Attribute-Based Signatures without Pairings by the Fiat-Shamir Transform (Corrected Version)*

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Abstract. We propose an attribute-based signature scheme (ABS) with features of pairing-free, short signatures and security proof in the random oracle model. Our strategy is in the Fiat-Shamir paradigm; we first provide a concrete procedure of the Σ-protocol which enables a prover to prove possession of witnesses that satisfy a statement of a monotone boolean formula. Next, using a signature bundle scheme of the Fiat-Shamir signature as those witnesses in the Σ-protocol, we obtain a generic attribute-based identification scheme (ABID). Then, we apply the Fiat-Shamir transform to our ABID to obtain a scheme of ABS. The series of these generic constructions are obtained from a given Σ-protocol. Finally, we provide our ABID and ABS schemes concretely in the Discrete-Logarithm setting and the RSA setting. These concretions are pairing-free. Signatures of our ABS are linkable, hence attribute privacy does not hold; it holds only as a one-time signature.

Keywords: digital signature, access structure, attribute, boolean formula, proof system, Fiat-Shamir transform

1 Introduction

Since the invention of digital signature scheme by Diffie and Hellman in 1976, there have been significant evolutions in the area and many functional variants have been proposed. One of distinguished variant is attribute-based signature scheme (ABS), which has been developed since 2008 [22, 34, 29, 28, 30, 14, 31, 17, 26, 32, 23, 13, 12, 13, 18, 24]. In ABS, a message is associated with an access structure by which a signer can make a signature on it only when his attributes satisfy the access structure. The access structure is described as a boolean formula over their attributes, which is called access formula. A signer with a set of authorized attributes can make a legitimate signature on the message only when his attributes satisfy the access formula. Then, a verifier checks whether the pair of the message and the signature is valid or not in accordance with the

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access formula. If the access formula is given as a single identity, ABS coincides with identity-based signature scheme (IBS) [35]. In that sense, ABS is a natural extension of IBS.

To obtain such an ABS scheme, we work in the Fiat-Shamir paradigm [16] in this paper. The Fiat-Shamir transform [16,1] is now an established technique in interactive proof theory and cryptography. By replacing the message of the second move with the hash value of the message of the first move, it transforms a public-coin 3-move interactive proof system into a non-interactive zero-knowledge proof system (NIZK). If the initial interactive proof system is an identification scheme and if an input message is concatenated with the message of the first move, then the resulting NIZK turns into a digital signature scheme. One of such interactive proof systems is the \( \Sigma \)-protocol [9,11]. It allows a prover, in a 3-move, to convince a verifier that the prover knows a witness of a statement. That is, the \( \Sigma \)-protocol is a public-coin 3-move interactive proof of knowledge system. We can apply the Fiat-Shamir transform to the \( \Sigma \)-protocol and obtain a zero-knowledge proof of knowledge system (NIZKPoK) whose extractor is in the random oracle model. Another feature of the \( \Sigma \)-protocol is that we can construct the OR-proof protocol [10,11] from a \( \Sigma \)-protocol. *Hiding which witness he knows*, it allows a prover to convince a verifier that the prover knows at least one of two witnesses of two statements, respectively.

In the Fiat-Shamir paradigm we first look into the protocol introduced by Cramer, Damgård and Schoenmakers [10] in CRYPTO ’94. Given a boolean formula \( f \), the protocol provides a witness hiding protocol [15]. That is, in \( f \), a value ‘1’ (TRUE) is substituted for boolean variables at which a prover has the corresponding witnesses; a value ‘0’ (FALSE) is substituted for the remaining boolean variables. Then the prover in the protocol can show to a verifier that he knows a set of witnesses that satisfies the boolean formula \( f \) (that is, a set of witnesses that makes a whole evaluation of \( f \) TRUE), but a cheating verifier does not learn anything that is enough to compute a satisfying set of witnesses. We call this notion boolean proof. In [10], the notion of the boolean proof was provided at a high level, and the OR-proof , which was a special case where \( f \) is a single “OR”, was explained with a concrete procedure. In this paper, we give a concrete procedure, \( \Sigma_f \), for a generic monotone boolean formula \( f \). (Here a monotone boolean formula means a boolean formula without negation.) Besides, we will show that \( \Sigma_f \) is also a \( \Sigma \)-protocol.

Then we will proceed further into the Fiat-Shamir paradigm. By using a signature bundle scheme from the Fiat-Shamir signature as those witnesses of our boolean proof system, we obtain a generic attribute-based identification scheme (ABID). Finally, we apply the Fiat-Shamir transform to our ABID to obtain a generic ABS which can be proved to be existentially unforgeable against chosen-message attacks in the random oracle model. A remarkable property is that the series of constructions are obtained from a given single \( \Sigma \)-protocol, and our ABS can be constructed without pairing maps ([7]) (pairings, for short).

### 1.1 Our Construction Idea of Concrete Procedure of Boolean Proof

To provide a concrete procedure for the above boolean proof system from a given \( \Sigma \)-protocol and a monotone boolean formula \( f \), we look into the technique employed in the OR-proof [10] and expand it so that it can treat any monotone boolean formula, as follows.

First express the boolean formula \( f \) as a binary tree \( T_f \). That is, we put leaf nodes each of which corresponds to each term in \( f \). We connect two leaf nodes by an \( \wedge \)-node or an \( \vee \)-node according to an AND-gate or an OR-gate that is between two corresponding terms in \( f \). Then

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4 In the preliminary version [3] the authors could not refer to this previous work. Now we refer this work and state the relation.
we connect the resulting nodes by an $\wedge$-node or an $\vee$-node in the same way, until we reach to the only $\wedge$-node or $\vee$-node called the root node. A verification equation of the $\Sigma$-protocol $\Sigma$ is assigned to every leaf node. If a challenge string $\text{Cha}$ of $\Sigma$ is given, then assign the string $\text{Cha}$ to the root node. If the root node is an $\wedge$-node, then assign the same string $\text{Cha}$ to two children. Else if the root node is an $\vee$-node, then divide $\text{Cha}$ into two random strings $\text{Cha}_L$ and $\text{Cha}_R$ under the constraint that $\text{Cha} = \text{Cha}_L \oplus \text{Cha}_R$, and assign $\text{Cha}_L$ and $\text{Cha}_R$ to the left child and the right child, respectively. Here $\oplus$ means a bitwise exclusive-OR operation. Then continue to apply this rule at each height, step by step, until we reach to every leaf node. Then, basically, the OR-proof technique assures that we can either honestly execute the $\Sigma$-protocol $\Sigma$ or execute the simulator of $\Sigma$. Only when a set of witnesses satisfies the binary tree $T_f$, the above procedure succeeds in satisfying verification equations for all leaf nodes.

1.2 Our Contributions

Our first contribution is to provide a concrete procedure of the boolean proof system [10]. That is, given a $\Sigma$-protocol $\Sigma$ and a monotone boolean formula $f$, we construct a concrete procedure $\Sigma_f$ in a recursive form that is suitable for implementation. Then we show that $\Sigma_f$ is certainly a $\Sigma$-protocol. Especially we show that $\Sigma_f$ is a protocol to prove knowledge of witnesses that satisfy the boolean formula $f$.

Our second contribution is to provide an attribute-based signature scheme (ABS) without pairings. We provide schemes of ABID and ABS concretely in both the Discrete-Logarithm setting and the RSA setting by employing the Schnorr identification scheme and the GQ identification scheme [33, 5] as $\Sigma$, respectively. These constructions are pairing-free. We note that signatures of our ABS are linkable, and attribute privacy holds only as a one-time signature. Moreover, the reduction of advantages of adversaries to the corresponding number theoretic hardnesses is loose.

1.3 Related Work

The $\Sigma$-protocol was formalized in the Cramer’s Ph.D thesis [9]. The $\Sigma$-protocol of OR-proof was described in Cramer, Damgård and Schoenmakers [10] (see also [11]). The $\Sigma$-protocol of their boolean proof system, an extension of the OR-proof system to any monotone boolean formula, was also discussed in the work [10] in an abstract manner. We provide a concrete procedure of the boolean proof system.

At a high level, our ABS is captured as a result of the Fiat-Shamir transform of our boolean proof system in which a set of witnesses is a signature bundle (credential bundle [30]). This construction can be compared with the generic construction of a scheme of ABS by Maji et al. [30]. They started with a signature bundle (of Boneh-Boyen signatures [8], for instance). Then they employed a non-interactive witness-indistinguishable proof of knowledge system (NI-WIPoK) of Groth and Sahai [21] to prove the knowledge of a signature bundle which satisfies a given (monotone) access formula, in the standard model.

Okamoto and Takashima (OT11) [31] gave a scheme of ABS with full-security; security against adaptive target in the standard model under a non-$q$-type assumption. It can treat non-monotone access formula and multi-use of attributes, and possesses attribute privacy in the information-theoretic sense. The construction is based on their Dual Pairing Vector Space.

Herranz [23] provided the first ABS with both collusion resistance (against collecting private secret keys) and (computationally secure) attribute privacy without pairings in the RSA setting.
In the work [23], the concrete procedure was described for monotone access formulas. Our ABS can also be constructed without pairings and provide a concrete procedure for monotone access formulas, but does not achieve attribute privacy.

Ghadafi et al. [12] proposed the functionality of “User-Controlled Linkability” (UCL) in the case of attribute-based signatures. UCL property in the work [12] can be captured as a kind of public linkability. In general, public linkability is achieved with the expense of loosing attribute privacy in ABS, and hence the scheme [12] and our scheme do not possess attribute privacy. It is notable that the scheme [12] uses pairings and can be set up in the multi-authorities setting [32, 13, 18].

**More on Attribute Privacy** Intuitively, ABS is called to have attribute privacy if any cheating verifier cannot distinguish two distributions of signatures each of which is generated by different satisfying attribute set. Originally, attribute privacy was introduced as information theoretic security [30, 31]. That is, even an adversary with unconditional computational power should not distinguish two distributions. It is notable that in the work of Herranz [23], attribute privacy was expanded to broader sense; that is, as computational security. There, an adversary with polynomial-time computational power should not distinguish two distributions.

Our ABS does not have attribute privacy even in this broader sense because of its linkability. The linkability is caused by our construction that a part of a private secret key is included in signatures [25]. Hence, our ABS merely has attribute privacy as a one-time signature scheme.

**1.4 Technical and Efficiency Comparison: Security, Functionality and Signature Length**

We compare our scheme with the above previously proposed schemes from the view point of security, functionality and signature length. The comparison is summarized in Table 1 with notations as follows. A prime of bit length $\lambda$ (the security parameter) is denoted by $p$. Though a pairing map $e$ should be analysed for the asymmetric bilinear groups [20], we simply evaluate for the symmetric case in which both source groups are $\mathbb{G}_p$ of order $p$. We assume that an element of $\mathbb{G}_p$ is represented by $2\lambda$ bits. $l$ and $r$ mean the number of rows and columns of the share-generating matrix for monotone access formula $f$ (that is, an access structure), respectively. CR means the collision resistance of an employed hash function. $q$-SDH means the Strong Diffie-Hellman assumption with $q$-type input for bilinear groups [7]. DLIN means the Decisional Linear assumption for bilinear groups [31]. DDH means the Decisional Diffie-Hellman assumption for a cyclic group [12]. DLog means the Discrete Logarithm assumption for a cyclic group [12]. $q$-SRSA means the strong RSA assumption with $q$-type input [23]. DDH in $QR(N)$ means the Decisional Diffie-Hellman assumption for quadratic residues modulo $N$ (the RSA modulus) [23]. RSAInv means the RSA Inversion assumption for integers modulo $N$ [5]. (info.) means information-theoretic security and (comp.) means computational security.

