Efficient Confirmer Signatures from the “Signature of a Commitment” Paradigm

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Abstract. Generic constructions of designated confirmer signatures follow one of the following two strategies; either produce a digital signature on the message to be signed, then encrypt the resulting signature, or produce a commitment on the message, encrypt the string used to generate the commitment and finally sign the latter. We study the second strategy by determining the exact security property needed in the encryption to achieve secure constructions. This study infers the exclusion of a useful type of encryption from the design due to an intrinsic weakness in the paradigm. Next, we propose a simple method to remediate to this weakness and we get efficient constructions which can be used with *any* digital signature.

Keywords: Designated Confirmer signatures, “Signature of a commitment” paradigm, Generic construction, Reduction/meta-reduction, Zero Knowledge.

1 Introduction

Digital signatures were introduced in [17] as an analogous to signatures in the paper world to seize most properties needed in a signature, for instance, the universal verification. However, in some applications, the signer might want to restrain the holder of a signature from convincing other parties of the validity of the signature in question. A typical example is a software vendor willing to embed signatures in his products such that only paying customers are entitled to check the authenticity of these products. Undeniable signatures, introduced in [12], provide a good solution to this problem as they are: 1. only verified with the help of the signer, 2. non transferable, 3. binding in the sense that a signer cannot deny a signature he has actually issued. The only drawback of these signatures is that unavailability of the signer obstructs the entire process. To overcome this problem, designated confirmer signatures were introduced in [10], where the confirmation/denial of a signature is delegated to a *designated confirmer*. With this solution, the signer can confirm only signatures he has just generated, whilst the confirmer can confirm/deny any signature. Actually, in the literature, there is a clear separation between confirmer signatures and *directed signatures* [27], which share the same concept as confirmer signatures with the exception of allowing both the signer and the confirmer to confirm/deny signatures. Finally, a desirable property in designated confirmer signatures is the convertibility of the signatures to ordinary ones. Indeed, such a property turned out to play a central role in fair payment protocols [6].

1.1 Related work

Since the introduction of confirmer signatures, a number of attempts have been made to produce them from basic primitives. Most such proposals fall into one of the following two categories:

“Encryption of a signature” approach. This approach consists in first producing a digital signature on the message to be signed, then encrypting the produced signature using a suitable cryptosystem. The construction was first formally [1] described in [8], and required the components to meet the highest security notions (EUF-CMA signatures and IND-CCA encryption). The main weakness of the construction lies in the resort to concurrent zero knowledge (ZK) protocols of general NP statements in the confirmation/denial protocol. Later, the construction in [23] managed to circumvent the problem by encrypting

[1] The idea without proof was already known, for instance, it was mentioned in [15].

the digital signature during the confirmation protocol. With this trick, the authors managed to get rid of concurrent ZK proofs of general NP statements in the confirmation protocol (the denial protocol still suffers the recourse to such proofs), but at the expense of the security and the length of the resulting signatures. Another construction implementing this principle is given in [14]; the construction uses cryptosystems with labels and is analyzed in a more elaborate security model. However, it is supplied with only one efficient instantiation as the confirmation/denial protocols still resort to concurrent ZK protocols of general NP statements. Finally, the last proposal in this category is given in [26] , where the author proposes a construction using certain cryptosystems that are required to be only IND-CPA secure. As a consequence, the confirmation/denial protocols are rendered efficient in case the construction is instantiated from a specific class of signature schemes (similar to the one considered in [23]). Moreover, the resulting confirmer signatures are very efficient (small generation/verification/conversion cost and short signatures due to IND-CPA encryption) and they enjoy a reasonable security [2]. However, although the considered class of digital signatures includes most proposals that appeared in the literature, there exists some schemes which do not seem to belong to it, e.g., the PSS signature scheme [3].

"Signature of a commitment" approach. This technique consists in generating a commitment on the message to be signed, then signing the produced commitment using a digital signature scheme. The confirmer signature is comprised of both the commitment and the signature. The first proposal that realizes this principle is [29] where a construction of confirmer signatures from digital signatures obtained from the Fiat-Shamir paradigm is presented. Thus, the resulting confirmer signatures can be only proven secure in the random oracle model (ROM), inheriting this property from the use of the Fiat-Shamir paradigm, which constitutes their major shortcoming. Actually, it is well known, according to [36], that most discrete-logarithm-based signatures obtained from the Fiat-Shamir technique are very unlikely to preserve the same level of security in the standard model. Moreover, the construction does not support the conversion of the confirmer signatures. In [20] and [39], a construction which supports the conversion of the signatures and applies to *any* digital signature scheme was proposed. The key idea behind the proposal resides in augmenting the confirmer signature (comprised of the commitment and a signature on it) by the encryption of the random string used to generate the commitment. Although the confirmation/denial protocols involve general ZK proofs since the confirmer has to prove in concurrent ZK the knowledge of the decryption of an IND-CCA encryption and of a string used for commitment, the construction accepts an efficient instantiation using Camenisch-Shoup's verifiable encryption scheme [9] and Pedersen's commitment scheme. It is worth mentioning that the idea underlying the constructions in [20] and [39] already existed under the name "Commit then Encrypt then Sign Paradigm", and was used in the context of signcryption in [1].

To finish the exhaustive list of constructions of confirmer signatures, we must cite the first construction due to Okamoto [32], which was used to prove equivalence between confirmer signatures and public key encryption with respect to existence. Thus, efficiency was not taken into account in the framework. There is also the construction [30] which uses an undeniable signature among its building blocks and provides a restricted security (under lunch time attacks) in the ROM. Finally, In [39], the authors proposed a second construction which does not require any encryption, but at the expense of the underlying security assumption. In fact, it has its invisibility resting on the decisional Diffie-Hellman assumption, which rules out using the scheme in bilinear groups and thus benefiting from the attractive features they present such as achieving short group elements. Moreover, the construction suffers also the recourse to the ROM.

[2] The signatures in [26] are hard to distinguish from random elements in the signature space for an adversary knowing only the public key of the signer (outside security model). However, the signatures in [8] are only difficult to distinguish based on the underlying message, but in the insider security model, e.g., for an adversary who might get hold of the private key of the signer.

In this paper, we revisit the second approach, namely the "signature of a commitment" method. In fact, efficient as the first approach is, it still applies only to a restricted class of signatures. This is clearly manifested in the constructions in [23] or [26] which do not seem to be plausible with the signature PSS [3]. Our goal is to further improve the "commit then sign" method in terms of efficiency (signature length and cost) and security by allowing more efficient instantiations of the encryption and commitment schemes used as building blocks.

1.2 Contributions and key ideas

We make three contributions. First, we revisit the constructions implementing the "signature of a commitment" paradigm, namely those provided in [20, 39]. We prove that indistinguishable cryptosystems under a plaintext checking attack (IND-PCA secure) are necessary and sufficient to obtain secure confirmer signatures. Our approach is similar to the one provided in [26] to study the "encryption of a signature" technique. In fact, we first exclude OW-CCA encryption from the design, which rules out the weaker notions that are OW-CPA and OW-PCA. We do this by means of an efficient tool, called meta-reductions, which was used in a number of important cryptographic results [5, 36, 35, 34]. Then, we exclude the NM-CPA notion by a similar technique, which again rules out the IND-CPA notion. The notion that has to be considered next is IND-PCA which luckily turns out to be sufficient to achieve secure constructions. We conclude that, although we mange to weaken the assumption on the encryption from IND-CCA (needed in [1, 20, 39]) to IND-PCA, the construction still cannot allow homomorphic encryption in the design since a homomorphic cryptosystem can never be IND-PCA secure. This is unfortunate since such an encryption proved to be efficient decryption verifiable (see [26] for an illustration), i.e., possesses efficient ZK protocols for proving the knowledge of the plaintext underlying a given ciphertext, and such a property in profoundly needed in the confirmation/denial protocols.

