

# The Locality of Searchable Symmetric Encryption

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## Abstract

This paper proves a lower bound on the trade-off between server storage size and the locality of memory accesses in searchable symmetric encryption (SSE). Namely, when encrypting an index of  $N$  identifier/keyword pairs, the encrypted index must have size  $\omega(N)$  *or* the scheme must perform searching with  $\omega(1)$  non-contiguous reads to memory *or* the scheme must read many more bits than is necessary to compute the results. Recent implementations have shown that non-locality of server memory accesses create a throughput-bottleneck on very large databases. Our lower bound shows that this is due to the security notion and not a defect of the constructions. An upper bound is also given in the form of a new SSE construction with an  $O(N \log N)$  size encrypted index that performs  $O(\log N)$  reads during a search.

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# 1 Introduction

Searchable symmetric encryption (SSE) [24, 15, 13] enables a client to encrypt an index of record/keyword pairs and later issue tokens allowing an untrusted server to retrieve the (identifiers of) all records matching a keyword. SSE aims to hide statistics about the index to the greatest extent possible while maintaining practical efficiency for large indexes like email repositories or personal document stores. These schemes employ only fast symmetric primitives and recent implementations [10] have shown that, in contrast to most applications of advanced cryptography, cryptographic processing like encryption is not the bottleneck for scaling. Instead, lower-level issues dealing with memory layouts required by the schemes are the limiting factor for large indexes.

This work studies how the security definitions for SSE inherently hamper scaling for large indexes. It proves an unconditional lower bound on the trade-off between server storage space and the *spatial locality* of its accesses to the encrypted index during a search. At a high level, the bound says that, for an index with  $N$  pairs, any secure SSE must either pad the encrypted index to an impractical (super-linear,  $\omega(N)$ ) size *or* perform searching in a very non-local way (with  $\omega(1)$  contiguous accesses or by reading far more bits than is necessary). Either of these options is likely to incur a large slow-down over a properly designed plaintext searching system with an  $O(N)$ -size index that can search with  $O(1)$  contiguous accesses.

The issue of locality in SSE surfaced in recent works [13, 10] where implementations showed that the non-local use of external storage was a bottleneck preventing scaling to large indexes. The only works with a highly local access pattern generated very large (roughly  $O(N^2)$ ) encrypted databases that also prevented scaling. This paper explains this dichotomy of padding versus spatial locality by proving it is an unavoidable consequence of the SSE security definition. As more cryptographic applications are developed for securely outsourcing large amounts of data (while maintaining either authenticity or secrecy), lower-level issues like locality may become more relevant. While in some contexts (like secure multiparty computation) it is clear that the entire input must be touched during computation, this work appears to be the first to study of the effect of security on locality in detail.

The lower bound suggests the question of a matching upper bound. We give a new scheme with an  $O(N \log N)$  size encrypted index and  $O(\log N)$  locality via a different padding strategy, which compares to a scheme with a  $O(N^2)$  size encrypted index and  $O(1)$  locality.<sup>1</sup> This scheme may not be competitive with prior highly-optimized implementations, but it serves as intermediate point in the trade-off curve implied by the lower bound. The interesting question of closing the gap is left open.

SSE AND LOCALITY. Let us describe the issue in more detail, starting with SSE and its security goals. An input to an SSE scheme, denoted  $\text{DB}$ , is an index (usually called an *inverted index* in information retrieval literature) associating with each keyword  $w$  a set of identifiers (bit strings)  $\text{DB}(w) = \{\text{id}_1, \dots, \text{id}_{t_w}\}$ , where the number  $t_w$  can vary. “Searching” means retrieving those identifiers, given  $w$ . An SSE scheme is a system for storing the index in encrypted form and delegating the ability to retrieving these sets while hiding statistics about the identifier sets matching un-searched keywords such as their number, size, the size of their intersections, and so on. Security is formally parameterized with a leakage function  $\mathcal{L}$  that describes an upper bound on what a server learns. One example of good leakage is  $N = \sum_w |\text{DB}(w)|$ , along with the identifier sets  $\text{DB}(w)$  for each keyword that is searched for. Other statistics like  $\max_w |\text{DB}(w)|$  and the number of unique keywords are also usually considered acceptable leakage. In any case, the defined leakage is *all* the server should learn, so plaintext keywords, identifiers, and anything else other than the output of  $\mathcal{L}$  must be hidden.

A scheme of Curtmola et al. [13] forms the basis for most subsequent SSE schemes. This scheme leaks only  $N$  and the number of unique keywords by placing all  $N$  of the identifier/keyword pairs in random order into a large array (along with some auxiliary tables) and enabling retrieval with encrypted linked lists that could be opened for the server. Performing an encrypted search for  $w$  requires walking

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<sup>1</sup>There are various ways to achieve a smaller index (see Figure 1), but these will achieve a slightly different notion of security.

| Scheme                  | Leakage        | EDB Size      | Locality        | Read Efficiency |
|-------------------------|----------------|---------------|-----------------|-----------------|
| CGKO'06-1 [13]          | $m, N$         | $O(N + m)$    | $O(t_w)$        | $O(1)$          |
| CGKO'06-2 [13]          | $M \cdot n$    | $O(Mn)$       | $O(t_w)$        | $O(1)$          |
| CK'10 [12]              | $m, n, M$      | $O(Mn)$       | $O(1)$          | $O(1)$          |
| LSDHJ'10 [25]           | $m, n$         | $O(mn)$       | $O(t_w)$        | $O(1)$          |
| KO'12 [20]              | $n, M$         | $O(Mn)$       | $O(t_w)$        | $O(1)$          |
| KPR'12 [19]             | $m, N$         | $O(N + m)$    | $O(t_w)$        | $O(1)$          |
| KP'13 [18]              | $m, n$         | $O(mn)$       | $O(t_w \log n)$ | $O(n \log n)$   |
| CJKRS'13 [10]           | $N$            | $O(N)$        | $O(t_w)$        | $O(1)$          |
| This paper: Scheme      | $N$            | $O(N \log N)$ | $O(\log N)$     | $O(1)$          |
| This paper: Lower Bound | Any from above | $\omega(N)$   | $O(1)$          | $O(1)$          |

**Figure 1:** Comparison of some SSE schemes. Legend: Leakage is leakage from EDB only, and all schemes also leak search results and access pattern.  $n$  = total # of unique identifiers,  $N = \sum_w |\text{DB}(w)|$ ,  $m$  = total # of unique keywords,  $M = \max_w |\text{DB}(w)|$ ,  $t_w = |\text{DB}(w)|$  for the query  $w$ . For [12] we mean the scheme in Section 5.2 of the full version there. The lower bound is achieved with  $\alpha = 0$  in Theorem 4.1. If a scheme support updates or more advanced searches then we consider a simplified static version for keyword searching as formalized in Section 2. Differences in security (simulation versus indistinguishability, adaptivity) are ignored here but explain why some schemes appear to be strictly worse than others.

through  $|\text{DB}(w)|$  pseudorandom locations in this large array. When the array is stored on disk, it means that each retrieved pair requires a disk read at a random location. Since each identifier is on the order of several bytes but the disk block size is now often 4KB, this searching will sacrifice throughput and latency compared to a plaintext search system which can store the identifiers together in sectors, and moreover in contiguous sectors which can be read together more efficiently without additional seeking. Naive modifications, like packing several identifiers from a single  $\text{DB}(w)$  set into one sector, render the scheme insecure for the leakage function they consider.

One work [12] addressed locality by enlarging the index to  $\omega(N)$  size. This scheme pads every set  $\text{DB}(w)$  to size  $\max_w |\text{DB}(w)|$  and then store these padded sets in the own contiguous rows. For very large indexes (with billions of pairs as in [10]), even doubling the plaintext size may be unreasonable, so these works do not appear to scale for realistic datasets where the padding will be large.

**RESULTS.** In the sections that follow a precise model is given for measuring and comparing the memory usage of SSE schemes. The parameters of locality, read-efficiency, and read overlaps are defined and discussed in relation to lower bounds. Briefly, *locality* is the number of non-contiguous memory accesses made by the server and *read-efficiency* is the number of bits read beyond the minimum necessary. In Figure 1 we compare the leakage, locality, and read efficiency of prior SSE constructions. (For read efficiency, the number listed is a multiplicative factor over the binary encoding of the identifiers matching the query.)

This paper’s primary results are summarized below. For the following two theorems, let  $\mathcal{L}$  be any of the leakage functions in Figure 1 (or any function efficiently computable from them). Below we write  $\text{BinEnc}(\text{DB})$  to mean an encoding on  $\text{DB}$  as a binary string formed by concatenating the lists of identifiers matching each keyword. Moreover, we say that a scheme has  $\alpha$ -overlapping reads if reads for a new search overlaps in at most  $\alpha$  bits with all previous reads. See Sections 2 and 3 for definitions.

**Theorem 1.1** *If  $\Pi$  is an  $\mathcal{L}$ -IND-secure SSE scheme with locality  $r$  as well as  $\alpha$ -overlapping reads, then  $\Pi$  has  $\omega\left(\frac{|\text{BinEnc}(\text{DB})|}{r \cdot (\alpha+1)}\right)$  server storage.*

**Theorem 1.2** *Assuming one-way functions exist, there exists an  $\mathcal{L}$ -IND-secure SSE scheme with locality  $O(\log N)$ ,  $O(1)$  read efficiency and  $O(N \log N)$  storage.*

We remark that the  $\alpha$ -overlapping read condition in the first theorem allows multiple queries to read a shared portion of the encrypted database of length independent of the actual result set. We remark that a very weak read efficiency requirement is implicit in the condition on overlapping reads, and all existing schemes (including the one in the proof of the second theorem) have highly non-overlapping

reads. We conjecture that information-theoretic techniques can be used to obtain a quantitatively similar lower bound when allowing arbitrary overlaps, but were unable to prove such a strong result. We leave this as an open question.

The bulk of the paper is spent proving the first theorem. We now start with an intuitive sketch of how one might prove a weak lower bound. See Section 4 for a detailed sketch of the actual proof, which is more complicated.

**LOWER BOUND APPROACH.** Intuitively, if a scheme is very local, then after some searching the server can look at what is *not* read after several searches and infer statistics about what has not been opened. In particular, if one of the sets  $\text{DB}(w)$  is very large, then good locality means there is a very large region of the encrypted index that will not be touched by other searches, and the server will notice that this happens after several searches with small number of results.

The lower bound develops this intuition, but requires further ideas to achieve a lower bound of  $\omega(N)$  on the server storage. For now let us sketch how one shows the server must approximately double the size of a plaintext index if it is to be *perfectly* local and read-efficient, meaning it processes a search by reading exactly the required number of bits from a single contiguous section of EDB, and moreover that the reads for all searches are disjoint. This seems highly restrictive, but later will we be able to weaken all of these assumptions to realistic versions.

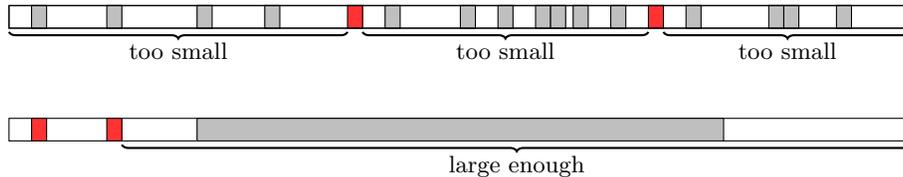
Now suppose we have a perfectly local SSE scheme. Consider two index inputs,  $\text{DB}_0$  and  $\text{DB}_1$ , where  $\text{DB}_0$  consists of  $N$  keywords each matching a single unique document and  $\text{DB}_1$  consists of 2 keywords matching a single unique document and a third keyword matching  $N - 2$  documents. If two random keywords matching single documents are searched for then the server learns which locations of the encrypted index are read in order to respond. If  $\text{DB}_0$  was encrypted, then pigeon-hole argument shows that with constant probability, there is no remaining contiguous interval large enough to contain the bits that would be read for the third keyword (it is here that we use an assumed bound on the server storage). This is diagrammed in the top part of Figure 2, where when the red regions are read there is no longer space between them for a larger interval. This is in contrast to the case when  $\text{DB}_1$  is encrypted, because after observing the two small reads, a perfectly local scheme there will always be a contiguous unread region large enough to hold the  $N - 2$  identifiers for the third keyword.

The full lower bound is an extension of this idea to consider a family of indexes with result sets of several sizes. Later it is argued that the technique above is limited to showing a factor 2 overhead in server storage, and that the complexity of the main attack seems necessary. We also address several extensions, such as when the server does not perform a single contiguous read but up to  $O(1)$  reads, the leakage function parameter varies, and the reads are allowed to partially overlap.

**RELATED WORK ON SECURE SEARCHING.** Following the initial work of [24] that suggested searchable encryption, Curtmola et al. [13] formalized the version of SSE that we consider in this paper. Subsequently SSE schemes were given with different efficiency properties [15, 11, 12], support for data updates [19, 18], authenticity [20] and support more advanced searches [10]. These improvements are rthogonal to the lower bound, which applies to these schemes when used for basic (non-dynamic, non-authenticated) SSE.

The problem of searching on encrypted data can be addressed in several ways using generic multi-party computation protocols, oblivious RAM schemes [16] or fully homomorphic encryption [14]. These approaches achieve slightly levels of functionality and different notions of security, meaning that the lower bound does not seem to apply. Order-preserving encryption [6, 7] takes a different approach to searching that achieves high efficiency for rich queries but is less secure than SSE. Implementations that use order-preserving encryption, notably CryptDB [22], inherit these properties. Our lower bound does not apply to them.

