to the view that $\mathcal{A}_{\mathsf{NMZK}}$ has in $\mathcal{H}_6(1^\lambda, z)$. From the Special HVZK of LS_L it follows that $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_6}(1^\lambda, z)\}_{\lambda\in\mathbb{N}, z\in\{0,1\}^*}$ and $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_7}(1^\lambda, z)\}_{\lambda\in\mathbb{N}, z\in\{0,1\}^*}$ are computationally indistinguishable. Note that $\mathcal{H}_7(1^\lambda, z)$ corresponds to the simulated experiment, that is $\{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_7}(1^\lambda, z)\}_{\lambda\in\mathbb{N}, z\in\{0,1\}^*} = \{S^1(1^\lambda, z)\}_{\lambda\in\mathbb{N}, z\in\{0,1\}^\star}$.

The proof ends with the observation that for all $\lambda \in \mathbb{N}, z \in \{0,1\}^*$ it holds that: $\{\mathsf{View}^{\mathcal{A}}_{\mathsf{NMZK}}(1^\lambda, z)\}_{\lambda,z} = \{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_0}(1^\lambda, z)\}_{\lambda,z} \approx \cdots \approx \{\mathsf{View}^{\mathcal{A}_{\mathsf{NMZK}}}_{\mathcal{H}_7}(1^\lambda, z)\}_{\lambda,z} = \{S^1(1^\lambda, z)\}_{\lambda,z}$

Lemma 2. *Let $\tilde{x}_1, \ldots, \tilde{x}_{\mathsf{poly}(\lambda)}$ be the right-session statements appearing in $\mathsf{View} = {\mathsf{Sim}_{\mathsf{NMZK}}}^1(1^\lambda, z)$ and let $\mathtt{id}$ be the identity of the left session and $\tilde{\mathtt{id}}_1, \ldots, \tilde{\mathtt{id}}_{\mathsf{poly}(\lambda)}$ be the identities of right sessions appearing in View. If the i-th right session is accepting and $\mathtt{id} \neq \tilde{\mathtt{id}}_i$ for $i \in \{1, \ldots, \mathsf{poly}(\lambda)\}$, then except with negligible probability, the second output of $\mathsf{Sim}_{\mathsf{NMZK}}(1^\lambda, z)$ is $\tilde{w}_i$ such that $(\tilde{x}_i, \tilde{w}_i) \in \mathsf{Rel_L}$ for $i \in \{1, \ldots, \mathsf{poly}(\lambda)\}$.*

In order to simplify the security proof, here we actually consider the notion of *multi-SHVZK*, *multi-trapdoorness* and *multi-hiding* instead of Special HVZK, trapdoorness and hiding. The only difference between multi-SHVZK and the classical notion of Special HVZK is the following. Let $(\mathsf{ls}^1, \mathsf{ls}^3, x)$ be a challenge of the Special HVZK security game. The challenger of multi-SHVZK picks a random bit b and computes an accepting transcript $t = (\mathsf{ls}^1, \mathsf{ls}^2, \mathsf{ls}^3, \mathsf{ls}^4)$ for x. If $b = 0$ then t has been computed using the prover procedure, otherwise it has been computed using the Special HVZK simulator. The adversary, upon receiving t, either outputs his guess $b' \in \{0,1\}$, or asks to receive another transcript t according to a new possible challenge $(\mathsf{ls}^{1'}, \mathsf{ls}^{3\prime}, x')$. The adversary can ask a polynomial number of transcripts according to different challenges before that he outputs b'. The adversary is successful if $\mathrm{Prob}[\, b = b' \,] - 1/2$ is non-negligible in the security parameter. It is easy to see that a protocol is Special HVZK iff it is multi-SHVZK.

Analogously, the only difference with the definition of trapdoorness given in Definition 4 and multi-trapdoorness is the following. Let (x, ρ, m) and $\mathtt{tk}$ s.t. $(x, \mathtt{tk}) \in \mathsf{Rel}_{L_\Sigma}$ be the challenge message used by the adversary to interact with the challenger. The challenger of multi-trapdoorness picks a random bit b and computes a commitment $\mathtt{com}$ and the corresponding decommitment information $(m, \mathtt{dec})$. If $b = 0$ then both $\mathtt{com}$ and $(m, \mathtt{dec})$ are computed using the trapdoor procedure $\mathsf{TFake}_{\boldsymbol{\Sigma}}$, otherwise they are computed using the honest procedure $\mathsf{Sen}_{\boldsymbol{\Sigma}}$. The adversary, upon receiving $\mathtt{com}$ and $(m, \mathtt{dec})$, either outputs his guess $b' \in \{0,1\}$, or asks to start another interaction against the challenger by using possible different (x', ρ', m') and $\mathtt{tk}'$. Note that the adversary can start a polynomial number of other interactions using different (x', ρ', m') before he outputs b'. The adversary is successful if $\mathrm{Prob}[\, b = b' \,] - 1/2$ is non-negligible in the security parameter. It is easy to see that a commitment scheme enjoys the multi-trapdoorness property iff it enjoys the trapdoorness property.

Analogously, we define the notion of multi-hiding. The only difference with the classical definition of hiding is the following. Let m_0 and m_1 be the challenge messages. The challenger of multi-hiding picks a random bit b and computes the commitment of m_b. The adversary, upon receiving the commitment, either outputs his guess $b' \in \{0,1\}$, or sends (m'_0, m'_1) and asks to receive a commitment of m'_b where (m'_0, m'_1) represents a new pair of challenge messages (the latter step can be executed up to a polynomial number of times). The adversary is successful if Prob $[\, b = b' \,] - 1/2$ is non-negligible in the security parameter. It is easy to see that a commitment scheme is hiding iff it is multi-hiding. Let us now proceed with the security proof of Lemma 2. We start the proof assuming that the adversary is synchronous. In the end of the security proof we show how to deal with an asynchronous adversary as well.

We now consider the hybrid experiments $\mathcal{H}_7, \dots, \mathcal{H}_0$ described in Lemma 1. We show that if the adversary completes at least one right session in $\mathcal{H}_7$ with non-negligible probability then, for each hybrid experiment $\mathcal{H}_i$ with $i = 0, \dots, 7$ there exists a procedure that extracts (except with negligible probability) the signatures for one of the verification keys used in the right sessions, thus reaching a contradiction in $\mathcal{H}_0$.[16] In the hybrid experiments $\mathcal{H}_7, \dots, \mathcal{H}_0$ we denote as the *main thread* the execution of the hybrid that comes before that the adversary is rewound.

- The hybrid experiment $\mathcal{H}_7$ corresponds to the simulated game. Assume by contradiction that Lemma 2 does not hold, then with non-negligible probability there exists a right session j that is accepting but the simulator $\mathsf{Sim}_{\mathsf{NMZK}}$ does not output a valid witness $\tilde{w}_j$ for the statement $\tilde{x}_j$ proved by the adversary. Since the probability that the adversary provides an accepting transcript for $\tilde{x}_j$ is non-negligible then $\mathsf{Sim}_{\mathsf{NMZK}}$ rewinds the adversary by sending randomly generated third rounds for LS_L until another accepting transcript for NMZK is received[17]. Given that we are assuming (by contradiction) that $\mathsf{Sim}_{\mathsf{NMZK}}$ does not extract $\tilde{w}_j$, then the accepting transcript for $\tilde{x}_j$ contains, in the last round, a value $\tilde{\mathsf{ls}}^{2'}_{L,j} \neq \tilde{\mathsf{ls}}^{2}_{L,j}$, where $\tilde{\mathsf{ls}}^{2}_{L,j}$ is the second round of LS_L that has been sent in the fourth round of NMZK by the adversary in the main thread. In summary, the expected polynomial-time extraction of $\tilde{w}_j$ can fail only when the adversary is able to open $\tilde{\mathtt{tcom}}_{0,j}$ to different values. We recall that the only hope to extract $\tilde{w}_j$ is to have two accepting transcript for LS_L such that $(\tilde{\mathsf{ls}}^{1}_{L,j}, \tilde{\mathsf{ls}}^{2}_{L,j}, \tilde{\mathsf{ls}}^{3}_{L,j}, \tilde{\mathsf{ls}}^{4}_{L,j})$ and $(\tilde{\mathsf{ls}}^{1}_{L,j}, \tilde{\mathsf{ls}}^{2}_{L,j}, \tilde{\mathsf{ls}}^{3'}_{L,j}, \tilde{\mathsf{ls}}^{4'}_{L,j})$ with $\tilde{\mathsf{ls}}^{3}_{L,j} \neq \tilde{\mathsf{ls}}^{3'}_{L,j}$. If the adversary is able to open to different values of $\tilde{\mathsf{ls}}^{2}_{L,j}$ during the rewinding procedure then, by the Special Binding property of TC_0 it holds that the value committed in $\tilde{\mathtt{com}}_j$ is 0. Then, by the Instance-Dependent Binding property of TC_1, we obtain that $\tilde{\mathtt{tcom}}_{1,j}$ is statistically binding. This chain of implications in turn implies that during the rewinds the value opened with respect to $\tilde{\mathtt{tcom}}_{1,j}$ in the fourth round stays the same.

  The crucial observation is that in this right session it is possible to run the adaptive-input PoK extractor of $\mathsf{LS}_{\mathsf{nm}}$ thus obtaining two accepting transcripts for $\mathsf{LS}_{\mathsf{nm}}$ that share the same first two rounds ($\tilde{\mathsf{ls}}^{1}_{\mathsf{nm},j}$ and $\tilde{\mathsf{ls}}^{2}_{\mathsf{nm},j}$). Therefore, with non-negligible probability, it is possible to extract the decommitment information of the non-malleable commitment NM and compute $\tilde{s}^j_0 \oplus \tilde{s}^j_1 = \tilde{\sigma}^j_1 || \tilde{\sigma}^j_2$, where $(\tilde{\sigma}^j_1, \tilde{\sigma}^j_2)$ are two valid signatures with respect to the verification key $\tilde{\mathsf{vk}}_j$. Thus we can conclude this part of the security proof claiming that if $\mathsf{Sim}_{\mathsf{NMZK}}$ fails in extracting $\tilde{w}_j$ then 1) two signatures for $\tilde{\mathsf{vk}}_j$ can be extracted with non-negligible probability and 2) the non-malleable commitment is well formed.

- In the hybrid experiment $\mathcal{H}_6$ the prover procedure is used instead of the Special HVZK simulator to compute $(\mathsf{ls}^1_L, \mathsf{ls}^2_L, \mathsf{ls}^3_L, \mathsf{ls}^4_L)$. We observe that, with non-negligible probability, there still exists a right session j where the signatures for the verification key $\tilde{\mathsf{vk}}_j$ can be extracted using the adaptive-input PoK extractor of $\mathsf{LS}_{\mathsf{nm}}$ and moreover the adaptive-PoK extractor of LS_L fails, otherwise a reduction to the multi-SHVZK of LS_L can be done.

  We now define a different extraction procedure $\mathtt{ExtCom}$ that works as follows.

[16]We recall that we have already proven in Lemma 1 that the probabilities of $\mathcal{A}_{\mathsf{NMZK}}$ to complete a right session in $\mathcal{H}_0$ and $\mathcal{H}_7$ are negligible close.

[17]We assume that the adaptive-input PoK extractor of LS_L works by rewinding the adversary by sending randomly generated third round ls^3_L. See the beginning of the security proof (the part referred to *(Delayed-Input) NMZK)* of Theorem 1 for the precise description of $\mathsf{Sim}_{\mathsf{NMZK}}$.

- In the **right sessions** $\mathtt{ExtCom}$ runs the adaptive-input PoK extractor for both LS_L and $\mathsf{LS}_{\mathsf{nm}}$. More precisely, $\mathtt{ExtCom}$ keeps fixed the first two rounds of the main thread and sends multiple random values $\tilde{\mathsf{ls}}^3_L$ and $\tilde{\mathsf{ls}}^3_{\mathsf{nm}}$.
- In the **left session** $\mathtt{ExtCom}$ acts similarly to $\mathsf{Sim}_{\mathsf{NMZK}}$ differing only in the way ls^2_L is computed during the rewinds. More precisely, for any new message ls^3_L received by the adversary, $\mathtt{ExtCom}$ opens $\mathtt{tcom}_0$ always to the same ls^2_L and provides an accepting transcript $(\mathsf{ls}^1_L, \mathsf{ls}^2_L, \mathsf{ls}^3_L, \mathsf{ls}^4_L)$ using the honest prover procedure (as it has been done in the main thread).