In the second contribution, we tackle the problem of homomorphic encryption in the design; we show that using a small trick that consists in producing the digital signature on the commitment *concatenated with the encryption of the string used in the commitment* suffices to make the security needed in the encryption drop drastically to being only IND-CPA secure. The key idea is to remark that the original construction is not strongly unforgeable, i.e., one can produce a valid confirmer signature without the help of the signer, which explains the need for a plaintext checking oracle (PCA security) to handle such signatures. With the small trick, we are able to annihilate this weakness and allow a weak encryption in the design without compromising the overall security. As a result, we achieve better performances that manifest in a short signature, a small signature generation, verification and conversion cost, and finally more efficient instantiations of the construction (instead of using only Camenisch-Shoup encryption and Pedersen commitment) by allowing homomorphic encryption.

Finally, our last contribution sheds light on a particular sub-case of the "signature of a commitment" paradigm, which consists in using IND-CPA encryption instead of the commitment scheme. In fact, it is well known that IND-CPA encryption yields secure commitment schemes, which makes such an instantiation plausible. However, the bright side of this technique consists in not requiring the encryption of the random string anymore. Thus, a confirmer signature on a given message can be achieved by encrypting the message to be signed, then producing a digital signature on this encryption. The pair consisting of the encryption and the resulting signature forms the confirmer signature on the message. This method clearly improves the original paradigm, however it necessitates efficient non-interactive proofs of knowledge. This is no longer a problem nowadays due to the progress of research made recently in this area, e.g., [16, 25].

We stress that all our constructions of confirmer signatures in the present paper investigate the invisibility (the hardness of distinguishing signatures based on the underlying messages) of the resulting signatures in the *outsider* security model. I.e., we disallow the adversary to know the private key of the signer, oppositely to the *insider* security model considered in [1, 20, 39]. In other words, we supplement the study of the "signature of a commitment" paradigm given in [1, 20, 39], by providing a security analysis in the outsider security model, exactly as the work in [26] explores the security of "encryption of a signature" paradigm in the outsider model, after it has been considered in [8, 1] in the insider model. However, our constructions contrarily to those in [1, 20, 39] allow the signer to sign the same message many times without loss of invisibility. This property is deeply needed in liscencing software, which is one of the main applications of confirmer signatures.

2 Convertible Designated Confirmer Signatures (CDCS)

In this section, we present the model of CDCS we adhere to in our constructions. We refer to Appendix A for the necessary cryptographic primitives that will come into use, that are, digital signatures, public key encryption schemes, commitment schemes, and finally Σ protocols.

2.1 Syntax

A CDCS scheme consists of the following procedures:

Key generation. Generates probabilistically key pairs $(\mathsf{sk}_S, \mathsf{pk}_S)$ and $(\mathsf{sk}_C, \mathsf{pk}_C)$ for the signer and for the confirmer respectively, consisting of the private and of the public key.

ConfirmedSign. On input sk_S, pk_C and a message m, outputs a confirmer signature signature μ, then interacts with the signature recipient to convince him of the validity of the just generated signature.

Confirmation/Denial protocol. These are interactive protocols between the confirmer and a verifier. Their common input consists of, in addition to pk_S and pk_C, the alleged signature μ, and the message m in question. The confirmer uses his private key sk_C to convince the verifier of the validity (invalidity) of the signature μ on m. At the end, the verifier either accepts or rejects the proof.

Selective conversion. This is an algorithm run by the confirmer using sk_C, in addition to pk_C and pk_S. The result is either $\perp$ or a string which allows the signature to be universally verified as a valid digital signature.

Selective verification. This is an algorithm for verifying converted signatures. It inputs the converted signature, the message and pk_S and outputs either 0 or 1.

Remark 1. In [20, 39], the authors give the possibility of obtaining *directly* digital signatures on a given message. We find this unnecessary since it is already enough that a CDCS scheme supports the convertibility feature. Moreover, in [14], the author considers a further protocol used by the confirmer to prove the correctness of the conversion. We show in Appendix H that all the constructions provided in this paper extend readily to this augmented model.

2.2 Security model.

The above algorithms and protocols must be complete. Moreover, a CDCS scheme should meet the following properties:

Security for the verifier. This property informally means that an adversary who compromises the private keys of both the signer and the confirmer cannot convince the verifier of the validity (invalidity) of an invalid (a valid) confirmer signature. That is, the protocols confirmedSign, confirmation and denial are *sound*. We refer to [20, 39] for the formal definition of such a requirement.

Non Transferability of the ConfirmedSign/confirmation/denial protocols. This property requires that the transcript resulting from the interaction of the verifier with the signer/confirmer during these protocols is indistinguishable from the transcript resulting from the interaction of the verifier with a simulator (which can be rewound) which does not have the private inputs of the signer/confirmer but is allowed to one oracle call to learn the validity/invalidity of the alleged signature w.r.t. the message in question. We refer to [8] for the formal definition (after considering the fix proposed by [14], namely, the possibility of rewinding the simulator).

Security for the signer (unforgeability). It is defined through the following game: the adversary $\mathcal{A}$ is given the public parameters of the CDCS scheme, namely pk_S and pk_C, the public key of the signer and of the confirmer resp, in addition to the private key sk_C of the confirmer. $\mathcal{A}$ is further allowed to query the signer on polynomially many messages, say q_s. At the end, $\mathcal{A}$ outputs a pair consisting of a message m, that has not been queried yet, and a string μ. $\mathcal{A}$ wins the game if μ is a valid confirmer signature on m. We say that a CDCS scheme is (t, ϵ, q_s)-EUF-CMA secure if there is no adversary, operating in time t, that wins the above game with probability greater than ϵ, where the probability is taken over the random choices of both $\mathcal{A}$ and his challenger.

Security for the confirmer (invisibility). Invisibility against a chosen message attack (INV1-CMA) is defined through the following game between an attacker $\mathcal{A}$ and his challenger $\mathcal{R}$: after $\mathcal{A}$ gets the public parameters of the scheme from $\mathcal{R}$, he starts **Phase 1** where he queries the confirmedSign, confirmation/denial, selective conversion oracles in an adaptive way. Once $\mathcal{A}$ decides that **Phase 1** is over, he outputs two messages m_0, m_1 and requests a challenge signature $\mu^\star$. $\mathcal{R}$ picks uniformly at random a bit $b \in \{0, 1\}$. Then $\mu^\star$ is generated using the confirmedSign oracle on the message m_b. Next, $\mathcal{A}$ starts adaptively querying the previous oracles (**Phase 2**), with the exception of not querying $(m_i, \mu^\star)$, $i = 0, 1$, to the confirmation/denial and selective conversion oracles. At the end, $\mathcal{A}$ outputs a bit b'. He wins the game if $b = b'$. We define $\mathcal{A}$'s advantage as $\mathsf{adv}(\mathcal{A}) = |\Pr[b = b'] - \frac{1}{2}|$, where the probability is taken over the random coins of both $\mathcal{A}$ and his challenger. We say that a CDCS scheme is $(t, \epsilon, q_s, q_v, q_{sc})$-INV1-CMA secure if no adversary operating in time t, issuing q_s queries to the confirmedSign oracle, q_v queries to the confirmation/denial oracles and q_{sc} queries to the selective conversion oracle wins the above game with advantage greater that ϵ.