The scheme of [31] has advantages in the security-proof model, access formula and information-theoretically secure attribute privacy, whereas our ABS realizes shorter length of signature (less than a half). The scheme of [23] is in the RSA setting and its security parameter $\lambda_{rsa}$ is almost 10 times longer than $\lambda$ in the DLog setting. For example, $\lambda_{rsa} = 2048$ is almost equivalent to $\lambda = 224$-bit security [36]. ($\theta$ is the threshold value of the threshold-type access structure. $\kappa$ is

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5 We thank to Javier Herranz for his sincere comment for our error on the statement of attribute privacy in the first version of this ePrint.
Table 1. Comparison of Security, Functionality and Signature Length. \( \lambda \) is the security parameter and \( l \) is the number of attributes.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Security Model</th>
<th>Assumption</th>
<th>Adaptive Target</th>
<th>Access Formula</th>
<th>Pairing Attribute -Free Privacy</th>
<th>Length of Signature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maji et al. [30]</td>
<td>Std. ( \wedge ) DLIN</td>
<td>✓</td>
<td>Monotone</td>
<td>-</td>
<td>✓ (info.)</td>
<td>((2\lambda)(5l + 2r + 18\lambda))</td>
</tr>
<tr>
<td>Okamoto-Takashima [31]</td>
<td>Std. ( \wedge ) CR</td>
<td>✓</td>
<td>Non-mono.</td>
<td>-</td>
<td>✓ (info.)</td>
<td>((2\lambda)(9l + 11))</td>
</tr>
<tr>
<td>Herranz [23]</td>
<td>R.O. ( q)-SRSA ( \wedge ) DDH in QR(( N )) ( \wedge ) CR</td>
<td>✓</td>
<td>Monotone</td>
<td>✓ (comp.)</td>
<td>✓</td>
<td>((\lambda_{rsa}(5 + \frac{n}{\lambda_{rsa}})l + \lambda_{rsa}3 - \kappa(\theta - 1)))</td>
</tr>
<tr>
<td>Ghadafi et al. [12]</td>
<td>R.O. ( q)-SDH ( \wedge ) DDH ( \wedge ) DLog ( \wedge ) CR</td>
<td>✓</td>
<td>Monotone</td>
<td>✓</td>
<td>✓</td>
<td>((2\lambda)(3l + r + 3) + \lambda(8l + 4))</td>
</tr>
<tr>
<td><strong>Our proposal</strong></td>
<td>R.O. ( \langle DLog \circ RSAInv \rangle ) ( \wedge ) CR</td>
<td>✓</td>
<td>Monotone</td>
<td>✓</td>
<td>✓</td>
<td>((2\lambda)l + \lambda(4l - 1))</td>
</tr>
</tbody>
</table>

explained in the work [23].) Therefore, our ABS in the DLog setting realizes shorter length of a signature. Note that the scheme of [23] achieves computationally secure attribute privacy.

1.5 Organization of this Paper

In Section 2, we prepare for the required tools and notions. In Section 3, we describe a concrete procedure of the boolean proof system, \( \Sigma_f \). In Section 4, by using a signature bundle scheme of the Fiat-Shamir signature \( FS(\Sigma) \) as witnesses of our \( \Sigma_f \), we obtain our ABID. In Section 5, by applying the Fiat-Shamir transform to our ABID, we obtain our ABS. In Section 6, we conclude our work in this paper. In Appendix A, we show concrete instantiations of our ABS in the discrete-logarithm setting and the RSA setting. In Appendix B, we summarize the notion of NIWI proof of knowledge system. In Appendix C, we state a NIWIPOK system that is obtained from our \( \Sigma_f \).

2 Preliminaries

The security parameter is denoted by \( \lambda \). Bit length of a string \( x \) is denoted as \(|x|\). When an algorithm \( A \) with input \( a \) outputs \( z \), we denote it as \( z \leftarrow A(a) \), or, because of space limitation, \( A(a) \rightarrow z \). When a probabilistic polynomial-time (PPT, for short) algorithm \( A \) with a random tape \( R \) and input \( a \) outputs \( z \), we denote it as \( z \leftarrow A(a;R) \). When \( A \) has oracle-access to \( O \), we denote it as \( A^O \). When \( A \) has concurrent oracle-access to \( n \) oracles \( O_1, \ldots, O_n \), we denote it as \( A^{O_1 \cdots O_n} \). Here “concurrent” means that \( A \) accesses to oracles in arbitrarily interleaved order of messages. We denote a concatenation of a string \( a \) with a string \( b \) as \( a \parallel b \). The expression \( a \overset{?}{=} b \) returns a value 1 (TRUE) when \( a = b \) and 0 (FALSE) otherwise. The expression \( a \in S \) returns a value 1 when \( a \in S \) and 0 otherwise. A probability of an event \( E \) is denoted by \( \Pr[E] \). A probability of an event \( E \) on condition that events \( E_1, \ldots, E_m \) occur in this order is denoted as \( \Pr[E_1, \ldots, E_m : E] \).

2.1 Language and Proof of Knowledge [4, 10, 11]

**Language** Let \( R = \{(x, w)\} \subset \{1, 0\}^* \times \{1, 0\}^* \) be a binary relation. We say that \( R \) is polynomially bounded if there exists a polynomial \( poly \) such that \(|w| \leq poly(|x|)\) for all \((x, w) \in R\). If
There are a PPT algorithm \( \text{test} \) on \( \text{P} \), called a prover, and \( \text{A} \). If there exists a PPT algorithm \( \text{A} \), then \( \text{P} \) ChaSp from \( \text{first message} \), called a commitment \( \text{Cmt} \), with the witness \( \text{w} \). The witness \( \text{L} \), \( \text{A} \) Proof of Knowledge \( \text{L} \) membership in \( \text{R} \) is polynomially bounded and, in addition, there exists a polynomial-time algorithm for deciding \( \text{R} \). We introduce a relation-function \( \text{R}(\cdot, \cdot) \) associated with the relation \( \text{R} \) by:

\[
R(\cdot, \cdot) : \{1, 0\}^* \times \{1, 0\}^* \to \{1, 0\},
\]

\[
(x, w) \mapsto 1 \text{ if } (x, w) \in \text{R}, \ 0 \text{ otherwise.}
\]

**Proof of Knowledge** A proof of knowledge system (PoK for short) \( \Pi = (\mathcal{P}, \mathcal{V}) \) for a language \( \text{L}_R \) is a protocol between interactive PPT algorithms \( \mathcal{P} \) and \( \mathcal{V} \) on initial input \( (x, w) \in \text{R} \) for \( \mathcal{P} \) and \( x \) for \( \mathcal{V} \), where \( \mathcal{V} \) outputs 1 (accept) or 0 (reject) after finite rounds of interaction. \( \mathcal{P} \) is called a prover and \( \mathcal{V} \) is called a verifier. In general, a prover \( \mathcal{P} \) has unbounded computational power, but in this paper we only consider the case that \( \mathcal{P} \) is PPT.

\( \Pi \) must possess the following two properties.

**Completeness.** For any statement \( x \in \text{L}_R \) and for any witness \( w \) such that \( (x, w) \in \text{R} \), \( \mathcal{P} \) with the witness \( w \) can make \( \mathcal{V} \) accept for the statement \( x \) with probability 1:

\[
\Pr[(\mathcal{P}(x, w), \mathcal{V}(x)) = 1] = 1.
\]

**Knowledge Soundness.** There are a PPT algorithm \( \mathcal{KE} \) called a knowledge extractor, a function \( \kappa : \{1, 0\}^* \to [1, 0] \) called a knowledge error function and a constant \( c > 0 \) that satisfy the following:

If there exists a PPT algorithm \( \mathcal{A} \) that satisfies \( p(x) := \Pr[1 \leftrightarrow \langle \mathcal{A}(x), \mathcal{V}(x) \rangle] > \kappa(x) \), then \( \mathcal{KE}(x) \), employing \( \mathcal{A}(x) \) as a subroutine that allows to be rewinded \(^6\), outputs a witness \( w \) which satisfies \( (x, w) \in \text{R} \) within an expected number of steps bounded by: \( |x|^c/(p(x) - \kappa(x)) \).

**\( \Sigma \)-protocol** \([9, 11]\) A \( \Sigma \)-protocol on a relation \( \text{R} \) is a public coin 3-move protocol between interactive PPT algorithms \( \mathcal{P} \) and \( \mathcal{V} \) on initial input \( (x, w) \in \text{R} \) for \( \mathcal{P} \) and \( x \) for \( \mathcal{V} \). \( \mathcal{P} \) sends the first message called a commitment \( \text{Cmt} \), then \( \mathcal{V} \) sends a random bit string called a challenge \( \text{CHA} \), and \( \mathcal{P} \) answers with a third message called a response \( \text{RES} \). Then \( \mathcal{V} \) applies a decision test on \( (x, \text{Cmt}, \text{CHA}, \text{RES}) \) to return accept (1) or reject (0). If \( \mathcal{V} \) accepts, then the triple \( (\text{Cmt}, \text{CHA}, \text{RES}) \) is said to be an accepting conversation. \( \text{CHA} \) is chosen uniformly at random from \( \text{CHASP}(1^\lambda) := \{1, 0\}^{|\lambda|} \) with \( l(\cdot) \) being a super-log function.

This protocol is written by a PPT algorithm \( \Sigma \) as follows. \( \text{Cmt} \leftarrow \Sigma^1(x, w) \): the process of selecting the first message \( \text{Cmt} \) according to the protocol \( \Sigma \) on input \( (x, w) \in \text{R} \). Similarly we denote \( \text{CHA} \leftarrow \Sigma^2(1^\lambda) \), \( \text{RES} \leftarrow \Sigma^3(x, w; \text{Cmt}, \text{CHA}) \) and \( b \leftarrow \Sigma^{\text{vrfy}}(x, \text{Cmt}, \text{CHA}, \text{RES}) \).

\( \Sigma \)-protocol must possess the following three properties.

**Completeness.** A prover \( \mathcal{P} \) with a witness \( w \) can make \( \mathcal{V} \) accept with probability 1.

**Special Soundness.** Any PPT algorithm \( \mathcal{P}^* \) without any witness, a cheating prover, can only respond for one possible challenge \( \text{CHA} \). In other words, there is a PPT algorithm called a

\(^6\) In \([4]\), it is described as “oracle-access to \( \mathcal{A}_x \)” instead of rewinding, which is more general statement but we do not need the generality in this paper.
knowledge extractor, \(\Sigma^{\text{KE}}\), which, given a statement \(x\) and using \(\mathcal{P}^*\) as a subroutine, can compute a witness \(w\) satisfying \((x, w) \in R\) with at most a negligible error probability, from two accepting conversations of the form \((\text{Cmt}, \text{Cha}, \text{Res})\) and \((\text{Cmt}, \text{Cha}', \text{Res}')\) with \(\text{Cha} \neq \text{Cha}'\).

**Honest Verifier Zero-Knowledge.** Given a statement \(x\) and a random challenge \(\text{Cha} \leftarrow \Sigma^{2(1^\lambda)}\), we can produce in polynomial-time, without knowing the witness \(w\), an accepting conversation \((\text{Cmt}, \text{Cha}, \text{Res})\) whose distribution is the same as the real accepting conversation. In other words, there is a PPT algorithm called a simulator, \(\Sigma^{\text{sim}}\), such that \((\text{Cmt}, \text{Res}) \leftarrow \Sigma^{\text{sim}}(x, \text{Cha})\).

Any \(\Sigma\)-protocol can be proved to be a proof of knowledge system ([11]).