We now show that $\mathtt{ExtCom}$ is able to extract the decommitment information of the non-malleable commitment that allows to compute two valid signatures. More precisely, we argue that during the rewinds the adversary is forced to open $\tilde{\mathtt{tcom}}_{1,j}$ always to the same 2nd round of $\mathsf{LS}_{\mathsf{nm}}$ ($\tilde{\mathsf{ls}}^2_{\mathsf{nm},j}$) in some right-session j. This holds because of the following chain of implications. In $\mathcal{H}_7$ we have showed that there exists a right session j where, with non-negligible probability, the adaptive-input PoK extractor for $\mathsf{LS}_{\mathsf{nm}}$ can compute a valid decommitment information for the non-malleable commitment such that two valid signatures can be extracted. Moreover, we have proved that in this session j the extractor for LS_L fails. As discussed above this implies that $\tilde{\mathtt{com}}_j$ contains 0 and that $\tilde{\mathtt{tcom}}_{1,j}$ is statistically binding. Since $\mathtt{ExtCom}$ keeps fixed the first two rounds of the main thread during the rewinds, the adversary has no way to use the trapdoor procedure to compute $\tilde{\mathtt{tcom}}_{1,j}$ once that $\tilde{\mathtt{com}}_j$ contains 0. Moreover, the adversary cannot selectively abort some of the right sessions since the values $(\mathsf{ls}^1_L, \mathsf{ls}^2_L, \mathsf{ls}^3_L, \mathsf{ls}^4_L)$ are identically distributed to values that are sent in the main thread of $\mathcal{H}_6$. These observations make us able to claim that $\mathtt{ExtCom}$ can extract, with non-negligible probability, a valid decommitment information of the non-malleable commitment that allows to compute two signatures for $\tilde{\mathsf{vk}}_j$. We observe that $\mathtt{ExtCom}$ could be able to extract $\tilde{w}_j$ as well in this case, but it would just ignore the problem.

- The next hybrid that we consider is $\mathcal{H}_5$. This hybrid experiment is the same as $\mathcal{H}_6$ with the difference that $\mathtt{tcom}_0$ is computed using the commitment procedure (and not the trapdoor procedure). We observe that the transcript for LS_L in $\mathcal{H}_6$ is computed using the honest prover procedure, therefore the trapdoor procedure is not needed anymore to provide an accepting transcript with respect to LS_L. We claim that the output of $\mathtt{ExtCom}$ for $\mathcal{H}_5$ is the same as the output of $\mathtt{ExtCom}$ for $\mathcal{H}_6$. If this is not true then a reduction to the trapdoorness of $\mathtt{tcom}_0$ can be done.

- The next hybrid that we consider is $\mathcal{H}_4$. The difference between $\mathcal{H}_5$ and $\mathcal{H}_4$ is that the value committed in $\mathtt{com}$ is 1. We observe that also in this case the output of $\mathtt{ExtCom}$ stays the same otherwise a reduction to the hiding of $\mathtt{com}$ can be done.

- The next hybrid that we consider is $\mathcal{H}_3$. The difference between $\mathcal{H}_4$ and $\mathcal{H}_3$ is that $\mathtt{tcom}_1$ is computed using the trapdoor procedure. We claim that the output of $\mathtt{ExtCom}$ is the same as the output of $\mathtt{ExtCom}$ for $\mathcal{H}_4$. If this is not true then a reduction to the trapdoorness of $\mathtt{tcom}_1$ can be done.

- The next hybrid that we consider is $\mathcal{H}_2$. The difference between $\mathcal{H}_3$ and $\mathcal{H}_2$ is that the transcript for $\mathsf{LS}_{\mathsf{nm}}$ is computed using the Special HVZK simulator instead of the honest prover procedure. We observe that the simulator of $\mathsf{LS}_{\mathsf{nm}}$ can be used in $\mathcal{H}_2$ because $\mathtt{tcom}_1$ is computed using the trapdoor procedure. So, upon receiving $\mathsf{ls}^3_{\mathsf{nm}}$ the Special HVZK simulator of $\mathsf{LS}_{\mathsf{nm}}$ is invoked thus obtaining $(\mathsf{ls}^2_{\mathsf{nm}}, \mathsf{ls}^4_{\mathsf{nm}})$, and $\mathsf{ls}^2_{\mathsf{nm}}$ will be the value to which $\mathtt{tcom}_1$ will be opened using the trapdoor procedure.

  We now want to prove that there exists a right session j where the adversary is still computing a valid non-malleable commitment of two signatures for $\tilde{\mathsf{vk}}_j$. We recall that this statement is true for the hybrid experiment $\mathcal{H}_3$. Suppose by contradiction that this claim does not hold when considering $\mathcal{H}_2$, then we can construct a reduction to the Special HVZK of $\mathsf{LS}_{\mathsf{nm}}$. The reduction works as follows. Let $\mathcal{C}^{SHVZK}$ be the challenger for the Special HVZK of $\mathsf{LS}_{\mathsf{nm}}$.

1. The reduction interacts with the adversary in the left and in the right sessions according to $\mathcal{H}_3$ (and $\mathcal{H}_2$).
2. The reduction, upon receiving $\mathsf{ls}^1_{\mathsf{nm}}$ and $\mathsf{ls}^3_{\mathsf{nm}}$ in the left session forwards these messages to $\mathcal{C}^{SHVZK}$ together with $(x_{\mathsf{nm}}, w_{\mathsf{nm}})$.
3. The reduction, upon receiving $(\mathsf{ls}^2_{\mathsf{nm}}, \mathsf{ls}^4_{\mathsf{nm}})$ from $\mathcal{C}^{SHVZK}$, uses them to complete the left execution against the adversary.
4. When the adversary stops, the reduction rewinds the adversary in the j-th right session using the `ExtCom`'s strategy, and upon receiving $\mathsf{ls}^{3'}_{\mathsf{nm}}$ in the left session computes $(\mathsf{ls}^{2'}_{\mathsf{nm}}, \mathsf{ls}^{4'}_{\mathsf{nm}})$ using the honest prover procedure (so using w'_{nm}). Moreover, during all the rewinds $\mathtt{tcom}_1$ is always opened to the value $\mathsf{ls}^{2'}_{\mathsf{nm}}$.
5. The reduction now uses the output of `ExtCom` to check whether the non-malleable commitment computed in the main thread contains a value $\tilde{s}^j_0$ such that $\tilde{s}^j_0 \oplus \tilde{s}^j_1$ represents two signatures with respect to $\tilde{\mathsf{vk}}_j$.
6. If the non-malleable commitment of the main thread is well formed and two signatures are extracted then the reduction outputs 0 (to claim that the challenger has used the honest prover procedure for $\mathsf{LS}_{\mathsf{nm}}$), and a random bit otherwise.

We observe that the extractor will output the decommitment information for the non-malleable commitment computed in one of the rewinds made by `ExtCom` and not the decommitment information for the non-malleable commitment received in the main thread. Since we assume that the non-malleable commitment scheme fixes the decommitment information in the second round, then the decommitment information extracted can be used to check what is committed in the main thread as well. We recall that the extraction procedure used in the reduction corresponds exactly to the extraction procedure used in $\mathcal{H}_3$. Moreover, when $(\mathsf{ls}^2_{\mathsf{nm}}, \mathsf{ls}^4_{\mathsf{nm}})$ is computed using the honest prover procedure by the challenger, the view of the adversary in the main thread and during the rewinds is perfectly indistinguishable. Therefore the probability that the extractor is successful when $(\mathsf{ls}^2_{\mathsf{nm}}, \mathsf{ls}^4_{\mathsf{nm}})$ is computed using the honest prover procedure corresponds (except for a negligible difference) to the probability that the extractor succeeds in $\mathcal{H}_3$.

- The next hybrid that we consider is $\mathcal{H}_1$. The difference between $\mathcal{H}_2$ and $\mathcal{H}_1$ is that the non-malleable commitment contains a value s_0 such that $s_0 \oplus s_1$ is equal to a random string, whereas in the previous hybrid experiment it holds that $s_0 \oplus s_1 = \sigma_1||\sigma_2$. Since we are considering a synchronous adversary we can immediately rely on the security of the one-one synchronous non-malleable commitment NM to claim that the adversary in some session j, with non-negligible probability, is still computing a valid non-malleable commitment of a string $\tilde{s}^j_0$ such that $\tilde{s}^j_0 \oplus \tilde{s}^j_1 = \sigma^j_0||\sigma^j_1$ and $\sigma^j_0||\sigma^j_1$ represents two valid signatures for $\tilde{\mathsf{vk}}_j$. We recall that during the reduction to NM two signatures for two messages with respect to vk need to be extracted in the left session. Since NM is public coin the extraction of the signatures does not interfere with the reduction to the non-malleable commitment.

- The next hybrid that we consider is $\mathcal{H}_0$. The difference between $\mathcal{H}_1$ and $\mathcal{H}_0$ is that on the left session the procedure to extract two valid signatures for vk is not run anymore. Moreover, as proved in the first part of Lemma 1, the adversary aborting probabilities in $\mathcal{H}_0$ and $\mathcal{H}_1$ are negligibly close. From previous arguments it follows that in $\mathcal{H}_1$ there exists a session j where the adversary computes a valid commitment of two signatures using NM. Therefore, we can claim that also in $\mathcal{H}_0$ there exists a session j where, with non-negligible probability, the adversary is computing a well-formed non-malleable commitment such that the value extracted from it allows to compute two signatures for $\tilde{\mathsf{vk}}_j$. This implies that using the weak extractability of the non-malleable commitment NM we can extract from some right session j two signatures for the verification $\tilde{\mathsf{vk}}_j$ reaching a contradiction. Indeed, it is possible to contradict the security of the signature scheme because no more rewinds are needed in the left session between the third and the second round. We just rewind from the fourth to the third round in the right session j to extract the signatures and this rewinding procedure does not interfere with the reduction that breaks the security of the signature scheme.

During the security proof we have assumed that the adversary is synchronous. We now briefly explain why the lemma holds also against asynchronous adversaries. In all the reductions that we have made for the synchronous case we have showed that there is always a procedure to extract the signatures from some j-th right session, thus reaching a contradiction in $\mathcal{H}_0$. This part of the proof follows mostly the same arguments used above for the synchronous case, with the major difference that we cannot rely on the security of the synchronous non-malleable commitment. In more details, we first argue that in $\mathcal{H}_2$ the asynchronous $\mathcal{A}_{\mathsf{NMZK}}$ computes a valid commitment of two signatures with respect to the verification key vk_j in some $j-$th right session, and then we show an extractor for $\mathcal{H}_1$ relying on the multi-hiding instead of the non-malleability property of NM.

In the reductions considered for the synchronous case the adversary is rewound to check the validity and the content of the non-malleable commitments computed by $\mathcal{A}_{\mathsf{NMZK}}$ in the right sessions. Any of these rewinding procedures, when applied to an asynchronous adversary, might completely rewind the left session. In this case the adversary could send messages in the left session which are different from all the messages that appeared in the main thread. This is a problem when $\mathcal{A}_{\mathsf{NMZK}}$ is involved in a security reduction. Indeed, the reduction might need to ask the challenger to restart the security game from the beginning. This is the reason why we require the cryptographic primitives involved in our protocol to enjoy this notion of *multi* security. Therefore, the reductions described for the hybrids experiments $\mathcal{H}_7, \dots, \mathcal{H}_2$ work almost as showed for the synchronous, but they rely on the *multi* security of the primitives involved in the reductions.

The only thing that is left to argue is the existence of an extraction procedure that outputs two valid signatures for vk_j in $\mathcal{H}_1$ and $\mathcal{H}_0$ even when $\mathcal{A}_{\mathsf{NMZK}}$ is asynchronous. Given that NM retains the non-malleability only against one-one *synchronous* adversary, we need to use a different strategy to prove the existence of such an extractor.

Since in $\mathcal{H}_2$ the adversary commits to two valid signatures in some session j with non-negligible probability, we can distinguish two cases: 1) $\mathcal{A}_{\mathsf{NMZK}}$ commits to those signatures in a *synchronous* right session of $\mathcal{H}_2$; 2) $\mathcal{A}_{\mathsf{NMZK}}$ commits to those signatures in an *asynchronous* right session of $\mathcal{H}_2$. If case 1 happens with non-negligible probability, then the proof is over since this case has been already discussed previously. So let us assume that only case 2 happens with non-negligible probability. We now prove that if this is the case then a reduction to the multi-hiding of NM can be done as follows. Let $\mathcal{C}$ be the challenger for the multi-hiding security game. The reduction picks two random challenge messages $(m_0 \leftarrow \{0,1\}^\lambda, m_1 \leftarrow \{0,1\}^\lambda)$ and sends them to $\mathcal{C}$. The reduction now computes all the messages as in $\mathcal{H}_1$ and $\mathcal{H}_2$ and acts as a proxy between $\mathcal{C}$ and $\mathcal{A}_{\mathsf{NMZK}}$ with respect to the left-session messages of NM. Moreover, in the last round of the left session the reduction sets $s_1 = m_0 \oplus \sigma_1 || \sigma_2$. When the execution with $\mathcal{A}_{\mathsf{NMZK}}$ stops, the reduction runs the weak-extractor of NM on all the asynchronous right sessions. If there exists a right session j where the extracted value is a string $\tilde{s}_0^j$ such that $\tilde{s}_0^j \oplus \tilde{s}_1^j = \sigma_0^j || \sigma_1^j$ and $\sigma_0^j || \sigma_1^j$ represents two valid signatures for $\tilde{\mathsf{vk}}_j$ then the reduction outputs m_0, otherwise she outputs m_b, with $b \in \{0,1\}$.