We have the following remarks regarding our security model

Remark 2.
- Our definition of security for the verifier and non transferability of the confirmedSign, confirmation and denial protocols is the same provided in [8, 20, 39].
- We consider the *insider security model* in our definition for unforgeability. I.e., the unforgeability adversary has the private key of the confirmer at his disposal. This is justified by the need of preventing the confirmer from impersonating the signer by issuing valid signatures on his behalf.
- Our definition of invisibility, oppositely to the definitions in [8, 20, 39], is considered in the *outsider security model*. I.e., the adversary does not know the private key of the signer. We justify this by considering the CDCS scheme broken if the signer is corrupted or coerced. Actually, insider security might be needed in situations where we want to protect the invisibility of signatures issued by the genuine signer from an adversary who has stolen this signer's private key. However *outsider security might be all one needs* for invisibility as quoted by the authors in [1].
- Our definition of invisibility, oppositely to the definitions in [20, 39], allows the signer to sign the same message many times without loss of invisibility, which is profoundly needed in liscencing software.
- Finally our definition of invisibility, like the definitions in [8, 20, 39] and unlike the definition in [19], does not guarantee the non transferability of the signatures. I.e., the confirmer signature might convince the recipient that the signer was involved in the signature of some message. We refer to the discussion in [20] (Section 3) for techniques that can be used by the signer to camouflage the presence of valid signatures.

3 The Plain "Signature of a Commitment" Paradigm

This paradigm was first considered in [29] to build confirmer signatures from signatures obtained using the Fiat-Shamir paradigm. The main criticism to such a construction lies in the resort to the ROM (resulting from the use of the Fiat-Shamir Paradigm) and the non support of the convertibility feature. In [1] upgraded this technique to the "Encrypt then Commit then Sign" method, which consists in first generating a random string, say r and encrypting it in e, then using r to generate a commitment c on the message to be signed, and finally produce a digital signature on the commitment c. This approach was used in the context of signcryption in [1] and was analyzed in the insider security model. Later in [20], the authors used it to build confirmer signatures and provided an efficient instantiation using Camenisch-Shoup [9]'s encryption and and Pedersen's commitment. The resulting construction was shown to be invisible in the insider security model if the underlying commitment is hiding and the underlying encryption is IND-CCA secure. However, the authors in [39] disproved this claim by exhibiting an attack against the invisibility of the construction regardless of the underlying encryption: Given the challenge signature (e, c, σ) on the message m_b, where $b \in \{0, 1\}$ and m_0, m_1 are the challenge messages output by the invisibility adversary $\mathcal{A}$, the latter computes a commitment c' such that the underlying message m' is meaningfully related to m_0, m_1 ($m' = k + m_b - m_0$, where k is known to $\mathcal{A}$) and the underlying random string is the same used to create c. Such a construction is possible using Pedersen's commitment. Next, $\mathcal{A}$ produces a digital signature σ' on c' (this is possible in the insider security model) and queries the conversion oracle on (e, c', σ') and the message k; if the oracle answers $r \neq \perp$, then $\mathcal{A}$ outputs $b = 0$, otherwise if the oracle answers $\perp$, the oracle answers $b = 1$. The authors in [39] proposed a fix to this construction which consists in using *cryptosystems with labels* (see Appendix A).

In the rest of this section, we describe the construction of [39] and we analyze its invisibility in the outsider security model.

3.1 The construction in [39]

Setup. Consider a digital signature scheme Σ, an encryption scheme Γ with labels and a commitment scheme Ω.

Key generation. The signer key pair consists of $(\Sigma.\mathsf{pk}, \Sigma.\mathsf{sk})$, corresponding to the key pair of the signature scheme Σ, whereas the confirmer key pair consists of $(\Gamma.\mathsf{sk}, \Gamma.\mathsf{sk})$ which corresponds to the key pair related to Γ.

ConfirmedSign. To sign a message m, the signer first computes a commitment c on the message, then encrypts in e, under the label $m \| \Sigma.\mathsf{pk}$, the random string used for the commitment, say r, and finally, signs the commitment c using $\Sigma.\mathsf{sk}$. The confirmer signature consists of the triple $(e, c, \Sigma.\mathsf{sign}_{\Sigma.\mathsf{sk}}(c))$. Next, the signer interacts with the verifier in a protocol where he (the signer) proves in ZK the knowledge of r such that $r = \Gamma.\mathsf{decrypt}_{\Gamma.\mathsf{pk}, m\|\Sigma.\mathsf{pk}}(e)$ and $c = \Omega.\mathsf{commit}(m, r)$. Such a proof is plausible to issue using the randomness used to encrypt r in e. In fact, the encryption and commitment algorithms in a cryptosystem and a commitment scheme resp define an NP language that accepts a zero knowledge proof system.

Confirmation/Denial protocol. To confirm/deny a signature $\mu = (\mu_1, \mu_2, \mu_3)$ on a given message m, the confirmer first checks whether μ_3 is a valid digital signature on μ_2 w.r.t. $\Sigma.\mathsf{pk}$, if so, he provides a concurrent ZK proof (using his private key $\Gamma.\mathsf{sk}$) of the equality/inequality of the decryption of μ_1 and the opening value of the commitment μ_2 w.r.t. m. Again this proof is possible since every NP (co-NP in case of inequality) language accepts a zero knowledge proof system

Selective conversion. Selective conversion of a signature $\mu = (\mu_1, \mu_2, \mu_3)$ is achieved by releasing the decryption of μ_1, in case μ is valid, or the symbol $\perp$ otherwise.

This construction was shown, in [39], to provide security for the verifiers, non-transferability of the involved protocols, insider security for the signer if it uses a EUF-CMA secure digital signature and a binding commitment scheme, and finally it provides insider security for the confirmer if it uses IND-CCA secure encryption and a hiding commitment.

In the rest of this section, we prove that IND-PCA cryptosystems with labels are necessary and sufficient to obtain outsider security for the confirmer if the underlying commitment scheme is secure and the underlying signature is SEUF-CMA secure.

3.2 The exact invisibility of the construction

In this subsection, we prove that IND-PCA cryptosystems with labels are necessary and sufficient to achieve invisible signatures. Our study is similar to the one provided in [26] which analyzes the plain "encryption of a signature" paradigm. Thus, we will first exclude OW-CCA secure cryptosystems with labels from use, which will rule out automatically OW-CPA and OW-PCA cryptosystems. We do this using an efficient algorithm (a *meta-reduction*) which transforms an algorithm (*reduction*), reducing the invisibility of the confirmer signatures to the OW-CCA security of the underlying cryptosystem, to an algorithm breaking the OW-CCA security of the same cryptosystem. Hence, such a result suggests that under the assumption of the underlying cryptosystem being OW-CCA secure, there exists no such a reduction, or if it (the cryptosystem) is not OW-CCA secure, such a reduction will be useless. Next, we exclude similarly NM-CPA cryptosystems from the design, which will rule out IND-CPA encryption. The next security notion that has to be considered is IND-PCA, which turns out to be sufficient to achieve invisibility. Likewise, our impossibility results are in a first stage partial in the sense that they apply only to *key preserving* reductions, i.e., reductions which, trying to attack a property of a cryptosystem given by the public key pk, feed the invisibility adversary with the confirmer public key pk. Next, we extend the result to arbitrary reductions under some complexity assumptions on the cryptosystem in question.

Lemma 1. *Assume there exists a key-preserving reduction $\mathcal{R}$ that converts an INV1-CMA adversary $\mathcal{A}$ against the above construction into a OW-CCA adversary against the underlying cryptosystem. Then, there exists a meta-reduction $\mathcal{M}$ that OW-CCA breaks the cryptosystem in question.*

As mentioned in the discussion above, the lemma claims that under the assumption of the underlying cryptosystem being OW-CCA secure, there exists no key-preserving reduction $\mathcal{R}$ that reduces OW-CCA breaking the cryptosystem in question to INV1-CMA breaking the construction, or if there exists such an algorithm, the underlying cryptosystem is not OW-CCA secure, thus rendering such a reduction useless.

Proof. Let $\mathcal{R}$ be the key-preserving reduction that reduces the invisibility of the construction to the OW-CCA security of the underlying cryptosystem. We construct an algorithm $\mathcal{M}$ that uses $\mathcal{R}$ to OW-CCA break the same cryptosystem by simulating an execution of the INV1-CMA adversary $\mathcal{A}$ against the construction.