We will need in this paper a property called *unique answer property* [6] that for legitimately produced commitment Cmt and challenge Cha, there exists one and only one response Res := \(w\) that is accepted by a verifier. Known \(\Sigma\)-protocols such as the Schnorr protocol and the Guillou-Quisquater protocol [33, 5] possess this property. For such a unique answer \(w\) we consider a statement \(x'\) such that \((x', w') \in R\). Then, we further assume that both a prover and a verifier can compute, in polynomial-time, such an \(x'\) from \((x, \text{Cmt}, \text{Cha})\). We denote the PPT algorithm as \(\Sigma^{\text{stmtgen}}\). That is,

\[
\Sigma^{\text{stmtgen}}(x, \text{Cmt}, \text{Cha}) : \\
\text{Compute } x' \text{ s.t.} \\
\exists! w \text{ s.t. } [(x', w') \in R \land (\text{Cmt}, \text{Cha}, \text{Res} := w') \text{ is an accepting conversation}] \\
\text{Return } x'
\]

Known \(\Sigma\)-protocols [33, 5] possess this *statement generation property* (see Appendix A).

**The OR-proof [11]** Consider the following relation for a boolean formula \(f(X_1, X_2) = X_1 \lor X_2\).

\[
R_{\text{OR}} = \{(x = (x_0, x_1), w = (w_0, w_1)) \in \{1, 0\}^* \times \{1, 0\}^*; \\
R(x_0, w_0) \lor R(x_1, w_1) = 1\}.
\]

The corresponding language for the relation \(R_{\text{OR}}\) is given as follows.

\[
L_{R_{\text{OR}}} = \{x \in \{1, 0\}^*; \exists w, (x, w) \in R_{\text{OR}}\}.
\]

The OR-proof is defined as an interactive proof system for the language \(L_{R_{\text{OR}}}\).

Suppose that a \(\Sigma\)-protocol \(\Sigma\) on a relation \(R\) is given. Then we can construct a new protocol, \(\Sigma_{\text{OR}}\), on a relation \(R_{\text{OR}}\) as follows. For instance, suppose \((x_0, w_0) \in R\) holds. \(\mathcal{P}\) computes \(\text{Cmt}_0 \leftarrow \Sigma^1(x_0, w), \text{Cha}_1 \leftarrow \Sigma^{2(1^\lambda)}\), \((\text{Cmt}_1, \text{Res}_1) \leftarrow \Sigma^{\text{sim}}(x_1, \text{Cha}_1)\) and sends \((\text{Cmt}_0, \text{Cmt}_1)\) to \(\mathcal{V}\). Then \(\mathcal{V}\) sends \(\text{Cha} \leftarrow \Sigma^{2(1^\lambda)}\) to \(\mathcal{P}\). Then, \(\mathcal{P}\) computes \(\text{Cha}_0 := \text{Cha} \oplus \text{Cha}_1, \text{Res}_0 \leftarrow \Sigma^3(x_0, w_0, \text{Cmt}_0, \text{Cha}_0)\) answers to \(\mathcal{V}\) with \((\text{Cha}_0, \text{Cha}_1)\) and \((\text{Res}_0, \text{Res}_1)\). Here \(\oplus\) denotes a bitwise exclusive-OR operation. Then both \((\text{Cmt}_0, \text{Cha}_0, \text{Res}_0)\) and \((\text{Cmt}_1, \text{Cha}_1, \text{Res}_1)\) are accepting conversations and have the same distribution as real accepting conversations. This protocol \(\Sigma_{\text{OR}}\) can be proved to be a \(\Sigma\)-protocol. We often call this \(\Sigma\)-protocol \(\Sigma_{\text{OR}}\) the *OR-proof*.

**Boolean Proof [10, 3]** We revisit the notion of a public coin interactive proof of knowledge system for the language \(L_f\) introduced by Cramer, Damgård and Schoenmakers [10], which we call a *boolean proof system*. Then we restate the definitions for the sake of clarity.

Let \(R\) be a binary relation. Let \(f(X_{i_1}, \ldots, X_{i_a})\) be a boolean formula over boolean variables \(U = \{X_1, \ldots, X_u\}\).
Definition 1 ([10], Rewritten Form) A relation $R_f$ is defined by:

\[
R_f \overset{\text{def}}{=} \{(x = (x_1, \ldots, x_{i_0}), w = (w_1, \ldots, w_{i_0})) \in \{1, 0\}^* \times \{1, 0\}^*; \\
f(R(x_1, w_1), \ldots, R(x_{i_0}, w_{i_0})) = 1\}.
\]

$R_f$ is a generalization of the relation $R_{OR}$ for the OR-proof [10, 11], where $f$ is a boolean formula with the single boolean connective: $X_1 \lor X_2$. Note that, if $R$ is an NP relation, then $R_f$ is also an NP relation under the assumption that $a$, the arity of $f$, is bounded by a polynomial in $\lambda$.

The corresponding language for the relation $R_f$ is given as follows.

\[
L_f = \{x \in \{1, 0\}^*; \exists w, (x, w) \in R_f\}.
\]

Finally, we achieve the following definition.

Definition 2 A boolean proof system is an interactive proof system for the language $L_f$.

We will provide a concrete procedure of a $\Sigma$-protocol $\Sigma_f$ that is a boolean proof system.

The Fiat-Shamir Transform [1] Suppose that a cryptographic hash function with collision resistance, $\text{Hash}_\mu(\cdot) : \{1, 0\}^* \rightarrow \{1, 0\}^{l(\lambda)}$, is given. We fix a hash key $\mu$ hereafter. A $\Sigma$-protocol $\Sigma$ on a relation $R$ can be transformed into a non-interactive zero-knowledge proof of knowledge system (NIZKPoK) with its knowledge extractor in the random oracle model. Hence a non-interactive witness-indistinguishable proof of knowledge system (NIWIPoK) can be obtained.

When a $\Sigma$-protocol $\Sigma$ is an identification scheme, the resulting scheme is a digital signature scheme. The transform is described as follows. (Here, a message $m$ is omitted in the case of a NIWIPoK.) Given a message $m \in \{1, 0\}^*$, execute: $a \leftarrow \Sigma^1(x, w), c \leftarrow \text{Hash}_\mu(a \parallel m), z \leftarrow \Sigma^3(x, w, a, c)$. Then $\sigma := (a, z)$ is a signature on $m$. We denote the above signing algorithm as $\text{FS}(\Sigma)^{\text{sign}}(x, w, m) \rightarrow (a, z) =: \sigma$. The verification algorithm $\text{FS}(\Sigma)^{\text{vrfy}}(x, m, \sigma)$ is given as: $c \leftarrow \text{Hash}_\mu(a \parallel m)$, Return $b \leftarrow \Sigma^{\text{vrfy}}(x, a, c, z)$.

The signature scheme $\text{FS}(\Sigma) = (R, \text{FS}(\Sigma)^{\text{sign}}, \text{FS}(\Sigma)^{\text{vrfy}})$ can be proved, in the random oracle model, to be existentially unforgeable against chosen-message attacks if and only if the underlying $\Sigma$-protocol $\Sigma$ is secure against passive attacks as an identification scheme [1]. More precisely, let $q_H$ denote the maximum number of hash queries issued by the adversary on $\text{FS}(\Sigma)$. Then, for any PPT algorithm $\mathcal{F}$, there exists a PPT algorithm $\mathcal{B}$ which satisfies the following inequality (neg$(\cdot)$ means a negligible function).

\[
\text{Adv}_{\mathcal{F}, \text{FS}(\Sigma)}^{\text{euf-cma}}(\lambda) \leq q_H \text{Adv}_{\Sigma}^{\text{pa}}(\lambda) + \text{neg}(\lambda).
\]

2.2 Signature Bundle Scheme (Credential Bundle Scheme [30])

A signature bundle scheme $\text{SB}$ is an extended notion of signature scheme. It consists of three PPT algorithms: $\text{SB} = (\text{SB.KG}, \text{SB.Sign}, \text{SB.Vrfy})$. Below $n$ is bounded by a polynomial in $\lambda$.

$\text{SB.KG}(1^\lambda) \rightarrow (PK, SK)$. Given $1^\lambda$ as input, it outputs a public key $PK$ and a secret key $SK$.

$\text{SB.Sign}(PK, SK, (m_1, \ldots, m_n)) \rightarrow (\tau, (\sigma_1, \ldots, \sigma_n))$. Given PK, SK and $n$ messages $m_1, \ldots, m_n$, it outputs a tag $\tau$ and $n$ signatures $\sigma_1, \ldots, \sigma_n$.

$\text{SB.Vrfy}(PK, (m_1, \ldots, m_n), (\tau, (\sigma_1, \ldots, \sigma_n))) \rightarrow 1/0$. Given PK, $n$ messages $m_1, \ldots, m_n$, a tag $\tau$ and $n$ signatures $\sigma_1, \ldots, \sigma_n$, it outputs 1 or 0.
Suppose that we are given a digital signature scheme \((\text{KG}, \text{Sign}, \text{Vrfy})\). Then we can construct a signature bundle scheme as follows (according to [30]). \text{SB.KG} takes as input \(1^\lambda\) and it runs \(\text{KG}(1^\lambda)\) to get \((\text{PK}, \text{SK})\). It outputs \((\text{PK}, \text{SK})\). \text{SB.Sign} takes as input \(\text{PK}, \text{SK}\) and a set of messages \((m_i)_{1 \leq i \leq n}\). It chooses a tag \(\tau\) of length \(\lambda\) at random. Then it executes \text{Sign} on each tagged message \(\tau \parallel m_i, i = 1, \ldots, n\) and outputs signatures \(\sigma_i, i = 1, \ldots, n\), respectively. \text{SB.Vrfy} takes as input \(\text{PK}, (m_i)_{1 \leq i \leq n}, \tau\) and \((\sigma_i)_{1 \leq i \leq n}\). Then it executes \text{Vrfy} on each tagged message and signature, \(((\tau \parallel m_i), \sigma_i), i = 1, \ldots, n\). It returns 1 if and only if \text{Vrfy} returns 1 for all \(i, i = 1, \ldots, n\).

### 2.3 Pseudorandom Function Family [27]

A pseudorandom function family, \(\{\text{PRF}_k\}_{k \in \text{PRFkeysp}(\lambda)}\), is a function family in which each function \(\text{PRF}_k : \{1, 0\}^* \rightarrow \{1, 0\}^*\) is an efficiently-computable function that looks random to any polynomial-time distinguisher, where \(k\) is called a key and \(\text{PRFkeysp}(\lambda)\) is called a key space. (See more details in, for example, the book [27].)

### 2.4 Access Structure [19]

Let \(\mathcal{U} = \{1, \ldots, u\}\) be an attribute universe. We must distinguish two cases: the case that \(\mathcal{U}\) is small (that is, \(|\mathcal{U}| = u\) is bounded by a polynomial in \(\lambda\)) and the case that \(\mathcal{U}\) is large (that is, \(u\) is not necessarily bounded). We assume the small case in this paper.