There is also a line of work on searching on *public-key* ciphertexts. Public-key encryption with keyword search [9, 17, 1, 2] In these schemes and subsequent work, the server performing the search by testing each encrypted record individually, resulting in a scheme that is trivial from the point of view of



**Figure 2:** Intuition for a basic lower bound.

the lower bound. The line of work on deterministic public-key encryption [4, 8, 5] enables fast searching but achieves different, weaker security meaning our lower bound does not apply.

**RELATED WORK ON LOCALITY.** Algorithmic performance with data stored on disk has been studied extensively in *external memory models* (c.f. [26, 23, 3]). These models usually consider block-oriented devices with varying degrees of precision (e.g., including modeling parallelism, drive geometry, memory hierarchies, caching, locality of blocks, etc.). Typically one measures the external memory efficiency of an algorithm by counting the number of blocks it accesses, and a wide array of techniques have been developed to optimize disk utilization at the algorithmic level.

Interestingly, matching lower and upper bounds are known for many natural problems like, e.g., dictionary retrieval, sorting, range searching – see, e.g., Chapter 6 of [26]. Our lower bound is fundamentally different from these results. There, one can give an information theoretic argument that a certain number of disk accesses are necessary in the worst case, with a flavor similar to the classic  $O(n \log n)$  comparison-based sorting lower bound. Our lower bound, however, will proceed by showing that any SSE scheme that meets a certain level of efficiency will be *insecure* (rather than *incorrect* as in traditional external memory lower bounds). That is, our lower bound comes in the form of an attack. Due to the nature of our lower bound we opt for an extremely simplified version of locality and leave its adaptation to fine-grained external memory models to future work.

We are not aware of any prior similar lower bounds on cryptographic primitives, other than the folklore observation that security forces many primitives to touch every bit their inputs (e.g., homomorphic encryption [14], multiparty computation).

**ORGANIZATION.** Preliminaries and definitions are recalled in Section 2. Definitions relating to locality are given and discussed in Section 3. The lower bound is stated and proved in Section 4, and the upper bound is in Section 5.

## 2 Preliminaries

Throughout this paper the security parameter is denoted  $\lambda$  and all algorithms (and adversaries) are assumed to run in time polynomial in  $\lambda$ . We write  $[n]$  for the set  $\{1, \dots, n\}$ . For a vector  $\mathbf{v}$  we write  $|\mathbf{v}|$  for the dimension (length) of  $\mathbf{v}$  and for  $i \in [|\mathbf{v}|]$  we write  $\mathbf{v}[i]$  for the  $i$ -th component of  $\mathbf{v}$ . For a bitstring  $s$ , we write  $s[a, b]$  for the substring starting with the bit in position  $a$  and ending in position  $b$ . For integers  $a \leq b$  we write  $[a, b]$  for  $\{a, a + 1, \dots, b\}$  and  $(a, b)$  for  $\{a + 1, a + 2, \dots, b - 1\}$ .

**DATABASES AND SSE SCHEMES.** An index (or database)  $\text{DB} = (\text{id}_i, \mathbf{W}_i)_{i=1}^n$  is a list of identifier/keyword-set pairs, where each  $\text{id}_i \in \{0, 1\}^\lambda$  and each  $\mathbf{W}_i$  is a set of bitstrings. When the DB under consideration is clear, we will write  $\mathbf{W} = \bigcup_{i=1}^n \mathbf{W}_i$ . For a keyword  $w \in \mathbf{W}$ , we write  $\text{DB}(w)$  for  $\{\text{id}_i : w \in \mathbf{W}_i\}$ . We will always use  $N = \sum_{w \in \mathbf{W}} |\text{DB}(w)| = \sum_{i=1}^n |\mathbf{W}_i|$  to mean the total number of keyword/identifier pairs in DB,  $n$  to mean the number of unique identifiers, and  $m = |\mathbf{W}|$  to mean the number of unique keywords. We will also denote as  $\text{BinEnc}(\text{DB}(w))$  the binary *representation* of  $\text{DB}(w)$ , i.e., the concatenation of all identifiers represented as bit strings, and  $\text{BinEnc}(\text{DB})$  to be the concatenation of all the  $\text{BinEnc}(\text{DB}(w))$  for each  $w$  in the database. Under our assumption that all identifiers are in  $\{0, 1\}^\lambda$ , we have  $|\text{BinEnc}(\text{DB}(w))| = \lambda |\text{DB}(w)|$  and  $|\text{BinEnc}(\text{DB})| = \lambda N$ .

A *searchable symmetric encryption (SSE) scheme*  $\Pi$  consists of algorithms  $(\text{KeyGen}, \text{EDBSetup},$

TokGen, Search) that satisfy the following syntax. The key generation algorithm **KeyGen** takes as input the security parameter and outputs a key  $K$ . The algorithm **EDBSetup** takes as input a key  $K$  and a database  $\text{DB}$  and outputs an encrypted database  $\text{EDB}$ . The token generation protocol takes as input a string  $w$  and key  $K$  and outputs a token  $\tau$ . Finally, the searching algorithm **Search** takes as input  $\tau$  and  $\text{EDB}$  and outputs as set  $L$  of results.

We note that formalization of an SSE scheme does not model the storage of actual document payloads, but only of metadata encoded in a keyword index. This simplifies the definition and makes it modular, but some care must be taken when combining an SSE scheme with a document storage scheme (see e.g. [13] for an example of how to store the payloads).

An SSE scheme is *correct* if the natural usage returns the correct results for the keyword being searched (i.e.,  $\text{DB}(w)$ ), except with negligible probability. Formally, for every database  $\text{DB}$ , consider an experiment where  $K \stackrel{\$}{\leftarrow} \text{KeyGen}(1^\lambda)$  and  $\text{EDB} \stackrel{\$}{\leftarrow} \text{EDBSetup}(K, \text{DB})$  are initially sampled. Then, an attacker learns  $\text{EDB}$  and can issue adaptive queries  $w_i$ , which are answered by first generating a token  $\tau_i \leftarrow \text{TokGen}(K, w_i)$  and then returning it together with  $S_i \leftarrow \text{Search}(\tau_i, \text{EDB})$  to the attacker. The scheme is correct if for all polynomial-time attackers,  $S_i = \text{DB}(w_i)$  for all  $i$ , except with negligible probability.

Finally, we say that the scheme  $\Pi$  has *server storage*  $s = s(N, \lambda, \ell, m)$  if on input a key  $K \stackrel{\$}{\leftarrow} \text{KeyGen}(1^\lambda)$  and a database  $\text{DB}$  with  $N$  keyword/identifier pairs, bit length  $\ell = |\text{BinEnc}(\text{DB})|$ , and  $m = |\mathbf{W}|$  keywords, **EDBSetup** outputs  $\text{EDB}$  with bit length  $|\text{EDB}|$  at most  $s(N, \lambda, \ell, m)$ . (Note that by the above discussion we typically have  $\ell = N\lambda$ , and  $\ell$  is hence somewhat redundant, and only serves the purpose of better readability.)

**SECURITY.** We recall the non-adaptive indistinguishability-based version of security from [13] which will be considered in the lower bound.

**Definition 2.1** Let  $\Pi = (\text{KeyGen}, \text{EDBSetup}, \text{TokGen}, \text{Search})$  be an SSE scheme and let  $\mathcal{L}$  be a leakage function and  $\mathcal{A}$  be an adversary. For  $b \in \{0, 1\}$  we define the game  $\text{IND-SSE}_{\Pi, \mathcal{L}, \mathcal{A}}^b(\lambda)$  as follows: The adversary chooses  $\text{DB}_0, \text{DB}_1, \mathbf{w}$ . The game runs  $K \stackrel{\$}{\leftarrow} \text{KeyGen}(1^\lambda)$ ,  $\text{EDB} \stackrel{\$}{\leftarrow} \text{EDBSetup}(K, \text{DB}_b)$  and  $\mathbf{t}[i] \leftarrow \text{TokGen}(K, \mathbf{w}[i])$  for each  $i \in [|\mathbf{w}|]$ . It gives  $(\text{EDB}, \mathbf{t})$  to  $\mathcal{A}$ , which outputs a bit  $b$ . Finally, if  $\mathcal{L}(\text{DB}_0, \mathbf{w}) \neq \mathcal{L}(\text{DB}_1, \mathbf{w})$ , the game outputs  $\perp$  and otherwise it outputs  $\hat{b}$ .

We define the  $\mathcal{L}$ -IND advantage of  $\mathcal{A}$  to be

$$\mathbf{Adv}_{\Pi, \mathcal{L}, \mathcal{A}}^{\text{ind-sse}}(\lambda) = |\Pr[\text{IND-SSE}_{\Pi, \mathcal{L}, \mathcal{A}}^0(\lambda) = 1] - \Pr[\text{IND-SSE}_{\Pi, \mathcal{L}, \mathcal{A}}^1(\lambda) = 1]|,$$

and we say that  $\Pi$  is  $\mathcal{L}$ -IND-secure if  $\mathbf{Adv}_{\Pi, \mathcal{L}, \mathcal{A}}^{\text{ind-sse}}(\lambda)$  is negligible for every  $\mathcal{A}$ .

Our construction will achieve the stronger (adaptive, simulation-based) definition from [13], which we recall here. (A non-adaptive version is such that in both games  $\mathcal{A}$  must choose all of its queries beforehand.)

**Definition 2.2** Let  $\Pi = (\text{KeyGen}, \text{EDBSetup}, \text{TokGen}, \text{Search})$  be an SSE scheme and let  $\mathcal{L}$  be a leakage function. For algorithms  $\mathcal{A}$  and  $\mathcal{S}$ , we define the two games  $\text{SIM-SSE}_{\Pi, \mathcal{L}, \mathcal{A}}^0(\lambda)$  and  $\text{SIM-SSE}_{\Pi, \mathcal{L}, \mathcal{A}, \mathcal{S}}^1(\lambda)$  as follows:

$\text{SIM-SSE}_{\Pi, \mathcal{L}, \mathcal{A}}^0(\lambda)$ :  $\mathcal{A}(1^\lambda)$  chooses  $\text{DB}, \mathbf{w}$ . The game then runs  $(K, \text{EDB}) \leftarrow \text{EDBSetup}(\text{DB})$  and  $\mathbf{t}[i] \leftarrow \text{TokGen}(K, \mathbf{w}[i])$  for each  $i \in [|\mathbf{w}|]$ . It gives  $\text{EDB}, \mathbf{t}$  to  $\mathcal{A}$ , which eventually returns a bit that the game uses as its own output.

$\text{SIM-SSE}_{\Pi, \mathcal{L}, \mathcal{A}, \mathcal{S}}^1(\lambda)$ :  $\mathcal{A}(1^\lambda)$  chooses  $\text{DB}, \mathbf{w}$ . The game then runs  $(\text{EDB}, \mathbf{t}) \leftarrow \mathcal{S}(\mathcal{L}(\text{DB}, \mathbf{w}))$  and gives  $\text{EDB}, \mathbf{t}$  to  $\mathcal{A}$ , which eventually returns a bit that the game uses as its own output.

We define the  $\mathcal{L}$ -SIM-advantage of  $\mathcal{A}$  and  $\mathcal{S}$  to be

$$\mathbf{Adv}_{\Pi, \mathcal{L}, \mathcal{A}, \mathcal{S}}^{\text{sim-sse}}(\lambda) = |\Pr[\text{SIM-SSE}_{\Pi, \mathcal{L}, \mathcal{A}}^0(\lambda) = 1] - \Pr[\text{SIM-SSE}_{\Pi, \mathcal{L}, \mathcal{A}, \mathcal{S}}^1(\lambda) = 1]|,$$

and we say that  $\Pi$  is  $\mathcal{L}$ -SIM-secure if for all adversaries  $\mathcal{A}$  there exists an algorithm  $\mathcal{S}$  such that  $\mathbf{Adv}_{\Pi, \mathcal{L}, \mathcal{A}, \mathcal{S}}^{\text{sim-sse}}(\lambda)$  is negligible.

LEAKAGE FUNCTIONS. Below we will consider two leakage functions  $\mathcal{L}_{\min}$  and  $\mathcal{L}_{\max}$ . The first is called the *size minimal leakage function*,<sup>2</sup> which is defined as follows:  $\mathcal{L}_{\min}(\text{DB}, \mathbf{w})$  outputs  $N = \sum_{w \in \mathcal{W}} |\text{DB}(w)|$  and the sets  $(\text{DB}(\mathbf{w}[1]), \dots, \text{DB}(\mathbf{w}[|\mathbf{w}|]))$ . The second is called the *maximal leakage function* which outputs  $(N, n, m, M)$  as well as  $(\text{DB}(\mathbf{w}[1]), \dots, \text{DB}(\mathbf{w}[|\mathbf{w}|]))$ , where  $N$  is defined as before,  $n$  is the number of unique identifiers in  $\text{DB}$ ,  $m = |\mathcal{W}|$  (the number of unique keywords), and  $M = \max_w |\text{DB}(w)|$ . It is of course possible to consider “more” leakage, but this is more than any existing scheme leaks, meaning our lower bound will apply to all of them.

### 3 Read Efficiency and Locality Metrics for SSE Schemes

This section introduces the notions of locality and read efficiency of SSE schemes.

