# A 128-bit Block Cipher Based on Three Group Arithmetics * 

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#### Abstract

Enlightened by the IDEA block cipher, the authors put forward a symmetric key cryptosystem called REESSE3+ based on three group arithmetics: addition modulo 2 (bit XOR), addition modulo $2^{\wedge} 16$, and multiplication modulo $2 \wedge 16+1$. Different from IDEA, REESSE3+ uses a 128 -bit block, a 256 -bit key, and a renovative round function. The authors describe the REESSE3+ cipher algorithm in the graph, and expound the encryption subkeys, encryption operation, decryption subkeys, and decryption operation. Further, demonstrate the correctness of the REESSE3+ cipher algorithm, and analyze the security of REESSE3+ from four aspects. The measures for assuring the security of REESSE3+ cover those for assuring the security of IDEA, which indicates that the ability of REESSE3+ in resisting differential cryptanalysis should be at least equivalent to that of IDEA. Moreover, experiments show that a mini-version of REESSE3+ is immune to differential cryptanalysis, thus it may be expected that REESSE3+ is secure against differential attack after 8 rounds.


Keywords: Block cipher algorithm; Symmetric key; Round function; Group arithmetic; Security; Markov cipher

## 1 Introduction

In April 1997, America National Institute of Standard Technology (NIST) began to collect symmetrical key cryptosystems for the advanced encryption standard (AES) in the concerned countries. NIST required the candidates of the AES algorithm to have faster speed, security no less than that of the triplicate DES, blocks of 128 bits, and a key of 128, 192 or 256 bits. It is well known that the several schemes were submitted to NIST.

Although the IDEA block cipher (a symmetric key cryptosystem) for 64-bit plaintext block encryption announced in 1990 [1][2] does not satisfy the NIST's requirements, the thought and structure of IDEA is still enlightening cryptographic researchers.

An opportune block cipher called REESSE3+ is proposed in this paper. It is the extension and renovation of IDEA. The block length of REESSE3+ is 128 bits being double that of IDEA, the key length of REESSE3+ is 256 bits being also double that of IDEA, the round function of REESSE3+ has 26 operations individually in three groups, and the security of REESSE3+ inherits all the good characteristics of IDEA. The 16 rounds of iteration will be needed in the renovated cipher.

## 2 Description of the Renovated Cryptosystem

### 2.1 Cipher Algorithm

The renovated block cipher is a symmetric key cryptosystem, and it employs one single algorithm called a cipher algorithm for encryption and decryption as well as one key called a session key for encryption and decryption. There are differences between the encryption

[^0]subkeys and the decryption subkeys, but the decryption subkeys can be derived from the encryption subkeys. The cipher algorithm consists of 8 rounds of the iteration followed by an output transformation (see Fig. 1).

Assume that $X$ is a 128 -bit plaintext block which is partitioned into 8 subblocks $X_{1}, X_{2}, X_{3}$, $X_{4}, X_{5}, X_{6}, X_{7}$, and $X_{8}$. Every $X_{i}$ as an input is 16 bits long.

Assume that $Y$ is a related 128 -bit ciphertext block which consists of 8 subblocks $Y_{1}, Y_{2}, Y_{3}$, $Y_{4}, Y_{5}, Y_{6}, Y_{7}$, and $Y_{8}$. Every $Y_{i}$ as an output is 16 bits long.

Assume that $Z$ is a 256 -bit initial key from which the encryption subkeys or decryption subkeys $Z_{i}^{(j)}$ s are derived, and every $Z_{i}^{(j)}$ is a 16 -bit subkey, where $i(1 \leq i \leq 8)$ represents the ordinal number of a subkey, and $j(1 \leq j \leq 9)$ represents the ordinal number of a round.

In Fig. 1, the operation $(a \oplus b)$ represents the bitwise XOR of two 16-bit subblocks $a$ and $b$, the operation $(a[+] b)$ represents the addition modulo $2^{16}$ of two 16 -bit subblocks $a$ and $b$, and the operation $(a \odot b)$ represents the multiplication modulo $\left(2^{16}+1\right)$ of two 16 -bit subblocks $a$ and $b$ with 0 corresponding to $2^{16} \in \mathbb{Z}_{2^{16}+1}$.


Fig. 1 REESSE3+ Cipher Structure

In Fig. 1, the symbol $\circ$ denotes that two straight lines are not intersectant.