Let Γ be the cryptosystem $\mathcal{M}$ is trying to attack w.r.t. a public key $\Gamma.\mathsf{pk}$. $\mathcal{M}$ launches $\mathcal{R}$ over Γ with the same public key $\Gamma.\mathsf{pk}$. After $\mathcal{M}$ gets the label L on which $\mathcal{R}$ wishes to be challenged, he ($\mathcal{M}$) forwards it to his own challenger. Finally, $\mathcal{M}$ gets a challenge ciphertext c, that he forwards to $\mathcal{R}$. Note that $\mathcal{M}$ is allowed to query the decryption oracle on any pair (ciphertext,label) except on the pair (c, L). Thus, all decryption queries made by $\mathcal{R}$, which are by definition different from the challenge (c, L), can be forwarded to $\mathcal{M}$'s own challenger. At some point, $\mathcal{M}$, acting as an INV1-CMA attacker against the construction, will output two messages m_0, m_1 such that $L \notin \{m_0\|\Sigma.\mathsf{pk}, m_1\|\Sigma.\mathsf{pk}\}$, where $\Sigma.\mathsf{pk}$ is the public key of the digital signature underlying the construction. $\mathcal{M}$ gets as response a challenge signature $\mu^\star = (\mu_1^\star, \mu_2^\star, \mu_3^\star)$ which he is required to tell to which message it corresponds. Since the messages m_0 and m_1 were chosen such that the label under which is created the encryption $\mu_1^\star$ (either $m_0\|\Sigma.\mathsf{pk}$ or $m_1\|\Sigma.\mathsf{pk}$) is different from the challenge label L, $\mathcal{M}$ can query his decryption oracle on both pairs $(\mu_1^\star, m_0\|\Sigma.\mathsf{pk})$ or $(\mu_1^\star, m_1\|\Sigma.\mathsf{pk})$. Result of such queries will enable $\mathcal{M}$ to open the commitment $\mu_2^\star$, and thus check the validity of the signature $\mu^\star$ w.r.t. to one of messages m_0 or m_1. Finally, when $\mathcal{R}$ outputs his answer, decryption of the challenge (c, L), $\mathcal{M}$ will simply forward this result to his challenger. □

Lemma 2. *Assume there exists a key-preserving reduction $\mathcal{R}$ that converts an INV1-CMA adversary $\mathcal{A}$ against the above construction to an NM-CPA adversary against the underlying cryptosystem. Then, there exists a meta-reduction $\mathcal{M}$ that NM-CPA breaks the cryptosystem in question.*

We provide the proof in Appendix B.

Thus, when the considered notions are obtained from pairing a security goal GOAL $\in \{\mathrm{OW}, \mathrm{IND}, \mathrm{NM}\}$ and an attack model ATK $\in \{\mathrm{CPA}, \mathrm{PCA}, \mathrm{CCA}\}$, we have

Theorem 1. *The cryptosystem underlying the above construction must be at least IND-PCA secure, in case the considered reduction is key-preserving, in order to achieve INV1-CMA secure signatures.* $\square$

Similarly to the study in [26], we generalize the above theorem to arbitrary reductions if the cryptosystem underlying the construction has a *non malleable key generator* (See Appendix C.1)

Theorem 2. *If the cryptosystem underlying the above construction has a non malleable key generator, then it must be at least IND-PCA secure in order to achieve INV1-CMA secure confirmer signatures.*

We provide the proof in Appendix C.2.

Remark 3. Note that the above impossibility result holds regardless of the used commitment and digital signature schemes.

One way to explain this result is to remark that the above construction is not *strongly unforgeable*. In fact, an adversary $\mathcal{A}$, given a valid signature $\mu = (\mu_1, \mu_2, \mu_3)$ on a message m, can create another valid signature μ' on m without the help of the signer as follows; $\mathcal{A}$ will first request the selective conversion of μ to obtain the decryption of μ_1, say r, which he will re-encrypt in μ_1' under the same label $m\|\Sigma.\mathsf{pk}$ ($\Sigma.\mathsf{pk}$ is the public key of the digital signature underlying the construction). Obviously $\mu' = (\mu_1', \mu_2, \mu_3)$ is also a valid confirmer signature on m that the signer did not produce, and thus cannot confirm/deny or convert without having access to a decryption oracle of the cryptosystem underlying the construction. This explains the insufficiency of notions like IND-CPA. However, we observe that an IND-CCA secure encryption is too much than needed in this framework since a query of the type μ' is not completely uncontrolled by the signer. In fact, its first component μ_1' is an encryption of some data already disclosed by the signer, namely r, and thus a plaintext checking oracle is sufficient to deal with such a query if the used digital signature is SEUF-CMA secure.

Theorem 3. *The above construction is $(t, \epsilon, q_s, q_v, q_{sc})$-INV1-CMA secure if it uses a (t, ϵ', q_s)-SEUF-CMA secure digital signature, a secure commitment and a $(t+q_s q_{sc}(q_{sc}+q_v), \epsilon\cdot(1-\epsilon')^{(q_{sc}+q_v)}, q_{sc}(q_{sc}+q_v))$-IND-PCA secure cryptosystem with labels.*

We provide the proof in Appendix D.

4 An Efficient Construction from the "Signature of a Commitment" Paradigm

A simple way to eliminate the strong forgeability in signatures from the plain "signature of a commitment" technique consists in producing a digital signature on both the commitment and the encryption of the random string used in it. In this way, the attack discussed after Remark 3 no longer applies, since an adversary will have to produce a digital signature on the commitment and the re-encryption of the random string used in it. Note that such a fix already appears in the construction of [20]. However it was not exploitable as the invisibility was considered in the insider model.

We describe the full construction in the following paragraph.

4.1 Construction

Let Σ be a signature scheme given by $\Sigma.\mathsf{keygen}$ that generates $(\Sigma.\mathsf{pk}, \Sigma.\mathsf{sk})$, $\Sigma.\mathsf{sign}$ and $\Sigma.\mathsf{verify}$. Let further Γ denote a cryptosystem given by $\Gamma.\mathsf{keygen}$ that generates $(\Gamma.\mathsf{pk}, \Gamma.\mathsf{sk})$, $\Gamma.\mathsf{encrypt}$ and $\Gamma.\mathsf{decrypt}$. We note that Γ does need to support labels in our construction. Finally let Ω denote a commitment scheme given by $\Omega.\mathsf{commit}$ and $\Omega.\mathsf{open}$. We assume that Γ produces ciphertexts of length at most some n. In the rest of this section, the expression $e\|c$, where e is a ciphertext produced by Γ and c is a commitment produced by Ω, denotes the string whose n-bit prefix is e if the bit-length of e is exactly n, otherwise the n-bit prefix is e padded with ones until the result has bit-length n. As a result, the first bit of c will always be at the $(n+1)$-st position in $e\|c$. Such a technical detail will play an important role in the unforgeability and invisibility of the construction.

The construction of confirmer signatures from Σ, Γ and Ω is given as follows.

Key generation. The signer key pair is $(\Sigma.\mathsf{pk}, \Sigma.\mathsf{sk})$ and the confirmer key pair is $(\Gamma.\mathsf{pk}, \Gamma.\mathsf{sk})$.

ConfirmedSign. On input message m, produce a commitment c on m using a random string r, encrypt this string in e and then produce a digital signature $\sigma = \Sigma.\mathsf{sign}_{\Sigma.\mathsf{sk}}(e\|c)$. Output $\mu = (e, c, \sigma)$ as a confirmer signature on m, and prove in ZK the equality of the decryption of e and the string used for the commitment c. This proof is possible using the randomness used to encrypt r in e.