READ PATTERNS. First, we observe that the searching procedure of any SSE scheme can be decomposed into a sequence of *contiguous* reads from the encrypted database. To formalize this point of view, fix an SSE scheme  $\Pi$ , an EDB output by  $\text{EDBSetup}$  and a token  $\tau$  output by  $\text{TokGen}$ . Viewing EDB as a bitstring of length  $M$ , we may express the computation of  $\text{Search}(\tau, \text{EDB})$  as follows: It starts by computing an interval  $[a_1, b_1]$  that depends only on  $\tau$ . It then computes another interval  $[a_2, b_2]$  that depends only on  $\tau$  and  $\text{EDB}[a_1, b_1]$ , and continues computing intervals to read based on  $\tau$  and all previously read intervals from EDB. We write  $\text{RdPat}(\tau, \text{EDB})$  for these intervals.

LOCALITY OF AN SSE SCHEME. We put forward the notion of locality of an SSE scheme, capturing the fact that every read pattern consists of at most a bounded number of intervals.

**Definition 3.1 (Locality)** *An SSE scheme  $\Pi$  is  $r$ -local (or has locality  $r$ ) if for any  $\lambda$ ,  $\text{DB}$ , and  $w \in \mathcal{W}$ , we have that  $\text{RdPat}(\tau, \text{EDB})$  consists of at most  $r$  intervals with probability 1 when  $\text{EDB}, \tau$  are computed as  $K \xleftarrow{\$} \text{KeyGen}(1^\lambda)$ ,  $\text{EDB} \xleftarrow{\$} \text{EDBSetup}(K, \text{DB})$ ,  $\tau \leftarrow \text{TokGen}(K, w)$ . If  $r = 1$ , we say  $\Pi$  has perfect locality.*

In particular, the value  $r$  can depend both on the security parameter  $\lambda$  and the index size  $|\text{DB}|$ .

READ EFFICIENCY. The notion of locality alone is not very meaningful. Of course, we can just make every scheme perfectly local by reading the whole EDB. This is why the notion of locality is directly tied to the notion of *read efficiency*, which measures the overall size of the portion read by a search operation.

**Definition 3.2 (Read Efficiency)** *An SSE scheme  $\Pi$  is  $c$ -read efficient (or has read efficiency  $c$ ) if for any  $\lambda$ ,  $\text{DB}$  and  $w \in \mathcal{W}$ , we have that  $\text{RdPat}(\tau, \text{EDB})$  consists of intervals of total length at most  $c \cdot |\text{BinEnc}(\text{DB}(w))|$  bits.*

We allow  $c$  to depend on the security parameter here.

READ DISJOINTNESS. The above definition of read efficiency is very general. In particular, for sufficiently large  $c$ , it allows multiple queries to read *exactly* the same bits. Our lower bound below will apply to a more restricted class of  $r$ -local schemes which read sufficiently many new bits. We feel this class is natural, and moreover it contains all prior constructions.

**Definition 3.3 (Overlapping reads)** *An SSE scheme  $\Pi$  has  $\alpha$ -overlapping reads if for all  $\lambda$  and all  $\text{DB}$ , the read pattern induced by the search of each keyword in  $\text{DB}$  has an overlap of at most  $\alpha$  bits with the read patterns induced by the searches of all other (previous or future) keywords (with probability 1 over the computation of  $K \leftarrow \text{KeyGen}(1^\lambda)$ ,  $\text{EDB} \leftarrow \text{EDBSetup}(K, \text{DB})$ , and the computation of the tokens). When  $\alpha = 0$  we say  $\Pi$  has disjoint reads.*

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<sup>2</sup>It appears to be impossible to define a true “minimal” amount of leakage, as we could consider a leakage function that leaks only some upper bound on  $N$ .

In general, the value  $\alpha$  is independent of  $N$ , but may additionally depend on  $\lambda$  or possibly on the number of words  $|W|$  in order for example to take into account a common portion of EDB which can be read at every search operation. Typically, a scheme will read some metadata like hash table entries and then perform reads to retrieve the actual results. (Of course we make no assumption on what computation the scheme actually does, beyond being of the form above.)

## 4 Lower Bounds

### 4.1 Theorem Statement and Proof Overview

In this section we sketch our proof that a secure SSE scheme cannot simultaneously achieve  $O(1)$  locality and  $O(|\text{BinEnc}(\text{DB})|)$  server storage. Concretely, we are going to prove the following theorem, where  $\mathcal{L}_{\max}$  was defined at the end of Section 2 and the locality metrics were defined in the previous section.

**Theorem 4.1** *If  $\Pi$  is an  $\mathcal{L}_{\max}$ -IND-secure SSE scheme with locality  $r$  as well as  $\alpha$ -overlapping reads, then  $\Pi$  has  $\omega\left(\frac{|\text{BinEnc}(\text{DB})|}{r \cdot (\alpha+1)}\right)$  server storage.*

We note that we consider  $\mathcal{L}_{\max}$  for the lower bound as this strengthens the result by considering schemes that are “very leaky.” In the theorem statement, we assume that  $\alpha$  does not depend on  $N$ , but may additionally depend on  $\lambda$  or possibly on the number of words  $|W|$ .

PROOF APPROACH. We will first sketch our lower bound with a few simplifications. First, we assume the SSE scheme is has perfect locality and read efficiency, meaning the server always performs exactly one contiguous read for exactly  $|\text{BinEnc}(\text{DB}(w))| = \lambda \cdot |\text{DB}(w)|$  bits from EDB when searching for a word  $w$ , the minimum required for the response. Second, we assume all reads are perfectly disjoint. Third, we consider the lower bound against SSE schemes achieving security with leakage function  $\mathcal{L}_{\min}$  instead of  $\mathcal{L}_{\max}$  (thus making the result easier). It turns out that this case encompasses most of the technical difficulties for the general result, which we derive afterwards.

The principle behind the attack extends the idea sketched in Section 1: When an SSE scheme fits the above assumptions, a curious server can observe the sizes of *unread* intervals after several searches and (as we will prove) infer some information about the parts of the index that have not been searched for.

We will formalize this by giving an adversary that distinguishes the two games in Definition 2.1 for any  $\Pi$  meets the restrictions. The adversary will choose two indexes  $\text{DB}_0, \text{DB}_1$  of the same size in a careful way so that  $\text{DB}_1$  has keywords that match a large number of documents while  $\text{DB}_0$  does not. Then it will query for tokens for several keywords matching relatively small numbers of documents. Using the tokens, it will compute the read pattern of the server when searching for those keywords, and then look at the *unread* portions of EDB. Since we are assuming perfect locality, if  $\text{DB}_1$  was encrypted, there must be large regions that go untouched by any query. On the other hand, we will show that this is often not the case if  $\text{DB}_0$  is encrypted, allowing the adversary to distinguish.

To describe the proof it will be useful to introduce a compact notation for the *shape* of a DB input.

**Definition 4.2** *We write*

$$\text{DB} \leftarrow (n_1 \times s_1; n_2 \times s_2; \dots; n_t \times s_t)$$

when  $\text{DB} = (\text{id}_i, W_i)$  has shape  $(n_1 \times s_1; n_2 \times s_2; \dots; n_t \times s_t)$ , which means that it satisfies the following:

- $\text{DB}$  has a keyword set  $W$  of size  $\sum_{j=1}^t n_j$  comprised of  $\lambda$ -bit strings.
- For each  $j \in [t]$ , there are  $n_j$  keywords  $w \in W$  such that  $|\text{DB}(w)| = s_j$ .
- For all  $w \neq w'$ , the sets  $\text{DB}(w)$  and  $\text{DB}(w')$  are disjoint.

Our attack sketched in the introduction corresponded to picking indexes  $\text{DB}_0, \text{DB}_1$  with shapes  $\text{DB}_0 \leftarrow (N \times 1)$  and  $\text{DB}_1 \leftarrow (2 \times 1 ; 1 \times (N - 2))$ , meaning that  $\text{DB}_0$  consists of  $N$  “singletons” and  $\text{DB}_1$  consists of two singletons and one large set of results. It is possible to formalize that attack and show that a secure perfectly local SSE scheme must produce an EDB that is at least twice as large as the bit representation of  $\text{DB}$ , but as we observe at the end of this section, any attack that uses indexes with such simple shapes will not be able to prove a better lower bound.

We now proceed to extend that attack. Let  $\Pi$  be perfectly local scheme with server storage  $k\lambda N$  for some constant  $k \geq 1$ . Our attack against the security  $\Pi$  will select two random inputs  $\text{DB}_0, \text{DB}_1$  with shapes

$$\text{DB}_0 \leftarrow (n_1 \times \varepsilon_1 N; n_2 \times \varepsilon_2 N; \dots ; n_{k-1} \times \varepsilon_{k-1} N; \hat{n}_k \times \varepsilon_k N) \quad (1)$$

$$\text{DB}_1 \leftarrow (n_1 \times \varepsilon_1 N; n_2 \times \varepsilon_2 N; \dots ; n_{k-1} \times \varepsilon_{k-1} N; n_k \times \varepsilon_k N; \hat{n}_{k+1} \times \varepsilon_{k+1} N) \quad (2)$$

where  $n_1 > n_2 > \dots > n_k > 1$  and  $\varepsilon_1 < \varepsilon_2 < \dots < \varepsilon_k < \varepsilon_{k+1} < 1$  are appropriately chosen constants. Intuitively,  $\text{DB}_0$  consists only of many small result sets  $\text{DB}_0(w)$  while  $\text{DB}_1$  consists of many small result sets and some relatively large sets of  $\varepsilon_{k+1}N$  keywords.

The attack will query for tokens for all  $\sum_{i=1}^k n_i$  keywords matching  $\varepsilon_i N$  documents with  $i \in [k]$  (in order to not lose the game, the attack ensures that  $\text{DB}_0$  and  $\text{DB}_1$  are consistent on these queries). It then calculates the read patterns of the server. Since  $\Pi$  is perfectly local, the reads are for  $\varepsilon_i \lambda N$  bit intervals respectively, and moreover they are all disjoint. Thus if  $\text{DB}_1$  is encrypted, then the perfect locality of  $\Pi$  means there must exist an interval of  $\varepsilon_{k+1} \lambda N$  bits in EDB that was not touched by the observed reads (actually, there will be at least  $\hat{n}_{k+1}$  such intervals) – These are where the large result sets are stored (note that the adversary does not query for the keyword corresponding to any of large sets, but only notices the presence of a suspiciously large untouched interval). However, as we will show, if  $\text{DB}_0$  was encrypted then the security of  $\Pi$  will mean there is a noticeable probability that there is no such interval remaining untouched. We stress that this will be due to the (forced) distribution of the reads, and not simply because there is no room, as the same number of bits is read when either  $\text{DB}_0$  or  $\text{DB}_1$  is encrypted.

Proving the latter claim on  $\text{DB}_0$  is the main technical part of the proof. Intuitively, it holds because security forces the small intervals (say of size  $\varepsilon_i \lambda N$ ) to be located in random-looking locations which do not leave large gaps (say of size  $\varepsilon_{i+1} \lambda N$ ) between them too often, which implies that large intervals cannot fit between them while remaining disjoint, effectively “killing” that space for large intervals.

We will show that, for a specific choice of the constants, that for each  $i = 1, \dots, k$ , the queried sets of size  $\varepsilon_i \lambda N$  will each kill at least  $\lambda N$  bits of the EDB from storing any larger intervals (in particular, intervals of size  $\varepsilon_j \lambda N$  for  $j > i$ ) with constant probability. Since we have  $k$  different read pattern sets each killing  $\lambda N$  bits from storing anything larger with constant probability, we get that  $k\lambda N$  bits are killed with constant probability. But this means the entire EDB has been killed with constant probability (here we use that  $k$  is a constant), and when that happens the adversary can conclude that  $\text{DB}_0$  was encrypted – If  $\text{DB}_1$  had been encrypted this would happen with probability 0.

We now sketch how to show that queries for a constant number of sets of size  $\varepsilon_i N$  will kill  $\lambda N$  bits (which is much larger than the actual number of bits –  $\varepsilon_i \lambda N$  – read by the server during its perfectly-local searching). To prove this we use hybrid argument, which can be understood as a sequence of adversaries  $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_k$  - the purpose of  $\mathcal{A}_i$  is to show that the sets of size  $\varepsilon_i N$  kill enough space with constant probability, assuming that the smaller sets each do so. The first adversary  $\mathcal{A}_1$  is the simplest to describe, and resembles our original attack from the Introduction. Adversary  $\mathcal{A}_1$  draws  $\text{DB}_0, \text{DB}_1$  of size  $N$  with shapes

$$\text{DB}_0 \leftarrow (\hat{n}_1 \times \varepsilon_1 N) \quad (3)$$

$$\text{DB}_1 \leftarrow (n_1 \times \varepsilon_1 N; \hat{n}_2 \times \varepsilon_2 N), \quad (4)$$

where  $n_1 < \hat{n}_1 = \varepsilon_1^{-1}$  and  $\varepsilon_1 < \varepsilon_2$  are constants. It populates the two databases with a consistent set of keywords, meaning that the  $n_1$  keywords matching  $\varepsilon_1 N$  documents  $\text{DB}_1$  are a random subset of the  $\hat{n}_1$  such keywords in  $\text{DB}_0$ . Intuitively,  $\text{DB}_0$  has a large number of keywords matching  $\varepsilon_1 N$  documents each, and searching for each keyword induces a read by the server for a disjoint interval of  $\lambda \varepsilon_1 N$  bits. Thus searching for a random subset of  $n_1 < \hat{n}_1$  of those keywords will reveal the location of a random subset of the disjoint intervals. In  $\text{DB}_1$ , however, there are only  $n_1$  of these keywords, but we can show that security forces their distributions to be as they are in  $\text{DB}_0$ .

