Feasible Attack on the 13-round AES-256

Alex Biryukov and Dmitry Khovratovich

University of Luxembourg

alex.biryukov@uni.lu, dmitry.khovratovich@uni.lu

Abstract. In this note we present the first attack with feasible complexity on the 13-round AES-256. The attack runs in the related-subkey scenario with four related keys, in 2^{76} time, data, and memory.

1 Introduction

The year 2009 saw significant improvements in the cryptanalysis of Advanced Encryption Standard. The following results were presented: practical distinguisher for AES-256 in the chosen-key model[3], boomerang attacks on the full-round AES-192 and AES-256 [2], practical complexity attacks on AES-256 with up to 10 rounds [1].

In this paper we consider related-key boomerang attacks in the secret-key model and exploit the related-key weaknesses in AES, that were extensively described in previous works.

We advance to the following results. First, we provide the first attack on a 13-round AES-256 with complexity feasible in the real world. The best feasible attack so far was given on a 10-round version and hypothesized on a 11-round version. Our attack has 2^{76} time and data complexity, which is also significantly lower than $2^{99.5}$ complexity of the attack on the full 14-round AES-256.

Attack	Rounds	# keys	Data	Time	Memory	Source
Partial sums	9	256	2^{85}	2^{226}	2^{32}	[4]
Related-key differential	10	2	2^{44}	2^{45}	2^{33}	[1]
Related-key differential	11	2	2^{70}	2^{70}	2^{33}	[1]
Related-key boomerang	13	4	2^{76}	2^{76}	2^{76}	This paper
Related-key differential	14	2^{35}	2^{131}	2^{131}	2^{65}	[3]
Related-key boomerang	14	4	$2^{99.5}$	$2^{99.5}$	2^{77}	[2]

Table 1. Best attacks on AES-256 in the secret-key model.

2 Attack on AES-256

In this section we present a related key boomerang attack on AES-256.

2.1 The trail

The boomerang trail is depicted in Figure 2, and the actual values are listed in Tables 3 and 2. It consists of two subtrails: the first one covers rounds 1–8, and the second one covers rounds 8–13. The switching state is the state A^8 (internal state after the SubBytes in round 8) and a special key state K_S, which is the concatenation of the last four columns of K^4 and the first four columns of K^5. Although there is an active S-box in the first round of the key schedule, we do not impose conditions on it. As a result, the difference in column 0 of K^0 is partly unknown.

Related keys We define the relation between four keys as follows (see Figure 1 for the illustration). For a secret key K_A, which the attacker tries to find, compute its second subkey K_A^1 and apply the difference ΔK^1 to get a subkey K_B^1, from which the key K_B is computed. The switch into the keys K_C, K_D happens between the 3rd and the 4th subkeys in order to minimize the number of active S-boxes in the key-schedule using the *Ladder switch* idea described above. We compute subkeys K^3 and K^4 for both K_A and K_B. We add the difference ∇K^3 to K_A^3 and compute the upper half (four columns) of K_C^3. Then we add the difference ∇K^4 to K_A^4 and compute the lower half (four columns) of K_C^4. From these eight consecutive columns we compute the full K_C. The key K_D is computed from K_B in the same way.

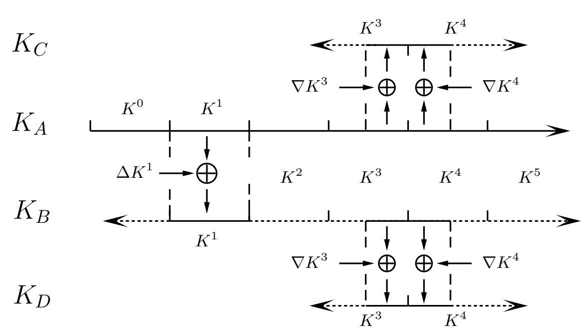


Fig. 1. Computing K_B, K_C, and K_D from K_A.