Confirmation/Denial protocol. On a message m and an alleged signature $\mu = (\mu_1, \mu_2, \mu_3)$, check the validity of μ_3 on $\mu_1\|\mu_2$. In case it is not valid, produce $\perp$. Otherwise, compute the decryption r of μ_1 and check whether $\mu_2 \stackrel{?}{=} \Omega.\mathsf{commit}(m, r)$, according to the result give a ZK of the equality/inequality of the decryption of c and the string used to create μ_2.

Selective conversion. Proceed as in the confirmation/denial protocol with the exception of issuing the decryption of μ_1 in case the signature is valid or the symbol $\perp$ otherwise.

4.2 Security analysis

First we note that the security for the verifier property and the non transferability of the confirmedSign, confirmation and denial protocols are ensured by using zero knowledge proofs of knowledge. Furthermore, the construction is EUF-CMA secure and INV1-CMA secure if the underlying components are secure.

Theorem 4. *The construction depicted above is (t, ϵ, q_s)-EUF-CMA secure if it uses a binding commitment scheme and a (t, ϵ, q_s)-EUF-CMA secure digital signature scheme.*

Theorem 5. *The construction depicted above is $(t, \epsilon, q_s, q_v, q_{sc})$-INV1-CMA secure if it uses a (t, ϵ', q_s)-SEUF-CMA secure digital signature, a secure commitment and a $(t + q_s(q_v + q_{sc}), \frac{\epsilon}{2}(1 - \epsilon')^{q_v + q_{sc}})$-IND-CPA secure cryptosystem.*

We provide the proofs of both theorems in Appendix E.

4.3 Efficiency analysis

We show in this paragraph that requesting the cryptosystem to be only IND-CPA secure improves the efficiency of constructions from the plain "signature of a commitment" paradigm from many sides. First, it enhances the signature generation, verification and conversion cost as encryption and decryption is usually faster in IND-CPA secure encryption than in IND-CCA secure encryption (e.g., ElGamal vs Cramer-Shoup or Paillier vs Camenisch-Shoup). Next, we achieve also a shorter signature since ciphertexts produced using IND-CPA schemes are standardly shorter than their similars produced using IND-CCA secure cryptosystems. Finally, we allow homomorphic encryption in the design, which will render the confirmedSign/confirmation/denial protocols more efficient. In fact, in [20, 39], the signer/confirmer has to prove in ZK the equality/inequality of the decryption of an IND-CCA encryption and an opening value of a commitment scheme. Thus, the only efficient instantiation, that was provided, used Camenisch-Shoup encryption and Pedersen commitment. In the rest of this subsection, we enlarge the category of encryption/commitment schemes that yield efficient instantiations thanks to the allowance of homomorphic encryption in the design.

Definition 1. ***(The class $\mathbb{C}$ of commitments)*** *$\mathbb{C}$ is the set of all commitment schemes for which there exists an algorithm* $\mathsf{Compute}$ *that on the input: the commitment public key* pk*, the message m and the commitment c on m, computes a description of a* one-way function $f : (\mathbb{G}, *) \to (\mathbb{H}, \circ_s)$*:*

- *where $(\mathbb{G}, *)$ is a group and $\mathbb{H}$ is a set equipped with the binary operation $\circ_s$,*
- *$\forall r, r' \in \mathbb{G}$: $f(r * r') = f(r) \circ_s f(r')$.*

and an $I \in \mathbb{H}$, such that $f(r) = I$, where r is the opening value of c w.r.t. m.

It is easy to check that Pedersen's commitment scheme is in this class. Actually, most commitment schemes have this built-in property because it is often the case that the committer wants to prove efficiently that a commitment is produced on some message. This is possible if the function f is homomorphic as shows Figure 1.

1. The prover chooses $r' \xleftarrow{R} \mathbb{G}$, computes and sends $t_1 = I \circ_s f(r')$ to the verifier.
2. The verifier chooses $b \xleftarrow{R} \{0, 1\}$ and sends it to the prover.
3. If $b = 0$, the prover sends r'.
   Otherwise, he sends $r * r'$.
4. If $b = 0$, the verifier checks that t_1 is computed as in Step 1.
   Otherwise, he accepts if $f(r * r') = t_1$.

1

Fig. 1. Proof system for membership to the language $\{r\colon\ f(r) = I\}$ Common input: I and Private input : r

Theorem 6. *The protocol depicted in Figure 1 is an efficient Σ protocol for proving knowledge of preimages of the function f described in Definition 1. Moreover, it (the protocol) is ZK close under parallel composition if the number of rounds is logarithmically bounded.*

The proof will be given in Appendix F.1.

For encryption, we use the same class $\mathbb{E}$ that was defined in [26], with the exception of not requiring the cryptosystems to be derived from the hybrid encryption paradigm.

Definition 2. ***(The class $\mathbb{E}$ of cryptosystems)*** *$\mathbb{E}$ is the set of encryption schemes Γ that have the following properties:*

1. *The message space is a group $\mathcal{M} = (\mathbb{G}, *)$ and the ciphertext space $\mathcal{C}$ is a set equipped with a binary operation $\circ_e$.*
2. *Let $m \in \mathcal{M}$ be a message and c its encryption with respect to a key* pk*. On the common input m and c, there exists an efficient zero knowledge proof of m being the decryption of c with respect to* pk*. The private input of the prover is either the private key* sk*, corresponding to* pk *or the randomness used to encrypt m in c.*
3. *$\forall m, m' \in \mathcal{M}$, $\forall \mathsf{pk}$: $\Gamma.\mathsf{encrypt}_{\mathsf{pk}}(m * m') = \Gamma.\mathsf{encrypt}_{\mathsf{pk}}(m) \circ_e \Gamma.\mathsf{encrypt}_{pk}(m')$. Moreover, given the randomness used to encrypt m in $\Gamma.\mathsf{encrypt}_{\mathsf{pk}}(m)$ and m' in $\Gamma.\mathsf{encrypt}_{pk}(m')$, one can deduce (using only the public parameters) the randomness used to encrypt $m * m'$ in $\Gamma.\mathsf{encrypt}_{\mathsf{pk}}(m) \circ_e \Gamma.\mathsf{encrypt}_{pk}(m')$.*

Examples of cryptosystems in the above class are ElGamal's encryption [18], the cryptosystem defined in [4] which uses the linear Diffie-Hellman KEM or Paillier's [33] cryptosystem. In fact, these cryptosystems are homomorphic and possess an efficient protocol for proving that a ciphertext decrypts to a given plaintext: the proof of equality of two discrete logarithms [11], in case of ElGamal or the cryptosystem in [4], or the proof of knowledge on an N-th root in case of Paillier's encryption.

1. The prover chooses $r' \xleftarrow{R} \mathbb{G}$, computes and sends $t_2 = \Gamma.\mathsf{encrypt}(r') \circ_e e$ to the verifier
2. The verifier chooses $b \xleftarrow{R} \{0,1\}$ and sends it to the signer.
3. If $b = 0$, the prover sends r' and the randomness used to encrypt it in $\Gamma.\mathsf{encrypt}(r')$.
   Otherwise, he sends $r' * r$ and proves that t_2 is an encryption of $r' * r$. 1
4. If $b = 0$, the verifier checks that t_2 is computed as in Step 1.
   Otherwise, he checks the proof of decryption of t_2:
      It it fails, he rejects the proof.

Fig. 2. Proof system for membership to the language $\{e\colon \exists m : m = \Gamma.\mathsf{decrypt}(e)\}$ Common input: $(e, \Gamma.\mathsf{pk})$ and Private input: $\Gamma.\mathsf{sk}$ or randomness encrypting m in e

Theorem 7. *Let Γ be a cryptosystem from the above class $\mathbb{E}$. Let furthermore e be an encryption of some message under some public* pk*. The protocol depicted in Figure 2 is an efficient Σ protocol for proving knowledge of the decryption of e and is ZK close under parallel composition if the number of rounds is logarithmically bounded.*

The proof is similar to the one given in [26]. □

The confirmation/denial protocol The confirmedSign, confirmation and denial protocols of the construction in Subsection 4.1 are depicted below.