Specifically, we have  $\mathcal{A}_1$  query for the  $n_1$  keywords matching  $\varepsilon_1 N$  documents in either database, and then it computes their read patterns. If  $\text{DB}_0$  was encrypted, then the intervals read by the server are chosen randomly from amongst  $\hat{n}_1$  intervals of that size. We show (unconditionally) with good probability there is a lot of space (about  $\lambda N$  bits) in EDB where intervals of size  $\varepsilon_2 N$  or larger cannot fit after the  $n_1$  intervals have been read. This happens because, for randomly chosen intervals, the gap between them cannot be larger than  $\varepsilon_2 N$  too often. Thus the larger intervals must go elsewhere in EDB. And since the scheme is secure,  $\text{DB}_1$  must also exhibit this behavior (despite the read intervals not being chosen from a larger set of intervals). In fact, this shows that when any database contains  $n_1$  keywords with  $\varepsilon N$  results each, then the resulting reads for those keywords must be laid out in a way that eliminates a large amount of space for larger intervals even though the actual bits read for them is very small, namely  $n_1 \varepsilon_1 \lambda N \ll \hat{n}_1 \varepsilon_1 \lambda N$ .

We then iterate this approach; The next adversary  $\mathcal{A}_2$  queries  $\text{DB}_0, \text{DB}_1$  with shapes

$$\text{DB}_0 \leftarrow (n_1 \times \varepsilon_1 N; \hat{n}_2 \times \varepsilon_2 N) \tag{5}$$

$$\text{DB}_1 \leftarrow (n_1 \times \varepsilon_1 N; n_2 \times \varepsilon_2 N; \hat{n}_3 \times \varepsilon_3 N). \tag{6}$$

(So  $\text{DB}_0$  now has the shape that  $\mathcal{A}_1$  chose for  $\text{DB}_1$ .) The adversary  $\mathcal{A}_2$  then queries for tokens for all  $n_1$  keywords matching  $\varepsilon_1 N$  documents, and then a random subset of the  $\hat{n}_2$  keywords matching  $\varepsilon_2 N$  documents in  $\text{DB}_1$ . (As before these databases are made with consistent keywords and identifiers.) We show that when  $\text{DB}_0$  is encrypted, conditioned on the read intervals of size  $\varepsilon_1 \lambda N$  disallowing  $\lambda N$  bits for larger intervals, a random subset of intervals of size  $\varepsilon_2 \lambda N$  will disallow about another  $\lambda N$  bits for larger intervals. Security again forces this is to be true when  $\text{DB}_1$  is encrypted despite not being forced statistically. The result is that with constant probability about  $2\lambda N$  bits of EDB can no longer accommodate larger intervals.

By considering the sequence of  $\mathcal{A}_1, \dots, \mathcal{A}_k$  of adversaries and applying this reasoning  $k$  times, we have that the entire database has been disallowed by a relatively small number of small reads intervals with good probability, and then we can finish the proof as sketched above.

EXTENSIONS TO MORE GENERAL LOCALITY. The argument above worked for *perfect* locality, meaning the server search algorithm for keyword  $w$  worked with a single, contiguous read from EDB for exactly  $\lambda \cdot |\text{DB}(w)|$  bits that is disjoint from the read for any other search. It is easy to extend the lower bound to when the server works with  $r = O(1)$  contiguous reads that total exactly  $\lambda \cdot |\text{DB}(w)|$  bits and are disjoint from all other reads by observing that one of the  $r$  reads must have size at least  $\lambda \cdot |\text{DB}(w)|/r$  contiguous bits, and then adjusting the parameters of the above argument to ensure that intervals of that size can be be disallowed with good probability by the final adversary.

Other relaxations will be given. For instance, to adaptive the attack to work with leakage function  $\mathcal{L}_{\max}$ , we need to additionally arrange for the submitted databases to always have the same number of documents, keywords, maximum size result set  $\text{DB}(w)$ . This only introduces minor technicalities.

WOULD A SIMPLER ATTACK WORK? It is fair to ask if the complexity of this attack is necessary, and specifically if an attack like  $\mathcal{A}_1$ , which only queries for keyword with  $|\text{DB}(w)|$  equal to two possible sizes (either  $\varepsilon_1 N$  or  $\varepsilon_2 N$ ) could give the lower bound and avoid the iterative argument.

While it is always possible in principle to simplify proofs, we *can* argue that no such simple adversary could prove a lower bound better than  $M \geq 2\lambda N$  by observing read patterns alone. This is because an SSE scheme, knowing that it will only be queried for keywords with two different sizes  $|\text{DB}(w)|$ , could

have EDB reserve  $\lambda N$  bits for the first size, and another  $\lambda N$  bits for the second size. Then it could simply store sets  $\text{DB}(w)$  of the first size in random order first half of EDB with padding, then the sets  $\text{DB}(w)$  with the second size in the second half.

This reasoning generalizes to show that any attack proving  $M \geq k\lambda N$  must query keywords with  $|\text{DB}(w)|$  having at least  $k + 1$  different sizes, as our attack does.

## 4.2 Tools for the Proof of Theorem 4.1

The proof will use technical, but elementary, facts about how intervals of integers can be packed together. The intervals in the proof will be the read patterns of the searches, and whether or not they can be arranged in certain ways will allow the attack to work. We start with some tools to formalize the meaning of intervals “killing” space for larger intervals. This is captured in the definition of closure below. At first reading, it may be helpful to proceed without worrying about the details of the definitions and lemmas below because it is possible to understand the proof structure without them.

Recall that we write  $[a, b]$  for  $\{a, a + 1, \dots, b\}$  and  $(a, b)$  for  $\{a + 1, \dots, b - 1\}$ .

**Definition 4.3** *Let  $S \subseteq I \subseteq [M]$  be a subset of an interval  $I = [\alpha, \beta]$ , and write  $S$  uniquely as a disjoint union of increasing intervals,  $S = \bigcup_{i=1}^t [a_i, b_i]$  (with  $b_i < a_{i+1}$ ), and let  $d > 0$ . We define the  $d$ -closure of  $S$  in  $I$  to be the set*

$$\text{cl}_{d,I}(S) = S \cup \bigcup \{(b_i, a_{i+1}) : 0 \leq i \leq t, a_{i+1} - b_i \leq d\}$$

where we define  $b_0 = \alpha - 1$  and  $a_{t+1} = \beta + 1$ .

More generally, if  $S \subseteq F \subseteq [M]$  where  $F = \bigcup_{i=1}^s I_i$  is the union of non-adjacent disjoint intervals, then we define the  $d$ -closure of  $S$  in  $F$  to be

$$\text{cl}_{d,F}(S) = \bigcup_{i=1}^s \text{cl}_{d,I_i}(S \cap I_i).$$

In words, the  $d$ -closure of  $S$  in  $F$  consists of  $S$  along with all of the intervals between its members or endpoints of intervals  $F$  that are strictly smaller than  $d$ . (The condition  $a_{i+1} - b_i \leq d$  implies there are at most  $d - 1$  positions between  $x$  and  $y$ .) We also observe that  $|\text{cl}_{d,F}(S)| \geq |S|$  always. To give some further intuition, in our attack below,  $[M]$  are the indices of the bits of the given encrypted database, whereas  $F$  is the set of live indices which can still be used, and  $S \subseteq F$  is the read pattern associated with a set of search operation.

It is easy to check that the notion of closure is well-defined over  $S$  in the sense that any choice of an interval-decomposition of  $S$  will result in the same  $d$ -closure. This is *not* true for the decomposition of  $F$ , which is why we write it to be decomposed uniquely as the union of *non-empty, non-adjacent* intervals.

The motivation for the prior definition is that intervals of size  $d$  or larger that are disjoint from a set must also be disjoint from its  $d$ -closure, as summarized by the following lemma.

**Lemma 4.4** *Let  $S \subseteq F \subseteq [M]$  and  $J \subseteq F$  be an interval. If an interval  $J$  is disjoint from  $S$  then it is also disjoint from  $\text{cl}_{d,F}(S)$  for any  $d \geq |J|$ .*

**Proof:** Suppose that  $J$  is disjoint from  $S$  but intersects  $\text{cl}_{d,F}(S)$ . Since  $J \subseteq F$ , we have  $J \subseteq I_k$  where  $I_k$  is some interval in the unique decomposition of  $F$ . Since  $J$  is disjoint from  $S$ , it must be *contained* one of the extra intervals added when forming  $\text{cl}_{d,F}(S)$ . But these intervals all contain at most  $d - 1 < |J|$  elements, a contradiction. ■

A HELPING LEMMA. We discuss an important helping lemma about closures that we will use in the attack. A first-time reader may want to jump ahead to the discussion of our attacks to better identify the context in which they are used.

It says that for any set  $S = \bigcup_{i=1}^t J_i$  expressed as a union of intervals, we can always pick relatively few of the intervals the union of which will have the same  $d$ -closure.

**Lemma 4.5** *If  $S = \bigcup_{i=1}^t J_i \subseteq F \subseteq [M]$  is written as a union of (possibly adjacent) intervals, then we can select a set  $T \subseteq [t]$  of size at most  $3M/d$  such that  $\text{cl}_{d,F}(S) = \text{cl}_{d,F}(\bigcup_{i \in T} J_i)$ .*

**Proof:** Now, we are going to choose a subset  $T \subseteq [t]$  such that  $\text{cl}_{d,F}(\bigcup_{i \in T} J_i) = \text{cl}_{d,F}(S)$ .

From now on, we can assume without loss of generality that  $F = [M]$ . To see that this is true, assume that it is not the case and let  $F = \bigcup_{i=1}^s I_i$ , where  $I_i$  are disjoint intervals in increasing order. We now define  $\bar{F} := [M]$  and let  $\bar{S} := S \cup S_\Delta$ , where  $S_\Delta$  is the union of disjoint intervals chosen as follows: For any two consecutive intervals  $I_i = [a_i, b_i]$  and  $I_{i+1} = [a_{i+1}, b_{i+1}]$ , we add the interval  $J_{i,i+1} = (a_i, b_{i+1})$  to  $S_\Delta$ , together with  $[1, a_1]$  (if  $a_1 > 1$ ) and  $(b_s, M]$  (if  $b_s < M$ ). It is easy to verify that  $\text{cl}_{d,\bar{F}}(\bar{S}) \cap F = \text{cl}_{d,F}(S)$ . Now, if we can find a subset  $S' \subseteq \bar{S}$  consisting of at most  $\frac{3M}{d}$  of the intervals of  $\bar{S}$  and such that  $\text{cl}_{d,\bar{F}}(\bar{S}) = \text{cl}_{d,\bar{F}}(S')$ , we also note that by removing the intervals in  $S_\Delta$  from  $S'$ , obtaining  $S'' \subseteq F$ , we have

$$\text{cl}_{d,F}(S'') = \text{cl}_{d,\bar{F}}(S') \cap F = \text{cl}_{d,\bar{F}}(\bar{S}) \cap F = \text{cl}_{d,F}(S) .$$

Write now  $\text{cl}_{d,F}(S) = \bigcup_{i=1}^u C_i$  as the union of disjoint non-adjacent intervals. We will choose the indices for  $T$  by showing how to choose the elements from each partition  $S \cap C_i$ . For each  $C_i$ , we let  $[a_1, b_1] \leq [a_2, b_2] \leq \dots \leq [a_\ell, b_\ell]$  be the subset of the (possibly adjacent) intervals among  $J_1, \dots, J_t$  which are in  $S \cap C_i$ , and we will take a subset of these, denoted  $[a'_1, b'_1] < \dots < [a'_{\ell'}, b'_{\ell'}]$ . We start by taking  $[a'_1, b'_1] = [a_1, b_1]$ . Then we take  $[a'_2, b'_2]$  to be the interval with largest  $a_j$  such that  $a_j - b'_1 \leq d$ . Then we pick the next interval  $[a'_3, b'_3]$  to have the largest  $a_j$  such that  $a_j - b'_2 \leq d$ , and continue in this way until we have taken  $[a'_{\ell'}, b'_{\ell'}] = [a_\ell, b_\ell]$ . We can always pick another element because there is no interval of size  $d$  between  $[a_i, b_i]$  and  $[a_{i+1}, b_{i+1}]$  by Lemma 4.4.

We define  $T$  to be the indices of intervals chosen in this way. To bound the size of  $T$ , we first observe that the number  $u$  of intervals composing  $\text{cl}_{d,F}(S)$  is at most  $|F|/d < M/d$  because the space between those intervals is at least  $d$ . Next, we have that all the  $[a'_j, b'_j]$  are strictly increasing, and moreover that for  $j \in [\ell' - 2]$ ,

$$a'_{j+2} > a'_j + d,$$

because otherwise  $a'_{j+2} > a'_{j+1}$  would have been chosen as  $a'_{j+1}$ . Let  $T_i$  be the set of indices  $j \in T$  such that  $J_j \subseteq C_i$ . Then,

$$|T_i| \leq \lceil 2|C_i|/d \rceil ,$$

and we have

$$|T| = \sum_{i=1}^u |T_i| \leq \sum_{i=1}^u \left\lceil \frac{2|C_i|}{d} \right\rceil \leq u + \sum_{i=1}^u \frac{2|C_i|}{d} \leq \frac{M}{d} + \frac{2|\text{cl}_{d,F_d}(S)|}{d} \leq \frac{3M}{d} .$$

Finally, it is easy to check that the selected intervals have same  $d$ -closure in  $F$  as  $S$ . **■**

### 4.3 The Basic Attack

We are now ready to state our attack for the case of perfect locality and read efficiency with disjoint reads. We will ignore round-off errors computing values that not necessarily integers. To address these formally we can simply choose constants that divide  $N$  and satisfy other divisibility conditions. We discuss how to extend the result to the more general case in the next section.