### 2.2 Explanation of Encryption and Decryption

### 2.2.1 Encryption Subkeys

The leftmost 128 bits of the 256 -bit key $Z$ are divided into 8 blocks in sequence, and the 8 blocks are assigned directly to the 8 subkeys $Z_{1}{ }^{(1)}, Z_{2}{ }^{(1)}, Z_{3}{ }^{(1)}, Z_{4}{ }^{(1)}, Z_{5}{ }^{(1)}, Z_{6}{ }^{(1)}, Z_{7}{ }^{(1)}$, and $Z_{8}{ }^{(1)}$, of which each is 16 bits long, and regarded as one key input element of the first round of the iteration. When beginning the second round of the iteration, we shift cyclically $Z$ left 25 bits (note that $25=16+9$ and $16 \times 8=256-128$ ), partition the leftmost 128 bits of $Z$ into 8 blocks, and assign these blocks to the next 8 subkeys $Z_{1}{ }^{(2)}, Z_{2}{ }^{(2)}, Z_{3}{ }^{(2)}, Z_{4}{ }^{(2)}, Z_{5}{ }^{(2)}, Z_{6}{ }^{(2)}, Z_{7}{ }^{(2)}$, and $Z_{8}{ }^{(2)}$. The rest subkeys may be deduced by analogy. Resultantly, we obtain the 72 subkeys for the 8 -round iteration and the output transformation as follows:

| Round | Encryption subkeys |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | $\mathrm{Z}_{1}{ }^{(1)}$ | $\mathrm{Z}_{2}{ }^{(1)}$ | $\mathrm{Z}_{3}{ }^{(1)}$ | $\mathrm{Z}_{4}{ }^{(1)}$ | $\mathrm{Z}_{5}{ }^{(1)}$ | $\mathrm{Z}_{6}{ }^{(1)}$ | $\mathrm{Z}_{7}{ }^{(1)}$ | $\mathrm{Z}_{8}{ }^{(1)}$ |
| 2 | $\mathrm{Z}_{1}{ }^{(2)}$ | $\mathrm{Z}_{2}{ }^{(2)}$ | $\mathrm{Z}_{3}{ }^{(2)}$ | $\mathrm{Z}_{4}{ }^{(2)}$ | $\mathrm{Z}_{5}{ }^{(2)}$ | $\mathrm{Z}_{6}{ }^{(2)}$ | $\mathrm{Z}_{7}{ }^{(2)}$ | $\mathrm{Z}_{8}{ }^{(2)}$ |
| 3 | $\mathrm{Z}_{1}{ }^{(3)}$ | $\mathrm{Z}_{2}{ }^{(3)}$ | $\mathrm{Z}_{3}{ }^{(3)}$ | $\mathrm{Z}_{4}{ }^{(3)}$ | $\mathrm{Z}_{5}{ }^{(3)}$ | $\mathrm{Z}_{6}{ }^{(3)}$ | $\mathrm{Z}_{7}{ }^{(3)}$ | $\mathrm{Z}_{8}{ }^{(3)}$ |
| 4 | $\mathrm{Z}_{1}{ }^{(4)}$ | $\mathrm{Z}_{2}{ }^{(4)}$ | $\mathrm{Z}_{3}{ }^{(4)}$ | $\mathrm{Z}_{4}{ }^{(4)}$ | $\mathrm{Z}_{5}{ }^{(4)}$ | $\mathrm{Z}_{6}{ }^{(4)}$ | $\mathrm{Z}_{7}{ }^{(4)}$ | $\mathrm{Z}_{8}{ }^{(4)}$ |
| 5 | $\mathrm{Z}_{1}{ }^{(5)}$ | $\mathrm{Z}_{2}{ }^{(5)}$ | $\mathrm{Z}_{3}{ }^{(5)}$ | $\mathrm{Z}_{4}{ }^{(5)}$ | $\mathrm{Z}_{5}{ }^{(5)}$ | $\mathrm{Z}_{6}{ }^{(5)}$ | $\mathrm{Z}_{7}{ }^{(5)}$ | $\mathrm{Z}_{8}{ }^{(5)}$ |
| 7 | $\mathrm{Z}_{1}{ }^{(6)}$ | $\mathrm{Z}_{2}{ }^{(6)}$ | $\mathrm{Z}_{3}{ }^{(6)}$ | $\mathrm{Z}_{4}{ }^{(6)}$ | $\mathrm{Z}_{5}{ }^{(6)}$ | $\mathrm{Z}_{6}{ }^{(6)}$ | $\mathrm{Z}_{7}{ }^{(6)}$ | $\mathrm{Z}_{8}{ }^{(6)}$ |
| 8 | $\mathrm{Z}_{1}{ }^{(7)}$ | $\mathrm{Z}_{2}{ }^{(7)}$ | $\mathrm{Z}_{3}{ }^{(7)}$ | $\mathrm{Z}_{4}{ }^{(7)}$ | $\mathrm{Z}_{5}{ }^{(7)}$ | $\mathrm{Z}_{6}{ }^{(7)}$ | $\mathrm{Z}_{7}{ }^{(7)}$ | $\mathrm{Z}_{8}{ }^{(7)}$ |
| Output Tra | $\mathrm{Z}_{1}{ }^{(9)}$ | $\mathrm{Z}_{2}{ }^{(9)}$ | $\mathrm{Z}_{3}{ }^{(9)}$ | $\mathrm{Z}_{4}{ }^{(9)}$ | $\mathrm{Z}_{5}{ }^{(9)}$ | $\mathrm{Z}_{6}{ }^{(9)}$ | $\mathrm{Z}_{7}{ }^{(9)}$ | $\mathrm{Z}_{8}{ }^{(9)}$. |