In this reduction there is an issue that we address by requiring NM to be multi-hiding. Indeed, it could happen that the weak-extractor completely rewinds the left session. In this case the adversary might start a new session with $\mathcal{C}$ by sending a different first round of NM nm_1, and this is the reason why we require NM to be multi-hiding.

We observe that what we have proved only guarantees that it is possible to extract valid signatures for vk_j for some j-th session of $\mathcal{H}_1$ with non-negligible probability. That is, we cannot claim that the non-malleable commitment computed by $\mathcal{A}_{\mathsf{NMZK}}$ in the j-th right session of $\mathcal{H}_1$ is well formed. Anyway, the extraction guarantee is sufficient to complete our security proof. Indeed, for the same arguments used for the synchronous case the probability that $\mathcal{A}_{\mathsf{NMZK}}$ aborts in $\mathcal{H}_0$ is negligibly close to the aborting probability of $\mathcal{A}_{\mathsf{NMZK}}$ in $\mathcal{H}_1$. Therefore the weak-extractor of NM can be used to extract two valid signatures from some j-th right section of $\mathcal{H}_0$ thus reaching a contradiction.

□

Theorem 2. *If OWFs exists, then* NMZK *is a synchronous delayed-input many-many NMZK AoK for* $\mathcal{NP}$.

Proof. The proof proceeds very similarly to the one showed for Theorem 1. The main difference between these two proofs is that we have to consider also polynomially many synchronous left sessions played in parallel. Therefore the only difference between this proof and the one of Theorem 1 is that in the reductions we need to rely on the security of a many-one non-malleable commitment scheme and on the security of the other primitives under parallel composition. We notice that using the same arguments of the security proof of Proposition 1 provided in [LPV08], it is possible to claim that a synchronous (one-one) non-malleable commitment is also synchronous many-one non-malleable. Therefore no additional assumption are required in order to prove that NMZK is also many-many parallel delayed-input NMZK. Note also that, the simulator needs to extract the trapdoor (the signatures of two different messages) in all the left (synchronous) sessions completed in the main thread. The extraction succeeds except with negligible probability for the same arguments used in the security proof of Theorem 1.

□

Common input: security parameter λ, identity $\mathtt{id} \in \{0,1\}^\lambda$, lengths for LS_L and $\mathsf{LS}_{\mathsf{nm}}$: ℓ_x, ℓ_{nm}.
Input to $\mathcal{P}_{\mathsf{NMZK}}$: (x, w) s.t. $(x, w) \in \mathsf{Rel_L}$, with (x, w) available only in the 4th round.
Commitment phase:

1. $\mathcal{V}_{\mathsf{NMZK}} \rightarrow \mathcal{P}_{\mathsf{NMZK}}$
   1. Run $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{Gen}(1^\lambda)$.
   2. Run $\mathcal{V}_L$ on input 1^λ and ℓ_x thus obtaining the 1st round ls_L^1 of LS_L.
   3. Run $\mathcal{V}_{\mathsf{nm}}$ on input 1^λ and ℓ_{nm} thus obtaining the 1st round $\mathsf{ls}_{\mathsf{nm}}^1$ of $\mathsf{LS}_{\mathsf{nm}}$.
   4. Run Dec on input 1^λ thus obtaining ρ.
   5. Run Rec_0 on input 1^λ thus obtaining ρ_0.
   6. Run Rec_1 on input 1^λ thus obtaining ρ_1.
   7. Run $\mathcal{R}$ on input 1^λ and $\mathtt{id}$ thus obtaining nm_1.
   8. Send $(\mathsf{vk}, \mathsf{ls}_L^1, \mathsf{ls}_{\mathsf{nm}}^1, \mathsf{nm}_1, \rho, \rho_0, \rho_1)$ to $\mathcal{P}_{\mathsf{NMZK}}$.
2. $\mathcal{P}_{\mathsf{NMZK}} \rightarrow \mathcal{V}_{\mathsf{NMZK}}$
   1. Run Com on input ρ and message 1 in order to compute the pair $(\mathtt{com}, \mathtt{dec})$.
   2. Run $\mathcal{P}_L$ on input 1^λ, ℓ_x and ls_L^1 thus obtaining the 2nd round ls_L^2 of LS_L.
   3. Run Sen_0 on input 1^λ, $((\rho, \mathtt{com}), 0)$, ρ_0 and message ls_L^2 to compute the pair $(\mathtt{tcom}_0, \mathtt{tdec}_0)$.
   4. Run TFake_1 on input 1^λ, $((\rho, \mathtt{com}), 1)$, ρ_1 to compute the pair $(\mathtt{tcom}_1, \mathtt{aux})$.
   5. Pick $s_0 \leftarrow \{0,1\}^\lambda$ and run $\mathcal{S}$ on input 1^λ, $\mathtt{id}$, nm_1, s_0 (in order to commit to the message s_0) thus obtaining nm_2.
   6. Pick a message $\mathtt{msg} \leftarrow \{0,1\}^\lambda$ and send $(\mathtt{com}, \mathtt{tcom}_0, \mathtt{tcom}_1, \mathtt{msg}, \mathsf{nm}_2)$ to $\mathcal{V}_{\mathsf{NMZK}}$.
3. $\mathcal{V}_{\mathsf{NMZK}} \rightarrow \mathcal{P}_{\mathsf{NMZK}}$
   1. Run $\mathcal{V}_L$ thus obtaining the 3rd round ls_L^3 of LS_L.
   2. Run $\mathcal{V}_{\mathsf{nm}}$ thus obtaining the 3rd round $\mathsf{ls}_{\mathsf{nm}}^3$ of $\mathsf{LS}_{\mathsf{nm}}$.
   3. Run $\mathcal{R}$ thus obtaining nm_3 of NM.
   4. Run $\mathsf{Sign}(\mathsf{sk}, \mathtt{msg})$ to obtain a signature σ of $\mathtt{msg}$.
   5. Send $(\mathsf{ls}_L^3, \mathsf{ls}_{\mathsf{nm}}^3, \mathsf{nm}_3, \sigma)$ to $\mathcal{P}_{\mathsf{NMZK}}$.
4. $\mathcal{P}_{\mathsf{NMZK}} \rightarrow \mathcal{V}_{\mathsf{NMZK}}$
   1. If $\mathsf{Ver}(\mathsf{vk}, \mathtt{msg}, \sigma) \neq 1$ then abort, continue as follows otherwise.
   2. Run $\mathcal{P}_L$ on input x, w and ls_L^3 thus obtaining the 4th round ls_L^4 of LS_L.
   3. Run $\mathcal{S}$ on input nm_3 thus obtaining $(\mathsf{nm}_4, \mathtt{dec}_{\mathsf{nm}})$.
   4. Pick $s_1 \leftarrow \{0,1\}^\lambda$ and set $x_{\mathsf{nm}} = (\mathsf{vk}, \mathsf{nm}_1, \mathsf{nm}_2, \mathsf{nm}_3, \mathsf{nm}_4, s_1)$ and run the Special HVZK simulator of $\mathsf{LS}_{\mathsf{nm}}$ on input $(x_{\mathsf{nm}}, \mathsf{ls}_{\mathsf{nm}}^1, \mathsf{ls}_{\mathsf{nm}}^3)$ thus obtaining $(\mathsf{ls}_{\mathsf{nm}}^2, \mathsf{ls}_{\mathsf{nm}}^4)$.
   5. Run TFake_1 on input $\mathtt{tk} = (\mathtt{dec})$, ρ_1, $\mathtt{tcom}_1$, $\mathtt{aux}$ and $\mathsf{ls}_{\mathsf{nm}}^2$ to compute $\mathtt{tdec}_1$.
   6. Send $((\mathtt{tdec}_1, \mathsf{ls}_{\mathsf{nm}}^2), \mathsf{ls}_{\mathsf{nm}}^4, (\mathtt{tdec}_0, \mathsf{ls}_L^2), \mathsf{ls}_L^4, x, x_{\mathsf{nm}})$ to $\mathcal{V}_{\mathsf{NMZK}}$.
5. $\mathcal{V}_{\mathsf{NMZK}}$: output 1 iff the following conditions are satisfied.
   1. Rec_0 on input $(\rho_0, (\rho, \mathtt{com}), 0), \mathtt{tcom}_0, \mathtt{tdec}_0, \mathsf{ls}_L^2$ accepts $(\mathsf{ls}_L^2, \mathtt{tdec}_0)$ as a decommitment of $\mathtt{tcom}_0$.
   2. Rec_1 on input $(\rho_1, (\rho, \mathtt{com}), 1), \mathtt{tcom}_1, \mathtt{tdec}_1, \mathsf{ls}_{\mathsf{nm}}^2$ accepts $(\mathsf{ls}_{\mathsf{nm}}^2, \mathtt{tdec}_1)$ as a decommitment of $\mathtt{tcom}_1$.
   3. $(\mathsf{ls}_L^1, \mathsf{ls}_L^2, \mathsf{ls}_L^3, \mathsf{ls}_L^4)$ is accepting for $\mathcal{V}_L$ with respect to the instance x.
   4. $(\mathsf{ls}_{\mathsf{nm}}^1, \mathsf{ls}_{\mathsf{nm}}^2, \mathsf{ls}_{\mathsf{nm}}^3, \mathsf{ls}_{\mathsf{nm}}^4)$ is accepting for $\mathcal{V}_{\mathsf{nm}}$ with respect to the instance x_{nm}.

Figure 2: Formal construction of our delayed-input NMZK AoK

4 Multi-Party Coin-Tossing Protocol

4.1 4-Round Secure Multi-Party Coin Tossing: Π_{MPCT}

The high-level idea of our protocol Π_{MPCT} significantly differs from the one of [GMPP16b] (e.g., we use our 4-round delayed-input synchronous many-many NMZK instead of 3-round 3-robust parallel non-malleable commitment scheme). Our protocol simply consists of each party committing to a random string r, which is opened in the last round along with a simulatable proof of correct opening given to all parties independently. The output consists of the $\oplus$ of all opened strings. Let's see in more details how our Π_{MPCT} works. For our construction we use the following tools.

1. A non-interactive perfectly binding computationally hiding commitment scheme $\mathsf{PBCOM} = (\mathsf{Com}, \mathsf{Dec})$.
2. A Σ-protocol $\mathsf{BL}_L = (\mathcal{P}_L, \mathcal{V}_L)$ for the $\mathcal{NP}$-language $L = \{\mathtt{com} : \exists\ (\mathtt{dec}, m) \text{ s.t. } \mathsf{Dec}(\mathtt{com}, \mathtt{dec}, m) = 1\}$ with Special HVZK simulator Sim_L. We uses two instantiations of BL_L in order to construct the protocol for the OR of two statements Π_{OR} as described in App. C.2. Π_{OR} is a proof system for the $\mathcal{NP}$-language $L_{\mathtt{com}} = \{(\mathtt{com}_0, \mathtt{com}_1) : \exists\ (\mathtt{dec}, m) \text{ s.t. } \mathsf{Dec}(\mathtt{com}_0, \mathtt{dec}, m) = 1 \text{ OR } \mathsf{Dec}(\mathtt{com}_1, \mathtt{dec}, m) = 1\}$ [18]. Informally, by running Π_{OR}, one can prove the knowledge of the message committed in $\mathtt{com}_0$ or in $\mathtt{com}_1$.
3. A 4-round delayed-input synchronous many-many NMZK $\mathsf{NMZK} = (\mathcal{P}_{\mathsf{NMZK}}, \mathcal{V}_{\mathsf{NMZK}})$ for the following $\mathcal{NP}$-language

$$L_{\mathsf{NMZK}} = \{((\mathtt{com}_0, \mathtt{com}_1), m) :\ \forall i \in \{0, 1\}\ \exists\ \mathtt{dec}_i \text{ s.t. } \mathsf{Dec}(\mathtt{com}_i, \mathtt{dec}_i, m) = 1\}.$$

   Informally, by running NMZK, one can prove that 2 commitments contain the same message m.

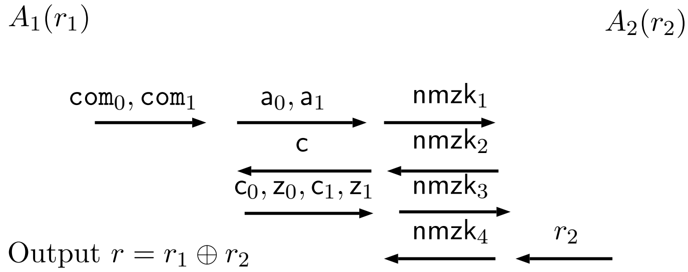
4.2 Π_{MPCT}: Informal Description and Security Intuition

The high level description of our protocol between just two parties (A_1, A_2) is given in Fig. 3. For a formal description of Π_{MPCT} we refer the reader to Sec. 4.3. In Fig. 3 we consider an execution of Π_{MPCT} that goes from A_1 to A_2 (the execution from A_2 to A_1 is symmetric). We recall that the protocol is executed simultaneously by both A_1 and A_2. The main idea is the following. Each party commits to his input using two instantiations of a non-interactive commitment. More precisely we have that A_1 computes two non-interactive commitments $\mathtt{com}_0$ and $\mathtt{com}_1$ (along with their decommitment information $\mathtt{dec}_0$ and $\mathtt{dec}_1$) of the message r_1. Each party also runs Π_{OR} for the $\mathcal{NP}$-language $L_{\mathtt{com}}$, from the first to the third round, in order to prove knowledge of the message committed in $\mathtt{com}_0$ or in $\mathtt{com}_1$. In the last round each party sends his own input (i.e. r_1 for A_1 and r_2 for A_2) and proves, using a delayed-input synchronous many-many non-malleable ZK for the $\mathcal{NP}$-language L_{NMZK}, that messages committed using PBCOM were actually equal to that input (i.e. r_1 for A_1 and r_2 for A_2). That is, A_1 sends r_1 and proves that $\mathtt{com}_0$ and $\mathtt{com}_1$ are valid commitments of the message r_1.