**Theorem 4.6 (Basic Attack)** *If  $\Pi$  is an  $\mathcal{L}_{\min}$ -IND-secure SSE scheme with perfect locality, perfect read efficiency, and disjoint reads, then  $\Pi$  has  $\omega(|\text{BinEnc}(\text{DB})|)$  server storage.*

Adversary  $\mathcal{A}_j$ : Computing  $\text{DB}_0, \text{DB}_1, \mathbf{w}$ :

- 01 Select random  $V \subset [\hat{n}_j]$  such that  $|V| = |n_j|$
- 02 Initialize  $\text{DB}_0, \text{DB}_1$  to empty lists
- 03 **for**  $i \in [j-1]$  and  $\ell \in [n_i]$  **do**:
- 04  $w_{i,\ell} \xleftarrow{\$} \{0,1\}^\lambda$ ;  $S_{i,\ell} \xleftarrow{\$} (\{0,1\}^\lambda)^{\varepsilon_i N}$ ; Add  $w_{i,\ell} \mapsto S_{i,\ell}$  to  $\text{DB}_0$  and  $\text{DB}_1$
- 05 **for**  $\ell \in [\hat{n}_j]$  **do**:
- 06  $w_{j,\ell} \xleftarrow{\$} \{0,1\}^\lambda$ ;  $S_{j,\ell} \xleftarrow{\$} (\{0,1\}^\lambda)^{\varepsilon_j N}$ ; Add  $w_{j,\ell} \mapsto S_{j,\ell}$  to  $\text{DB}_0$
- 07 **if**  $\ell \in V$  **then**: Add  $w_{j,\ell} \mapsto S_{j,\ell}$  to  $\text{DB}_1$
- 08 **for**  $\ell \in [\hat{n}_{j+1}]$  **do**:
- 09  $w_{j+1,\ell} \xleftarrow{\$} \{0,1\}^\lambda$ ;  $S_{j+1,\ell} \xleftarrow{\$} (\{0,1\}^\lambda)^{\varepsilon_{j+1} N}$ ; Add  $w_{j+1,\ell} \mapsto S_{j+1,\ell}$  to  $\text{DB}_1$
- 10  $\mathbf{w} = \{w_{i,\ell} : i \in [j-1], \ell \in [n_i]\} \cup \{w_{j,\ell} : \ell \in V\}$
- 11 Output  $\text{DB}_0, \text{DB}_1, \mathbf{w}$

Adversary  $\mathcal{A}_j$ : Computing output bit:

- 12 Input  $\text{EDB}, \mathbf{t} = \{\tau_{i,\ell} : i \in [j-1], \ell \in [n_i]\} \cup \{\tau_{j,\ell} : \ell \in V\}$
- 13  $L_1 \leftarrow [|\text{EDB}|]$
- 14 **for**  $i \in [j-1]$  **do** :
- 15 Compute read patterns induced by  $\tau_{i,1}, \dots, \tau_{i,n_i}$ ; Let  $R_i$  be their union
- 16 **if**  $|\text{cl}_{\varepsilon_{i+1}\lambda N, L_i}(R_i)| < \hat{n}_i \varepsilon_i \lambda N$  **then** Output 0
- 17 **else**  $L_{i+1} \leftarrow L_i \setminus \text{cl}_{\varepsilon_{i+1}\lambda N, L_i}(R_i)$
- 18 Compute read patterns induced by  $\{\tau_{j,\ell} : \ell \in V\}$ ; Let  $R_j$  be their union
- 19 **if**  $|\text{cl}_{\varepsilon_{j+1}\lambda N, L_j}(R_j)| < \hat{n}_j \varepsilon_j \lambda N$  **then** Output 0
- 20 **else** Output 1

**Figure 3:** Adversary  $\mathcal{A}_j$  for the proof of Theorem 4.6.

**Proof:** Suppose for contradiction that  $\Pi$  has perfect locality and perfect read efficiency and server storage  $M = k \cdot |\text{BinEnc}(\text{DB})|$  for some constant  $k$  and all sufficiently large  $\text{DB}$ . We are going to show that the scheme cannot be  $\mathcal{L}_{\min}$ -IND-secure. Without loss of generality, we will assume that  $k > 2$ ; this obviously implies impossibility for all  $k$ , as schemes with smaller storage can be padded to have  $k > 2$ .

Our attack will fix some sufficiently large  $N$  as well as constants  $D, n_1 > n_2 > \dots > n_D > 1, \hat{n}_1 > \hat{n}_2 > \dots > \hat{n}_{D+1}$  and  $\varepsilon_1 < \varepsilon_2 < \dots < \varepsilon_{D+1} < 1$ . Looking ahead, we will set these constants

$$D = k, \quad \varepsilon_j = \left(\frac{1}{3k^3}\right)^{D-j+2}, \quad n_j = \frac{3k}{\varepsilon_{j+1}}, \quad \text{and } \hat{n}_j = \frac{1 - \sum_{i=1}^{j-1} n_i \varepsilon_i}{\varepsilon_j}, \quad (7)$$

where  $N = \text{poly}(\lambda)$  is sufficiently large.

For each  $j \in [D]$  we define an adversary  $\mathcal{A}_j$  using the code in Figure 3. Our plan is to show, unconditionally, that at least one of these adversaries has non-negligible advantage in attacking  $\Pi$ . Each  $\mathcal{A}_j$  plays the game  $\text{IND-SSE}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_j}^b(\lambda)$ , which starts by having the adversary supply indices  $\text{DB}_0, \text{DB}_1$  and a list of keywords  $\mathbf{w}$  such that  $\mathcal{L}_{\min}(\text{DB}_0, \mathbf{w}) = \mathcal{L}_{\min}(\text{DB}_1, \mathbf{w})$ .

Let us give some intuition for how the adversaries are design.  $\mathcal{A}_j$  generates  $\text{DB}_0$  and  $\text{DB}_1$ , which will have shapes

$$\begin{aligned} \text{DB}_0 &\leftarrow (n_1 \times \varepsilon_1 N; \dots; n_{j-1} \times \varepsilon_{j-1} N; \hat{n}_j \times \varepsilon_j N) \\ \text{DB}_1 &\leftarrow (n_1 \times \varepsilon_1 N; \dots; n_{j-1} \times \varepsilon_{j-1} N; n_j \times \varepsilon_j N; \hat{n}_{j+1} \times \varepsilon_{j+1} N). \end{aligned}$$

These shapes are chosen so that  $\text{DB}_0$  has a large ( $\hat{n}_j$ , but still constant) number of keywords matching  $\varepsilon_j N$  random (i.e., unique w.h.p.) documents each, while  $\text{DB}_1$  has only a few,  $n_j < \hat{n}_j$  and instead has some keywords matching  $\varepsilon_{j+1} N$  documents each.

Formally, for each  $i = 1, \dots, j-1$ ,  $\mathcal{A}_j$  picks  $n_i$  random keywords  $w_{i,1}, \dots, w_{i,n_i}$ , and then it associates each of these with  $\varepsilon_i N$  random identifiers (we ignore the case where the identifiers repeat, which happens with negligible probability), which in the code are the sets  $S_{i,1}, \dots, S_{i,n_i}$ . It adds these keywords and identifier sets to both  $\text{DB}_0$  and  $\text{DB}_1$ .

Next it generates  $\hat{n}_j$  random keywords  $w_{j,1}, \dots, w_{j,\hat{n}_j}$  and associated identifier sets  $S_{j,1}, \dots, S_{j,\hat{n}_j}$  (each of size  $\varepsilon_j N$ ), and adds them to  $\text{DB}_0$ . Now it uses the randomly chosen subset  $V \subset [\hat{n}_j]$  to select  $n_j < \hat{n}_j$  of those keywords and sets to add to  $\text{DB}_1$ .

At this point  $\text{DB}_0$  is complete, but  $\text{DB}_1$  is smaller because not all of the keywords  $w_{j,1}, \dots, w_{j,\hat{n}_j}$  were added (only those determined by  $V$  were added to  $\text{DB}_1$ ). The next “for” loop adds  $\hat{n}_{j+1}$  random keywords, each matching  $\varepsilon_{j+1} N$  random identifiers, to  $\text{DB}_1$ . By our choice of parameters, this will bring  $\text{DB}_1$  to have the same size of  $\text{DB}_0$ .

This completes the computation of  $\text{DB}_0$  and  $\text{DB}_1$ . The adversary sets its query vector  $\mathbf{w}$  to be

$$\mathbf{w} = \{w_{i,\ell} : i \in [j-1], \ell \in [n_i]\} \cup \{\hat{w}_{j,\ell} : \ell \in V\}.$$

This means  $\mathcal{A}_j$  queries for tokens for all keywords  $w$  with  $|\text{DB}(w)| = \varepsilon_i N$  with  $i \in [j-1]$ , and then for the  $n_j$  keywords  $w \in V$  that have  $|\text{DB}(w)| = \varepsilon_j N$  and are common to both  $\text{DB}_0$  and  $\text{DB}_1$ . In  $\text{DB}_1$ , these are *all* of the keywords matching  $\varepsilon_j N$  documents, but in  $\text{DB}_0$  this will be a *strict subset* of the  $\hat{n}_j$  such keywords. This will be crucial in our analysis because it means in  $\text{DB}_0$  we will observe statistical patterns when choosing a subset of keywords to search, and security will force the searching to exhibit the same patterns when searching  $\text{DB}_1$ . We will reason along these lines to conclude that  $\text{DB}_1$  must read regions of memory in an inefficient way in order to “look like”  $\text{DB}_0$ .

The adversary  $\mathcal{A}_j$  gives  $(\text{DB}_0, \text{DB}_1, \mathbf{w})$  to its game, and receives  $\text{EDB}$  with  $M := |\text{EDB}| = k \cdot |\text{BinEnc}(\text{DB})| = k\lambda N$  and the tokens corresponding to the queries.

Finally  $\mathcal{A}_j$  determines its output bit by performing a series of checks which ensure that the read patterns of the searched keywords “kill” large regions of the memory, meaning they prevent those regions from storing intervals of a given size. It starts by setting  $L_1 = [M]$ . During  $i$ -th round of checking,  $L_i$  will be the set of “live” regions of  $\text{EDB}$  where intervals of size  $\varepsilon_i \lambda N$  may fit without overlapping with any of the previously read regions.

There is one detail to verify in  $\mathcal{A}_j$ . For the checks involving closures to make sense, we need that  $R_i \subseteq L_i$  for each  $i \in [j]$  so that the closures are defined. Perfect locality says that the intervals in  $R_i$  must avoid  $R_1, R_2, \dots, R_{i-1}$ , and so by Lemma 4.4 the intervals comprising  $R_i$  (which are of size  $\varepsilon_i N$ ) must avoid  $\text{cl}_{\varepsilon_2 \lambda N, L_1}(R_1), \dots, \text{cl}_{\varepsilon_i \lambda N, L_{i-1}}(R_{i-1})$ . This implies  $R_i \subseteq L_i$ .

For  $i = j$  we can prove something slightly stronger that will be used below. If we let  $T_j$  be the union of read patterns of  $\{w_{j,\ell} : \ell \in [\hat{n}_j]\}$ , then obviously  $R_j \subseteq T_j$ . We also have  $T_j \subseteq L_j$  by an argument nearly identical to the one in the previous paragraph.

ANALYSIS. Theorem 4.6 will follow by combining the the next two Lemmas. After stating them we show how to derive the theorem, and then we prove them in turn.

**Lemma 4.7** *Let  $\Pi$  be an SSE scheme with perfect locality, perfect read efficiency and  $k \cdot |\text{BinEnc}(\text{DB})|$  server storage, and  $\mathcal{A}_D$  the adversary defined in Figure 3 with parameter settings in (7). Then*

$$\Pr[\text{IND-SSE}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_{D+1}}^1(\lambda) \Rightarrow 1] = 0.$$

**Lemma 4.8** *Let  $\Pi$  be an  $\mathcal{L}_{\min}$ -IND-secure SSE scheme with perfect locality, perfect read efficiency and  $k \cdot |\text{BinEnc}(\text{DB})|$  server storage, and  $\mathcal{A}_j$  the adversary defined in Figure 3 with parameter settings in (7). Then for each  $j \in [D]$  and  $b \in \{0, 1\}$ ,*

$$\Pr[\text{IND-SSE}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_j}^b(\lambda) \Rightarrow 1] \geq \frac{1}{\prod_{i=1}^j \binom{\hat{n}_j}{n_j}} - \text{negl}(\lambda).$$

Moreover, for our setting of parameters in (7), this probability is at least some positive constant independent of  $\lambda$  of  $N$  and dependent on  $k$ .

**Proof of Theorem 4.6 using Lemmas 4.7 and 4.8:** Let  $\Pi$  be an SSE scheme satisfying the conditions in the theorem and  $\mathcal{A}_j$  the the previously defined adaveraries with the specified parameters.