Let \(f = f(X_{i_1}, \ldots, X_{i_u})\) be a boolean formula over boolean variables \(U = \{X_1, \ldots, X_u\}\). That is, variables \(X_{i_1}, \ldots, X_{i_u}\) are connected by boolean connectives; AND-gate (\(\land\)) and OR-gate (\(\lor\)). For example, \(f = X_{i_1} \land ((X_{i_2} \land X_{i_3}) \lor X_{i_4})\) for some \(i_1, i_2, i_3, i_4, 1 \leq i_1 < i_2 < i_3 < i_4 \leq u\). Note that there is a bijective map between boolean variables and attributes:

\[
\psi : U \rightarrow \mathcal{U}, \, \psi(X_i) \overset{\text{def}}{=} i.
\]

For \(f(X_{i_1}, \ldots, X_{i_u})\), we denote the set of indices (that is, attributes) \(\{i_1, \ldots, i_u\}\) by \(\text{Att}(f)\). We note the arity of \(f\) as \(\text{arity}(f)\). Hereafter we use the symbol \(i_j\) to mean the following:

\[
i_j \overset{\text{def}}{=} \text{the index } i \text{ of a boolean variable that is the } j \text{-th argument of } f.
\]

Suppose that we are given an access structure as a boolean formula \(f\). For \(S \in 2^\mathcal{U}\), we evaluate the boolean value of \(f\) at \(S\) as follows:

\[
f(S) \overset{\text{def}}{=} f(X_{i_j} \leftarrow [\psi(X_{i_j}) \in S]; j = 1, \ldots, \text{arity}(f)) \in \{1, 0\}.
\]

Under this definition, a boolean formula \(f\) can be seen as a map: \(f : 2^\mathcal{U} \rightarrow \{1, 0\}\). We call a boolean formula \(f\) with this map an access formula over \(\mathcal{U}\). In this paper, we assume that no NOT-gate (\(\neg\)) appears in \(f\). In other words, we only consider a monotone access formula \(f\).\(^7\)

\(^7\) This limitation can be removed by adding negation attributes to \(\mathcal{U}\) for each attribute in the original \(\mathcal{U}\) though the size of the attribute universe \(|\mathcal{U}|\) doubles.

9
**Access Tree** A monotone access formula $f$ can be represented by a finite binary tree $T_f$. Each inner node represents a boolean connective, $\land$-gate or $\lor$-gate, in $f$. Each leaf node corresponds to a term $X_i$ (not a variable $X_i$) in $f$ in one-to-one way. For a finite binary tree tree $T$, we denote the set of all nodes, the root node, the set of all leaf nodes, the set of all inner nodes (that is, all nodes excluding leaf nodes) and the set of all tree-nodes (that is, all nodes excluding the root node) as Node($T$), $r(T)$, Leaf($T$), iNode($T$) and tNode($T$), respectively. Then an attribute map $\rho(\cdot)$ is defined as:

$$\rho : \text{Leaf}(T) \to U, \quad \rho(l) \overset{\text{def}}{=} (\text{the attribute } i \text{ that corresponds to } l \text{ through } \psi).$$

If $\rho$ is not injective, then we call the case multi-use of attributes.

If $T$ is of height greater than 0, $T$ has two subtrees whose root nodes are two children of $r(T)$. We denote the two subtrees by Lsub($T$) and Rsub($T$), which mean the left subtree and the right subtree, respectively.

### 2.5 Attribute-Based Identification Scheme [2]

An attribute-based identification scheme consists of four PPT algorithms:

(\text{ABID.Setup}, \text{ABID.KG}, \mathcal{P}, \mathcal{V}) [2].

\text{ABID.Setup}(1^\lambda, U) \rightarrow (PK, MSK). \text{ABID.Setup} takes as input the security parameter $1^\lambda$ and an attribute universe $U$. It outputs a public key PK and a master secret key MSK.

\text{ABID.KG}(PK, MSK, S) \rightarrow SK_S. A key-generation algorithm \text{ABID.KG} takes as input the public key PK, the master secret key MSK and an attribute set $S \subset U$. It outputs a signing key SK$_S$ corresponding to $S$.

$\mathcal{P}(PK, SK_S, f)$ and $\mathcal{V}(PK, f)$. $\mathcal{P}$ and $\mathcal{V}$ are interactive algorithms called a prover and a verifier, respectively. $\mathcal{P}$ takes as input the public key PK, the secret key SK$_S$ and an access formula $f$. Here the secret key SK$_S$ is given to $\mathcal{P}$ by an authority that runs \text{ABID.KG}(PK, MSK, S). $\mathcal{V}$ takes as input the public key PK and an access formula $f$. $\mathcal{P}$ and $\mathcal{V}$ interact with each other for at most constant rounds. Then, $\mathcal{V}$ returns its decision 1 or 0. When it is 1, we say that $\mathcal{V}$ accepts $\mathcal{P}$ for $f$. When it is 0, we say that $\mathcal{V}$ rejects $\mathcal{P}$ for $f$. We demand correctness of ABID that for any $\lambda$, and if $f(S) = 1$, then $\Pr[(PK, MSK) \leftarrow \text{ABID.Setup}(1^\lambda, U), SK_S \leftarrow \text{ABID.KG}(PK, MSK, S), b \leftarrow (\mathcal{P}(PK, SK_S), \mathcal{V}(PK, f)) : b = 1] = 1$.

**Attacks on ABID and Security** An adversary $\mathcal{A}$’s objective is impersonation. $\mathcal{A}$ tries to make a verifier $\mathcal{V}$ accept with an access formula $f^\ast$. The following experiment Exprmt$^{\mathcal{A}}_{\text{ABID}}(\lambda, U)$ of an adversary $\mathcal{A}$ defines the game of concurrent attack on ABID.

\begin{align*}
\text{Exprmt}^{\mathcal{A}}_{\text{ABID}}(\lambda, U) : \\
(PK, MSK) &\leftarrow \text{ABID.Setup}(1^\lambda, U) \\
(f^*, st) &\leftarrow \mathcal{A}^{\mathcal{G}(PK, MSK, \cdot)}_{\lambda, U}(PK, MSK, \cdot)^{0_1}(PK, U) \\
b &\leftarrow (\mathcal{A}(st), \mathcal{V}(PK, f^*)) \\
\text{If } b = 1 \text{ then Return Win else Return Lose}
\end{align*}

In the experiment, $\mathcal{A}$ issues key-extraction queries to its key-generation oracle $\mathcal{G}$. Giving an attribute set $S_i$, $\mathcal{A}$ queries $\mathcal{G}(PK, MSK, \cdot)$ for the secret key SK$_S$. In addition, $\mathcal{A}$ invokes provers
\( \mathcal{P}_j(\text{PK}, \text{SK}, \cdot), \ j = 1, \ldots, q'_p, \ldots, q_p, \) by giving a pair \((S_j, f_j)\) of an attribute set and an access formula. Acting as a verifier with an access formula \(f_j\), \(\mathcal{A}\) interacts with each \(\mathcal{P}_j(\text{PK}, \text{SK}_{S_j}, f_j)\) concurrently.

The access formula \(f^*\) declared by \(\mathcal{A}\) is called a **target access formula**. Here we consider the adaptive target in the sense that \(\mathcal{A}\) is allowed to choose \(f^*\) after seeing PK, issuing key-extraction queries and interacting with of provers. Two restrictions are imposed on \(\mathcal{A}\) concerning \(f^*\). In key-extraction queries, each attribute set \(S_i\) must satisfy \(f^*(S_i) = 0\). In interactions with each prover, \(f^*(S_j) = 0\). The number of key-extraction queries and the number of invoked provers are at most \(q_k\) and \(q_p\) in total, respectively, which are bounded by a polynomial in \(\lambda\).

The **advantage** of \(\mathcal{A}\) over \(\text{ABID}\) in the game of concurrent attack is defined as

\[
\text{Adv}^\text{ca}_{\mathcal{A}, \text{ABID}}(\lambda) \overset{\text{def}}{=} \Pr[\text{Exprmt}^\text{ca}_{\mathcal{A}, \text{ABID}}(\lambda, \mathcal{U}) \text{ returns } \text{WIN}].
\]

\(\text{ABID}\) is called **secure against concurrent attacks** if, for any PPT \(\mathcal{A}\) and for any \(\mathcal{U}\), \(\text{Adv}^\text{ca}_{\mathcal{A}, \text{ABID}}(\lambda)\) is negligible in \(\lambda\).

The game of **passive attack** on \(\text{ABID}\) is obtained by replacing concurrent provers \(\mathcal{P}_j(\text{PK}, \text{SK}, \cdot)\) with transcript oracle \(\text{Transc}\) in \(\text{Exprmt}^\text{ca}_{\mathcal{A}, \text{ABID}}(\lambda, \mathcal{U})\). The following experiment \(\text{Exprmt}^\text{pa}_{\mathcal{A}, \text{ABID}}(\lambda, \mathcal{U})\) of an adversary \(\mathcal{A}\) defines the game of passive attack on \(\text{ABID}\).

\[
\begin{align*}
\text{Exprmt}^\text{pa}_{\mathcal{A}, \text{ABID}}(\lambda, \mathcal{U}) : \\
(PK, MSK) &\leftarrow \text{ABID.Setup}(1^\lambda, \mathcal{U}) \\
(f^*, st) &\leftarrow \mathcal{A}(\mathcal{K}^{G}(PK, MSK, \cdot), \text{Transc}(\mathcal{P}(PK, SK, \cdot), \mathcal{V}(PK, \cdot))(PK, \mathcal{U}) \\
b &\leftarrow (\mathcal{A}(st), \mathcal{V}(PK, f^*)) \\
\text{If } b = 1 \text{ then Return } \text{WIN} \text{ else Return } \text{LOSE}
\end{align*}
\]

In the experiment, \(\mathcal{A}\) issues key-extraction queries to its key-generation oracle \(\mathcal{K}^G\) and transcript queries to its transcript oracle \(\text{Transc}\). In a transcript query, giving a pair \((S_j, f_j)\) of an attribute set and an access formula, \(\mathcal{A}\) queries \(\text{Transc}(\mathcal{P}(PK, SK, \cdot), \mathcal{V}(PK, \cdot))\) for a whole transcript of messages interacted between \(\mathcal{P}(PK, SK_{S_j}, f_j)\) and \(\mathcal{V}(PK, f_j)\).

The advantage \(\text{Adv}^\text{pa}_{\mathcal{A}, \text{ABID}}(\lambda)\) and security are defined in the same way as the concurrent case. Concurrent security means passive security; for any PPT \(\mathcal{A}\), there exists a PPT \(\mathcal{B}\) that satisfies the following inequality.

\[
\text{Adv}^\text{pa}_{\mathcal{A}, \text{ABID}}(\lambda) \leq \text{Adv}^\text{ca}_{\mathcal{B}, \text{ABID}}(\lambda). \quad (1)
\]

### 2.6 Attribute-Based Signature Scheme [30, 31]

An attribute-based signature scheme, \(\text{ABS}\), consists of four PPT algorithms: \(\text{ABS} = (\text{ABS.Setup, ABS.KG, ABS.Sign, ABS.Vrfy})\).

\(\text{ABS.Setup}(1^\lambda, \mathcal{U}) \rightarrow (PK, MSK)\). It takes as input the security parameter \(\lambda\) and an attribute universe \(\mathcal{U}\). It outputs a public key \(PK\) and a master secret key \(MSK\).

\(\text{ABS.KG}(PK, MSK, S) \rightarrow SK_S\). It takes as input the public key \(PK\), the master secret key \(MSK\), and an attribute set \(S \subseteq \mathcal{U}\). It outputs a signing key \(SK_S\) corresponding to \(S\).

\(\text{ABS.Sign}(PK, SK_S, (m, f)) \rightarrow \sigma\). It takes as input a public key \(PK\), a private secret key \(SK_S\) corresponding to an attribute set \(S\), a pair \((m, f)\) of a message \(\in \{1, 0\}^*\) and an access formula. It outputs a signature \(\sigma\).
**ABS.Vrfy**(PK, (m, f), σ). It takes as input a public key PK, a pair (m, f) of a message and an access formula, and a signature σ. It outputs a decision 1 or 0. When it is 1, we say that ((m, f), σ) is *valid*. When it is 0, we say that ((m, f), σ) is *invalid*. We demand correctness of ABS that, for any λ, any U, any S ⊂ U and any (m, f) such that f(S) = 1, Pr[(PK, MSK) ← ABS.Setup(1^λ, U), SK_S ← ABS.KG(PK, MSK, S), σ ← ABS.Sign(PK, SK_S, (m, f)), b ← ABS.Vrfy(PK, (m, f), σ) : b = 1] = 1.