1. The prover and verifier, given the public input, compute I as defined in Definition 1.
2. The prover chooses $r' \xleftarrow{R} \mathbb{G}$, computes and sends $t_1 = f(r') \circ_s I$ and
$t_2 = \Gamma.\mathsf{encrypt}(r') \circ_e e$ to the verifier.
3. The verifier chooses $b \xleftarrow{R} \{0,1\}$ and sends it to the prover.
4. If $b = 0$, the prover sends r' and the randomness used to encrypt it in $\Gamma.\mathsf{encrypt}(r')$.
   Otherwise, he sends $r' * r$ and proves that t_2 is an encryption of $r' * r$. 1
5. If $b = 0$, the verifier checks that t_1 and t_2 are computed as in Step 1.
   Otherwise, he checks the proof of decryption of t_2:
      It it fails, he rejects the proof.
      Otherwise:
         If the prover is confirming the signature, the verifier accepts if $f(r' * r) = t_1$.
         If the prover is denying the given signature, the verifier accepts the proof if $f(r' * r) \neq t_1$.

Fig. 3. Proof system for membership (non membership) to the language $\{(e, c)\colon \exists r : r = \Gamma.\mathsf{decrypt}(e) \wedge r = (\neq)\Omega.\mathsf{open}(c, m)\}$ Common input: $(e, c, m, \Gamma.\mathsf{pk}, \Omega.\mathsf{pk})$ and Private input: $\Gamma.\mathsf{sk}$ or randomness encrypting r in e

Remark 4. The prover in Figure 3 is either the confirmer who can run the above protocols with the knowledge of his private key, or the signer who wishes to confirm the validity of a just generated signature. In fact, with the knowledge of the randomness used to encrypt s in e, the signer can issue the above confirmation protocol thanks to the properties satisfied by Γ.

Theorem 8. *The confirmation protocol described in Figure 3 is a Σ protocol which is ZK close under parallel composition if the number of rounds is logarithmically bounded..*

Theorem 9. *The denial protocol described in Figure 3 is a Σ protocol under the assumption of the underlying cryptosystem being IND-CPA-secure. Moreover, it is ZK close under parallel composition if the number of rounds is logarithmically bounded.*

The proofs of both theorems are given in Appendices F.2 and F.3 respectively.

5 The "signature of an encryption" paradigm

We have seen that confirmer signatures realizing the "signature of a commitment" paradigm are comprised of a commitment on the message to be signed, an encryption of the random string used to produce the commitment, and a digital signature on the commitment. Since IND-CPA encryption can be easily used to get secure commitments, one can use instead of the commitment in the previous constructions an IND-CPA secure cryptosystem. With this choice, there will be no need of encrypting the string used to produce the encryption of the message, since the private key of the cryptosystem is sufficient to check the validity of a ciphertext w.r.t. to a given message. Note that this construction already appeared in [1] in the context of signcryption. We give below the full description of the construction.

Key generation. The signer key pair is $(\Sigma.\mathsf{pk}, \Sigma.\mathsf{sk})$ and the confirmer key pair is $(\Gamma.\mathsf{pk}, \Gamma.\mathsf{sk})$ where Σ and Γ are the digital signature and the cryptosystem underlying the construction resp.

ConfirmedSign. On input message m, compute an encryption $c = \Gamma.\mathsf{encrypt}_{\Gamma.\mathsf{pk}}(m)$ of m, then a digital signature $\sigma = \Sigma.\mathsf{sign}_{\Sigma.\mathsf{sk}}(c)$. Finally output (c, σ) and a ZK proof that c decrypts in m. Such a proof is possible given the randomness used to encrypt m in c.

Confirmation/Denial protocol. On a message m and an alleged signature $\mu = (\mu_1, \mu_2)$, check the validity of μ_2 on μ_1. In case it not valid, produce $\perp$. Otherwise, compute the decryption $\tilde{m}$ of μ_1 and check whether $\tilde{m} \stackrel{?}{=} m$, according to the result give a ZK of the equality/inequality of the decryption of μ_1 and m. These proofs are possible using the private key of Γ.

Selective conversion. Proceed as in the confirmation/denial protocol with the exception of issuing $\perp$ is case the signature is invalid, and a *non-interactive* proof that m is the decryption of the first field of the signature otherwise.

We notice that the construction depicted above achieves better performances than all previously cited constructions in terms of signature length, generation/verification and conversion cost. In fact, the signature contains only an IND-CPA encryption and signature on it. Moreover, verification or conversion of the signature are simpler as they do not involve anymore checking whether a commitment is correctly computed. Besides, the proofs underlying the confirmedSign/confirmation/denial protocols are reduced in case of Discrete-Logarithm-based cryptosystems to proofs of equality/inequality of discrete logarithms for which there exists efficient protocols [11, 9]. The only problem with this technique is the resort to non-interactive ZK (NIZK) proofs of knowledge. In fact, we know how to produce such proofs from their interactive variants using the Fiat-Shamir paradigm, which is known to provide security only in the ROM. However, the recent results in [16, 25, 24] exhibit efficient NIZK proofs of knowledge in some settings. We provide in Appendix G.1 efficient instantiations of such a method.

Concerning the security analysis, we first note that completeness, soundness and the ZK property of the confirmedSign/confirmation/denial protocols is ensured by the use of ZK proofs. Next, we prove that the construction resists existential forgeries and is invisible if the underlying digital signature and cryptosystem are EUF-CMA and IND-CPA secure resp. Details are provided in Appendix G.2.

6 Summary

We analyzed the invisibility of confirmer signatures from the "signature of commitment" paradigm in the outsider security model. The plain paradigm was shown to necessitate strong encryption which makes it quite impractical, or at least allows very limited instantiations. However, a small variation results in a tremendous improvement in the efficiency. We also shed light on a particular construction, which can be seen as a special sub-case of the latter paradigm, namely the "signature of an encryption" technique. The advantage of this technique consists in achieving better performances than the original technique (short signature, small generation, verification and conversion cost) , yet applying to any signature scheme. Its sole limitation resides in requiring efficient non interactive proofs of knowledge. This motivates research to further tackle this problem as was started recently in [16, 25, 24].

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A Preliminaries

A.1 Digital signatures

A signature scheme Σ comprises three algorithms, namely the key generation algorithm keygen, the signing algorithm sign, and the verification algorithm verify. The standard security notion for a signature scheme is existential unforgeability under chosen message attacks (EUF-CMA), which was introduced in [22]. Informally, this notion refers to the hardness of, given a signing oracle, producing a valid pair of message and corresponding signature such that message has not been queried to the signing oracle. There exists also the stronger notion, SEUF-CMA (strong existential unforgeability under chosen message attack), which allows the adversary to produce a forgery on a previously queried message, however the corresponding signature must not be obtained from the signing oracle.

A.2 Public key encryption schemes

A public key encryption (PKE) scheme consists of the key generation algorithm keygen, the encryption algorithm $\mathsf{encrypt}$ and the decryption algorithm $\mathsf{decrypt}$. The typical *security goals* a cryptosystem should attain are: one-wayness (OW) which corresponds to the difficulty of recovering the plaintext from a ciphertext, indistinguishability (IND) which refers to the hardness of distinguishing ciphertexts based on the messages they encrypt, and finally non-Malleability (NM) which corresponds to the hardness of deriving from a given ciphertext another ciphertext such that the underlying plaintexts are meaningfully related. Conversely, the typical *attack models* an adversary against an encryption scheme is allowed to are: Chosen Plaintext Attack (CPA) where the adversary can encrypt any message of his choice. This is inevitable in public key settings, Plaintext Checking Attack (PCA) in which the adversary is allowed to query an oracle on pairs (m, c) and gets answers whether m is really encrypted in c or not, and finally Chosen Ciphertext Attack (CCA) where the adversary is allowed to query a decryption oracle. Pairing the mentioned goals with these attack models yields nine *security notions*: GOAL-ATK for GOAL $\in \{\mathrm{OW}, \mathrm{IND}, \mathrm{NM}\}$ and ATK $\in \{\mathrm{CPA}, \mathrm{PCA}, \mathrm{CCA}\}$. We refer to [2] for the formal definitions of these notions as well as for the relations they satisfy.