Intuition about the security of Π_{MPCT}. Let A_1^* be the corrupted party.

Informally the simulator Sim works as follows. Sim starts an interaction against A_1^* using as input a random string y until the third round of Π_{MPCT} is received by A_1^*. More precisely, in the first round he computes two commitments $\mathtt{com}_0$ and $\mathtt{com}_1$ (along with their decommitment information $\mathtt{dec}_0$ and $\mathtt{dec}_1$) of y, and runs $\mathcal{P}_{\mathsf{OR}}$ using as a witness $(\mathtt{dec}_1, y)$. After the 3rd round Sim extracts the input r_1^* of the corrupted party A_1^* using the extractor E_{OR} of Π_{OR} (that exists from the PoK property of Π_{OR}) and sends r_1^* to the ideal world functionality. At this point Sim receives r from the ideal-world functionality, and completes the execution of the 4th round by sending $r_2 = r \oplus r_1^*$. We observe that Sim, in order to send a string r_2 that

[18] We use Π_{OR} in a non-black box way, but for ease of exposition sometimes we will refer to entire protocol Π_{OR} in order to invoke the proof of knowledge property enjoyed by Π_{OR}.



- $\mathtt{com}_0$ and $\mathtt{com}_1$ are two non-interactive commitments of the message r_1 computed using PBCOM.
- $(\mathsf{a}_0, \mathsf{a}_1, \mathsf{c}_0, \mathsf{c}_1, \mathsf{z}_0, \mathsf{z}_1)$ is the transcript generated from an execution of the WIPoK Π_{OR} in which $\mathcal{P}_{\mathsf{OR}}$ proves the knowledge of either the message committed in $\mathtt{com}_0$ or in $\mathtt{com}_1$.
- $(\mathsf{nmzk}_1, \mathsf{nmzk}_2, \mathsf{nmzk}_3, \mathsf{nmzk}_4)$ in the transcript generated from an execution of the delayed-input synchronous many-many NMZK NMZK in which $\mathcal{P}_{\mathsf{NMZK}}$ proves that both $\mathtt{com}_0$ and $\mathtt{com}_1$ are valid commitments of the message r_1.

Figure 3: Π_{MPCT}: Informal description of the execution from A_1 to A_2. The execution from A_2 to A_1 is symmetric.

differs from y in the 4th round, has to cheat in NMZK. This is done by simply running the simulator of NMZK. To prove the security of our scheme we will go through a sequence of hybrid experiments in order to show that the output view of the adversary in the real world can be simulated in the ideal world by Sim. The security proof strongly relies on the non-malleable zero knowledge property of NMZK. Indeed the aim of NMZK is to ensure that the adversary does not maul the messages received from Sim. That is, the behavior of A_1^* allows to extract, in every hybrid experiments that we will consider, the correct input of A_1^*. This holds even in case the commitments sent by Sim to A_1^* are commitments of a random string y, and the value sent in the 4th round is inconsistent with the value committed in the first round.

4.3 Formal Description

Let $P = \{P_1, \dots, P_n\}$ be the set of parties. Furthermore, denote by $(\mathtt{id}_1, \dots, \mathtt{id}_n)$[19] the unique identities of parties $\{P_1, \dots, P_n\}$, respectively. Let us denote by $\mathsf{F}_{\mathsf{MPCT}} : (1^\lambda)^n \to \{0,1\}^\lambda$ the function $\mathsf{F}_{\mathsf{MPCT}}(r_1, \dots, r_n) = r_1 \oplus \cdots \oplus r_n$. The protocol starts with each party P_i choosing a random string r_i for $i = 1, \dots, n$. It consists of four rounds, i.e., all parties send messages in each round and the messages of all executions are seen by every party. Following [GMPP16b] we describe the protocol between two parties (A_1, A_2) observing that the real protocol actually consists of n simultaneous executions of a two-party coin-tossing protocol $\Pi_{\mathsf{MPCT}} = (A_1, A_2)$ between parties (P_i, P_j) where P_i acts as A_1 with input r_i and P_j acts as A_2 with input r_j (both are symmetric). Let the input of A_1 be r_1, and the input of A_2 be r_2. The set of messages enabling A_1 to learn the output are denoted by (m_1, m_2, m_3, m_4) where (m_1, m_3) are sent by A_1 and (m_2, m_4) are sent by A_2. Likewise, the set of messages enabling A_2 to learn the output are denoted by $(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3, \tilde{m}_4)$ where $(\tilde{m}_1, \tilde{m}_3)$ are sent by A_2 and $(\tilde{m}_2, \tilde{m}_4)$ are sent by A_1. Therefore, messages $(m_l, \tilde{m}_l)$ are simultaneously exchanged in the l-th round for $l = 1, \dots, 4$.

Protocol Π_{MPCT}. *Common input:* security parameter λ, instances length: ℓ_{NMZK}, $\ell_{\mathtt{com}}$.
Round 1. We first describe how A_1 constructs m_1.

1. Compute $(\mathtt{com}_0, \mathtt{dec}_0) \leftarrow \mathsf{Com}(r_1)$ and $(\mathtt{com}_1, \mathtt{dec}_1) \leftarrow \mathsf{Com}(r_1)$.
2. Compute $\mathsf{a}_0 \leftarrow \mathcal{P}_L(1^\lambda, \mathtt{com}_0, (\mathtt{dec}_0, r_1))$.

[19] As discuss in the Definition 2 the use of the identifiers can be avoid, we use them, to uniformity of notation.

3. Pick $\mathsf{c}_1 \leftarrow \{0,1\}^\lambda$ and compute $(\mathsf{a}_1, \mathsf{z}_1) \leftarrow \mathsf{Sim}_L(1^\lambda, \mathtt{com}_1, \mathsf{c}_1)$.
4. Run $\mathcal{V}_{\mathsf{NMZK}}$ on input 1^λ and ℓ_{NMZK} thus obtaining the 1st round nmzk_1 of NMZK.
5. Message m_1 is defined to be $(\mathtt{com}_0, \mathtt{com}_1, \mathsf{a}_0, \mathsf{a}_1, \mathsf{nmzk}_1)$.

Likewise, A_2 performs the same action as A_1 in order to construct $\tilde{m}_1 = (\tilde{\mathtt{com}}_0, \tilde{\mathtt{com}}_1, \tilde{\mathsf{a}}_0, \tilde{\mathsf{a}}_1, \tilde{\mathsf{nmzk}}_1)$.
Round 2. In this round A_2 sends message m_2 and A_1 sends $\tilde{m}_2$. We first describe how A_2 constructs m_2.
1. Run $\mathcal{P}_{\mathsf{NMZK}}$ on input 1^λ, $\mathtt{id}_2$, ℓ_{NMZK} and nmzk_1 thus obtaining the 2nd round nmzk_2 of NMZK.
2. Pick $\mathsf{c} \leftarrow \{0,1\}^\lambda$.
3. Define message $m_2 = (\mathsf{c}, \mathsf{nmzk}_2)$.

Likewise, A_1 performs the same actions as A_2 in the previous step to construct the message $\tilde{m}_2 = (\tilde{\mathsf{c}}, \tilde{\mathsf{nmzk}}_2)$.
Round 3. In this round A_1 sends message m_3 and A_2 sends $\tilde{m}_3$. A_1 prepares m_3 as follows.

1. Compute $\mathsf{c}_0 = \mathsf{c} \oplus \mathsf{c}_1$ and $\mathsf{z}_0 \leftarrow \mathcal{P}_L(\mathsf{c}_0)$.
2. Run $\mathcal{V}_{\mathsf{NMZK}}$ on input nmzk_2 thus obtaining the 3rd round nmzk_3 of NMZK.
3. Define $m_3 = (\mathsf{nmzk}_3, \mathsf{c}_0, \mathsf{c}_1, \mathsf{z}_0, \mathsf{z}_1)$.

Likewise, A_2 performs the same actions as A_1 in the previous step to construct the message $\tilde{m}_3 = (\tilde{\mathsf{nmzk}}_3, \tilde{\mathsf{c}}_0, \tilde{\mathsf{c}}_1, \tilde{\mathsf{z}}_0, \tilde{\mathsf{z}}_1)$.
Round 4. In this round A_2 sends message m_4 and A_1 sends $\tilde{m}_4$. A_2 prepares m_4 as follows.
1. Check that the following conditions are satisfied: a) $\mathsf{c} = \mathsf{c}_0 \oplus \mathsf{c}_1$; b) the transcript $\mathsf{a}_0, \mathsf{c}_0, \mathsf{z}_0$ is accepting w.r.t. the instance $\mathtt{com}_0$; c) the transcript $\mathsf{a}_1, \mathsf{c}_1, \mathsf{z}_1$ is accepting w.r.t. the instance $\mathtt{com}_1$. If one of the check fails then output $\perp$, otherwise continue with the following steps.
2. Set $x_{\mathsf{NMZK}} = (\tilde{\mathtt{com}}_0, \tilde{\mathtt{com}}_1, r_2)$ and $w_{\mathsf{NMZK}} = (\tilde{\mathtt{dec}}_0, \tilde{\mathtt{dec}}_1)$.
3. Run $\mathcal{P}_{\mathsf{NMZK}}$ on input nmzk_3, the statement to be proved x_{NMZK} and the witness w_{NMZK} s.t. $(x_{\mathsf{NMZK}}, w_{\mathsf{NMZK}}) \in \mathsf{Rel}_{L_{\mathsf{NMZK}}}$, thus obtaining the 4th round nmzk_4 of NMZK.
4. Define $m_4 = (r_2, x_{\mathsf{NMZK}}, \mathsf{nmzk}_4)$.

Likewise, A_1 performs the same actions as A_2 in the previous step to construct the message $\tilde{m}_4 = (r_1, \tilde{x}_{\mathsf{NMZK}}, \tilde{\mathsf{nmzk}}_4)$.
Output computation of Π_{MPCT}. Check, for each party, if $(\mathsf{nmzk}^i_1, \mathsf{nmzk}^i_2, \mathsf{nmzk}^i_3, \mathsf{nmzk}^i_4)$ is accepting for $\mathcal{V}_{\mathsf{NMZK}}$ with respect to the instance x^i_{NMZK} $(i = 1, \dots, n)$ and that all pairs of parties used the same inputs $(r_1, \dots, r_n)$. If so, output $r = r_1 \oplus \dots \oplus r_n$.

Theorem 3. *If one-to-one OWFs exist, then the multi-party protocol Π_{MPCT} securely computes the multi-party coin-tossing functionality with black-box simulation.*

Proof. Let $P = \{P_1, \dots, P_n\}$ be the set of parties participating in the execution of Π_{MPCT}. Also let $P^* \subseteq P$ be the set of parties corrupted by the adversary $\mathcal{A}$. The simulator Sim only generates messages on behalf of parties $P \setminus P^*$. In particular, we show that for every adversary $\mathcal{A}$ there exists an "ideal" world adversary Sim such that

$$\mathsf{REAL}_{\Pi_{\mathsf{MPCT}}, \mathcal{A}(z)}(1^\lambda) \approx \mathsf{IDEAL}_{\mathsf{F}_{\mathsf{MPCT}}, \mathsf{Sim}(z)}(1^\lambda).$$

We prove this claim by considering hybrid experiments $\mathcal{H}_1, \dots, \mathcal{H}_7$ as described below. Without loss of generality we will assume that party P_1 is the only honest party since our protocol is secure against $n-1$ corruptions. We denote the output of the parties in the hybrid experiment $\mathcal{H}_i$ with $\{\mathsf{OUT}_{\mathcal{H}_i, \mathcal{A}(z)}(1^\lambda)\}$.