**Chosen-Message Attack on ABS and Security** An adversary F’s objective is to make an *existential forgery*. F tries to make a forgery ((m^*, f^*), σ^*) that consists of a message, a target access structure and a signature. The following experiment Exprmt_{F, ABS}^{euf-cma}(λ, U) of a forger F defines the *chosen-message attack on ABS to make an existential forgery*.

\[
\text{Exprmt}_{F, ABS}^{euf-cma}(λ, U) : \\
\text{(PK, MSK) ← ABS.Setup}(1^λ, U) \\
\text{((m^*, f^*), σ^*) ← F}^\text{KG}(PK, MSK, ·), \text{SIG}^*(PK, SK, ·)(PK) \\
\text{If ABS.Vrfy}(PK, (m^*, f^*), σ^*) = 1 \text{ then Return } \text{WIN} \\
\text{else Return } \text{LOSE}
\]

In the experiment, F issues key-extraction queries to its key-generation oracle KG and signing queries to its signing oracle SIG*. Giving an attribute set S_i, F queries KG(PK, MSK, ·) for the secret key SK_{S_i}. In addition, giving an attribute set S_j and a pair (m, f) of a message and an access formula, F queries SIG*(PK, SK, ·)(·) for a signature σ that satisfies ABS.Vrfy(PK, (m, f), σ) = 1 when f(S_j) = 1.

The access formula f* declared by F is called a *target access formula*. Here we consider the *adaptive* target in the sense that F is allowed to choose f* after seeing PK and issuing some key-extraction queries and signing queries. Two restrictions are imposed on F concerning f*. In key-extraction queries, S_i that satisfies f*(S_i) = 1 was never queried. In signing queries, (m^*, f^*) was never queried. The number of key-extraction queries and the number of signing queries are at most q_k and q_a in total, respectively, which are bounded by a polynomial in λ.

The *advantage* of F over ABS in the game of chosen-message attack to make existential forgery is defined as

\[\text{Adv}_{F, ABS}^{euf-cma}(λ) \overset{\text{def}}{=} \text{Pr[Exprmt}_{F, ABS}^{euf-cma}(λ, U) \text{ returns } \text{WIN}].\]

ABS is called *existentially unforgeable against chosen-message attacks* if, for any PPT F and for any U, Adv_{F, ABS}^{euf-cma}(λ) is negligible in λ.

**Attribute Privacy of ABS** Roughly speaking, ABS is called to have attribute privacy if any unconditional cheating verifier cannot distinguish two distributions of signatures each of which is generated by different attribute set. The following definition is due to Maji et al. and Okamoto-Takashima.

**Definition 3 (Attribute Privacy (Perfect Privacy [30, 31]))** ABS is called to have attribute privacy if, for all (PK, MSK) ← ABS.Setup(1^λ, U), for all message m, for all attribute sets S_1 and S_2, for all signing keys SK_{S_1} ← ABS.KG(PK, MSK, S_1) and SK_{S_2} ← ABS.KG(PK, MSK, S_2) and for all access formula f such that f(S_1) = 1 and f(S_2) = 1 or f(S_1) ≠ 1 and f(S_2) ≠ 1, two distributions ABS.Sign(PK, SK_{S_1}, (m, f)) and ABS.Sign(PK, SK_{S_2}, (m, f)) are identical.
Basically \( \Sigma \) is a 3-move protocol between interactive PPT algorithms \( P \) and \( V \) on initial input \((x := (x_{i_j})_{1 \leq j \leq \text{arity}(f)}, w := (w_{i_j})_{1 \leq j \leq \text{arity}(f)}) \in R_f\) for \( P \) and \( x \) for \( V \). In our prover-algorithm \( P \), there are three PPT subroutines; \( \Sigma^1_f \) and \( \Sigma^3_f \). On the other hand, in our verifier-algorithm \( V \), there are two PPT subroutines; \( \Sigma^2_f \) and \( \Sigma^\text{vrfy}_f \). \( \Sigma^\text{vrfy}_f \) has two subroutines; \( \text{VrfyCha} \) and \( \text{VrfyRes} \). Fig. 1 shows our procedure of boolean proof, \( \Sigma_f \).

**Evaluation of Satisfiability.** The prover \( P \) begins with evaluation concerning whether and how \( S \) satisfies \( f \). We label each node of \( T \) with a value \( v = 1 \) (True) or 0 (False). For each leaf node \( l \), we label \( v_l \) with \( v_l = 1 \) if \( \rho(l) \in S \) and \( v_l = 0 \) otherwise. For each inner node \( n \), we label \( n \) with \( v_n = v_{n_L} \land v_{n_R} \) or \( v_n = v_{n_L} \lor v_{n_R} \) according to AND/OR evaluation of two labels of its two children \( n_L, n_R \). The computation is executed for every node from the root node to each leaf node, recursively, in the following way.

\[
P(x, w, f): \quad \forall(x, f):
\]

\[
\Sigma^\text{val}_f(T, S) : \quad \Sigma^\text{val}_f(T_f, S) \rightarrow (v_n)_n
\]

If \( v_{r(T)} \neq 1 \), then abort
else \( \text{CHA}_f(T_f) := s, \eta \in \mathbb{Z} \)
\[
\Sigma^1_f(x, w, v_n, \text{CHA}_f(T_f)) \rightarrow ((\text{Cmt}_1)_l, (\text{CHA}_n)_n, (\text{Res}_1)_l) \quad (\text{Cmt}_1)_l
\]

\[
\text{CHA}_f(T_f) := \text{CHA} \quad \text{CHA} \leftarrow \Sigma^2_f(1^3)
\]

\[
\Sigma^1_f(x, w, T_f, (v_n)_n), \quad (\text{Cmt}_1)_l, (\text{CHA}_n)_n, (\text{Res}_1)_l \quad \rightarrow \quad \Sigma^\text{vrfy}_f(x, T_f, \text{CHA},)
\]

\[
(\text{Cmt}_1)_l, (\text{Res}_1)_l \quad \rightarrow \quad b, \text{Return } b
\]

**Commitment.** Prover’s computation of a commitment value for each leaf node is described in Fig. 2. Basically, the algorithm \( \Sigma^1_f \) runs for every node from the root node to each leaf node, recursively. As a result, \( \Sigma^1_f \) generates for each leaf node \( l \) a value \( \text{Cmt}_l \); if \( v_l = 1 \), then \( \text{Cmt}_l \) is
computed honestly according to $\Sigma^1$. Else if $v_t = 0$, then $\text{CMT}_I$ is computed in the simulated way according to $\Sigma^{\text{sim}}$. Other values, $(\text{CHA}_t)_l$ and $(\text{RES})_l$, are needed for the simulation. Note that a distinguished symbol ‘$*$’ is used for those other values to indicate the honest computation.

$$\Sigma^1_f(x, w, T, (v_n)_n, \text{CHA}) :$$
$$T_L := \text{Lsub}(T), T_R := \text{Rsub}(T)$$
If $r(T)$ is a node, then $\text{CHA}_v(T_L) := \text{CHA}, \text{CHA}_v(T_R) := \text{CHA}$
Return($\text{CHA}_v(T_L), \Sigma^1_f(x, w, T_L, (v_n)_n, \text{CHA}_v(T_L), \text{CHA}_v(T_R), \Sigma^1_f(x, w, T_R, (v_n)_n, \text{CHA}_v(T_R))$)
else if $r(T)$ is a leaf-node, then
If $u_r(T_L) = 1 \land u_r(T_R) = 1$, then $\text{CHA}_v(T_L) := *$, $\text{CHA}_v(T_R) := *$
else if $u_r(T_L) = 1 \land u_r(T_R) = 0$, then $\text{CHA}_v(T_L) := *$, $\text{CHA}_v(T_R) := \Sigma^2(1^\lambda)$
else if $u_r(T_L) = 0 \land u_r(T_R) = 1$, then $\text{CHA}_v(T_L) := \Sigma^2(1^\lambda), \text{CHA}_v(T_R) := *$
else if $u_r(T_L) = 0 \land u_r(T_R) = 0$, then $\text{CHA}_v(T_L) := \Sigma(1^\lambda), \text{CHA}_v(T_R) := \text{CHA} \oplus \text{CHA}_v(T_L)$
Return($\text{CHA}_v(T_L), \Sigma^1_f(x, w, T_L, (v_n)_n, \text{CHA}_v(T_L), \text{CHA}_v(T_R), \Sigma^1_f(x, w, T_R, (v_n)_n, \text{CHA}_v(T_R))$)
else if $r(T)$ is a leaf-node, then
If $u_r(T) = 1$, then $\text{CMT}_r(T) := \Sigma^1(x_{\rho_r(T)}, w_{\rho_r(T)}), \text{RES}_r(T) := *$
else if $u_r(T) = 0$, then $(\text{CMT}_r(T), \text{RES}_r(T)) := \Sigma^{\text{sim}}(x_{\rho_r(T)}, \text{CHA})$
Return($\text{CMT}_r(T), \text{RES}_r(T)$)

Fig. 2. The subroutine $\Sigma^1_f$ of our $\Sigma_f$.

Challenge. Verifier picks up a challenge value by using $\Sigma^2$.

$$\Sigma^2_f(1^\lambda) : \text{CHA} \leftarrow \Sigma^2(1^\lambda), \text{Return(CHA)}$$

Response. Prover’s computation of a response value for each leaf node is described in Fig. 3. Basically, the algorithm $\Sigma^3_f$ runs for every node from the root node to each leaf node, recursively. As a result, $\Sigma^3_f$ generates values, $(\text{CHA}_t)_l$ and $(\text{RES}_l)_l$. Note that all challenge values $(\text{CHA}_t)_l$ are completed according to the “division rule” described in Section 1.1.

$$\Sigma^3_f(x, w, T, (v_n)_n, (\text{CMT}_l)_l, (\text{CHA}_n)_n, (\text{RES}_l)_l) :$$
$$T_L := \text{Lsub}(T), T_R := \text{Rsub}(T)$$
If $r(T)$ is a node, then $\text{CHA}_v(T_L) := \text{CHA}, \text{CHA}_v(T_R) := \text{CHA}$
Return($\text{CHA}_v(T_L), \Sigma^3_f(x, w, T_L, (v_n)_n, (\text{CMT}_l)_l, (\text{CHA}_n)_n, (\text{RES}_l)_l), \text{CHA}_v(T_R), \Sigma^3_f(x, w, T_R, (v_n)_n, (\text{CMT}_l)_l, (\text{CHA}_n)_n, (\text{RES}_l)_l)$)
else if $r(T)$ is a leaf-node, then
If $u_r(T_L) = 1 \land u_r(T_R) = 1$, then $\text{CHA}_v(T_L) := \Sigma^2(1^\lambda)$, $\text{CHA}_v(T_R) := \text{CHA} \oplus \text{CHA}_v(T_L)$
else if $u_r(T_L) = 1 \land u_r(T_R) = 0$, then $\text{CHA}_v(T_L) := \text{CHA} \oplus \text{CHA}_v(T_R), \text{CHA}_v(T_R) := \text{CHA}_v(T_R)$
else if $u_r(T_L) = 0 \land u_r(T_R) = 1$, then $\text{CHA}_v(T_L) := \text{CHA}_v(T_L) \oplus \text{CHA}_v(T_L)$, $\text{CHA}_v(T_R) := \text{CHA}_v(T_R)$
else if $u_r(T_L) = 0 \land u_r(T_R) = 0$, then $\text{CHA}_v(T_L) := \text{CHA}_v(T_L) \oplus \text{CHA}_v(T_L)$, $\text{CHA}_v(T_R) := \text{CHA}_v(T_R)$
Return($\text{CHA}_v(T_L), \Sigma^3_f(x, w, T_L, (v_n)_n, (\text{CMT}_l)_l, (\text{CHA}_n)_n, (\text{RES}_l)_l), \text{CHA}_v(T_R), \Sigma^3_f(x, w, T_R, (v_n)_n, (\text{CMT}_l)_l, (\text{CHA}_n)_n, (\text{RES}_l)_l)$)
else if $r(T)$ is a leaf-node, then
If $u_r(T) = 1$, then $\text{RES}_r(T) := \Sigma^3(x_{\rho_r(T)}, w_{\rho_r(T)}, \text{CMT}_r(T), \text{CHA})$
else if $u_r(T) = 0$, then $\text{RES}_r(T) := \text{RES}_r(T)$
Return($\text{RES}_r(T)$)

Fig. 3. The subroutine $\Sigma^3_f$ of our $\Sigma_f$. 