### 2.2.2 Encryption Operation

At the beginning, the 128 -bit plaintext block $X$ is partitioned into 816 -bit subblocks $X_{1}, X_{2}$, $X_{3}, X_{4}, X_{5}, X_{6}, X_{7}$, and $X_{8}$ which are regarded as the input elements of the first round of the iteration.

Every round of the iteration performs three types of operations between the plaintext subblocks and the encryption subkeys: addition $\bmod 2^{16}$ denoted by $[+]$, multiplication mod $\left(2^{16}+1\right)$ denoted by $\odot$, and addition mod 2 , namely bitwise XOR denoted by $\oplus$. The operation order is described in Section 3.

The output elements of each of the previous 7 rounds, where the 2 nd subblock and 3 rd subblock are exchanged for each other, and the 6th subblock and 7th subblock are exchanged for each other, are regarded as the input elements of the next round. The exchange between some two output elements of Round 8 is not needed (See Fig.1).

After finishing the 8 -round iteration, we do an extra output transformation, and then acquire 8 output subblocks $Y_{1}, Y_{2}, Y_{3}, Y_{4}, Y_{5}, Y_{6}, Y_{7}$, and $Y_{8}$, which are joined to a 128-bit ciphertext block $Y$.

### 2.2.3 Decryption Subkeys

The decryption subkeys are deduced from the encryption subkeys while the encryption subkeys are derived directly from the 256-bit initial key $Z$. In terms of the inverse operation
rules, the 72 decryption subkeys corresponding to the 8 -rounds iteration and the output transformation are as follows:

| Round | Decryption subkeys |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | $Z_{1}{ }^{(9)-1}$ | $-Z_{2}{ }^{(9)}$ | $-Z_{3}{ }^{(9)}$ | $Z_{4}{ }^{(9)-1}$ | $-Z_{5}{ }^{(9)}$ | $Z_{6}{ }^{(9)-1}$ | $Z_{7}{ }^{(9)-1}$ | $-Z_{8}{ }^{(9)}$ |
| 2 | $Z_{1}{ }^{(8)-1}$ | $-Z_{3}{ }^{(8)}$ | $-Z_{2}{ }^{(8)}$ | $Z_{4}{ }^{(8)-1}$ | $-Z_{5}{ }^{(8)}$ | $Z_{7}{ }^{(8)-1}$ | $Z_{6}{ }^{(8)-1}$ | $-Z_{8}{ }^{(8)}$ |
| 3 | $Z_{1}{ }^{(7)-1}$ | $-Z_{3}{ }^{(7)}$ | $-Z_{2}{ }^{(7)}$ | $Z_{4}{ }^{(7)-1}$ | $-Z_{5}{ }^{(7)}$ | $Z_{7}{ }^{(7)-1}$ | $Z_{6}{ }^{(7)-1}$ | $-Z_{8}{ }^{(7)}$ |
| 4 | $Z_{1}{ }^{(6)-1}$ | $-Z_{3}{ }^{(6)}$ | $-Z_{2}{ }^{(6)}$ | $Z_{4}{ }^{(6)-1}$ | $-Z_{5}{ }^{(6)}$ | $Z_{7}{ }^{(6)-1}$ | $Z_{6}{ }^{(6)-1}$ | $-Z_{8}{ }^{(6)}$ |
| 5 | $Z_{1}{ }^{(5)-1}$ | $-Z_{3}{ }^{(5)}$ | $-Z_{2}{ }^{(5)}$ | $Z_{4}{ }^{(5)-1}$ | $-Z_{5}{ }^{(5)}$ | $Z_{7}{ }^{(5)-1}$ | $Z_{6}{ }^{(5)-1}$ | $-Z_{8}{ }^{(5)}$ |
| 6 | $Z_{1}{ }^{(4)-1}$ | $-Z_{3}{ }^{(4)}$ | $-Z_{2}{ }^{(4)}$ | $Z_{4}{ }^{(4)-1}$ | $-Z_{5}{ }^{(4)}$ | $Z_{7}{ }^{(4)-1}$ | $Z_{6}{ }^{(4)-1}$ | $-Z_{8}{ }^{(4)}$ |
| 7 | $Z_{1}{ }^{(3)-1}$ | $-Z_{3}{ }^{(3)}$ | $-Z_{2}{ }^{(3)}$ | $Z_{4}{ }^{(3)-1}$ | $-Z_{5}{ }^{(3)}$ | $Z_{7}{ }^{(3)-1}$ | $Z_{6}{ }^{(3)-1}$ | $-Z_{8}{ }^{(3)}$ |
| 8 | $Z_{1}{ }^{(2)-1}$ | $-Z_{3}{ }^{(2)}$ | $-Z_{2}{ }^{(2)}$ | $Z_{4}{ }^{(2)-1}$ | $-Z_{5}{ }^{(2)}$ | $Z_{7}{ }^{(2)-1}$ | $Z_{6}{ }^{(2)-1}$ | $-Z_{8}{ }^{(2)}$ |
| Output Tra | $Z_{1}{ }^{(1)-1}$ | $-Z_{2}{ }^{(1)}$ | $-Z_{3}{ }^{(1)}$ | $Z_{4}{ }^{(1)-1}$ | $-Z_{5}{ }^{(1)}$ | $Z_{6}{ }^{(1)-1}$ | $Z_{7}{ }^{(1)-1}$ | $-Z_{8}{ }^{(1)}$, | where $Z_{i}^{(j)-1}$ denotes the multiplication inverse of $Z_{i}^{(j)} \bmod \left(2^{16}+1\right)$, namely $Z_{i}^{(j)} \odot Z_{i}^{(j)-1} \equiv$ $1\left(\bmod 2^{16}+1\right)$, and $-Z_{i}^{(j)}$ denotes the addition inverse of $Z_{i}^{(j)} \bmod 2^{16}$, namely $Z_{i}^{(j)}[+]\left(-Z_{i}^{(j)}\right)$ $\equiv 0\left(\bmod 2^{16}\right)$.

In particular, for multiplication $\bmod \left(2^{16}+1\right)$, the inverse of $2^{16}$ is still $2^{16}$, and because the lower 16 bits of $2^{16}$ are all zero, we use the 16 -bit zero subblock to represent $2^{16}$ or its inverse, that is, the multiplication inverse of 0 is still 0 .

### 2.2.4 Decryption Operation

Decryption is the reverse operation of encryption.
The decryption employs the same cipher structure: the 8 -round iteration and the output transformation as what is illustrated by Fig.1. This cipher structure is just the cipher algorithm.

The 128-bit ciphertext block $Y$ and the decryption subkeys are the inputs of the cipher algorithm. Refer to the next section for the operation order.

## 3 Proof of Correctness of the Cipher Algorithm

Assume that there is only one round of the iteration before the output transformation. This does not influence the correctness of the algorithm because we only need to show that the round function is reversible.