Finally, we point out that difference between K_C and K_D can be computed in the backward direction deterministically since we apply the *Feistel trick*. The secret key K_A, and the three keys K_B, K_C, K_D computed from K_A as described above form a proper related key quartet. Moreover, due to a slow diffusion in the backward direction, as a bonus we can compute some values in ∇K^i even for $i = 0, 1, 2, 3$ (Table 2). Hence given the byte value $k^l_{i,j}$ for K_A we can partly compute K_B, K_C and K_D.

Internal state The plaintext difference is specified in 7 bytes. We require that all the active S-boxes in the internal state should output the difference `0x1f` so that the active S-boxes are passed with probability 2^{-6}. The only exception is the first round where the input differences in four of seven active bytes are not specified.

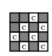
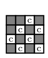
Let us start a boomerang attack with a random pair of plaintexts that fit the trail after two rounds. Active S-boxes in rounds 3–7 are passed with probability 2^{-6} each so the overall probability is 2^{-18}.

We switch the internal state in round 8 with the *Ladder switch* technique: the row 1 is switched before the application of S-boxes, and the other rows are switched after the S-box layer. As a result, we do not pay for the active S-boxes at all in this round.

The second part of the boomerang trail is quite simple. Three S-boxes in rounds 10–13 contribute to the probability, which is thus equal to 2^{-18}. Finally, a right pair after the second round produces a boomerang quartet with probability $2^{-18-18-18-18} = 2^{-72}$.

2.2 The attack

Repeat the following steps four times.

1. Prepare a structure of 2^{72} plaintexts each  .
2. Encrypt it on keys K_A and K_B and keep the resulting sets S_A and S_B in memory.
3. XOR Δ_C to all the ciphertexts in S_A and decrypt the resulting ciphertexts with K_C. Denote the new set of plaintexts by S_C.
4. Repeat previous step for the set S_B and the key K_D. Denote the set of plaintexts by S_D.
5. Compose from S_C and S_D all the possible pairs of plaintexts which are equal in 56 bits  .
6. For every remaining pair check if the difference in $p_{0,0}$ is equal on both sides of the boomerang quartet (8-bit filter). Note that $\nabla k^0_{1,7} = 0$ so $\Delta k^0_{0,0}$ should be equal for both key pairs (K_A, K_B) and (K_C, K_D).
7. For every remaining quartet try all 2^{28} values for ΔB^1 (2^{14} for each related-key pair):
   - Compute both ΔA^1. Check if ΔA^1 is admissible for ΔP (one-bit condition for each of 16 positions).

– Given ΔA^1 and ΔP, every plaintext row i proposes two candidates for each of the two key bytes in both related-key pairs. Since the ∇ difference is equal in all the row bytes, this is an 8-bit equation on the key bytes. Therefore, this is a 4-bit filter for each row, or a 16-bit filter in total. As a result, we get a four-bit filter on the quartets and leave with the only possible combination of ΔB^1.

8. Each remaining quartet proposes an 8-byte key candidate for K_A and, independently, a 4-byte key candidate for K_C.

Finally, choose the key candidate that is proposed by four quartets.

Each structure has all possible values in 9 bytes, and constant values in the other bytes. Of 2^{72} texts per structure we can compose 2^{144} ordered pairs. Of these pairs $2^{144-8\cdot 9} = 2^{72}$ pass the first round. Thus we expect one right quartet per structure, or four right quartets in total.

Let us compute the number of candidate quartets. We can compose 2^{146} quartets from the initial structures, of which 2^{80} quartets come out of step 6. Then we apply a 4-bit filter so that there remains 2^{76} candidates, each proposing a 12-byte key candidate. It is highly likely that only the right quartets propose the same candidate. We also point out, that each quartet propose two candidates for $k^0_{1,7}$, which defines $\Delta p_{0,0}$. The most time-consuming filtering part is the processing of 2^{80} candidate quartets, which is equivalent to about 2^{74} AES encryptions.