- The 1st hybrid experiment $\mathcal{H}_1$ is identical to the real execution. More specifically, $\mathcal{H}_1$ starts $\mathcal{A}$ with fresh randomness and interacts with it as P_1 would do using uniform randomness r_1 as input. The output of $\mathcal{H}_1$ consists of $\mathcal{A}$'s view. We observe that, by construction, the output of $\mathcal{A}$ in the real execution is identically distributed to $\mathcal{H}_1$. Moreover, all the messages generated on the behalf of P^* are honestly computed with overwhelming probability due to the soundness of NMZK.
- The 2nd hybrid experiment $\mathcal{H}_2$ is identical to $\mathcal{H}_1$ except that this hybrid experiment also extracts the P^*'s inputs $r^*_2, \dots, r^*_n$. In order to obtain $r^*_2, \dots, r^*_n$, $\mathcal{H}_2$ runs the extractor E_{OR} of Π_{OR} on each execution of Π_{OR} made by a malicious party. Note that the existence of E_{OR} is guaranteed from the adaptive-input

PoK property of Π_{OR}. If the extractor fails, then $\mathcal{H}_2$ aborts. At this point $\mathcal{H}_2$ completes the 4th round and prepares the output exactly as $\mathcal{H}_1$[20].

$\{\mathsf{OUT}_{\mathcal{H}_1,\mathcal{A}(z)}(1^\lambda)\}$ and $\{\mathsf{OUT}_{\mathcal{H}_2,\mathcal{A}(z)}(1^\lambda)\}$ are statistically close, and the extraction is successful in expected polynomial time, both claims follow from the PoK property of Π_{OR}. Observe that we are guaranteed that what E_{OR} outputs correspond to the input of the the malicious party, from the fact that with non-negligible probability $\mathcal{A}$ correctly computes all the steps of Π_{MPCT}. More precisely the soundness of NMZK ensures that the extracted values correspond to the $r_2^*, \ldots, r_n^*$ received in the last round.

- The 3rd hybrid experiment $\mathcal{H}_3$ differs from $\mathcal{H}_2$ in the way the transcript for the delayed-input synchronous many-many NMZK NMZK is computed. More precisely in this hybrid experiment the simulator $\mathsf{Sim}^{\mathsf{NMZK}}$ for NMZK is used. Following [GMPP16b, ACJ17] the extraction of NMZK's trapdoor and the extraction of P^*'s input are performed during the same steps. Observe that these two extraction procedures do not interfere with each other, indeed they just rewind from the third to the second round by sending a freshly generated second round.

  The first property of $\mathsf{Sim}^{\mathsf{NMZK}}$ (see Definition 2) ensures that $\{\mathsf{OUT}_{\mathcal{H}_2,\mathcal{A}(z)}(1^\lambda)\}$ is computationally indistinguishable from $\{\mathsf{OUT}_{\mathcal{H}_3,\mathcal{A}(z)}(1^\lambda)\}$. Moreover the second property enjoyed by $\mathsf{Sim}^{\mathsf{NMZK}}$ (simulation-extraction) ensures that in $\mathcal{H}_3$ the witnesses can be extracted from $\mathcal{A}$ (one witness for every execution of NMZK made by every malicious P_i^*), therefore we are guaranteed that $\mathcal{A}$ correctly computes all the steps of Π_{MPCT}. That is, the value $r_2^*, \ldots, r_n^*$ sent by the malicious party in the last round are actually committed in the second round sent by $\mathcal{A}$. It is important to observe that in this hybrid experiment the probability that $\mathcal{A}$ completes the third round is negligible close to the probability of completing the third round in $\mathcal{H}_2$ (otherwise the output of the two experiments would be distinguishable). Therefore the probability that E_{OR} works correctly in this experiment is negligibly close to the probability that E_{OR} works in $\mathcal{H}_2$. This holds because, following the Definition 9, the probability of E_{OR} to given in output a valid witness for the instance $(\mathtt{com}_0, \mathtt{com}_1)$ is negligible close to the probability that $\mathcal{A}$ completes an accepting third round.

- The 4th hybrid experiment $\mathcal{H}_4$ differs from $\mathcal{H}_3$ in the way $\mathtt{com}_1$ is computed. More precisely, instead of a committing to r_1 in $\mathtt{com}_1$ a commitment of a random string y is made. We claim that $\{\mathsf{OUT}_{\mathcal{H}_3,\mathcal{A}(z)}(1^\lambda)\}$ and $\{\mathsf{OUT}_{\mathcal{H}_4,\mathcal{A}(z)}(1^\lambda)\}$ are computationally indistinguishable due to the computationally hiding of PBCOM. We claim also that in $\mathcal{H}_4$ $\mathcal{A}$ still behaves correctly, indeed we can use the simulator extractor $\mathsf{Sim}^{\mathsf{NMZK}}$ in order to check whether the theorem proved by every party controlled by $\mathcal{A}$ using NMZK are still true. If it is not the case, then we can make a reduction to the hiding of $\mathtt{com}_1$[21].

- The 5th hybrid experiment $\mathcal{H}_5$ follows the same steps of $\mathcal{H}_4$ except that the honest prover procedure ($\mathcal{P}_L$), instead of the Special HVZK simulator (Sim_L), is used to compute the prover's messages $\mathsf{a}_1, \mathsf{z}_1$ of the transcript $\tau_1 = (\mathsf{a}_1, \mathsf{c}_1, \mathsf{z}_1)$ w.r.t. the instance $\mathtt{com}_1$.

  Suppose now by contradiction that the output distributions of the hybrid experiments are distinguishable, then we can show a malicious verifier $\mathcal{V}^\star$ that distinguishes between a transcript $\tau_1 = (\mathsf{a}_1, \mathsf{c}_1, \mathsf{z}_1)$ computed using Sim_L and one computed using the honest prover procedure. In more details, let $\mathcal{C}_{\mathsf{SHVZK}}$ be the challenger of the Special HVZK. $\mathcal{V}^\star$ picks $\mathsf{c}_1 \leftarrow \{0,1\}^\lambda$ and sends c_1 to $\mathcal{C}_{\mathsf{SHVZK}}$. Upon receiving $\mathsf{a}_1, \mathsf{z}_1$ from $\mathcal{C}_{\mathsf{SHVZK}}$ $\mathcal{V}^\star$ plays all the messages of Π_{MPCT} as in $\mathcal{H}_4$ ($\mathcal{H}_5$) except for the messages of τ_1 where he $\mathcal{V}^\star$ acts as a proxy between $\mathcal{C}_{\mathsf{SHVZK}}$ and $P^\star$. At the end of the execution $\mathcal{V}^\star$ runs the distinguisher D that distinguishes the output distribution of $\mathcal{H}_4$ from the output distribution of $\mathcal{H}_5$ and outputs what D outputs. We observe that if $\mathcal{C}_{\mathsf{SHVZK}}$ sends a simulated transcript then $P_2^\star$ acts as in $\mathcal{H}_4$ otherwise he acts as in $\mathcal{H}_5$.

[20] Also in this case we are considering an adversary that completes the execution of Π_{MPCT} against Sim with non-negligible probability. In the case that the abort probability of the adversary is overwhelming then the security proof is already over.

[21] In order to extract the witnesses for the theorems proved by every party controlled by $\mathcal{A}$, $\mathsf{Sim}^{\mathsf{NMZK}}$ needs to rewind also from the 4th to the 3rd round, but this does not affect the reduction.

There is a subtlety in the above reduction $\mathcal{V}^\star$ runs the $\mathsf{Sim}^{\mathsf{NMZK}}$ that rewinds from the third to the second round. This means that $\mathcal{V}^\star$ has to be able to complete during the rewinds the third round while receiving different challenges $\mathsf{c}^1, \dots, \mathsf{c}^{\mathsf{poly}(\lambda)}$ w.r.t. Π_{OR}. Since we are splitting the challenge c, $\mathcal{V}^\star$ can just keep fixed the value c_1 reusing the same z_1 (sent by $\mathcal{C}_{\mathsf{SHVZK}}$) and computing an answer to a_0 using the knowledge of the decommitment information of $\mathtt{com}_0$. To argue that $\mathcal{A}$ correctly computes all the steps of Π_{MPCT}, also in this hybrid experiment we can use the simulator-extractor $\mathsf{Sim}^{\mathsf{NMZK}}$ to check whether the theorem proved by $\mathcal{A}$ is still true. If it is not the case we can construct a reduction to the Special HVZK property of BL_L. Note that the rewinds of $\mathsf{Sim}^{\mathsf{NMZK}}$ from the fourth to the third round do not affect the reduction. Moreover, the fact that $\mathsf{Sim}^{\mathsf{NMZK}}$ extracts the witnesses for the theorems proved by every party controlled by $\mathcal{A}$ still ensures that $\mathcal{A}$ behaves honestly.

- $\mathcal{H}_6$ proceeds exactly as $\mathcal{H}_5$ except that the Special HVZK simulator (Sim_L), instead of honest procedure ($\mathcal{P}_L$), is used to compute the prover's messages $\mathsf{a}_0, \mathsf{z}_0$ of the transcript $\tau_0 = (\mathsf{a}_0, \mathsf{c}_0, \mathsf{z}_0)$ w.r.t. the instance $\mathtt{com}_0$.

  We claim that $\{\mathsf{OUT}_{\mathcal{H}_5,\mathcal{A}(z)}(1^\lambda)\}$ and $\{\mathsf{OUT}_{\mathcal{H}_6,\mathcal{A}(z)}(1^\lambda)\}$ are computationally indistinguishable due the same arguments used to prove that $\{\mathsf{OUT}_{\mathcal{H}_4,\mathcal{A}(z)}(1^\lambda)\} \approx \{\mathsf{OUT}_{\mathcal{H}_5,\mathcal{A}(z)}(1^\lambda)\}$. Furthermore we claim that $\mathcal{A}$ still behaves honestly for the same arguments given in $\mathcal{H}_5$.
- The 7th hybrid experiment $\mathcal{H}_7$ differs from $\mathcal{H}_6$ in the way $\mathtt{com}_0$ is computed. More precisely, instead of committing to r_1 in $\mathtt{com}_0$, a commitment of a random string y is computed. For the same arguments used to prove that $\{\mathsf{OUT}_{\mathcal{H}_3,\mathcal{A}(z)}(1^\lambda)\} \approx \{\mathsf{OUT}_{\mathcal{H}_4,\mathcal{A}(z)}(1^\lambda)\}$, we claim that $\{\mathsf{OUT}_{\mathcal{H}_6,\mathcal{A}(z)}(1^\lambda)\} \approx \{\mathsf{OUT}_{\mathcal{H}_7,\mathcal{A}(z)}(1^\lambda)\}$ and that $\mathcal{A}$ still behaves honestly. We observe that r_1 appears only in the 4th round. More precisely there is no relation between r_1 and the values committed in $\mathcal{H}_1$. Therefore the security proof is almost over. Indeed our simulator Sim proceeds as $\mathcal{H}_7$ until the 3rd round, then invokes the functionality thus obtaining a value r and completes the 4th round of $\mathcal{H}_7$ setting $r_1 = r \oplus \cdots \oplus r_n^*$.

□

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A Standard Definitions

Definition 5 (One-way function (OWF))**.** *A function $f : \{0,1\}^\star \rightarrow \{0,1\}^\star$ is called one way if the following two conditions hold:*

- *there exists a deterministic polynomial-time algorithm that on input y in the domain of f outputs $f(y)$;*
- *for every* PPT *algorithm $\mathcal{A}$ there exists a negligible function ν, such that for every auxiliary input $z \in \{0,1\}^{\mathsf{poly}(\lambda)}$:*

$$\text{Prob}\left[\, y \leftarrow \{0,1\}^\star : \mathcal{A}(f(y), z) \in f^{-1}(f(y)) \,\right] < \nu(\lambda).$$

We say that a OWF f is a 1-to-1 OWF *if $f(x) \neq f(y)$ $\forall (x,y) \in \{0,1\}^\star$.*

Definition 6 (Following the notation of [CPS13])**.** *A triple of* PPT *algorithms* $(\mathsf{Gen}, \mathsf{Sign}, \mathsf{Ver})$ *is called a* signature scheme *if it satisfies the following properties.*

Validity: *For every pair $(s, v) \leftarrow \mathsf{Gen}(1^\lambda)$, and every $m \in \{0,1\}^\lambda$, we have that*

$$\mathsf{Ver}(v, m, \mathsf{Sign}(s, m)) = 1.$$

Security: *For every* PPT *$\mathcal{A}$, there exists a negligible function ν, such that for all auxiliary input $z \in \{0,1\}^\star$ it holds that:*

$$\Pr[(s, v) \leftarrow \mathsf{Gen}(1^\lambda); (m, \sigma) \leftarrow \mathcal{A}^{\mathsf{Sign}(s,\cdot)}(z, v) \wedge \mathsf{Ver}(v, m, \sigma) = 1 \wedge m \notin Q] < \nu(\lambda)$$

where Q denotes the set of messages whose signatures were requested by $\mathcal{A}$ to the oracle $\mathsf{Sign}(s, \cdot)$.