Cryptosystems with labels. Encryption with labels was first introduced in [38]. In these schemes, the encryption algorithm takes as input, in addition to the public key pk and the message m intended to be encrypted, a label L. Similarly, the decryption algorithm takes additionally to the ciphertext and private key the label under which the ciphertext was created. Security notions are then defined as usual except that the adversary specifies always the label, to be used in the challenge ciphertext, to his challenger, and in case he (the adversary) is allowed to query oracles, then he cannot query them on the pair formed by the challenge and the label used to form it.

A.3 Commitment schemes

A commitment scheme [7] consists of the following algorithms:

- setup: the setup algorithm that generates the public parameters of the system.
- keygen: generates probabilistically a public commitment key pk.
- commit: a probabilistic algorithm that, on input a public key pk and a message m, produces a pair (c, r): c serves as the commitment value (locked box), and r as the opening value.
- open: this is a deterministic algorithm that given a commitment (c, r), w.r.t. a public key pk, on a alleged message m, checks whether $c \stackrel{?}{=} \mathsf{commit}_{\mathsf{pk}}(m, r)$.

The algorithm open must succeed if the commitment was correctly formed (correctness). Moreover, we require the following security properties:

1. **Hiding.** It is hard for an adversary A to generate two messages m_0, m_1 such that he can distinguish between their corresponding locked boxes c_o, c_1. That is, c reveals no information about m.
2. **Binding.** It is hard for an adversary A to come up with a *collision* (c, d, d') such that (c, d) and (c, d') are valid commitments for m and m' resp and $m \neq m'$.

We call a commitment scheme *secure* if it meets the previous properties.

It is easy to see the similarity between public key encryption and commitment schemes. In fact, one can easily check that IND-CPA encryption implies a secure commitment scheme. The main difference between encryption and commitment is that the former requires the decryption algorithm to be based on a "universal" secret key (independent of the message) whilst commitment allows to decrypt with a "message-dependent" secret key, namely the opening value r of the message in question. Another difference is that in encryption, the message is always derived from the ciphertext. This is not always the case in commitments, as the following example shows:

- setup and keygen choose a multiplicative group $(\mathbb{G}, \cdot)$ of order d and generated by an element g. Choose further an element $y \in \mathbb{G}$ of unknown discrete logarithm with respect to g, and a collision resistant hash function $h\colon \{0,1\}^\star \to \mathbb{Z}_d$. The public commitment key is y.
- commit on a message $m \in \{0,1\}^\star$ is the pair (r, c) where $r \stackrel{R}{\longleftarrow} \mathbb{Z}_d$ and $c = g^r y^{h(m)}$.
- open an alleged commitment (c, r) on a message m is achieved by checking whether $c \stackrel{?}{=} g^r y^{h(m)}$.

It is easy to check that the above commitment, referred to as *Pedersen-based* commitment scheme, is correct. Moreover it is statistically hiding because r is random in $\mathbb{Z}_d$ and so is $c = g^r y^{h(m)}$, regardless of m. Besides the biding property is achieved under the discrete logarithm assumption in $\mathbb{G}$ and the collision resistance assumption on the hash function h.

Finally, it is worth noting that given an alleged commitment value c on a message m, one can use the opening value r to prove (disprove) in zero knowledge that c is (is not) a commitment on m. In fact, the last assertion corresponds to an NP (co-NP) language which accepts a zero knowledge proof system (see [21]).

A.4 Σ protocols

A Σ protocol is an argument of knowledge which is complete, has the special soundness property and is Zero Knowledge (ZK). We refer to [21] for more information.

B Proof of Lemma 2

Proof. Let $\mathcal{R}$ be a key-preserving reduction that reduces the invisibility of the construction to the NM-CPA security of its underlying cryptosystem. We will construct an algorithm $\mathcal{M}$ that uses $\mathcal{R}$ to NM-CPA break the same cryptosystem by simulating an execution of the INV1-CMA adversary $\mathcal{A}$ against the construction.

Let Γ be the cryptosystem with labels $\mathcal{M}$ is trying to attack. $\mathcal{M}$ launches $\mathcal{R}$ over Γ with the same public key, say $\Gamma.\mathsf{pk}$. $\mathcal{M}$, acting as the INV1-CMA adversary against the construction, queries $\mathcal{R}$ on $m_0, m_1 \xleftarrow{R} \{0,1\}^\star$ for confirmer signatures. Then he queries the resulting strings $\mu_0 = (\mu_0^1, \mu_0^2, \mu_0^3)$ and $\mu_1 = (\mu_1^1, \mu_1^2, \mu_1^3)$ (corresponding to the confirmer signatures on m_0 and m_1 respectively) for a selective conversion. Let r_0 and r_1 be the output decryption of μ_0^1 and μ_1^1 resp (i.e., the randomnesses used generate the commitments μ_0^2 and μ_1^2 on m_0 and m_1 resp). With overwhelming probability, we have $r_0 \neq r_1$ [3], and if it is not the case, $\mathcal{M}$ will repeat the experiment until he obtains two different r_0 and r_1. Then, $\mathcal{M}$ inputs $\mathcal{D} = \{r_0, r_1\}$ to his own challenger as a distribution probability from which the plaintexts will be drawn. Moreover, he chooses uniformly at random a bit $b \xleftarrow{R} \{0,1\}$ and outputs to his challenger the challenge label $m_b \| \Sigma.\mathsf{pk}$, where $\Sigma.\mathsf{pk}$ is the public key of the digital signature underlying the construction. $\mathcal{M}$ will receive as a challenge encryption $\mu_b^\star$. At that point, $\mathcal{M}$ will query $\mathcal{R}$ on the string $(\mu_b^\star, \mu_b^2, \mu_b^3)$ and the message m_b for a selective conversion. If the result of such a query is different from $\perp$, then, $\mu_b^\star$ is a valid encryption of the random string used to generate the commitment μ_b^2, namely r_b. $\mathcal{M}$ will then output to his challenger an encryption μ of $\overline{r_b}$ under the same challenge label $m_b \| \Sigma.\mathsf{pk}$, where $\overline{r_b}$ refers to the bit-complement of the element r_b, and the relation R: $R(r, r') = (r' = \overline{r})$. Otherwise, he will output an encryption of $\overline{r_{1-b}}$ (under the same challenge label) and the same relation R. Finally $\mathcal{M}$ aborts the game (stops simulating an INV1-CMA attacker against the generic construction). $\square$

C Generalization to Arbitrary Reductions

C.1 Non malleable key generators

We define the notion of *non malleability of a cryptosystem key generator* through the following two games:

In **Game 0**, we consider an algorithm $\mathcal{R}$ trying to break a cryptosystem Γ , w.r.t. a public key $\Gamma.\mathsf{pk}$, in the sense of NM-CPA (or OW-CCA) using an adversary $\mathcal{A}$ which solves a problem A, perfectly reducible to OW-CPA breaking the cryptosystem Γ (w.r.t. the public key $\Gamma.\mathsf{pk}$). In this game, $\mathcal{R}$ lunches $\mathcal{A}$ over his own challenge key $\Gamma.\mathsf{pk}$ and some other parameters chosen freely by $\mathcal{R}$. We will denote by $\mathsf{adv}_0(\mathcal{R}^{\mathcal{A}})$ the success probability of $\mathcal{R}$ in such a game, where the probability is taken over the random tapes of both $\mathcal{R}$ and $\mathcal{A}$. We further define $\mathsf{succ}_\Gamma^{\mathsf{Game0}}(\mathcal{A}) = \max_{\mathcal{R}} \mathsf{adv}_0(\mathcal{R}^{\mathcal{A}})$ to be the success in **Game 0** of the best reduction $\mathcal{R}$ making the best possible use of the adversary $\mathcal{A}$. Note that the goal of **Game 0** is to include all key-preserving reductions $\mathcal{R}$ from NM-CPA (or OW-CCA) breaking the cryptosystem in question to solving a problem A, which is reducible to OW-CPA breaking the same cryptosystem.