Therefore, we recover 71 key bits with 2^{74} chosen plaintexts and ciphertexts, and time equivalent to 2^{76} encryptions. The remaining key bits can be found using our partial knowledge of the key and using slightly different key relations.

References

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ΔK^i					
0	? 00 00 00 3e 3e 3e 3e ? 01 01 01 ? 1f 1f 1f ? 00 00 00 1f 1f 1f 1f ? 00 00 00 21 21 21 21	1	00 00 00 00 3e 00 3e 00 00 01 00 01 1f 00 1f 00 00 00 00 00 1f 00 1f 00 00 00 00 00 21 00 21 00	2	00 00 00 00 3e 3e 00 00 00 01 01 00 1f 1f 00 00 00 00 00 00 1f 1f 00 00 00 00 00 00 21 21 00 00
3	00 00 00 00 3e 00 00 00 00 01 00 00 1f 00 00 00 00 00 00 00 1f 00 00 00 00 00 00 00 21 00 00 00	4	01 01 01 01 ? ? ? ? 00 00 00 00 1f 1f 1f 1f 00 00 00 00 1f 1f 1f 1f 00 00 00 00 21 21 21 21		

∇K^i					
0	X X X X ? ? ? 00 Y Y Y Y 01 01 01 00 Z Z Z Z 01 01 01 00 T T T T 03 03 03 00	1	X ab X 00 ? ? 00 00 Y 00 Y 00 01 01 00 00 Z 00 Z 00 01 01 00 00 T 00 T 00 03 03 00 00	2	X X 00 00 ? 00 00 00 Y Y 00 00 01 00 00 00 Z Z 00 00 01 00 00 00 T T 00 00 03 00 00 00
3	X ab ab ab 02 02 02 02 Y 00 00 00 01 01 01 01 Z 00 00 00 01 01 01 01 T 00 00 00 03 03 03 03	4	ab 00 ab 00 02 00 02 00 00 00 00 00 01 00 01 00 00 00 00 00 01 00 01 00 00 00 00 00 03 00 03 00	5	ab ab 00 00 02 02 00 00 00 00 00 00 01 01 00 00 00 00 00 00 01 01 00 00 00 00 00 00 03 03 00 00
6	ab 00 00 00 02 00 00 00 00 00 00 00 01 00 00 00 00 00 00 00 01 00 00 00 00 00 00 00 03 00 00 00	7	ab ab ab ab ? ? ? ? 00 00 00 00 01 01 01 01 00 00 00 00 01 01 01 01 00 00 00 00 03 03 03 03		

Table 2. Subkey difference in the 13-round trail.

ΔP	? ? 3e ? ? 1f ? 1f 1f ? 1f ? ? 21 ? 21	ΔA^1	00 ? 00 ? ? 00 ? 00 00 ? 00 ? ? 00 ? 00	ΔA^2	00 00 00 00 00 1f 00 1f 00 00 00 00 00 00 00 00	ΔA^3 ΔA^5 ΔA^7	00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
ΔA^4	00 00 00 00 00 1f 1f 00 00 00 00 00 00 00 00 00	ΔA^6	00 00 00 00 00 1f 00 00 00 00 00 00 00 00 00 00	ΔA^8	00 00 00 00 00 ? ? ? 00 00 00 00 00 00 00 00	∇A^9 ∇A^{11} ∇A^{13}	00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
∇A^8	01 00 01 00 00 00 00 00 00 00 00 00 00 00 00 00	∇A^{10}	01 01 00 00 00 00 00 00 00 00 00 00 00 00 00 00	∇A^{12}	01 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00	ΔC	00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00

Table 3. Internal state difference in the 13-round trail.