The plaintext subblocks $X_{i}$ and the subkeys $Z_{i}^{(1)}$ and $Z_{i}^{(2)}(i=1, \ldots, 8)$ are regarded as inputs, and according to Fig. 1, the encryption operation is as follows:
(01) $A=X_{1} \odot Z_{1}{ }^{(1)}$
(02) $B=X_{2}[+] Z_{2}^{(1)}$
(03) $C=X_{3}[+] Z_{3}{ }^{(1)}$
(04) $D=X_{4} \odot Z_{4}{ }^{(1)}$
(05) $E=X_{5}[+] Z_{5}{ }^{(1)}$
(06) $F=X_{6} \odot Z_{6}{ }^{(1)}$
(07) $G=X_{7} \odot Z_{7}{ }^{(1)}$
(08) $H=X_{8}[+] Z_{8}{ }^{(1)}$
(10) $J=B \oplus D$
(12) $L=F \oplus H$
(11) $K=E \oplus G$
(14) $N=K[+] M$
(16) $P=M[+] \Gamma$
(15) $\Gamma=L \odot N$
(18) $\Omega=L \odot P$
(20) $R=C \oplus P$
(21) $S=E \oplus \Omega$
(22) $T=G \oplus \Omega$
(23) $U=B \oplus \Phi$
(24) $V=D \oplus \Phi$
(25) $W=F \oplus \Gamma$
(26) $\Lambda=H \oplus \Gamma$

Notice that exchange is not done between some two subblocks of the iteration output.
Further, doing the output transformation:
(01) $Y_{1}=Q \odot Z_{1}{ }^{(2)}$
(02) $Y_{2}=U[+] Z_{2}^{(2)}$
(03) $Y_{3}=R[+] Z_{3}{ }^{(2)}$
(04) $Y_{4}=V \odot Z_{4}{ }^{(2)}$
(05) $Y_{5}=S[+] Z_{5}^{(2)}$
(06) $Y_{6}=W \odot Z_{6}{ }^{(2)}$
(07) $Y_{7}=T \odot Z_{7}^{(2)}$
(08) $Y_{8}=\Lambda[+] Z_{8}{ }^{(2)}$

The decryption employs the same algorithm as what is illustrated by Fig 1. The ciphertext subblocks $Y_{i}$ and the inverses of the subkeys $Z_{i}^{(2)}$ and $Z_{i}^{(1)}(i=1, \ldots, 8)$ are regarded as inputs. According to Fig 1, the decryption operation is as follows:

$$
\begin{aligned}
& \text { (01) } A^{\prime}=Y_{1} \odot\left(Z_{1}^{(2)}\right)^{-1}=Q \\
& \text { (02) } B^{\prime}=Y_{2}[+]\left(-Z_{2}^{(2)}\right)=U \\
& \text { (03) } C^{\prime}=Y_{3}[+]\left(-Z_{3}^{(2)}\right)=R \\
& \text { (04) } D^{\prime}=Y_{4} \odot\left(Z_{4}^{(2)}\right)^{-1}=V \\
& \text { (05) } E^{\prime}=Y_{5}[+]\left(-Z_{5}^{(2)}\right)=S \\
& \text { (06) } F^{\prime}=Y_{6} \odot\left(Z_{6}^{(2)}\right)^{-1}=W \\
& \text { (07) } G^{\prime}=Y_{7} \odot\left(Z_{7}^{(2)}\right)^{-1}=T \\
& \text { (08) } H^{\prime}=Y_{8}[+]\left(-Z_{8}^{(2)}\right)=\Lambda \\
& \text { (09) } I^{\prime}=A^{\prime} \oplus C^{\prime}=Q \oplus R=A \oplus C=I \\
& \text { (10) } J^{\prime}=B^{\prime} \oplus D^{\prime}=U \oplus V=B \oplus D=J \\
& \text { (11) } K^{\prime}=E^{\prime} \oplus G^{\prime}=S \oplus T=E \oplus G=K \\
& \text { (12) } L^{\prime}=F^{\prime} \oplus H^{\prime}=W \oplus \Lambda=F \oplus H=L \\
& \text { (13) } M^{\prime}=I^{\prime} \odot J^{\prime}=I \odot J=M \\
& \text { (14) } N^{\prime}=K^{\prime}[+] M^{\prime}=K[+] M=N \\
& \text { (15) } \Gamma^{\prime}=L^{\prime} \odot N^{\prime}=L \odot N=\Gamma \\
& \text { (16) } P^{\prime}=M^{\prime}[+] \Gamma^{\prime}=M[+] \Gamma=P \\
& \text { (17) } \Phi^{\prime}=I^{\prime}[+] \Gamma^{\prime}=I[+] \Gamma=\Phi \\
& \text { (18) } \Omega^{\prime}=L^{\prime} \odot P^{\prime}=L \odot P=\Omega \\
& \text { (19) } Q^{\prime}=A^{\prime} \oplus P^{\prime}=Q \oplus P=A \oplus P \oplus P=A \\
& \text { (20) } R^{\prime}=C^{\prime} \oplus P^{\prime}=R \oplus P=C \oplus P \oplus P=C \\
& \text { (21) } S^{\prime}=E^{\prime} \oplus \Omega^{\prime}=S \oplus \Omega=E \oplus \Omega \oplus \Omega=E \\
& \text { (22) } T^{\prime}=G^{\prime} \oplus \Omega^{\prime}=T \oplus \Omega=G \oplus \Omega \oplus \Omega=G \\
& \text { (23) } U^{\prime}=B^{\prime} \oplus \Phi^{\prime}=U \oplus \Phi=B \oplus \Phi \oplus \Phi=B \\
& \text { (24) } V^{\prime}=D^{\prime} \oplus \Phi^{\prime}=V \oplus \Phi=D \oplus \Phi \oplus \Phi=D \\
& \text { (25) } W^{\prime}=F^{\prime} \oplus \Gamma^{\prime}=W \oplus \Gamma=F \oplus \Gamma \oplus \Gamma=F \\
& \text { (26) } \Lambda^{\prime}=H^{\prime} \oplus \Gamma^{\prime}=\Lambda \oplus \Gamma=H \oplus \Gamma \oplus \Gamma=H
\end{aligned}
$$