Definition 7 (Computational indistinguishability)**.** *Let $X = \{X_\lambda\}_{\lambda \in \mathbb{N}}$ and $Y = \{Y_\lambda\}_{\lambda \in \mathbb{N}}$ be ensembles, where X_λ's and Y_λ's are probability distribution over $\{0,1\}^l$, for same $l = \mathsf{poly}(\lambda)$. We say that $X = \{X_\lambda\}_{\lambda \in \mathbb{N}}$ and $Y = \{Y_\lambda\}_{\lambda \in \mathbb{N}}$ are* computationally indistinguishable, *denoted $X \approx Y$, if for every* PPT *distinguisher $\mathcal{D}$ there exists a negligible function ν such that for sufficiently large $\lambda \in \mathbb{N}$,*

$$\left|\text{Prob}\left[\; t \leftarrow X_\lambda : \mathcal{D}(1^\lambda, t) = 1 \;\right] - \text{Prob}\left[\; t \leftarrow Y_\lambda : \mathcal{D}(1^\lambda, t) = 1 \;\right]\right| < \nu(\lambda).$$

We note that in the usual case where $|X_\lambda| = \Omega(\lambda)$ and λ can be derived from a sample of X_λ, it is possible to omit the auxiliary input 1^λ. In this paper we also use the definition of *Statistical Indistinguishability*. This definition is the same as Definition 7 with the only difference that the distinguisher $\mathcal{D}$ is unbounded. In this case use $X \equiv_s Y$ to denote that two ensembles are statistically indistinguishable.

Definition 8 (Witness Indistinguishable (WI))**.** *An argument/proof system* $\Pi = (\mathcal{P}, \mathcal{V})$*, is* Witness Indistinguishable (WI) *for a relation* Rel *if, for every malicious* PPT *verifier* $\mathcal{V}^\star$*, there exists a negligible function* ν *such that for all* x, w, w' *such that* $(x, w) \in \mathsf{Rel}$ *and* $(x, w') \in \mathsf{Rel}$ *it holds that:*

$$\left| \mathrm{Prob}\left[\, \langle \mathcal{P}(w), \mathcal{V}^\star \rangle(x) = 1 \,\right] - \mathrm{Prob}\left[\, \langle \mathcal{P}(w'), \mathcal{V}^\star \rangle(x) = 1 \,\right] \right| < \nu(|x|).$$

Obviously one can generalize the above definitions of WI to their natural adaptive-input variants, where the adversarial verifier can select the statement and the witnesses adaptively, before the prover plays the last round.

Definition 9 (Proof of Knowledge [LP11b])**.** *A protocol* $\Pi = (\mathcal{P}, \mathcal{V})$ *that enjoys completeness is a* proof of knowledge *(PoK) for the relation* $\mathsf{Rel_L}$ *if there exists a probabilistic expected polynomial-time machine* E*, called the extractor, such that for every algorithm* $\mathcal{P}^\star$*, there exists a negligible function* ν*, every statement* $x \in \{0,1\}^\lambda$*, every randomness* $r \in \{0,1\}^\star$ *and every auxiliary input* $z \in \{0,1\}^\star$*,*

$$\mathrm{Prob}\left[\, \langle \mathcal{P}_r^\star(z), \mathcal{V} \rangle(x) = 1 \,\right] \le \mathrm{Prob}\left[\, w \leftarrow \mathsf{E}^{\mathcal{P}_r^\star(z)}(x) : (x, w) \in \mathsf{Rel_L} \,\right] + \nu(\lambda).$$

We also say that an argument system Π *is a* argument of knowledge *(AoK) if the above condition holds w.r.t. any* PPT $\mathcal{P}^\star$*.*

In our security proofs we make use of the following observation. An interactive protocol Π that enjoys the property of completeness and PoK (AoK) is a proof (an argument) system. Indeed suppose by contradiction that is not. By the definition of PoK (AoK) it is possible to extract the witness for every theorem $x \in \{0,1\}^\lambda$ proved by $\mathcal{P}_r^\star$ with probability greater than $\mathrm{Prob}[\, \langle \mathcal{P}_r^\star(z), \mathcal{V} \rangle(x) = 1 \,]$; contradiction.

In this paper we also consider the *adaptive-input* PoK/AoK property for all the protocols that enjoy delayed-input completeness. Adaptive-input PoK/AoK ensures that the PoK/AoK property still holds when a malicious prover can choose the statement adaptively at the last round.

A *3-round protocol* $\Pi = (\mathcal{P}, \mathcal{V})$ for a relation $\mathsf{Rel_L}$ is an interactive protocol played between a prover $\mathcal{P}$ and a verifier $\mathcal{V}$ on common input x and private input w of $\mathcal{P}$ s.t. $(x, w) \in \mathsf{Rel_L}$. In a 3-round protocol the first message a and the third message z are sent by $\mathcal{P}$ and the second messages c is played by $\mathcal{V}$. At the end of the protocol $\mathcal{V}$ decides to accept or reject based on the data that he has seen, i.e. $x, \mathsf{a}, \mathsf{c}, \mathsf{z}$.

We usually denote the message c sent by $\mathcal{V}$ as a *challenge*, and as *challenge length* the number of bit of c.

Definition 10 (Σ-Protocol)**.** *A 3-round public-coin protocol* $\Pi = (\mathcal{P}, \mathcal{V})$ *for a relation* $\mathsf{Rel_L}$ *is a* Σ*-Protocol if the following properties hold:*

- *Completeness: if* $(\mathcal{P}, \mathcal{V})$ *follow the protocol on input* x *and private input* w *to* $\mathcal{P}$ *s.t.* $(x, w) \in \mathsf{Rel_L}$*,* $\mathcal{V}$ *always accepts.*

- *Special soundness: if there exists a polynomial time algorithm such that, for any pair of accepting transcripts on input* x*,* $(\mathsf{a}, \mathsf{c}_1, \mathsf{z}_1), (\mathsf{a}, \mathsf{c}_2, \mathsf{z}_2)$ *where* $\mathsf{c}_1 \neq \mathsf{c}_2$*, outputs witness* w *such that* $(x, w) \in \mathsf{Rel_L}$*.*

- *Special Honest Verifier Zero-knowledge (Special HVZK): there exists a* PPT *simulator algorithm* Sim *that for any* $x \in L$*, security parameter* λ *and any challenge* c *works as follow:* $(\mathsf{a}, \mathsf{z}) \leftarrow \mathsf{Sim}(1^\lambda, x, \mathsf{c})$*.*

Furthermore, the distribution of the output of Sim *is computationally indistinguishable from the distribution of a transcript obtained when* $\mathcal{V}$ *sends* c *as challenge and* $\mathcal{P}$ *runs on common input* x *and any* w *such that* $(x, w) \in \mathsf{Rel_L}$[22].

Definition 11. *A delayed-input 3-round protocol* $\Pi = (\mathcal{P}, \mathcal{V})$ *for relation* $\mathsf{Rel_L}$ *enjoys* adaptive-input special soundness *if there exists a polynomial time algorithm such that, for any pair of accepting transcripts* $(\mathsf{a}, \mathsf{c}_1, \mathsf{z}_1)$ *for input* x_1 *and* $(\mathsf{a}, \mathsf{c}_2, \mathsf{z}_2)$ *for input* x_2 *with* $\mathsf{c}_1 \neq \mathsf{c}_2$*, outputs witnesses* w_1 *and* w_2 *such that* $(x_1, w_1) \in \mathsf{Rel_L}$ *and* $(x_2, w_2) \in \mathsf{Rel_L}$*.*

Definition 12. *A delayed-input 3-round protocol* $\Pi = (\mathcal{P}, \mathcal{V})$ *for relation* $\mathsf{Rel_L}$ *enjoys* adaptive-input Special Honest Verifier Zero-knowledge (adaptive-input Special HVZK) *if there exists a two phases* PPT *simulator algorithm* Sim *that works as follow:*

1. $\mathsf{a} \leftarrow \mathsf{Sim}(1^\lambda, \mathsf{c}, \kappa; \rho)$*, where* 1^λ *is the security parameter,* c *is the challenge* κ *is the size of the instance to be proved and the randomness* ρ*;*
2. $\mathsf{z} \leftarrow \mathsf{Sim}(x, \rho)$[23]*, where* x *is the instance to be proved.*

Π *is adaptive-input Special HVZK if any* $x \in L$ *and for any* $\mathsf{c} \in \{0,1\}^\lambda$*, the distribution of the transcripts* $(\mathsf{a}, \mathsf{c}, \mathsf{z})$*, computed by* Sim*, is computationally indistinguishable from the distribution of a transcript obtained when* $\mathcal{V}$ *sends* c *as challenge and* $\mathcal{P}$ *runs on common input* x *and any* w *(available only in the third round) such that* $(x, w) \in \mathsf{Rel_L}$*.*

A.1 Commitment Schemes

Definition 13 (Commitment Scheme)**.** *Given a security parameter* 1^λ*, a commitment scheme* $\mathtt{CS} = (\mathsf{Sen}, \mathsf{Rec})$ *is a two-phase protocol between two* PPT *interactive algorithms, a sender* Sen *and a receiver* Rec*. In the commitment phase* Sen *on input a message* m *interacts with* Rec *to produce a commitment* com*, and the private output* d *of* Sen*.*

In the decommitment phase, Sen *sends to* Rec *a decommitment information* $(m, \mathtt{d})$ *such that* Rec *accepts* m *as the decommitment of* com*.*

Formally, we say that $\mathtt{CS} = (\mathsf{Sen}, \mathsf{Rec})$ *is a perfectly binding commitment scheme if the following properties hold:*

Correctness:

- *Commitment phase. Let* com *be the commitment of the message* m *given as output of an execution of* $\mathtt{CS} = (\mathsf{Sen}, \mathsf{Rec})$ *where* Sen *runs on input a message* m*. Let* d *be the private output of* Sen *in this phase.*
- *Decommitment phase*[24]*.* Rec *on input* m *and* d *accepts* m *as decommitment of* com*.*

Statistical (resp. Computational) Hiding([Lin10]): *for any adversary (resp.* PPT *adversary)* $\mathcal{A}$ *and a randomly chosen bit* $b \in \{0,1\}$*, consider the following hiding experiment* $\mathsf{ExpHiding}^b_{\mathcal{A},\mathtt{CS}}(\lambda)$*:*

- *Upon input* 1^λ*, the adversary* $\mathcal{A}$ *outputs a pair of messages* m_0, m_1 *that are of the same length.*
- Sen *on input the message* m_b *interacts with* $\mathcal{A}$ *to produce a commitment of* m_b*.*
- $\mathcal{A}$ *outputs a bit* b' *and this is the output of the experiment.*

For any adversary (resp. PPT *adversary)* $\mathcal{A}$*, there exist a negligible function* ν*, s.t.:*

$$\left|\mathrm{Prob}\left[\ \mathsf{ExpHiding}^0_{\mathcal{A},\mathtt{CS}}(\lambda) = 1\ \right] - \mathrm{Prob}\left[\ \mathsf{ExpHiding}^1_{\mathcal{A},\mathtt{CS}}(\lambda) = 1\ \right]\right| < \nu(\lambda).$$

[22] Note that we require that the two transcripts are computationally indistinguishable as in [GMY06], instead of following [CDS94] that requires the perfect indistinguishability between the two transcripts.

[23] To not overburden the notation we omit the randomness when we use the adaptive-input Special HVZK simulator

[24] In this paper we consider a non-interactive decommitment phase only.

Statistical (resp. Computational) Binding: *for every commitment* $\texttt{com}$ *generated during the commitment phase by a possibly malicious unbounded (resp. malicious* PPT*) sender* $\mathsf{Sen}^\star$ *there exists a negligible function* ν *such that* $\mathsf{Sen}^\star$*, with probability at most* $\nu(\lambda)$*, outputs two decommitments* (m_0, d_0) *and* (m_1, d_1)*, with* $m_0 \neq m_1$*, such that* Rec *accepts both decommitments.*

We also say that a commitment scheme is perfectly binding *iff* $\nu(\lambda) = 0$.

When a commitment scheme $(\mathsf{Com}, \mathsf{Dec})$ *is non-interactive, to not overburden the notation, we use the following notation.*

– *Commitment phase.* $(\texttt{com}, \texttt{dec}) \leftarrow \mathsf{Com}(m)$ *denotes that* $\texttt{com}$ *is the commitment of the message* m *and* $\texttt{dec}$ *represents the corresponding decommitment information.*
– *Decommitment phase.* $\mathsf{Dec}(\texttt{com}, \texttt{dec}, m) = 1$.

A.2 3-Round Honest-Extractable Commitment Schemes

Informally, a 3-round commitment scheme is honest-extractable if there exists an efficient extractor that having black-box access to any efficient honest sender that successfully performs the commitment phase, outputs the only committed string that can be successfully decommitted. We give now a definition that follows the one of [PW09].

Definition 14 (Honest-Extractable Commitment Scheme)**.** *A perfectly (resp. statistically) binding commitment scheme* $\texttt{ExCS} = (\mathrm{ExSen}, \mathrm{ExRec})$ *is an* honest-extractable commitment scheme *if there exists an expected* PPT *extractor* $\texttt{ExtCom}$ *that given oracle access to any honest sender* ExSen*, outputs a pair* (τ, m) *such that the following two properties hold:*

- **Simulatability:** τ *is identically distributed to the view of* ExSen *(when interacting with an honest* ExRec*) in the commitment phase.*
- **Extractability:** *the probability that there exists a decommitment of* τ *to a message* m'*, where* $m' \neq m$ *is 0 (resp. negligible).*

B Definition of Secure Computation

Here we recall some useful definitions for our application. Our Multi-Party Computation (MPC) protocol for coin tossing is secure in the same model used in [GMPP16b], therefore some definitions are taken almost verbatim from [GMPP16b]. Always following Garg et al. we only recall the security definition for the the two party case. The description naturally extends to multi party case as well (details can be found in [Gol09]).