14
**Verification.** Verifier’s computation is executed for each leaf node as follows.

\[ \Sigma_f^{\text{vrfy}}(x, \mathcal{T}, \text{CH}_A, \text{CMT}_l)_l, (\text{CH}_A)_l, (\text{RES}_l)_l) : \]

\[ \text{Return} (\text{VrfyCha}(\mathcal{T}, \text{CH}_A, (\text{CH}_A)_l) \land \text{VrfyRes}(x, \mathcal{T}, (\text{CMT}_l, \text{CH}_A, \text{RES}_l)_l)) \]

\[ \text{VrfyCha}(\mathcal{T}, \text{CH}_A, (\text{CH}_A)_l) : \]

\[ \mathcal{T}_L := \text{Lsub}(\mathcal{T}), \mathcal{T}_R := \text{Rsub}(\mathcal{T}) \]

If \( r(\mathcal{T}) \) is an \( \land \)-node,

\[ \text{then Return } ((\text{CH}_A \neq \text{CH}_A(\mathcal{T}_L)) \land (\text{CH}_A \neq \text{CH}_A(\mathcal{T}_R)) \land \text{VrfyCha}(\mathcal{T}_L, \text{CH}_A(\mathcal{T}_L), (\text{CH}_A)_l) \land \text{VrfyCha}(\mathcal{T}_R, \text{CH}_A(\mathcal{T}_R), (\text{CH}_A)_l)) \]

else if \( r(\mathcal{T}) \) is an \( \lor \)-node,

\[ \text{then Return } ((\text{CH}_A \neq \text{CH}_A(\mathcal{T}_L) \lor \text{CH}_A(\mathcal{T}_R)) \land \text{VrfyCha}(\mathcal{T}_L, \text{CH}_A(\mathcal{T}_L), (\text{CH}_A)_l) \land \text{VrfyCha}(\mathcal{T}_R, \text{CH}_A(\mathcal{T}_R), (\text{CH}_A)_l)) \]

else if \( r(\mathcal{T}) \) is a leaf node,

\[ \text{then Return } (\text{CH}_A \in \text{CHAsp}(1^\lambda)) \]

\[ \text{VrfyRes}(x, \mathcal{T}, (\text{CMT}_l, \text{CH}_A, \text{RES}_l)_l) : \]

For \( l \in \text{Leaf}(\mathcal{T}) \)

\[ \text{If } \Sigma^{\text{vrfy}}(\rho(l), \text{CMT}_l, \text{CH}_A, \text{RES}_l) = 0, \text{then Return } (0) \]

\[ \text{Return } (1) \]

Now we have to check that \( \Sigma_f \) is certainly a \( \Sigma \)-protocol for the language \( L_f \).

**Proposition 1 (Completeness)** Completeness holds for our \( \Sigma_f \). More precisely, Suppose that \( v_r(\mathcal{T}_f) = 1 \). Then, for every node in \( \text{Node}(\mathcal{T}_f) \), either \( v_n = 1 \) or \( \text{CH}_n \neq * \) holds after executing \( \Sigma_f^1 \).

**Proof.** Induction on the height of \( \mathcal{T}_f \). The case of height 0 follows from \( v_r(\mathcal{T}_f) = 1 \) and the completeness of \( \Sigma \). Suppose that the case of height \( k \) holds and consider the case of height \( k + 1 \). The construction of \( \Sigma_f^1 \) assures the case of height \( k + 1 \). \( \square \)

**Proposition 2 (Special Soundness)** Special soundness holds for our \( \Sigma_f \).

We can construct a knowledge extractor \( \Sigma_f^{\text{KE}} \) from a knowledge extractor \( \Sigma^{\text{KE}} \) of the underlying \( \Sigma \)-protocol \( \Sigma \) as follows.

\[ \Sigma_f^{\text{KE}}(x, (\text{CMT}_l, \text{CH}_A, \text{RES}_l)_l \in \text{Leaf}(\mathcal{T}_f), (\text{CMT}_l, \text{CH}_A', \text{RES}_l)_l \in \text{Leaf}(\mathcal{T}_f)) : \]

For \( 1 \leq j \leq \text{arity}(f) : w_{ij}^* := * \)

For \( l \in \text{Leaf}(\mathcal{T}_f) \)

If \( \text{CH}_A \neq \text{CH}_A' \), then \( w_{\rho(l)}^* \leftarrow \Sigma^{\text{KE}}(x_{\rho(l)}, \text{CMT}_l, \text{CH}_A, \text{RES}_l), (\text{CMT}_l, \text{CH}_A', \text{RES}_l)) \)

else if \( w_{\rho(l)}^* = * \), then \( w_{\rho(l)}^* \leftarrow \{1, 0\}^* \)

Return \( (w^* := (w_{ij}^*)_{1 \leq j \leq \text{arity}(f)}) \)

Then Lemma 1 assures the proposition.
Lemma 1 (Witness Extraction) The set $w^*$ output by $\Sigma_f^{KE}$ satisfies $(x, w^*) \in R_f$.

Proof. Induction on the number of all $\lor$-nodes in iNode$(T_f)$. First remark that $\text{CHA} \neq \text{CHA}'$.

Suppose that all nodes in iNode$(T_f)$ are $\land$-nodes. Then the above claim follows immediately because $\text{CHA}_I \neq \text{CHA}'_I$ holds for all leaf nodes.

Suppose that the case of $k$ $\lor$-nodes holds and consider the case of $k + 1$ $\lor$-nodes. Look at one of the lowest height $\lor$-node and name the height and the node as $h^*$ and $n^*$, respectively. Then $\text{CHA}_{n^*} \neq \text{CHA}'_{n^*}$ because all nodes with height less than $h^*$ are $\land$-nodes. So at least one of children of $n^*$, say $n^*_L$, satisfies $\text{CHA}_{n^*_L} \neq \text{CHA}'_{n^*_L}$. Divide the tree $T_f$ into two subtrees by cutting the branch right above $n^*$, and the induction hypothesis assures the claim.

Proposition 3 (Honest Verifier Zero-Knowledge) Honest verifier zero-knowledge property holds for our $\Sigma_f$.

Proof. This is the immediate consequence of honest verifier zero-knowledge property of $\Sigma$. That is, we can construct a polynomial-time simulator $\Sigma_f^{sim}$ which, on input $(\text{PK}, \text{CHA})$, outputs commitment and response message of $\Sigma_f$.

We summarize the above results into the following theorem and corollary.

Theorem 1 Our procedure $\Sigma_f$ obtained from a $\Sigma$-protocol $\Sigma$ on the relation $R$ and a boolean formula $f$ is a $\Sigma$-protocol on the relation $R_f$.

Corollary 1 Our procedure $\Sigma_f$ is an boolean proof for the language $L_f$.

4 Our Attribute-Based Identification Scheme

In this section, by combining our $\Sigma$-protocol of boolean proof in Section 3 with a signature bundle scheme of the Fiat-Shamir signature, we obtain a verifier-policy attribute-based identification scheme.

4.1 Our ABID

By combining our $\Sigma_f$ in Section 3 with a signature bundle scheme $\text{FS}(\Sigma)$, we obtain a scheme of VP-ABID, ABID, which has collusion resistance against collecting private secret keys. Our ABID has a feature that it can be constructed without pairings. Fig. 4 shows the construction of our ABID, $\text{ABID} = (\text{ABID.Setup}, \text{ABID.KG}, \mathcal{P}, \mathcal{V})$.

4.2 Security of Our ABID

Theorem 2 (Concurrent Security) If the employed signature scheme $\text{FS}(\Sigma)$ is existentially unforgeable against chosen-message attacks, then our ABID is secure against concurrent attacks. More precisely, for any PPT algorithm $A$, there exists a PPT algorithm $F$ which satisfies the following inequality ($\text{neg}(\cdot)$ means a negligible function).

\[
\text{Adv}^{\text{cuf-cma}}_{A,\text{ABID}}(\lambda) \leq (\text{Adv}^{\text{cuf-cma}}_{F,\text{FS}(\Sigma)}(\lambda))^{1/2} + \text{neg}(\lambda).
\]
\textbf{ABID Setup}(\(1^\lambda, \mathcal{U}\)):
\( (x_{\text{mat}}, w_{\text{mat}}) \leftarrow \text{Instance}_P(1^\lambda) \)
\( \mu \leftarrow \text{Hashkeys}(\lambda) \)
\( \text{PK} := (x_{\text{mat}}, \mathcal{U}, \mu), \text{MSK} := (w_{\text{mat}}) \)
Return(PK, MSK)

\textbf{ABID KG}(\text{PK}, \text{MSK}, S):
\( k \leftarrow \text{PRFkeys}(\lambda), \tau \leftarrow \{1, 0\}^\lambda \)
For \( i \in S \)
\( m_i := (\tau \parallel i), a_i \leftarrow \Sigma^1(x_{\text{mat}}, w_{\text{mat}}) \)
\( c_i \leftarrow \text{Hash}_U(a_i \parallel m_i), w_i \leftarrow \Sigma^3(x_{\text{mat}}, w_{\text{mat}}, a_i, c_i) \)
\( \text{SK}_S := (k, \tau, (a_i, w_i)_{i \in S}) \)
Return SK_S

\( \mathcal{P}(\text{PK}, \text{SK}_S, f): \)
\( \Sigma^\text{val}_f(T_f, S) \rightarrow (v_n)_n \)
If \( v_f(T_f) \neq 1 \), then abort
else \( \text{CHA}_r(T_f) := * \)
\( \text{Supp}(\text{PK}, \text{SK}_S, f) \rightarrow (a_{ij}, w_{ij})_{1 \leq j \leq \text{arity}(f)} \)
\( w := (w_{ij})_{1 \leq j \leq \text{arity}(f)} \)
\( \text{StmtGen}(\text{PK}, \tau, (a_{ij})_{1 \leq j \leq \text{arity}(f)}) \)
\( \rightarrow (x_{ij})_{1 \leq j \leq \text{arity}(f)} := x \)
\( \Sigma^f_1(x, w, T_f, (v_n)_n, \text{CHA}_r(T_f)) \rightarrow ((\text{CMT}_l)_i, (\text{CHA}_n)_n, (\text{Res}_l)_i) \)
\( \text{CHA}_r(T_f) := \text{CHA} \)
\( \Sigma^f_3(x, w, T_f, (v_n)_n, (\text{CMT}_l)_i, (\text{CHA}_n)_n, (\text{Res}_l)_i) \rightarrow ((\text{CHA}_n)_n, (\text{Res}_l)_i) \)
\( \text{CHA} \leftarrow \Sigma^2_f(1^\lambda) \)
\( \text{StmtGen}(\text{PK}, \tau, (a_{ij})_{1 \leq j \leq \text{arity}(f)}) \)
\( \rightarrow (x_{ij})_{1 \leq j \leq \text{arity}(f)} := x \)
\( \Sigma^\text{vrfy}_f(x, T_f, (\text{CMT}_l)_i, (\text{CHA}_n)_n, (\text{Res}_l)_i) \rightarrow b \)
\( \rightarrow \text{Return } b \)

\textbf{Fig. 4.} The scheme of our ABID.
Note that $FS(\Sigma)$ is only known to be secure in the random oracle model.