Notice that exchange is not done between some two subblocks of the iteration output.
Further, doing the output transformation of the decryption operation:
(01) $Y_{1}{ }^{\prime}=Q^{\prime} \odot\left(Z_{1}{ }^{(1)}\right)^{-1}=A \odot\left(Z_{1}{ }^{(1)}\right)^{-1}=X_{1}$
(02) $Y_{2}^{\prime}=U^{\prime}[+]\left(-Z_{2}^{(1)}\right)=B[+]\left(-Z_{2}^{(1)}\right)=X_{2}$
(03) $Y_{3}{ }^{\prime}=R^{\prime}[+]\left(-Z_{3}{ }^{(1)}\right)=C[+]\left(-Z_{3}{ }^{(1)}\right)=X_{3}$
(04) $Y_{4}{ }^{\prime}=V^{\prime} \odot\left(Z_{4}{ }^{(1)}\right)^{-1}=D \odot\left(Z_{4}{ }^{(1)}\right)^{-1}=X_{4}$

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(05) \(Y_{5^{\prime}}=S^{\prime}[+]\left(-Z_{5}^{(1)}\right)=E[+]\left(-Z_{5}^{(1)}\right)=X_{5}\)
(06) \(Y_{6}{ }^{\prime}=W^{\prime} \odot\left(Z_{6}{ }^{(1)}\right)^{-1}=F \odot\left(Z_{6}{ }^{(1)}\right)^{-1}=X_{6}\)
(07) \(Y_{7}{ }^{\prime}=T^{\prime} \odot\left(Z_{7}^{(1)}\right)^{-1}=G \odot\left(Z_{7}^{(1)}\right)^{-1}=X_{7}\)
(08) \(Y_{8}{ }^{\prime}=\Lambda^{\prime}[+]\left(-Z_{8}^{(1)}\right)=H[+]\left(-Z_{8}^{(1)}\right)=X_{8}\)
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Through the decryption operation, the original plaintext block $X$ is obtained. Therefore the cipher algorithm illustrated by Fig. 1 can decrypt a ciphertext correctly.

## 4 Security Analysis of the Renovated Cryptosystem

### 4.1 Inheriting the Some Characteristic of IDEA in Security

The design of the renovated cipher complies with the principle of "confusion" and "diffusion", and inherits the some characteristic of IDEA in security [3].

The confusion is implemented through the mixing operations over the three different groups of which any two cannot construct a field due to the inconsistency of the sets or the dissatisfaction of the distribution law. The diffusion is implemented through the extended Multiplication-Addition structure which makes the output of every operator be regarded as the input of another distinct operator, and each of output subblocks be involved in every input subblock and every subkey [2].

Besides, the exchange of the output subblock 2 and 3 as well as the output subblock 6 and 7 makes the mutual infiltration of the input subblocks and the subkeys faster.

Therefore, REESSE3+ bears the confusion and diffusion.

### 4.2 More Complex Round Function

In Fig.1, we see that the round function of the renovated cipher contains seven modular multiplications, seven modular additions, and twelve XOR operations. Evidently it is more complex than that of IDEA.