By Lemma 4.7,

$$\Pr[\text{IND-SSE}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_D}^1(\lambda) \Rightarrow 1] = 0. \quad (8)$$

However, since  $\Pi$  is  $\mathcal{L}_{\min}$ -IND-secure, by Lemma 4.8 with  $j = D$  and  $b = 0$ , we have

$$\Pr[\text{IND-SSE}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_D}^0(\lambda) \Rightarrow 1] \geq \frac{1}{\prod_{i=1}^D \binom{\hat{n}_D}{n_D}} - \text{negl}(\lambda), \quad (9)$$

where the right hand side is at least an absolute constant for sufficiently large  $\lambda$ . Subtracting (8) and (9), we get that  $\text{Adv}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_D}^{\text{ind-sse}}(\lambda)$  is a positive constant. Since  $\mathcal{A}_D$  is efficient this contradicts the assumed security of  $\Pi$  and finishes the proof.  $\blacksquare$

**Proof of Lemma 4.7:** If the  $\mathcal{A}_D$  outputs 1, then for all  $i = 1, \dots, D$ , we have  $|\text{cl}_{\varepsilon_{i+1}\lambda N, L_i}(R_i)| \geq \hat{n}_i \varepsilon_i \lambda N$  for all  $i = 1, \dots, D$ , where  $L_1 := [k\lambda N]$  and  $L_{i+1} := L_i \setminus \text{cl}_{\varepsilon_{i+1}\lambda N, L_i}(R_i)$ . We will show that this condition cannot hold when  $\Pi$  has perfect locality and read efficiency.

The key observation is that, for the  $\text{DB}_1$  chosen by  $\mathcal{A}_D$ , it must be that  $|L_{D+1}| > \hat{n}_{D+1} \cdot \varepsilon_{D+1} \cdot \lambda N$ . This is because perfect locality and perfect read efficiency mean the remaining  $\hat{n}_{D+1}$  keywords, if searched for, would result in reading overall a set  $I$  consisting of  $\hat{n}_{D+1} \cdot \varepsilon_{D+1} \cdot \lambda N$  bits within  $L_{D+1}$ .

We assume that  $\mathcal{A}_D$  outputs 1 and get a contradiction by showing that  $I$  is larger than  $L_{D+1}$ . Using (7),  $|I|$  is

$$\begin{aligned} |I| &= \hat{n}_{D+1} \varepsilon_{D+1} \lambda N = \frac{1 - \sum_{j=1}^k n_j \varepsilon_j}{\varepsilon_{k+1}} \cdot \varepsilon_{k+1} \lambda N = \left(1 - \sum_{j=1}^k \frac{3k}{\varepsilon_{j+1}} \varepsilon_j\right) \cdot \lambda N \\ &= \left(1 - 3k \sum_{j=1}^k \frac{1}{3k^3}\right) \cdot \lambda N = \left(1 - 3k \cdot k \cdot \frac{1}{3k^3}\right) \cdot \lambda N = \left(1 - \frac{1}{k}\right) \cdot \lambda N. \end{aligned}$$

The first equality uses the definition  $\hat{n}_{D+1}$ , the second uses the definition of  $n_i$ , and the third uses  $\varepsilon_j / \varepsilon_{j+1} = 1/3k^3$ .

Next we bound  $|L_{D+1}|$  when  $\mathcal{A}_D$  outputs 1. In this case,  $\mathcal{A}_D$  must have calculated  $|\text{cl}_{\varepsilon_{i+1}\lambda N, L_i}(R_i)| \geq \hat{n}_i \varepsilon_i \lambda N$  for each  $i \in [D]$ . This means, since the  $\text{cl}_{\varepsilon_{i+1}\lambda N, L_i}(R_i)$ 's are disjoint,

$$|L_{D+1}| = k\lambda N - \left| \bigcup_{i=1}^D \text{cl}_{\varepsilon_{i+1}\lambda N, L_i}(R_i) \right| \leq k\lambda N - \sum_{i=1}^D |\text{cl}_{\varepsilon_{i+1}\lambda N, L_i}(R_i)| \leq k\lambda N - \sum_{i=1}^D \hat{n}_i \varepsilon_i \lambda N.$$

For our setting of parameters (see (7)), we can simplify this as

$$\begin{aligned} k\lambda N - \sum_{i=1}^k \hat{n}_i \varepsilon_i \lambda N &= k\lambda N - \sum_{i=1}^k \left(1 - 3k \sum_{l=1}^{i-1} \frac{\varepsilon_l}{\varepsilon_{l+1}}\right) \lambda N = k\lambda N - k\lambda N + 3k\lambda N \sum_{i=1}^k (k-i) \frac{\varepsilon_i}{\varepsilon_{i+1}} \\ &= 3k\lambda N \sum_{i=1}^k (k-i) \frac{1}{3k^3} = \frac{\lambda N}{k^2} \cdot \sum_{i=1}^k (k-i) \leq \frac{\lambda N}{2} \end{aligned}$$

Recall that we assumed WLOG that  $k > 2$ , from which we get that  $|I| > |L_{D+1}|$ .  $\blacksquare$

**Proof of Lemma 4.8:** We use induction on  $j$ . We first observe that the game will never output  $\perp$  because  $\mathcal{L}_{\min}(\text{DB}_0, \mathbf{w}) = \mathcal{L}_{\min}(\text{DB}_1, \mathbf{w})$  by our construction of  $\mathcal{A}_j$ .

For  $j = 1, b = 0$ , we first examine the probability that  $\mathcal{A}_1$  outputs 1. Here  $\mathcal{A}_1$  chooses  $\text{DB}_0$  that consists of  $\hat{n}_1 = 1/\varepsilon_1$  keywords  $\{\hat{w}_{1,1}, \dots, \hat{w}_{1,\hat{n}_1}\}$ , each satisfying  $|\text{DB}_0(\hat{w}_{1,\ell})| = \varepsilon_1 N$ . Then it chooses a set  $V \subset \{\hat{w}_{1,1}, \dots, \hat{w}_{1,\hat{n}_1}\}$  of size  $n_1 < \hat{n}_1$  at random and produces a  $\text{DB}_1$  that is consistent with  $\text{DB}_0$  on those

keywords and has some additional keywords added. The adversary  $\mathcal{A}_1$  queries for tokens for the keywords in  $V$ , computes the read pattern  $R_1$  induced by them, and tests if  $\text{cl}_{\varepsilon_2 \lambda N, [M]}(R_1) < \hat{n}_1 \lambda \varepsilon_1 N = \lambda N$ .

Since  $\Pi$  is perfectly local, each of the  $\hat{n}_1$  keywords induces a disjoint read of  $\varepsilon_1 \lambda N$  bits out of the  $M = k \lambda N$  total bits in the EDB server memory. Let  $T_1$  be the set of read patterns for *all*  $\hat{n}_1$  keywords, queried or not. We have that  $|T_1| = \hat{n}_1 \varepsilon_1 \lambda N = \lambda N$ .

Now consider  $R_1$ , which is the union of the read patterns for keywords in  $V$ . This is a uniformly random set of  $n_1$  intervals chosen from the  $\hat{n}_1$  intervals making up  $T_1$ . We will show that

$$|\text{cl}_{\varepsilon_2 \lambda N, [M]}(R_1)| \geq |\text{cl}_{\varepsilon_2 \lambda N, [M]}(S_1)| \geq |T_1| = \lambda N$$

for at least one choice of the set  $V$ . Since the total number of choices for  $V$  is  $\binom{\hat{n}_1}{n_1}$  this will give the bound for the base case. This follows because using Lemma 4.5 we can choose a set of at most  $n_1 = 3k/\varepsilon_2$  of the intervals making up  $T_1$  that will have the same  $\varepsilon_2 \lambda N$ -closure in  $[M]$ . The adversary  $\mathcal{A}_1$  will output 1 when it selects  $V$  that contains this set. Thus we have

$$\Pr[\text{IND-SSE}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_1}^0(\lambda) \Rightarrow 1] \geq \frac{1}{\binom{\hat{n}_1}{n_1}},$$

proving the base case.

We now let  $j > 1$  be arbitrary. It enough to prove the statement for  $b = 0$  because, by the security of  $\Pi$ , we have that

$$\Pr[\text{IND-SSE}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_j}^1(\lambda) \Rightarrow 1] = \Pr[\text{IND-SSE}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_j}^0(\lambda) \Rightarrow 1] \pm \text{negl}(\lambda)$$

We claim that

$$\Pr[\text{IND-SSE}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_j}^0(\lambda) \Rightarrow 1] \geq \frac{1}{\binom{\hat{n}_j}{n_j}} \cdot \Pr[\text{IND-SSE}_{\Pi, \mathcal{L}_{\min}, \mathcal{A}_{j-1}}^1(\lambda) \Rightarrow 1].$$

We will prove this first by showing that  $\mathcal{A}_j$  selects  $\text{DB}_0$  from the same distribution that  $\mathcal{A}_{j-1}$  selects  $\text{DB}_1$  from, and then showing  $\mathcal{A}_j$  outputs 1 exactly when  $\mathcal{A}_{j-1}$  would have output 1 *and* an additional condition holds.

For the first part, by construction we have that  $\mathcal{A}_j$  picks  $\text{DB}_0$  with the same shape that  $\mathcal{A}_{j-1}$  uses for  $\text{DB}_1$ , and all that remains to check is that the identifiers and keywords are all uniform and independent in both cases. For  $\mathcal{A}_j$  this is obviously true when generating  $\text{DB}_0$ , and for  $\mathcal{A}_{j-1}$  it only complicated by the fact that the adversary first generates its own  $\text{DB}_0$  and then picks a subset of its identifiers along with some fresh ones to use in  $\text{DB}_1$ . But it is easy to check that this process also gives random identifiers and keywords, establishing the first part.

We now examine the extra condition  $\mathcal{A}_j$  checks before outputting 1. By construction we have that  $\mathcal{A}_j$  runs the same code as  $\mathcal{A}_{j-1}$  to determine if it outputs 1, except for two differences. First,  $\mathcal{A}_{j-1}$  will run its final test that  $|\text{cl}_{\varepsilon_j \lambda N, L_{j-1}}(R_{j-1})| < \hat{n}_{j-1} \varepsilon_{j-1} \lambda N$  with the read pattern  $R_{j-1}$  for keywords indexed by the set  $V$  of size  $n_j$  while  $\mathcal{A}_j$  will run the same test with the read pattern  $R_{j-1}$  calculated from all  $n_j$  keywords matching  $\varepsilon_j N$  documents. But in either case, the adversaries are checking the condition of the read pattern for the same keywords.

The second difference is that  $\mathcal{A}_j$  performs the additional check on the read pattern of  $n_j$  keywords that each match  $\varepsilon_j N$  documents chosen at random from the  $\hat{n}_j$  such keywords. We will show that, conditioned on the event that  $\mathcal{A}_j$  has not output 0 before this point, the probability that it outputs 1 is at least  $1/\binom{\hat{n}_j}{n_j}$ , which proves the claim.

This check selects  $n_j$  keywords  $\{\hat{w}_{j,\ell} : \ell \in V\}$  where each has  $|\text{DB}(\hat{w}_{j,\ell})| = \varepsilon_j N$ , and computes the union of their read patterns  $R_j$ . These keywords are chosen from amongst  $\hat{n}_j$  total such keywords

(namely,  $\{\hat{w}_{j,\ell} : \ell \in [\hat{n}_j]\}$ ). Let  $T_j$  be the read pattern induced by all  $\hat{n}_j$  such keywords, queries and unqueried. By the claim after the description of  $\mathcal{A}_j$ , we know that  $T_j \subseteq L_j$ . And by Lemma 4.5, there exists a subset of  $n_j = 3k/\varepsilon_{j+1}$  intervals out of the total  $\hat{n}_j$  comprising  $T_j$  that have the same  $\varepsilon_{j+1}\lambda N$ -closure in  $L_j$  as  $T_j$ . If the set of intervals corresponding to the keywords chosen in  $V$  contain this set, then the test will pass because then

$$|\text{cl}_{\varepsilon_{j+1}\lambda N, L_j}(R_j)| \geq |\text{cl}_{\varepsilon_{j+1}\lambda N, L_j}(T_j)| \geq |T_j| = \hat{n}_j \varepsilon_j \lambda N.$$

Thus this happens with probability at least  $1/\binom{\hat{n}_j}{n_j}$ . Moreover, this bound is also true conditioned on the prior checks passing, which gives the inductive step and completes the proof of Lemma 4.8.  $\blacksquare$

#### 4.4 Extending the Basic Attack

The above theorem considers perfect locality ( $r = 1$ ), perfect read efficiency  $\gamma$ , as well as disjoint reads. In other words, every search operations reads *exactly*  $\gamma \cdot |\text{DB}(w)|$  bits, and all of these have not been read before. We now extend our result to prove Theorem 4.1. Below, we also discuss extensions to other leakage metrics.

To this end, we present a generalization of the attack of the previous section which allows us to extend the result to more complex read patterns. To this end, let us denote the read pattern induced by querying a keyword  $w$  in some arbitrary execution as  $R_w$ . For the theorem below, we assume that for an SSE scheme  $\Pi$  there exists a value  $\rho$  (independent of  $N$ , but possibly dependent on  $\lambda$  and  $|\mathbf{W}|$ ) such that for every queried keyword  $w \in \mathbf{W}$  with  $|\text{DB}(w)| = c_w \cdot N$  for some constant  $c_w$ , there must a subset  $R'_w \subseteq R_w$  consisting of exactly  $\rho \cdot |\text{DB}(w)|$  contiguous bits which are not part of  $R_{w'}$  for any  $w' \neq w$ . Then, a lower bound of  $\omega(\rho \cdot N)$  can be computed using the same proof technique as above, as stated in the next lemma, for which we only sketch a proof. We stress that we are *not* asking for efficient computability of  $R'_w$ , only for its existence.