B.1 Two-Party Computation

A two-party protocol problem is cast by specifying a random process that maps pairs of inputs to pairs of outputs (one for each party). We refer to such a process as a functionality and denote it $F : \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}^* \times \{0,1\}^*$ where $F = (F_1, F_2)$. That is, for every pair of inputs (x, y), the output-pair is a random variable $(F_1(x, y), F_2(x, y))$ ranging over pairs of strings. The first party (with input x) wishes to obtain $F_1(x, y)$ and the second party (with input y) wishes to obtain $F_2(x, y)$.

Adversarial behavior. Loosely speaking, the aim of a secure two-party protocol is to protect an honest party against dishonest behavior by the other party. In this paper, we consider malicious adversaries who may arbitrarily deviate from the specified protocol. When considering malicious adversaries, there are certain undesirable actions that cannot be prevented. Specifically, a party may refuse to participate in the protocol, may substitute its local input (and use instead a different input) and may abort the protocol prematurely. One ramification of the adversary's ability to abort, is that it is impossible to achieve fairness. That is, the adversary may obtain its output while the honest party does not. In this work we consider a static corruption model, where one of the parties is adversarial and the other is honest, and this is fixed before the execution begins.

Communication channel. In our result we consider a secure simultaneous message exchange channel in which all parties can simultaneously send messages over the channel at the same communication round but allowing a rushing adversary. Moreover, we assume an asynchronous network[25] where the communication is open and delivery of messages is not guaranteed. For simplicity, we assume that the delivered messages are authenticated. This can be achieved using standard methods.

Execution in the ideal model. An ideal execution proceeds as follows. Each party obtains an input, denoted w ($w = x$ for P_1, and $w = y$ for P_2). An honest party always sends w to the trusted party. A malicious party may, depending on w, either abort or send some $w' \in \{0,1\}^{|w|}$ to the trusted party. In case it has obtained an input pair (x, y), the trusted party first replies to the first party with $F_1(x, y)$. Otherwise (i.e., in case it receives only one valid input), the trusted party replies to both parties with a special symbol $\perp$. In case the first party is malicious it may, depending on its input and the trusted party's answer, decide to stop the trusted party by sending it $\perp$ after receiving its output. In this case the trusted party sends $\perp$ to the second party. Otherwise (i.e., if not stopped), the trusted party sends $F_2(x, y)$ to the second party. Outputs: An honest party always outputs the message it has obtained from the trusted party. A malicious party may output an arbitrary (probabilistic polynomial-time computable) function of its initial input and the message obtained from the trusted party.

Let $F : \{0,1\}^* \times \{0,1\}^* \to \{0,1\}^* \times \{0,1\}^*$ be a functionality where $F = (F_1, F_2)$ and let $S = (S_1, S_2)$ be a pair of non-uniform probabilistic expected polynomial-time machines (representing parties in the ideal model). Such a pair is admissible if for at least one $i \in \{0,1\}$ we have that S_i is honest (i.e., follows the honest party instructions in the above-described ideal execution). Then, the joint execution of F under S in the ideal model (on input pair (x, y) and security parameter λ), denoted $\mathsf{IDEAL}_{F,S(z)}(1^\lambda, x, y)$ is defined as the output pair of S_1 and S_2 from the above ideal execution.

Execution in the real model. We next consider the real model in which a real (two-party) protocol is executed (and there exists no trusted third party). In this case, a malicious party may follow an arbitrary feasible strategy; that is, any strategy implementable by non-uniform probabilistic polynomial-time machines. In particular, the malicious party may abort the execution at any point in time (and when this happens prematurely, the other party is left with no output). Let F be as above and let Π be a two-party protocol for computing F. Furthermore, let $A = (A_1, A_2)$ be a pair of non-uniform probabilistic polynomial-time machines (representing parties in the real model). Such a pair is admissible if for at least one $i \in \{0,1\}$ we have that A_i is honest (i.e., follows the strategy specified by Π). Then, the joint execution of Π under A in the real model, denoted $\mathsf{REAL}_{\Pi,\mathcal{A}(z)}(1^\lambda)$, is defined as the output pair of A_1 and A_2 resulting from the protocol interaction.

Definition 15 (secure two-party computation)**.** *Let F and Π be as above. Protocol Π is said to securely compute F (in the malicious model) if for every pair of admissible non-uniform probabilistic polynomial-time machines $A = (A_1, A_2)$ that run with auxiliary input z for the real model, there exists a pair of admissible non-uniform probabilistic expected polynomial-time machines $S = (S_1, S_2)$ (that use z as auxiliary input) for the ideal model, such that:*

$$\mathsf{REAL}_{\Pi,\mathcal{A}(z)}(1^\lambda) \approx \mathsf{IDEAL}_{f,S(z)}(1^\lambda).$$

[25] The fact that the network is asynchronous means that the messages are not necessarily delivered in the order which they are sent.

C Special WIPoK

C.1 Improving the Soundness of LS

In this section we consider the 3-round WIPoK for the $\mathcal{NP}$-complete language of graph Hamiltonicity (HC), provided in [LS90], and we will refer to this construction as the *LS protocol*. An interesting property of this WIPoK is that only the size of the statement need to be known before the last round by both the prover and the verifier. We show that the LS protocol does not enjoys special soundness when the statement to be proved is adaptively chosen by the prover in the last round. That is, if two accepting transcripts (that share the first round) are provided w.r.t. to two different instances x_0 and x_1, then only the witness w for x_b is extracted (with $b \in \{0,1\}$). More precisely, given the accepting transcript $(\mathsf{ls}^1, \mathsf{ls}^2_0, \mathsf{ls}^3_0)$ for the statement x_0 and $(\mathsf{ls}^1, \mathsf{ls}^2_1, \mathsf{ls}^3_1)$ for the statement x_1 (with $\mathsf{ls}^2_0 \neq \mathsf{ls}^2_1$) then it could be that only w_b can be extracted. We provide a construction that overcomes this issue, allowing the extraction of the witnesses for both x_0 and x_1 thus obtaining a Σ-protocol where the special soundness holds even when the two accepting transcripts refer to different theorems adaptively chosen in the last round. Following [CPS+16b] we refer to this property as adaptive-input special soundness (see Definition 11).

Before showing why LS is not already adaptive-input special sound and how our construction works, we briefly describe the LS protocol with one-bit challenge following [OV12].

Let $\mathcal{P}$ be prover and $\mathcal{V}$ the verifier. The common input of $\mathcal{P}$ and $\mathcal{V}$ is κ, that represents the number of vertexes of the instance G to be proved. The graph G is represented by a $\kappa \times \kappa$ adjacency matrix MG where $\mathsf{MG}[i][j] = 1$ if there exists an edge between vertexes i and j in G. A non-edge position i,j is a pair of vertexes that are not connected in G and for which $\mathsf{MG}[i][j] = 0$.

- $\mathcal{P}$ picks a random κ-vertex cycle graph C and commits bit-by-bit to the corresponding adjacency matrix using a statistically binding commitment scheme.
- $\mathcal{V}$ responds with a randomly chosen bit b.
- $\mathcal{P}$ on input the graph G and the Hamiltonian cycle w executes the following steps. If $b = 0$, $\mathcal{P}$ opens all the commitments, showing that the matrix committed in the first round is actually an κ-vertex cycle. If $b = 1$, $\mathcal{P}$ sends a permutation π mapping the vertex of C in G. Then it opens the commitment of the adjacency matrix of C corresponding to the non-edges of the graph G.
- $\mathcal{V}$ accepts (outputs 1) if what he receives in the third round is consistent with the bit b that he was sent in the second round.

Getting the answer for both $b = 0$ and $b = 1$ (w.r.t. to the same graph G) allows the extraction of the cycle for G. The reason is the following. For $b = 0$ one gets the random cycle C. Then for $b = 1$ one gets the permutation mapping the random cycle in the actual cycle that is given to $\mathcal{P}$ before the last message of the protocol.

We now observe that a malicious prover $\mathcal{P}^\star$ could gives the answer for $b = 0$ w.r.t. to the graph G_0 and the answer for $b = 1$ w.r.t. the graph G_1 (due to the delayed-input nature of LS). This means that even knowing two accepting transcripts that share the first round, the permutation that maps the vertexes of C in G_0 it is not known. Therefore an efficient algorithm can only compute the cycle w_1 of G_1 and gets no information about the Hamiltonian cycle of G_0. Summing up, given the accepting transcripts $(\mathsf{ls}^1, 0, \mathsf{ls}^3_0)$ for the graph G_0 and $(\mathsf{ls}^1, 1, \mathsf{ls}^3_1)$ for the graph G_1, only the Hamiltonian cycle for G_1 can be computed. That is, only the cycle for the graph proved by $\mathcal{P}^\star$ to be Hamiltonian using as a second round the challenge 1 can be efficiently computed. Starting from this observation, in order to allow an efficient algorithm to compute cycles for both G_0 and G_1, we construct an improved version of LS that we denoted with $\mathsf{LS}^{\mathsf{imp}} = (\mathcal{P}^{\mathsf{imp}}, \mathcal{V}^{\mathsf{imp}})$. $\mathsf{LS}^{\mathsf{imp}}$ uses LS in a black-box way. For ease of exposition we use the following notation. $\mathsf{ls}^1 \leftarrow \mathcal{P}(1^\lambda, \kappa; \rho)$ denotes that $\mathcal{P}$ is executed on input the security parameter (in unary) 1^λ, κ and the randomness ρ and gives in output the first round of LS ls^1. $\mathsf{ls}^3 \leftarrow \mathcal{P}(G, w, \mathsf{ls}^2, \rho)$ denotes that $\mathcal{P}$ has computed the third round of LS by running on input the graph G, the cycle w for the graph G, the bit ls^2 and the randomness used to compute ls^1. $\mathcal{V}(\mathsf{ls}^1, \mathsf{ls}^2, \mathsf{ls}^3, G)$ denotes the output of $\mathcal{V}$ on input $\mathsf{ls}^1, \mathsf{ls}^2, \mathsf{ls}^3$ and the graph G. Let κ be the number of vertexes of the graph G to be proved, our $\mathsf{LS}^{\mathsf{imp}} = (\mathcal{P}^{\mathsf{imp}}, \mathcal{V}^{\mathsf{imp}})$ works as follows.

1. $\mathcal{P}^{\mathsf{imp}}$ on input the security parameter λ, κ and the randomness $\rho_0||\rho_1$ computes $\mathsf{ls}_0^1 \leftarrow \mathcal{P}(1^\lambda, \kappa; \rho_0)$, $\mathsf{ls}_1^1 \leftarrow \mathcal{P}(1^\lambda, \kappa; \rho_1)$ and sends $(\mathsf{ls}_1^0, \mathsf{ls}_1^1)$ to $\mathcal{V}^{\mathsf{imp}}$.
2. $\mathcal{V}^{\mathsf{imp}}$ picks and sends a random bit b.
3. $\mathcal{P}^{\mathsf{imp}}$, upon receiving b, on input the graph G and the Hamiltonian cycle w for G computes $\mathsf{ls}_0^3 \leftarrow \mathcal{P}(G, w, b, \rho_0)$, $\mathsf{ls}_1^3 \leftarrow \mathcal{P}(G, w, 1-b, \rho_1)$ and sends $(\mathsf{ls}_0^3, \mathsf{ls}_1^3)$.
4. $\mathcal{V}^{\mathsf{imp}}$ accepts iff $\mathcal{V}(G, \mathsf{ls}_0^1, b, \mathsf{ls}_0^3) = 1$ and $\mathcal{V}(G, \mathsf{ls}_1^1, 1-b, \mathsf{ls}_1^3) = 1$.