**Proof.** Employing any given adversary $A$ as subroutine, we construct a signature forger $F$ as follows. $F$ can answer to $A$’s key-extraction queries for a secret key $SK_\Sigma$ because $F$ can query his signing oracle about $(\tau \parallel i; i \in S)$, where $F$ choose $\tau$ at random. $F$ can simulate any concurrent prover with $SK_\Sigma$ which $A$ invokes because $F$ can generate $SK_\Sigma$ in the above way. After the learning phase above, $F$ simulates a verifier with which $A$ begins to interact as a prover. There $F$ rewinds $A$ once and $F$ can obtain a witness $w^*$ by running $\Sigma_f^{\text{KE}}$, as well as the set of attributes $S^*$ where the rewinding works. We use here the Reset Lemma [5], which reduces the advantage $\text{Adv}_{A,\text{ABID}}(\lambda)$ to $\text{Adv}_{\Sigma_f,FS(\Sigma)}(\lambda)$ with a loss of exponent by $1/2$. Finally $F$ converts $(w^*, S^*)$ into $SK_{S^*}$.

\[ \square \]

**More on Reduction of Concurrent Security** We mean by “a number theoretic problem” the discrete-logarithm problem or the RSA-inverse problem ([5]). There exists the following security reduction to a number theoretic problem.

\[ \text{Adv}_{A,\text{ABID}}(\lambda) \leq q_H^{1/2} (\text{Adv}_{S,\text{Grp}}^{\text{num.prob}}(\lambda))^{1/4} + \text{neg}(\lambda). \]  

(2)

Here we denote $q_H$ as the maximum number of hash queries issued by forger $F$ on $FS(\Sigma)$ in the random oracle model.

**Proof.** As is discussed in Section 2.1, we can reduce the advantage $\text{Adv}_{F,FS(\Sigma)}^{\text{euf-cma}}(\lambda)$ to the advantage $\text{Adv}_{S,\Sigma}^{\text{pa}}(\lambda)$ of passive security of the underlying $\Sigma$-protocol, in the random oracle model, with a loss factor $q_H$. Applying the Reset Lemma [5], we can reduce $\text{Adv}_{S,\Sigma}^{\text{pa}}(\lambda)$ to the advantage $\text{Adv}_{S,\text{Grp}}^{\text{num.prob}}(\lambda)$ of a PPT solver $S$ of a number theoretic problem, with a loss of exponent by $1/2$.

In Appendix A, we provide two instantiations of our $\Sigma_f$ and $\text{ABID}$; in the discrete-logarithm setting and in the RSA setting.

**5 Our Attribute-Based Signature Scheme**

In this section, by applying the Fiat-Shamir transform to our ABID in Section 4.1, we obtain an attribute-based signature scheme. Our ABS is collusion resistant against collecting private secret keys, EUF-CMA secure in the random oracle model, and has a feature that it can be constructed without pairings. We note that our ABS does not have attribute privacy.

**5.1 Our ABS**

By applying $FS(\cdot)$ to our $\text{ABID}$ in Section 4.1, we obtain an attribute-based signature scheme, $\text{ABS}$. Fig. 5 shows shows the construction of our ABS, $\text{ABS} = (\text{ABS.Setup}, \text{ABS.KG}, \text{ABS.Sign}, \text{ABS.Vrfy})$.

**ABS.Setup** chooses, on input $1^\lambda$ and $U$, a pair $(x_{\text{mst}}, w_{\text{mst}})$ at random from $R = \{(x, w)\}$ by running $\text{Instance}_R(1^\lambda)$, where $|x|$ and $|w|$ are bounded by a polynomial in $\lambda$. It also chooses a hash key $\mu$ at random from a hash-key space $\text{Hashkeysp}(\lambda)$. It outputs a public key $PK = (x_{\text{mst}}, U, \mu)$ and a master secret key $\text{MSK} = (w_{\text{mst}})$.

\[ \text{ABS.Setup}(1^\lambda, U) : \]
\[ (x_{\text{mst}}, w_{\text{mst}}) \leftarrow \text{Instance}_R(1^\lambda), \mu \leftarrow \text{Hashkeysp}(\lambda) \]
\[ \text{PK} := (x_{\text{mst}}, U, \mu), \text{MSK} := (w_{\text{mst}}) \]
\[ \text{Return}(\text{PK}, \text{MSK}) \]
**ABS.KG**, chooses, on input PK, MSK, S, a PRF key $k$ from $\text{PRFkeys}(\lambda)$ at random and a random string $\tau$ from \{1, 0\}^$\lambda$ at random. Then KG applies the signature bundle technique [30] for each message $m_i := (\tau \parallel i), i \in S.$ Here we employ the Fiat-Shamir signing algorithm $\text{FS}(\Sigma)^{\text{sign}}$ (see 2.1).

**ABS.KG**(PK, MSK, S):

$k \leftarrow \text{PRFkeys}(\lambda), \tau \leftarrow \{1, 0\}^$\lambda$

For $i \in S$:

$m_i := (\tau \parallel i), a_i \leftarrow \Sigma^2(x_{\text{mst}}, w_{\text{mst}})$

$c_i \leftarrow \text{Hash}_{\mu}(a_i \parallel m_i), w_i \leftarrow \Sigma^3(x_{\text{mst}}, w_{\text{mst}}, a_i, c_i)$

$\text{SK}_S := (k, \tau, (a_i, w_i)_{i \in S}), \text{Return} \text{SK}_S.$

**ABS.Sign** uses a supplementation algorithm **Supp** and a statement-generator algorithm **StmtGen**.

**Supp** runs for $j$, $1 \leq j \in \text{arity}(f)$, and generates keys $(a_{ij}, w_{ij})$ for $i_j \notin S$.

**Supp**(PK, SK$_S$, f):

For $j = 1$ to $\text{arity}(f)$:

If $i_j \notin S$, then

$m_{ij} := (\tau \parallel i_j), c_{ij} \leftarrow \text{PRF}_k(m_{ij} \parallel 0)$

$(a_{ij}, w_{ij}) \leftarrow \Sigma^\text{sim}(x_{\text{mst}}, c_{ij}; \text{PRF}_k(m_{ij} \parallel 1))$

$\text{Return} (a_{ij}, w_{ij})_{1 \leq j \leq \text{arity}(f)}$

**StmtGen** generates, for each $j$, $1 \leq j \in \text{arity}(f)$, a statement $x_{ij}$. Note that we employ here the algorithm $\Sigma^\text{stmtgen}$ which is associated with $\Sigma$, and whose existence is assured by our assumption (see Section 2.1).

**StmtGen**(PK, $\tau, (a_{ij})_{1 \leq j \leq \text{arity}(f)})$:

For $j = 1$ to $\text{arity}(f)$:

$m_{ij} := (\tau \parallel i_j), c_{ij} \leftarrow \text{Hash}_{\mu}(a_{ij} \parallel m_{ij})$

$x_{ij} \leftarrow \Sigma^\text{stmtgen}(x_{\text{mst}}, a_{ij}, c_{ij})$

$\text{Return} (x_{ij})_{1 \leq j \leq \text{arity}(f)}$

Note that $(x_i, w_i) \in R$ for $i \in S$ but $\text{Pr}[(x_i, w_i) \in R] = \text{neg}(\lambda)$ for $i \notin S$.

Then **ABS.Sign** is obtained by adding the following procedures.

**Supp**(PK, SK$_S$, f) $\rightarrow (a_{ij}, w_{ij})_{1 \leq j \leq \text{arity}(f)}$

$w := (w_{ij})_{1 \leq j \leq \text{arity}(f)}$

**StmtGen**(PK, $\tau, (a_{ij})_{1 \leq j \leq \text{arity}(f)})$

$\rightarrow (x_{ij})_{1 \leq j \leq \text{arity}(f)} := x$

The above procedures are needed to input a pair of statement and witness, $(x = (x_{ij})_{1 \leq j \leq \text{arity}(f)}, w = (w_{ij})_{1 \leq j \leq \text{arity}(f)})$, to $\Sigma^1_f$. Note here that $(x_{ij}, w_{ij}) \in R$ for any $i_j \in S$. On the other hand, $(x_{ij}, w_{ij}) \notin R$ for any $i_j \notin S$, without a negligible probability, $\text{neg}(\lambda)$. 

19
Hence the message on the first move has to include not only commitments (Cmt₁), but also a string τ and elements (aᵢⱼ)₁≤j≤arity(f) for the verifier V to be able to produce the same statement x.

Finally, ABS.Vrfy utilizes StmtGen and Σᵢᵥᵣ𝐟 to check validity of the pair of message and access formula, (m, f), and the signature σ, under the public key PK.

### ABS.Setup(1¹, U):
(xₘₐˢᵗ, wₘₐˢᵗ) ← Instanceₐ(1¹)
μ ← Hashkeys(λ)
PK := (xₘₐˢᵗ, U, μ), MSK := (wₘₐˢᵗ)
Return(PK, MSK)

### ABS.KG(PK, MSK, S):
k ← PRFkeys(λ), τ ← {1, 0}¹
For i ∈ S
mᵢ := (τ || i), aᵢ ← Σᵢ(xₘₐˢᵗ, wₘₐˢᵗ)
cᵢ ← Hashᵦ(aᵢ || mᵢ), wᵢ ← Σᵢ(xₘₐˢᵗ, wₘₐˢᵗ, aᵢ, cᵢ)
SKᵦ := (k, τ, (aᵢ, wᵢ)ᵢ∈S)
Return SKᵦ

### ABS.Sign(PK, SKᵦ, (m, f)):
Σᵢᵥᵣ𝐟(Tᵦ, S) → (v₀)n
If vᵦ(Tᵦ) ≠ 1, then abort
else CHₐᵦ(Tᵦ) := *

Supp(PK, SKᵦ, (m, f)) → (aᵢ, wᵢ)₁≤j≤arity(f)
w := wᵢ,₁≤j≤arity(f)
StmtGen(PK, τ, (aᵢ)₁≤j≤arity(f))
→ (xᵢ,₁≤j≤arity(f)) := x

Σᵢₗ(x, w, Tᵦ, (vᵦ)n, CHₐᵦ(Tᵦ))
→ ((Cmtᵦ)i,
(CHAᵦ)n,
(RESᵦ)ᵦ)

CHA ← Hashᵦ((aᵢ)₁≤j≤arity(f) || (Cmtᵦ)i || m)
CHₐᵦ(Tᵦ) := CHₐ

Σᵢₗ(x, w, Tᵦ, (vᵦ)n, CHₐᵦ(Tᵦ),
(Cmtᵦ)i,
(CHAᵦ)n,
(RESᵦ)ᵦ)
→ ((CHAᵦ)n,
(RESᵦ)ᵦ)

Return σ := (τ, (aᵢ)₁≤j≤arity(f),
(Cmtᵦ)i,
(CHAᵦ)n,
(RESᵦ)ᵦ)

### ABS.Vrfy(PK, (m, f), σ := (τ, (aᵢ)₁≤j≤arity(f),
(Cmtᵦ)i,
(CHAᵦ)n,
(RESᵦ)ᵦ))

Cha ← Hashᵦ((aᵢ)₁≤j≤arity(f) || (Cmtᵦ)i || m)

Σᵢᵥᵣ𝐟(xᵦ, Tᵦ,
(Cmtᵦ)i,
CHA, (CHAᵦ)n,
(RESᵦ)ᵦ)
→ b
Return b

#### Fig. 5. The scheme of our ABS.

### 5.2 Security of Our ABS

Applying the standard technique in the work of Abdalla et al. [1] shows that the security of our ABS is equivalent to the security of an attribute-based identification scheme, ABID, against passive attacks, where our ABID is obtained by combining our Σ-protocol Σᵢ with the signature
bundle scheme of the Fiat-Shamir signature $\text{FS}(\Sigma)$ in the same way as $\text{ABS}$ (See Appendix 4 for our $\text{ABID}$).