**Lemma 4.9** *Assume that  $\Pi$  is an  $\mathcal{L}_{\min}$ -IND-secure SSE scheme with the above property on its read patterns. Then, its server storage is  $\omega(\rho \cdot N)$ .*

**Proof:** The proof is very similar to the one of Theorem 4.6, and we only sketch the main modifications. In particular, the attackers use the same databases  $\text{DB}_0$  and  $\text{DB}_1$  used in the proof of Theorem 4.6. The main difference is that we cannot guarantee any more in the attacker  $\mathcal{A}_j$  that  $R_i \subseteq L_i$ . For this reason, the attacker  $\mathcal{A}_j$  is now going to do the following: It sets  $L_1 = [M]$  ( $L_i$  will be the set of “live” indices, where intervals of size  $\varepsilon_i N \rho$  may fit). Then it does the following for  $i = 1, \dots, j - 1$ :

- Compute the read pattern of the server for keywords  $\{w_{i,\ell} : \ell \in [n_j]\}$ . Call the union of their read patterns  $R_i \subseteq [M]$ .
- If  $|\text{cl}_{\varepsilon_{i+1}N\rho, L_i}(R_i \cap L_i)| < \hat{n}_i \varepsilon_i N \rho$  then output 0. Else set  $L_{i+1} = L_i \setminus \text{cl}_{\varepsilon_{i+1}N\rho, L_i}(R_i \cap L_i)$ .

Finally,  $\mathcal{A}_j$  computes the read patterns for the keywords  $\{w_{j,\ell} : \ell \in V\}$  and calls their union  $R_j$ . It tests if  $|\text{cl}_{\varepsilon_{j+1}N\rho, L_j}(R_j \cap L_j)| < \hat{n}_j \varepsilon_j N \rho$  and outputs 0 if so. Otherwise, it outputs 1.

In order to show that the proof still applies, we need to make the following observations:

1. For all words  $w_{i,\ell}$  ( $i \in [j]$ ), we have that the associated subset of the read pattern  $R'_{w_{i,\ell}} \subseteq R_{w_{i,\ell}}$  must be contained in  $L_i$  by construction, as it consist of  $\rho \cdot |\text{DB}(w_{i,\ell})|$  contiguous bits, which cannot be made to fit into  $[M] \setminus L_i$ .
2. We also need to modify the proof of Claim 4.8: We are going to replace  $R_j$  with  $R'_j$  consisting of the union of the  $R'_w$  for  $w \in V$ . As in the original proof, we show that there exists a subset of size

$n_j$  of the  $R'_w$ 's corresponding associated with the  $\hat{n}_j$  keywords with closure as large as  $\hat{n}_j \epsilon_j N \rho$ , and that therefore

$$|\text{cl}_{\epsilon_{j+1} N \rho, L_j}(R'_j)| \geq \hat{n}_j \epsilon_j N \rho$$

with probability at least  $1/\binom{\hat{n}_j}{n_j}$ . The attacker does not necessarily know  $R'_j$ , but since  $R'_j \subseteq R_j \cap L_j$ , we have also have  $|\text{cl}_{\epsilon_{j+1} N \rho, L_j}(R_j \cap L_j)| \geq \hat{n}_j \epsilon_j N \rho$ , as the closure can only become larger by taking supersets.

■

PROOF OF THEOREM 4.1 WITH  $\mathcal{L}_{\min}$ . We now use Lemma 4.9 to derive Theorem 4.1 with  $\mathcal{L}_{\min}$  in place of  $\mathcal{L}_{\max}$ . In contrast to the setting of Theorem 4.6, we make no a-priori assumption on the size of the read pattern when searching of a keyword  $w$ . A simple compression argument shows that every search operation for  $w$  should read at least  $B_0(w)$  bits, where for some universal constant  $C_0$ ,

$$B_0(w) \geq \lambda \cdot |\text{DB}(w)| - O(\lambda) \geq \lambda \cdot (|\text{DB}(w)| - C_0),$$

where the  $O(\lambda)$  term takes into account the secret-key length (assumed wlog to have length smaller than  $C_0 \cdot \lambda$ ) and some possible slackness. If the above were not true, then correctness of the SSE scheme  $\Pi$  implies that we could compress the random indices in  $\text{DB}(w)$  to less than  $\lambda \cdot |\text{DB}(w)|$  bits using the secret key and the portion of EDB read upon searching for  $w$  on that key. (We dispense with the standard proof of this simple fact.) Moreover, since the scheme has locality  $r$ , we can guarantee that when query of a keyword  $w$ , the read pattern must contain a set of contiguous locations of size at least  $B_1(w)$ , where

$$B_1(w) \geq \frac{B_0(w)}{r} \geq \frac{\lambda \cdot (|\text{DB}(w)| - C_0)}{r}.$$

Still, we cannot guarantee that these bits have not been read by previous queries or will be read by later queries. However, by the  $\alpha$ -overlapping assumption, we know that among these  $B_1(w)$  bits, at most  $\alpha$  can ever be contained in the read patterns associated with other search queries. By an elementary counting argument, this means that there must still exist an interval of  $B_2(w)$  contiguous locations which are read solely when searching  $w$ , where

$$B_2(w) \geq \frac{B_1(w) - \alpha}{\alpha + 1} \geq \frac{\lambda \cdot |\text{DB}(w)|}{r(\alpha + 1)} - \frac{C_0 \cdot \lambda + \alpha}{r(\alpha + 1)} \geq \frac{\lambda \cdot |\text{DB}(w)|}{2r(\alpha + 1)},$$

for sufficiently large  $N$ . The right-most inequality holds because for the databases used by the attacker in Lemma 4.9, we have  $|\text{DB}(w)| = \epsilon \cdot N$  for some constant  $\epsilon$ , and moreover  $\alpha$  does not grow with  $N$  (when keeping all other parameters constant). Hence, for sufficiently large  $N$ ,

$$\frac{C_0 \cdot \lambda + \alpha}{r(\alpha + 1)} = \frac{C_0 \cdot \lambda}{r(\alpha + 1)} + \frac{\alpha}{r(\alpha + 1)} \leq \frac{\lambda \cdot |\text{DB}(w)|}{2r(\alpha + 1)}.$$

To conclude the proof of Theorem 4.1, we invoke Lemma 4.9 with  $\rho = \frac{\lambda}{2r(\alpha + 1)}$ .

EXTENSION TO  $\mathcal{L}_{\max}$ . The proof of Theorem 4.6 constructs adversaries  $\mathcal{A}_j$  that always produce  $(\text{DB}_0, \mathbf{w}_0), (\text{DB}_1, \mathbf{w}_1)$  with  $\mathcal{L}_{\min}(\text{DB}_0, \mathbf{w}_0) = \mathcal{L}_{\min}(\text{DB}_1, \mathbf{w}_1)$ . We cannot immediately use these adversaries to prove Theorem 4.1 because they will immediately lose the game with leakage  $\mathcal{L}_{\max}$  because we will have  $\mathcal{L}_{\max}(\text{DB}_0, \mathbf{w}_0) \neq \mathcal{L}_{\max}(\text{DB}_1, \mathbf{w}_1)$ . In particular,  $\text{DB}_0, \text{DB}_1$  will have a different number of unique keywords, and they will also have different values for  $\max_{w \in \mathcal{W}} |\text{DB}_b(w)|$ . (They will contain the same number of unique identifiers,  $N$ , however.)

Thus in order to prove theorem 4.1 with leakage function  $\mathcal{L}_{\max}$  we must adapt the attack above so that all of the index/query choices by the  $\mathcal{A}_j$  satisfy the equality condition with  $\mathcal{L}_{\max}$ . This amounts

to modifying  $\mathcal{A}_j$  so that  $\text{DB}_0$  and  $\text{DB}_1$  have the contain the same number of unique keywords, and have the same maximum size set  $\max_{w \in \mathcal{W}} |\text{DB}_b(w)|$ .

We sketch the modifications to  $\mathcal{A}_j$ . We start by modifying the number of unique keywords. Originally,  $\mathcal{A}_j$  submits  $\text{DB}_0$  containing  $\left(\sum_{i=1}^{j-1} n_i\right) + \hat{n}_j$  keywords while  $\text{DB}_0$  contains  $\sum_{i=1}^j n_i$  keywords. To address this issue we change  $\mathcal{A}_j$  to always submit both  $\text{DB}_0$  and  $\text{DB}_1$  with  $\sum_{i=1}^D n_i$  keywords by padding with additional random keywords that match some previously used identifier (so that  $n$  will not change). We then adjust the parameters so that these additional keyword/identifier pairs will not affect the main argument. This is not difficult as  $m = \sum_{i=1}^D n_i$  is still a constant independent of  $N$ .

Finally we modify  $\mathcal{A}_j$  to construct  $\text{DB}_0$  and  $\text{DB}_1$  so that  $\max_w |\text{DB}_0(w)| = \max_w |\text{DB}_1(w)|$ . In  $\mathcal{A}_j$  these are  $\varepsilon_j N$  and  $\varepsilon_{j+1} N$  respectively. Now instead  $\mathcal{A}_j$  will insert into both  $\text{DB}_0$  and  $\text{DB}_1$  a set of identifiers of size  $\varepsilon' N$  that match a single new keyword, for a constant (say)  $\varepsilon' = 2\varepsilon_{D+1}$ . It then carries proceeds as in basic the attack with  $N$  replaced by  $(1 - \varepsilon')N$  everywhere. With these adjusted parameters, each step in the induction will “disallow” about  $(1 - \varepsilon')\lambda N$  bits for the next step (instead of above  $\lambda N$ ). Finally, when computing its output bit,  $\mathcal{A}_j$  will test if the remaining allowed area contain intervals large enough for *both* the set of  $\varepsilon' N$  identifiers and  $\varepsilon_{j+1} N$  identifiers, instead of only the latter.

## 5 A Positive Result: SSE with Logarithmic Locality

In the previous section, we have seen that any scheme with *constant* locality produces encrypted index of size  $\omega(N)$ . To complement this result, we provide a new scheme with logarithmic locality, at the cost of an asymptotically larger encrypted index of size roughly  $N \log N$ . At the same time, our scheme is going to *only* leak the database size  $N$ , i.e., it is going to be  $\mathcal{L}_{\min}$ -secure. None of the previous SSE schemes achieved such locality level without additional leakage or a larger worst-case blow-up of the encrypted database.

**HASH TABLES.** The scheme below relies on *hash tables*. Concretely, a hash table *implementation* consists of a pair of algorithms ( $\text{HTCreate}, \text{HTGet}$ ). The function  $\text{HTCreate}$  takes as input a list  $L = \{(l_i, d_i)\}_{1 \leq i \leq k}$  of pairs  $(l_i, d_i)$  of strings, where  $l_i \in \{0, 1\}^\ell$  is the *label* and  $d_i \in \{0, 1\}^r$  is the *data*, and outputs the *hash table*  $\text{HT}$ . After running  $\text{HT} \leftarrow \text{HTCreate}(L)$ , we have that  $\text{HTGet}(\text{HT}, l)$  returns  $d$  if and only if  $(l, d) \in L$ , and returns  $\perp$  otherwise.

There exist hash-table implementations (for example, via variants of cuckoo hashing [21]) with the following properties: The overall size of  $\text{HT}$  is  $O(k(r + \ell) + \log^2 k)$ , and the algorithm  $\text{HTGet}$  needs to read from the hash table  $\text{HT}$  a constant number (e.g. two) of blocks of  $\ell$  contiguous bits, as well as one  $r$ -bit block, when searching for a label  $l = l_i$ . Moreover,  $\text{HT}$  does not depend on the ordering of the list  $L$ .

**DESCRIPTION OF THE SCHEME.** We now proceed to specify our new SSE scheme  $\Pi = (\text{KeyGen}, \text{EDBSetup}, \text{TokGen}, \text{Search})$  with logarithmic locality. It relies on two keyed functions  $F$  and  $F'$ , where  $F : \mathcal{K} \times \{0, 1\}^* \rightarrow \mathcal{K} \times \mathcal{K}'$  and  $F' : \mathcal{K} \times \mathbb{N} \rightarrow \{0, 1\}^\ell$  (both later to be assumed as pseudorandom). Moreover, it uses a symmetric encryption scheme  $(\mathcal{E}, \mathcal{D})$  with key space  $\mathcal{K}'$  and  $m$ -bit ciphertexts. In particular, we are going to use the latter scheme to encrypt document identifiers and we are going to assume that all identifiers are in the message space of the scheme  $(\mathcal{E}, \mathcal{D})$  and their encryption results in ciphertext of *exactly* length  $s$ .

The four algorithms of  $\Pi$  now operate as follows:

**Key Generation.** Algorithm  $\text{KeyGen}$  simply generates a key  $K \xleftarrow{\$} \mathcal{K}$  for  $F$ .

**Setup.** Assume that we are given  $\text{DB}$  with size  $N = 2^t$  for some  $t \geq 1$ , and for every word  $w \in \mathcal{W}$ , we use the notation  $\text{DB}(w) = \{\text{id}_1, \dots, \text{id}_{n_w}\}$  to denote its  $n_w$  associated identifiers. We also need to consider the binary expansion  $n_w = \sum_{i=0}^{t-1} n_{w,i} \cdot 2^i$ . (If  $N$  is not a power of two, we need to pad  $\text{DB}$  to satisfy this by adding some dummy keyword-identifier pairs.)