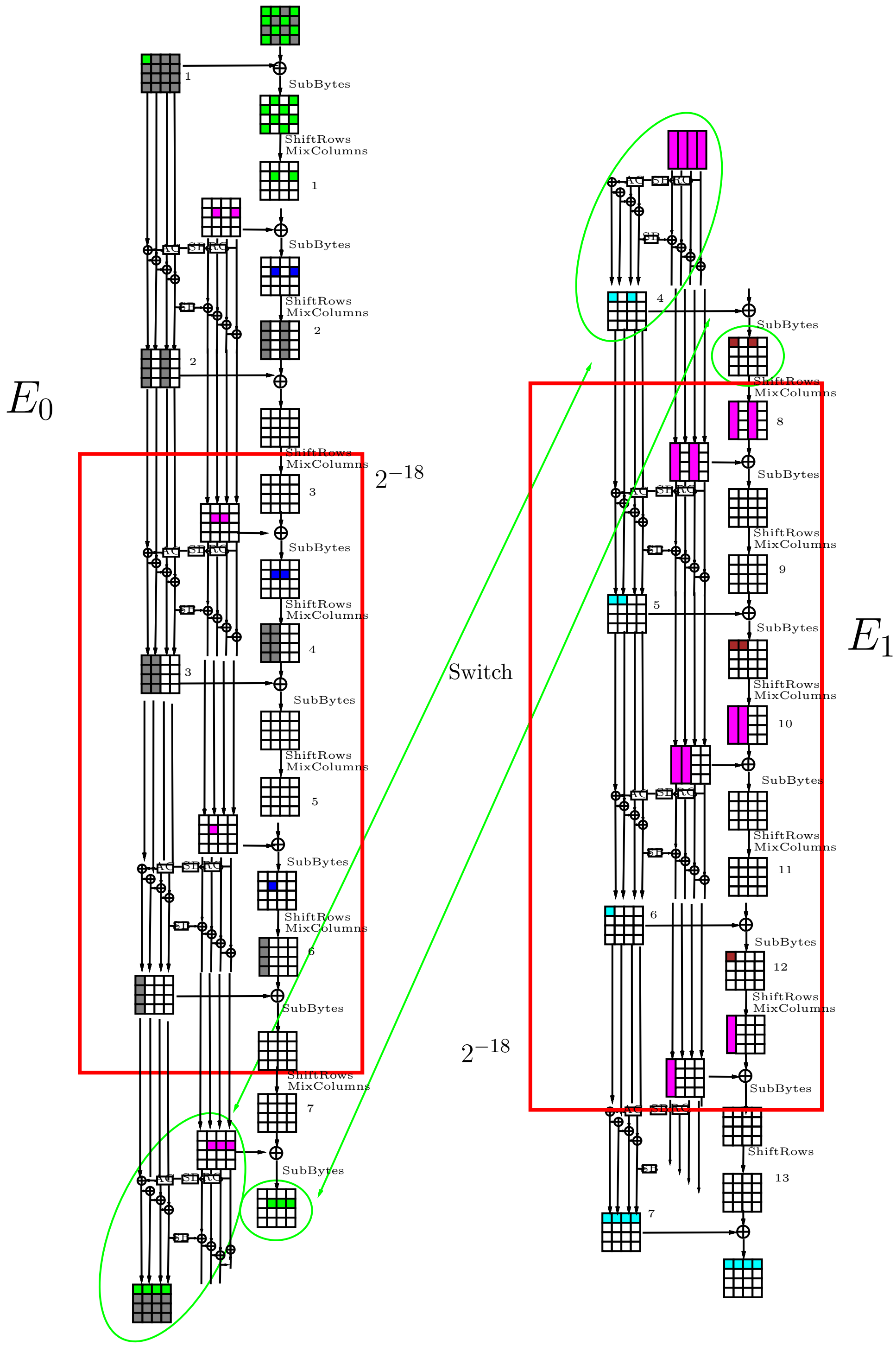

Fig. 2. AES-256 E_0 and E_1 trails. Green ovals show an overlap between the two trails where the switch happens.