**Theorem 3 (Unforgeability)** Our attribute-based signature scheme $\text{ABS}$ is existentially unforgeable against chosen-message attacks in the random oracle model, based on the passive security of $\text{ABID}$. More precisely, let $q_H$ denote the maximum number of hash queries issued by a forger $F$ on $\text{ABS}$. Then, for any PPT algorithm $F$, there exists a PPT algorithm $B$ which satisfies the following inequality (neg$(\cdot)$ means a negligible function).

$$\text{Adv}_{F,\text{ABS}}^{\text{euf-cma}}(\lambda) \leq q_H \text{Adv}_{B,\text{ABID}}^{\text{pa}}(\lambda) + \text{neg}(\lambda).$$  

(3)

**Proof.** First, our $\text{ABS}$ is considered to be obtained by applying the Fiat-Shamir transform to our $\text{ABID}$. This is because, in the first message of our $\text{ABID}$, the tag $\tau$ and the elements $(a_i)_{1 \leq j \leq \text{arity}(f)}$ are fixed even when the 3-move protocol is repeated between the prover $P$ with a secret key $\text{SK}_P$ and the verifier $V$ with an access structure $f$. So $\text{ABS} = \text{FS}(\text{ABID})$.

As is discussed in Section 2.1, we can reduce the advantage $\text{Adv}_{F,\text{ABS}}^{\text{euf-cma}}(\lambda)$ to the advantage $\text{Adv}_{B,\text{ABID}}^{\text{pa}}(\lambda)$ of passive security of the underlying $\text{ABID}$, in the random oracle model, with a loss factor $q_H$. This is because $B$ can simulate key-extraction queries of $F$ perfectly with the aid of the key-generation oracle of $B$. □

**More on Reduction of Unforgeability** Let $q_H$ denote the maximum number of hash queries issued by a forger $F$ on $\text{ABS}$ and a forger $F'$ on $\text{FS}(\Sigma)$. Combining the inequality (3) with the inequalities (1) and (2) in Section 2.5 and Section 4, we obtain the following security reduction of advantages.

$$\text{Adv}_{F,\text{ABS}}^{\text{euf-cma}}(\lambda) \leq q_H^{3/2} (\text{Adv}_{S,\text{Grp}}^{\text{num.prob.}}(\lambda))^{1/4} + \text{neg}(\lambda).$$

**Attribute Privacy** Our $\text{ABS}$ does not have attribute privacy defined in Section 2.6 because of its linkability; that is, the constant components $\tau, (a_{ij})_j$ make two signatures linkable (especially, $\tau$ is a component of a private secret key $\text{SK}_S$). Hence, our $\text{ABS}$ merely has attribute privacy as a one-time signature scheme.

### 6 Conclusions

We proposed an attribute-based signature scheme $(\text{ABS})$ with short signatures and security proof in the random oracle model. First we provided a concrete procedure of the $\Sigma$-protocol which enables a prover to prove possession of witnesses that satisfy a statement of a monotone boolean formula. Next, combining the $\Sigma$-protocol with a signature bundle scheme of the Fiat-Shamir signature, we obtained a generic attribute-based identification scheme $(\text{ABID})$. Then we applied the Fiat-Shamir transform to our $\text{ABID}$ to obtain a scheme of $\text{ABS}$. The series of these generic constructions were obtained from a given single $\Sigma$-protocol. Finally, we provided our $\text{ABID}$ and $\text{ABS}$ schemes concretely in the Discrete-Logarithm setting and the RSA setting. These concretions are pairing-free.

Signatures of our $\text{ABS}$ are linkable, and attribute privacy holds only as a one-time signature. To construct an efficient $\text{ABS}$ with attribute privacy without pairings in the way of the boolean proof system is a challenging problem.
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References

A Instantiations

In this section, we provide two instantiations of our $\Sigma_f$, ABID and ABS; that is, in the discrete-logarithm setting and the RSA setting.

A.1 Discrete-Logarithm Setting

A prime of bit length $\lambda$ is denoted by $p$. A multiplicative cyclic group of order $p$ is denoted by $\mathbb{G}_p$. We fix a base $g \in \mathbb{G}_p$, $(g) = \mathbb{G}_p$. The ring of the exponent domain of $\mathbb{G}_p$, which consists of integers from 0 to $p-1$ with modulo $p$ operation, is denoted by $\mathbb{Z}_p$.

**ABS.Setup** takes as input $(1^\lambda, \mathcal{U})$. Let $R_\lambda := \{ (\beta, \alpha) \in \mathbb{G}_p \times \mathbb{Z}_p; \beta = g^\alpha \}$. Then $\text{Instance}_R(1^\lambda)$ chooses an element $(\beta, \alpha) \in R_\lambda$ at random. **ABS.Setup** outputs a public key and a master secret key: $\text{PK} = ((g, \beta), \mathcal{U}, \mu)$, $\text{MSK} = \alpha$. 
ABS.KG outputs SK$_S$ with signatures, for $i \in S$, $\sigma_i = (a_i = g^{r_i}, w_i = r_i + c_i \alpha)$. Here we use a key $k$ obtained by $k \leftarrow Hash_\mu(a || \tau)$, put $m_i = \tau || i$, and $r_i \in \mathbb{Z}_p$ is chosen at random according to a random tape: $PRF_k(m_i)$, and $c_i$ is obtained by $c_i \leftarrow Hash_\mu(a_i || m_i)$.

$\Sigma^{\text{stmtgen}}(\beta, a_i, c_i)$ is an algorithm that computes $x_i := a_i \beta^{c_i} \in \mathbb{G}_p$.

The rest of protocol is executed according to $\Sigma_f$ on input $(x, w)$ and with the following setting.

$$CMT_l = g^{r_l}, \text{RES}_l = r_l + \text{CHA}_l \ w_{\rho(l)},$$

Verification Equation : $g^{\text{RES}_l} \overset{?}{=} CMT_l \ (x_{\rho(l)})^{\text{CHA}_l}$.

### A.2 RSA Setting

An RSA modulus of bit length $\lambda$ is denoted by $N$. An RSA exponent of odd prime is denoted by $e$.

**ABS.Setup** takes as input $(1^\lambda, \mathcal{U})$. Let $R_\lambda := \{(\beta, \alpha) \in \mathbb{Z}_N \times \mathbb{Z}_N; \beta = \alpha^e\}$. Then Instance$_R(1^\lambda)$ chooses an element $(\beta, \alpha) \in R_\lambda$ at random. **ABS.Setup** outputs a public key and a master secret key: $PK = (((N, e, \beta), \mathcal{U}, \mu), \text{MSK} = \alpha)$.

**ABS.KG** outputs SK$_S$ with signatures, for $i \in S$, $\sigma = (a_i = r_i^e, w_i = r_i \alpha^{c_i})$. Here we use a key $k$ obtained by $k \leftarrow Hash_\mu(\alpha || \tau)$, put $m_i = \tau || i$, and $r_i \in \mathbb{Z}_N$ is chosen at random according to a random tape: $PRF_k(m_i)$, and $c_i$ is obtained by $c_i \leftarrow Hash_\mu(a_i || m_i)$. $\Sigma^{\text{stmtgen}}(\beta, a_i, c_i)$ is an algorithm that computes $x_i := a_i \beta^{c_i} \in \mathbb{Z}_N$.

The rest of protocol is executed according to $\Sigma_f$ on input $(x, w)$ and with the following setting.

$$CMT_l = r_l^e, \text{RES}_l = r_l \ (w_{\rho(l)})^{\text{CHA}_l},$$

Verification Equation : $\text{RES}_l^e \overset{?}{=} CMT_l \ (x_{\rho(l)})^{\text{CHA}_l}$.

### B Non-interactive Witness-Indistinguishable Proof of Knowledge [21]

A *non-interactive witness-indistinguishable proof of knowledge system* (NIWIPoK, for short) $\Pi = (K, \mathcal{P}, \mathcal{V})$ for a language $L_R$ is a protocol, where a PPT algorithm $K$, on input $1^\lambda$, outputs crs called a *common reference string*; a PPT algorithm $\mathcal{P}$, on input $(x, w) \in R$ and crs, outputs $\pi$ called a *proof*; and a PPT algorithm $\mathcal{V}$, on input $(x, \pi)$ and crs, outputs 1 (accept) or 0 (reject).

$\Pi$ must possess the following three properties.

**Completeness.** For any statement $x \in L_R$ and for any witness $w$ such that $(x, w) \in R$, $\mathcal{P}$ with the witness $w$ can make $\mathcal{V}$ accept on the statement $x$ with probability 1:

$$\Pr[\pi \leftarrow \mathcal{P}(x, w) : \mathcal{V}(x, \pi) = 1] = 1.$$

**Knowledge Soundness.** There are an algorithm $KE$ called a *knowledge extractor*, a function $\kappa : \{0, 1\}^* \rightarrow [1, 0]$ called a *knowledge error function* and a constant $c > 0$ that satisfy the following:

If there exists a PPT algorithm $A$ that satisfies $p(x) := \Pr[\text{crs} \leftarrow K(1^\lambda), \pi \leftarrow A(\text{crs}) : \mathcal{V}(x, \pi) = 1] > \kappa(x)$, then $KE(x)$, employing $A(x)$ as a subroutine that allows to be rewinded, outputs a witness $w$ which satisfies $(x, w) \in R$ within an expected number of steps bounded by: $|x|^c/(p(x) - \kappa(x))$. 


**Witness-Indistinguishability.** There is a polynomial-time algorithm \( S \) called a simulator, such that for any non-uniform polynomial-time algorithm \( \mathcal{A} \) we have

\[
\Pr[\text{crs} \leftarrow K(1^\lambda) : \mathcal{A}(\text{crs}) = 1] \approx \Pr[\text{crs} \leftarrow S(1^\lambda) : \mathcal{A}(\text{crs}) = 1]
\]

(computationally indistinguishable)

and for any unbounded algorithm \( \mathcal{A} \), we have

\[
\Pr[\text{crs} \leftarrow S(1^\lambda), (x, w_0, w_1) \leftarrow \mathcal{A}(\text{crs}), \pi \leftarrow \mathcal{P}(\text{crs}, x, w_0) : \mathcal{A}(\pi) = 1] = \Pr[\text{crs} \leftarrow S(1^\lambda), (x, w_0, w_1) \leftarrow \mathcal{A}(\text{crs}), \pi \leftarrow \mathcal{P}(\text{crs}, x, w_1) : \mathcal{A}(\pi) = 1]
\]

where \((R(x, w_0) = 1 \land R(x, w_1) = 1) \lor (R(x, w_0) = 0 \land R(x, w_1) = 0)\) holds.

### C Our Non-interactive Witness-Indistinguishable Proof of Knowledge System

The Fiat-Shamir transform \( FS(\cdot) \) can be applied to any \( \Sigma \)-protocol \( \Sigma ([16, 1], \text{see Section 2.1}) \) to obtain a NIZKPoK system. As a NIZKPoK system is a NIWIPoK system, we obtain a NIWIPoK system. Here the generator \( K \) of common reference strings is becomes as follows.

\[
K(1^\lambda) : \mu \leftarrow \text{Hashkeys}(\lambda), \text{crs} := \mu, \text{ Return crs}
\]

Hence we obtain the following theorem.

**Theorem 4** \( FS(\Sigma_f) \) is a non-interactive witness-indistinguishable proof of knowledge system for the language \( L_f \). A knowledge extractor can be constructed in the random oracle model.