Algorithm **EDBSetup**, on input  $\text{DB}$  and  $K$ , proceeds as follows: It initially sets up  $t$  empty lists  $L_0, L_1, \dots, L_{t-1}$ . For every word  $w \in \mathcal{W}$ , it then computes two derived keys  $F_K(w) = (K_{w,0}, K_{w,1})$  and sets  $c = 0$ . Subsequently, for all  $i = 0, \dots, t-1$ , if  $n_{w,i} = 1$ , we define the  $\ell$ -bit label  $l = F'_{K_{w,0}}(i)$  and the  $(2^i \cdot s)$ -bit data

$$d = \mathcal{E}(K_{w,1}, \text{id}_c) \parallel \dots \parallel \mathcal{E}(K_{w,1}, \text{id}_{c+2^i}),$$

increase  $c$  by  $2^i$ , and add  $(l, d)$  to  $L_i$ . Once done with the iteration, for all  $i = 0, \dots, t-1$ , we first add pairs  $(l, d)$  to  $L_i$  until it contains *exactly*  $2^{t-i}$  elements, where  $l$  is a random label and  $d$  is a random  $(2^i \cdot s)$ -bit string, and then compute  $\text{HT}_i \leftarrow \text{HTCreate}(L_i)$ . The final output is

$$\text{EDB} = \text{HT}_0 \parallel \text{HT}_1 \parallel \dots \parallel \text{HT}_{t-1}.$$

**Token Generation.** Algorithm **TokGen**, on inputs  $K$  and  $w$ , computes and outputs the two derived keys  $(K_{w,0}, K_{w,1}) \leftarrow F_K(w)$ .

**Search.** The search algorithm **Search**, on input  $\text{EDB} = \text{HT}_0 \parallel \text{HT}_1 \parallel \dots \parallel \text{HT}_{t-1}$  and  $(K_0, K_1)$ , initially defines an empty response set  $R = \emptyset$ . Then, for all  $i = 0, \dots, t-1$ , it computes  $l \leftarrow F'_{K_0}(i)$  and  $d \leftarrow \text{HTGet}(\text{HT}_i, l)$ . If  $d = C_1 \parallel \dots \parallel C_{2^i} \neq \perp$ , it adds  $\mathcal{D}(K_1, C_1), \dots, \mathcal{D}(K_1, C_{2^i})$  to the response set  $R$ . At the end, it outputs  $R$ .

**CORRECTNESS, COMPLEXITY AND LOCALITY.** Correctness of the SSE scheme  $\Pi$  holds with high probability assuming pseudorandomness of  $F$  and  $F'$  – we dispense with a formal analysis.

Assume now that we use the space- and lookup-efficient hash-table implementation mentioned above. Note first that every  $L_i$  is going to always contain  $2^{t-i}$  elements consisting of a pair  $(l, d)$  where  $|l| = \ell$  and  $|d| = 2^i \cdot s$ . Indeed, we cannot add more than  $2^{t-i}$  pairs (before possibly filling up  $L_i$ ) because each such pair is associated with  $2^i$  keyword-identifier pairs, and overall there are  $N = 2^t$  such pairs. For this reason, the size of  $\text{HT}_i$  is going to be  $O(N(\ell + s) + \log(N)^2)$ , and thus the overall size of  $\text{EDB}$  is

$$|\text{EDB}| = O(N \log N \cdot (\ell + s) + \log(N)^3).$$

As for locality, by the property of the hash tables, we are going to read  $O(1)$  blocks of consecutive values for every  $i = 0, \dots, t-1$ , thus obtaining locality  $O(\log N)$ . Also, read efficiency is constant.

**SECURITY.** We turn to the security of the SSE scheme  $\Pi$ . We start with non-adaptive security, and below discuss the changes necessary in order to prove adaptive security in the random-oracle model. Here, we are going to prove that the scheme achieves the strong notion  $\mathcal{L}_{\min}$ -SIM-security. Recall that we say that  $(\mathcal{E}, \mathcal{D})$  has *pseudorandom ciphertxts* if no polynomial-time attacker can decide whether a given oracle is behaving as  $\mathcal{E}_K(\cdot)$  for random secret key  $K$  or whether it is returning a fresh random string upon each invocation, except with negligible advantage.

**Theorem 5.1 (Non-adaptive Security of  $\Pi$ .)** *The above SSE-scheme  $\Pi$  is  $\mathcal{L}_{\min}$ -SIM-secure against non-adaptive attacks if  $F$  and  $F'$  are pseudorandom functions and  $(\mathcal{E}, \mathcal{D})$  has pseudorandom ciphertxts.*

**Proof:** Recall that in a non-adaptive attack, the attacker  $\mathcal{A}$  first commits to key-word queries  $\mathbf{w}$  and a database  $\text{DB}$ . We also recall that in the real experiment  $\text{SIM-SSE}^0_{\Pi, \mathcal{L}_{\min}, \mathcal{A}}(\lambda)$ , **KeyGen** is run, resulting in a key  $K$ , and then **EDBSetup** is run, producing the encrypted database  $\text{EDB}$ . The attacker  $\mathcal{A}$  is then given  $\text{EDB}$ , together with the search tokens  $(K_{\mathbf{w}[i],0}, K_{\mathbf{w}[i],1}) = F_K(\mathbf{w}[i])$  for  $i \in [|\mathbf{w}|]$ . In contrast, in the ideal experiment  $\text{SIM-SSE}^1_{\Pi, \mathcal{L}_{\min}, \mathcal{A}, \mathcal{S}}(\lambda)$ , the simulator  $\mathcal{S}$  initially only obtains  $N = |\text{DB}|$  and  $\text{DB}(\mathbf{w}[i])$  for  $i \in [|\mathbf{w}|]$ , and needs to output  $\text{EDB}'$  as well as search tokens  $(K'_{\mathbf{w}[i],0}, K'_{\mathbf{w}[i],1})$  such that

$$\Pr[\text{SIM-SSE}^0_{\Pi, \mathcal{L}_{\min}, \mathcal{A}}(\lambda) \Rightarrow 1] - \Pr[\text{SIM-SSE}^1_{\Pi, \mathcal{L}_{\min}, \mathcal{A}, \mathcal{S}}(\lambda) \Rightarrow 1] = \text{negl}(\lambda).$$

Concretely, the simulator  $\mathcal{S}$  operates as follows, assuming  $N = 2^t$ . First, it creates random and independent tokens  $(K_{\mathbf{w}[i],0}, K_{\mathbf{w}[i],1})$  for  $i \in [|\mathbf{w}|]$  and initializes empty sets  $L_0, \dots, L_{t-1}$ . For every  $i \in [|\mathbf{w}|]$ , it then does the following, with  $\text{DB}(\mathbf{w}[i]) = \{\text{id}_1, \dots, \text{id}_{n_w}\}$  and  $n_{\mathbf{w}[i]} = \sum_{j=0}^{t-1} n_{\mathbf{w}[i],j} \cdot 2^j$ . It sets  $c = 0$ , and for every  $j = 0, \dots, t-1$ , if  $n_{\mathbf{w}[i],j} = 1$ , it computes

$$d = \mathcal{E}(K_{\mathbf{w}[i],1}, \text{id}_c) \parallel \dots \parallel \mathcal{E}(K_{\mathbf{w}[i],1}, \text{id}_{c+2^j}),$$

adds  $(F_{K_{\mathbf{w}[i],0}}(j), d)$  to  $L_j$ , and increases  $c$  by  $2^j$ . Once done with the iteration, for all  $j = 0, \dots, t-1$ , the simulator adds pairs  $(l, d)$  to  $L_j$  until it contains *exactly*  $2^{t-j}$  elements (where  $l$  is a random  $\ell$ -bit label and  $d$  is a random  $(2^j \cdot s)$ -bit string) and computes  $\text{HT}_j \leftarrow \text{HTCreate}(L_j)$ . The final output is  $\text{EDB}' = \text{HT}_0 \parallel \text{HT}_1 \parallel \dots \parallel \text{HT}_{t-1}$ , together with the tokens  $(K_{\mathbf{w}[i],0}, K_{\mathbf{w}[i],1})$  for  $i \in [|\mathbf{w}|]$ .

The proof now proceeds via a hybrid argument. The first hybrid experiment  $H_0$  behaves the real-world experiment, in particular returning the distribution  $[\text{EDB}, \{(K_{\mathbf{w}[i],0}, K_{\mathbf{w}[i],1})\}_{i \in [|\mathbf{w}|]}]$  to  $\mathcal{A}$ . In the second hybrid, the function  $F_K$  is replaced by a truly random function when running  $\text{EDBSetup}$  and when producing the tokens  $(K_{\mathbf{w}[i],0}, K_{\mathbf{w}[i],1})$  given to  $\mathcal{A}$ , i.e., every search token is replaced with a truly-random key pair. It is easy to see that  $\Pr[H_0 \Rightarrow 0] - \Pr[H_1 \Rightarrow 1] = \text{negl}(\lambda)$  by the pseudorandomness of  $F$ .

For the next hybrid  $H_2$ , when running  $\text{EDBSetup}$ , we are going to replace  $F'_{K_{\mathbf{w}[i],0}}$  with an independent random function for every  $i \in [|\mathbf{w}|]$ . In particular, this means that every label  $l$  of a pair  $(l, d)$  added to  $L_j$  when processing the key-word  $w$  in  $\text{EDBSetup}$  is independent and uniform. Similarly to the above,  $\Pr[H_1 \Rightarrow 0] - \Pr[H_2 \Rightarrow 1] = \text{negl}(\lambda)$  by the pseudorandomness of  $F'$ .

Finally, in  $H_3$ , for all  $i \in [|\mathbf{w}|]$ , we replace every data-block  $d$  containing encryptions of identifiers in  $\text{DB}(\mathbf{w}[i])$  produced in  $\text{EDBSetup}$  with a randomly chosen string of the appropriate length. It is not hard to see that  $H_3$  behaves exactly as  $\text{SIM-SSE}^1_{\Pi, \mathcal{L}_{\min}, \mathcal{A}, \mathcal{S}}$ , and moreover,  $\Pr[H_2 \Rightarrow 0] - \Pr[H_3 \Rightarrow 1] = \text{negl}(\lambda)$  by the pseudorandomness of  $(\mathcal{E}, \mathcal{D})$ . ■

**ADAPTIVE SECURITY.** We additionally propose an efficient instantiation of the above scheme which is actively secure *in the random oracle model*. Note that, in this case, the security notion allows the simulator to *program* the random oracle.

Concretely, we instantiate  $(\mathcal{E}, \mathcal{D})$  with the scheme encrypting  $M$  under secret key  $K$  as  $\mathcal{E}_K(M) = R \parallel (H(K \parallel R) \oplus M)$ , where  $R$  is a random  $\lambda$ -bit string, and  $H$  is a hash function with output length equal the message length, to be modeled as a random oracle in the proof. (As above, the total ciphertext length is denoted as  $s$ .) Moreover, we also instantiate  $F'$  using the same hash function  $H$ , letting  $F'(K_{w,0}, i) = H(K_{w,0} \parallel \langle i \rangle)$ , where  $\langle i \rangle$  is a binary encoding of the integer  $i \in \mathbb{N}$ .

In the proof, the simulator  $\mathcal{S}$  handles the random oracle queries, setting  $H(x)$  to a random value whenever handling a query on input  $x \in \{0, 1\}^*$ . Moreover, when the attacker chooses an index  $\text{DB}$ ,  $\mathcal{S}$  is given  $N = 2^t$  and for all  $i = 0, 1, \dots, t-1$ , adds  $2^{t-i}$  pairs  $(l, d)$  to the set  $L_i$ , where  $l$  is a random  $\ell$ -bit label and  $d$  is a random  $(2^i \cdot s)$ -bit string. It then generates  $\text{EDB}$  as the concatenation of the hash table created from  $L_0, \dots, L_{t-1}$ , and hands  $\text{EDB}$  over to the attacker. (Still,  $\mathcal{S}$  keeps  $L_0, L_1, \dots, L_{t-1}$  as its state.)

Later, upon each query  $w$  from the attacker, the simulator learns  $\text{DB}(w) = \{\text{id}_1, \dots, \text{id}_{n_w}\}$ , where  $n_w = \sum_{i=0}^{t-1} n_{w,i} \cdot 2^i$ . In this case, it generates random  $K_{w,0}$  and  $K_{w,1}$  as the corresponding token. Moreover, it sets  $c = 0$ , and for every  $i = 0, 1, \dots, t-1$ , if  $n_{w,i} = 1$ , the simulator picks a random pair  $(l, d) \in L_i$  where  $d = R_1 \parallel C_1 \parallel \dots \parallel R_{2^i} \parallel C_{2^i}$ , removes  $(l, d)$  from  $L_i$ , and *programs* the random oracle so that

$$H(K_{w,0} \parallel \langle i \rangle) = l, \quad H(K_{w,1} \parallel R_j) \oplus C_j = \text{id}_{c+j} \text{ for all } j = 1, \dots, 2^i,$$

and adds  $2^i$  to  $c$ . If the programming cannot succeed (because the corresponding values are already set for  $H$ ), the simulator aborts.

We omit a formal analysis that the above is a good simulation strategy, as it follows from standard techniques. Overall, we obtain the following theorem.

**Theorem 5.2 (Adaptive Security of  $\Pi$ .)** *The above hash-based instantiation of the SSE-scheme  $\Pi$  is  $\mathcal{L}_{\min}$ -SIM-secure in the random oracle model if  $F$  is a pseudorandom function.*

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