Theorem 4. *Assuming one-to-one OWFs, $\mathsf{LS}^{\mathsf{imp}}$ is a Σ-protocol with adaptive-input Special HVZK simulator and adaptive-input special soundness. Moreover $\mathsf{LS}^{\mathsf{imp}}$ is Zero Knowledge.*

Proof. **(Delayed-input) Completeness.** The (delayed-input) completeness of $\mathsf{LS}^{\mathsf{imp}}$ comes from the (delayed-input) completeness of LS.
Adaptive-input special soundness. Let us consider two accepting transcripts that share the first round for $\mathsf{LS}^{\mathsf{imp}}$: $\big((\mathsf{ls}_0, \mathsf{ls}_1), 0, (\mathsf{ls}_0^3, \mathsf{ls}_1^3)\big)$ for the statement G and $\big((\mathsf{ls}_0, \mathsf{ls}_1), 1, (\mathsf{ls}_1^{3'}, \mathsf{ls}_1^{3'})\big)$ for the statement G'. We can isolate the sub-transcripts $(\mathsf{ls}_0, 0, \mathsf{ls}_0^3)$ and $(\mathsf{ls}_0, 1, \mathsf{ls}_0^{3'})$ and observe that $\mathcal{V}(G, \mathsf{ls}_0^1, 0, \mathsf{ls}_0^3) = 1 = \mathcal{V}(G'\mathsf{ls}_0^1, 1, \mathsf{ls}_0^{3'})$. From what we discuss before about LS we know that in this case the witness w for G' can be extracted. Also let us now consider the two sub-transcripts $(\mathsf{ls}_1, 1, \mathsf{ls}_1^3)$ and $(\mathsf{ls}_1, 0, \mathsf{ls}_1^{3'})$. Also in this case, by observing that $\mathcal{V}(G, \mathsf{ls}_1, 1, \mathsf{ls}_1^3) = 1 = \mathcal{V}(G', \mathsf{ls}_1, 0, \mathsf{ls}_1^{3'})$, the cycle for G can be efficiently computed.
Adaptive-input Special HVZK. Following [MV16], we consider an adaptive-input Special HVZK simulator S associated to the LS's protocol. This is equal to a Special HVZK simulator with the additional property that the first round can be simulated without knowing the instance to be proved (see Definition 12). In more details S works in two phases. In the first phase just 1^λ, the challenge ls^2, the number of vertexes κ is used to output the first round ls^1. We denote this phase using: $\mathsf{ls}^1 \leftarrow S(1^\lambda, \mathsf{ls}^2, \kappa)$. In the second phase S takes as input the instance and output the third round ls^3. We denote this phase using $\mathsf{ls}^3 \leftarrow S(G)$. The adaptive-input Special HVZK simulator S^{imp} for $\mathsf{LS}^{\mathsf{imp}}$ just internally runs S two times, once using b and once using $1-b$ as a challenge. In more details the two phase of S^{imp} are the following.

1. S^{imp}, on input 1^λ, the challenge b, κ and the randomness $\rho_b||\rho_{1-b}$, computes $\mathsf{ls}_b^1 \leftarrow S(1^\lambda, b, \kappa; \rho_b)$, $\mathsf{ls}_{1-b}^1 \leftarrow S(1^\lambda, 1-b, \kappa; \rho_{1-b})$ and outputs $(\mathsf{ls}_b^1, \mathsf{ls}_{1-b}^1)$.
2. S^{imp}, on input the graph G, ρ_0 and ρ_1 computes $\mathsf{ls}_b^3 \leftarrow S(G, \rho_b)$, $\mathsf{ls}_{1-b}^3 \leftarrow S(G, \rho_{1-b})$ and outputs $(\mathsf{ls}_b^3, \mathsf{ls}_{1-b}^3)$.

The transcript $\big((\mathsf{ls}_b^1, \mathsf{ls}_{1-b}^1), b, (\mathsf{ls}_b^3, \mathsf{ls}_{1-b}^3)\big)$ output by S^{imp} is is computationally indistinguishable from a transcript computed by $\mathcal{P}^{\mathsf{imp}}$ (that uses as input an Hamiltonian cycle w of G) due to the security of the underlying adaptive-input Special HVZK simulator S.
Zero-Knowledge. The ZK simulator of $\mathsf{LS}^{\mathsf{imp}}$ just needs to guess the bit b chosen by the adversarial verifier and runs the adaptive-input Special HVZK simulator.

□

It is easy to see that (as for LS) if we consider λ parallel executions of $\mathsf{LS}^{\mathsf{imp}}$ then we obtain a protocol LS^λ that still enjoys adaptive-input completeness, adaptive-input special soundness, adaptive-input Special HVZK. Moreover LS^λ is WI. Formally, we can claim the following theorems.

Theorem 5. *Assuming one-to-one OWFs, LS^λ is a Σ-protocol with adaptive-input Special HVZK, adaptive-input special soundness and WI.*

Proof. Completeness, adaptive-input special soundness and adaptive-input Special HVZK come immediately from the adaptive-input special soundness and adaptive-input Special HVZK of $\mathsf{LS}^{\mathsf{imp}}$. The WI comes from the observation that $\mathsf{LS}^{\mathsf{imp}}$ is WI (due to the zero knowledge property), and that WI is preserved under parallel (and concurrent) composition. □

Theorem 6. *Assuming OWFs, LS^λ is a 4-round public-coin interactive protocol with adaptive-input Special HVZK, adaptive-input special soundness and WI.*

Proof. The proof of this theorem just relies on the observation that in order to instantiate a statistically binding commitment scheme using OWFs an additional round is required to compute the first round of Naor's commitment scheme [Nao91]. □

Observe that since Hamiltonicity is an $\mathcal{NP}$-complete language, the above constructions work for any $\mathcal{NP}$ language through $\mathcal{NP}$ reductions. For simplicity in the rest of the paper we will omit the $\mathcal{NP}$ reduction therefore assuming that the above scheme works directly on a given $\mathcal{NP}$-language L.

C.2 Combining (adaptive-input) Special HVZK PoK Through [CDS94]

In our paper we use the well known technique for composing two Σ-protocols to compute the OR for compound statement [CDS94, GMY06]. In more details, let $\Pi_0 = (\mathcal{P}_0, \mathcal{V}_0)$ and $\Pi_1 = (\mathcal{P}_1, \mathcal{V}_1)$ be Σ-protocols for the respective $\mathcal{NP}$-relation $\mathsf{Rel}_{\mathsf{L}_0}$ (with Special HVZK simulator Sim_0) and $\mathsf{Rel}_{\mathsf{L}_1}$ (with Special HVZK simulator Sim_1). Then it is possible to use Π_0 and Π_1 to construct $\Pi_{\mathsf{OR}} = (\mathcal{P}_{\mathsf{OR}}, \mathcal{V}_{\mathsf{OR}})$ for relation $\mathsf{Rel}_{\mathsf{OR}} = \{((x_0, x_1), w) : ((x_0, w) \in \mathsf{Rel}_{\mathsf{L}_0}) \text{ OR } ((x_1, w) \in \mathsf{Rel}_{\mathsf{L}_1})\}$ that works as follows.

Protocol $\Pi_{\mathsf{OR}} = (\mathcal{P}_{\mathsf{OR}}, \mathcal{V}_{\mathsf{OR}})$: Let w_b with $b \in \{0,1\}$ be s.t. $(x_b, w_b) \in \mathsf{Rel}_{\mathsf{L_b}}$. $\mathcal{P}_{\mathsf{OR}}$ and $\mathcal{V}_{\mathsf{OR}}$ on common input (x_0, x_1) and private input w_b compute the following steps.

- $\mathcal{P}_{\mathsf{OR}}$ computes $\mathsf{a}_b \leftarrow \mathcal{P}_b(1^\lambda, x_b, w_b)$. Furthermore he picks $\mathsf{c}_{1-b} \leftarrow \{0,1\}^\lambda$ and computes $(\mathsf{a}_{1-b}, \mathsf{z}_{1-b}) \leftarrow \mathsf{Sim}_{1-b}(1^\lambda, x_{1-b}, \mathsf{c}_{1-b})$. $\mathcal{P}_{\mathsf{OR}}$ sends $\mathsf{a}_0, \mathsf{a}_1$ to $\mathcal{V}_{\mathsf{OR}}$.
- $\mathcal{V}_{\mathsf{OR}}$, upon receiving $\mathsf{a}_0, \mathsf{a}_1$ picks $\mathsf{c} \leftarrow \{0,1\}^\lambda$ and sends c to $\mathcal{P}_{\mathsf{OR}}$.
- $\mathcal{P}_{\mathsf{OR}}$, upon receiving c computes $\mathsf{c}_b = \mathsf{c}_{1-b} \oplus \mathsf{c}$ and computes $\mathsf{z}_b \leftarrow \mathcal{P}_b(\mathsf{c}_b)$. $\mathcal{P}_{\mathsf{OR}}$ sends $\mathsf{c}_0, \mathsf{c}_1, \mathsf{z}_0\ \mathsf{z}_1$ to $\mathcal{V}_{\mathsf{OR}}$.
- $\mathcal{V}_{\mathsf{OR}}$ checks that the following conditions holds: $\mathsf{c} = \mathsf{c}_0 \oplus \mathsf{c}_1$, $\mathcal{V}_0(x_0, \mathsf{a}_0, \mathsf{c}_0, \mathsf{z}_0) = 1$ and $\mathcal{V}_1(x_1, \mathsf{a}_1, \mathsf{c}_1, \mathsf{z}_1) = 1$. If all the checks succeed then outputs 1, otherwise outputs 0.

Theorem 7. *([CDS94]) Let Σ_0 and Σ_1 be two Σ-protocols, then $\Pi_{\mathsf{OR}} = (\mathcal{P}_{\mathsf{OR}}, \mathcal{V}_{\mathsf{OR}})$ is a Σ-protocol for $\mathsf{Rel}_{\mathsf{L_{OR}}}$.*

Theorem 8. *([Dam10]) Let $\Pi = (\mathcal{P}, \mathcal{V})$ be a Σ-protocol for relation $\mathsf{Rel}_{\mathsf{L}}$ with negligible soundness error[26], then Π is a proof of knowledge for $\mathsf{Rel}_{\mathsf{L}}$.*

In our work we instantiate Π_{OR} using as Π_0 and Π_1 the Blum's protocol [Blu86b] for the $\mathcal{NP}$-complete language for graph Hamiltonicity (that also is a Σ-Protocol). Therefore Th. 7 (and Th. 8) can be applied.

We also consider an instantiation of Π_{OR} using as $\Pi = (\mathcal{P}, \mathcal{V})$ our LS^λ. If we instantiate Π_{OR} using LS^λ and the corresponding adaptive-input Special HVZK simulator LS^λ, then Π_{OR} is adaptive-input special soundness. More formally we can claim the following theorem.

Theorem 9. *If Π_{OR} is instantiated using LS^λ (and the corresponding adaptive-input Special HVZK simulator S^λ), then Π_{OR} enjoys the delayed-input completeness and adaptive-input special sound for the $\mathcal{NP}$-relation $\mathsf{Rel}_{\mathsf{L_{OR}}}$.*

Proof. The delayed-input completeness follows from the delayed-input completeness of LS^λ.

Adaptive-input special soundness. Let us consider two accepting transcripts that share the first round for Π_{OR}: $\left((\pi_0, \pi_1), \pi^2, (\pi_0^2, \pi_0^3, \pi_1^2, \pi_1^3)\right)$ for the statement (x_0, x_1) and $\left((\pi_0, \pi_1), \pi^{2'}, (\pi_0^{2'}, \pi_0^{3'}, \pi_1^{2'}\pi_1^{3'})\right)$ for the statement (x'_0, x'_1), where $\pi^2 \neq \pi^{2'}$. We observe that since $\pi^2 \neq \pi^{2'}$, $\pi^2 = \pi_0^2 \oplus \pi_1^2$ and $\pi^{2'} = \pi_0^{2'} \oplus \pi_1^{2'}$ it holds that either $\pi_0^2 \neq \pi_0^{2'}$ or $\pi_1^2 \neq \pi_1^{2'}$. Suppose w.l.o.g. that $\pi_0^2 \neq \pi_0^{2'}$. Then we are guaranteed from the adaptive-input special soundness of LS^λ that using the transcripts $(\pi_0, \pi_0^2, \pi_0^3)$ and $(\pi_0, \pi_0^{2'}, \pi_0^{3'})$ the values (w_a, w_b) s.t. $(x_0, w_a) \in \mathsf{Rel}_{\mathsf{L}_0}$ and $(x'_0, w_b) \in \mathsf{Rel}_{\mathsf{L}_0}$ can be extracted in polynomial-time. The same arguments can be used when $\pi_1^2 \neq \pi_1^{2'}$. □

[26]The soundness error represents the probability of a malicious prover to convince the verifier of a false statement.

Using a result of [CPS+16b] we can claim the following theorem.

Theorem 10. Π_{OR} *instantiated using* LS^{λ} *is adaptive-input PoK for the* $\mathcal{NP}$*-relation* $\mathsf{Rel}_{\mathsf{L_{OR}}}$.

It would be easy to prove that Π_{OR} is also WI, however in this paper we are not going to rely directly on the WI property of Π_{OR}, in order to deal with the rewinding issue that we have described earlier. More precisely, in the two main contributions of this paper we will use Π_{OR} (the one instantiated from Blum's protocol and the one instantiated using LS^{λ}) in a non-black box way in order to prove the security of our protocols. It will be crucial for our reduction to rely on the (adaptive-input) Special HVZK of Π_0 and Π_1 instead of using directly the WI property of Π_{OR}. The intuitively reason is that it is often easier in a reduction to rely on the security of a non-interactive primitive (like Special HVZK is) instead of an interactive primitive (like WI). This is the reason why we use the OR composition of [CDS94, GMY06] combined with the Blum's protocol (or the LS protocol) instead of relying on the (adaptive-input) WI provided by a Blum's protocol (LS protocol).

In the rest of the paper, in order to rely on OWFs only, we sometimes use a four round version of Blum's and LS protocols. In this case there is an additional initial round that goes from the verifier to the prover and corresponds to the first round of Naor's commitment scheme [Nao91].