The more complex the round function is, then the more expeditious diffusion of the bits in a block is, the more irregular confusion of the bits is, and the more ineffective the differential analysis method is [4][5].

### 4.3 More Extensive Variation Range

From Fig.1, it is not difficulty for us to understand that the confusion and diffusion in the renovated cipher are performed within the new range of 128 bits instead of the old range of 64 bits.

As a result of the range extension, the round trail of the iteration will be diluted, which indicates that differential analysis approach will become more futile [6].

### 4.4 Resisting Differential Cryptanalysis

### 4.4.1 Surveying from the Improved MA Structure

There exists the remarkable distinction between REESSE3+ and IDEA, that is, REESSE3+ bears an improved MA structure that has no the subkey inputs $\mathrm{Z}_{5}$ and $\mathrm{Z}_{6}$ which are substituted with the middle values $I$ and $L$ obtained respectively through the three different group
computations (see Fig.2). Even, it may be said that the improved MA structure is similar to the $S$ box in the DES cipher [4].


Fig. 2 Improved MA Structure

Assume that we define an appropriate difference [4], and then we can see that cryptanalysis needs a differential distribution table with four 16 -bit inputs and two 16 -bit outputs, which will take about $\left(2^{\left(4^{*} 16\right)}\right)^{2}=2^{128}$ arithmetic operations. This effort is equivalent to that effort which is needed by an exhaustive search attack on REESSE3+.

The original MA structure has only two 16 -bit inputs (for the fixed subkeys $Z_{5}$ and $Z_{6}$ do not affect analysis) and two 16 -bit outputs. We see that it will take roughly $\left(2^{\left(22^{* 16}\right)}\right)^{2}=2^{64}$ arithmetic operations which is equivalent to the exhaustive search effort to acquire the corresponding differential distribution table.

### 4.4.2 Surveying from a Markov Cipher

According to [7] and [8], the Markov chain technique can be used to analyze the efficacy of differential cryptanalysis. If the maximal probability in the involution of a differential probability matrix is less than or approximately equal to $1 / 2^{n}$, where $n$ is the bit-length of a plaintext block, the differential cryptanalysis of a related Markov cipher with sufficiently many rounds will be ineffective.
A Markov cipher is an iterated cipher whose round function makes the differential probabilities independent of the choice of one of the component plaintexts under an appropriate definition of a difference. Hence, a Markov chain may be formed by the sequence of round differentials of a Markov cipher with independent round subkeys [7][8].

IDEA is a Markov cipher as DES is [7][8]. Similarly, REESSE3+ is a Markov cipher. A differential cryptanalysis of an $r$-round Markov cipher may be reduced to the analysis of the transition probabilities created by its round function [7][8]. The transition probabilities can be computed through the involution of a differential probability matrix.

Because the involution of a differential probability matrix will take great effort, a mini-version of a Markov cipher is considered.

It is not difficulty to understand that the mini-versions of IDEA with block bit-length 8,16 and 32 are also Markov ciphers. It is shown that $\operatorname{IDEA}(8)$ and $\operatorname{IDEA}(16)$ are immune to differential cryptanalysis after sufficiently many rounds [7][8].
Similarly, a mini-version of REESSE3+ with block bit-length 16 is a Markov cipher. Our experiments in which the asymmetry, aperiodicity, and completely nonzero columns of the related probability matrix are detected show that REESSE3+(16) is immune to differential
cryptanalysis after 8 rounds. Further, it may be expected that REESSE3+ is secure against differential attacks after 8 rounds.

## 5 Conclusions

The block cipher 128-bit in length will be securer against attacks and fitter for interfaces of the future cryptographic applications.

To encrypt 128-bit plaintext block, the IDEA algorithm needs to be called two times, and executes 28 operations in total while the renovated REESSE3+ algorithm needs to be called only one time, and executes 26 operations.

Although the round function of the renovated cipher is some more complex, realizing the round function with hardware or software is still convenient.

Obviously, if we change the arrangement of the operators $\odot$ and $[+]$ at the two terminals of Fig. 1 to $\left(\odot\left[^{+}\right][+] \odot \odot[+][+] \odot\right)$ or $\left([+] \odot \odot\left[^{[+]}[+] \odot \odot[+]\right.\right.$ ), then the correctness of the algorithm will be not influenced, but the security of the algorithm will probably be influenced. Increased, decreased, or intact?

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