In **Game 1**, we consider the same entities as in **Game 0**, with the exception of providing $\mathcal{R}$ with, in addition to $\mathcal{A}$, a OW-CPA oracle (i.e. a decryption oracle corresponding to Γ) that he can query w.r.t. any public key $\Gamma.\mathsf{pk}' \neq \Gamma.\mathsf{pk}$, where $\Gamma.\mathsf{pk}$ is the challenge public key of $\mathcal{R}$. Similarly, we define $\mathsf{adv}_1(\mathcal{R}^{\mathcal{A}})$ to be the success of $\mathcal{R}$ in such a game, and $\mathsf{succ}_\Gamma^{\mathsf{Game1}}(\mathcal{A}) = \max_{\mathcal{R}} \mathsf{adv}_0(\mathcal{R}^{\mathcal{A}})$ the success in **Game 1** of the reduction $\mathcal{R}$ making the best possible use of the adversary $\mathcal{A}$ and of the decryption (OW-CPA) oracle.

Definition 3. *A cryptosystem Γ is said to have a non malleable key generator if* $\Delta = max_{\mathcal{A}} |\mathsf{succ}_\Gamma^{\mathsf{Game1}}(\mathcal{A}) - \mathsf{succ}_\Gamma^{\mathsf{Game0}}(\mathcal{A})|$ *is negligeable in the security parameter.*

[3] Actually, if $\mathcal{R}$ uses always the same string to produce the commitments, then the construction is clearly not invisible.

This definition informally means that a cryptosystem has a non malleable key generator if NM-CPA (or OW-CCA) breaking it w.r.t. a key pk is no easier when given access to a decryption (OW-CPA) oracle w.r.t. any public key $\mathsf{pk}' \neq \mathsf{pk}$.

C.2 Proof of Theorem 2

To prove Theorem 2, we first need the following Lemma (similar to Lemma 6 of [35])

Lemma 3. *Let $\mathcal{A}$ be an adversary solving a problem A, reducible to OW-CPA breaking a cryptosystem Γ, and let $\mathcal{R}$ be an arbitrary reduction $\mathcal{R}$ that NM-CPA (OW-CCA) breaks a cryptosystem Γ, given access to $\mathcal{A}$. We have*

$$\mathsf{adv}(\mathcal{R}) \leq \mathsf{succ}_{\Gamma}^{\mathsf{Game1}}(\mathcal{A})$$

Proof. We will construct an algorithm $\mathcal{M}$ that plays **Game 1** with respect to a perfect oracle for $\mathcal{A}$ and succeeds in breaking the NM-CPA (OW-CCA) security of Γ with the same success probability of $\mathcal{R}$. Algorithm $\mathcal{M}$ gets a challenge w.r.t. a public key pk and launches $\mathcal{R}$ over the same challenge and the same public key. If $\mathcal{R}$ calls $\mathcal{A}$ on pk, then $\mathcal{M}$ will call his own oracle for $\mathcal{A}$. Otherwise, if $\mathcal{R}$ calls $\mathcal{A}$ on $\mathsf{pk}' \neq \mathsf{pk}$, $\mathcal{M}$ will invoke his own decryption oracle for pk' (OW-CPA oracle) to answer the queries. In fact, by assumption, the problem A is reducible to OW-CPA solving Γ. Finally, when $\mathcal{R}$ outputs the result to $\mathcal{M}$, the latter will output the same result to his own challenger. □

Proof of Theorem 2 This proof is similar to the one of Theorem 5 in [35].

Proof. We first remark that the invisibility of the construction depicted in Section 3 is perfectly reducible to OW-CPA breaking the cryptosystem underlying the construction. In fact, an invisibility adversary $\mathcal{A}$, given a challenge confirmer signature can first decrypt its first component, then use the resulting string to check the validity of the second component (alleged commitment on the message in question).
Next, we note that the advantage of the meta-reduction $\mathcal{M}$ in the proof of Lemma 2 (Lemma 1) is the same as the advantage of any key-preserving reduction $\mathcal{R}$ reducing the invisibility of a given confirmer signature to the NM-CPA (OW-CCA) security of its underlying cryptosystem Γ. For instance, this applies to the reduction making the best use of an invisibility adversary $\mathcal{A}$ against the construction. Therefore we have:

$$\mathsf{succ}_{\Gamma}^{\mathsf{Game0}}(\mathcal{A}) \leq \mathsf{succ}(NM - CPA[\Gamma])$$

where $\mathsf{succ}(NM - CPA[\Gamma])$ is the success of breaking Γ in the NP-CPA sense. We also have

$$\mathsf{succ}_{\Gamma}^{\mathsf{Game0}}(\mathcal{A}) \leq \mathsf{succ}(OW - CCA[\Gamma])$$

.
Now, Let $\mathcal{R}$ be an arbitrary reduction from NM-CPA (OW-CCA) breaking a cryptosystem Γ, with a non malleable key generator, to INV1-CMA breaking the construction (using the same cryptosystem Γ). We have

$$\begin{aligned}
\mathsf{adv}(\mathcal{R}) &\leq \mathsf{succ}_{\Gamma}^{\mathsf{Game1}}(\mathcal{A}) \\
&\leq \mathsf{succ}_{\Gamma}^{\mathsf{Game0}}(\mathcal{A}) + \Delta \\
&\leq \mathsf{succ}(NM - CPA[\Gamma])(\mathsf{succ}(OW - CCA[\Gamma])) + \Delta
\end{aligned}$$

since Δ is negligeable, then under the assumption of Γ being NM-CPA (OW-CCA) secure, the advantage of $\mathcal{R}$ is also negligeable. □

D Proof of Theorem 3

Proof. Let $\mathcal{A}$ be an attacker against the construction. We will construct an attacker $\mathcal{R}$ against the underlying cryptosystem scheme as follows.

$\mathcal{R}$ gets the parameters of the cryptosystem Γ from his challenger. Then he will choose a signature scheme Σ (along with a key pair $(\Sigma.\mathsf{pk}, \Sigma.\mathsf{sk})$) and a suitable commitment scheme Ω. $\mathcal{R}$ will set the above entities as components of the construction $\mathcal{A}$ is trying to attack.

For a signature query on a message m_i, $\mathcal{R}$ will compute a commitment c_i on m_i using a random string r_i, which he will encrypt in e_i under the label $m_i \| \Sigma.\mathsf{pk}$, then he will produce a digital signature σ_i on c_i using $\Sigma.\mathsf{sk}$. Next, he outputs $\mu_i = (e_i, c_i, \sigma_i)$ as a confirmer signature on m_i and a ZK proof of knowledge of the equality of the decryption of e_i and the string used in the commitment c_i. Such a proof is possible using the randomness t_i used to encrypt r_i in e_i. Finally, $\mathcal{R}$ will add the record $R_i = (m_i, t_i, r_i, e_i, c_i, \sigma_i)$ to a history list $\mathcal{L}$.

To confirm/deny an alleged signature $\mu_i = (\mu_i^1, \mu_i^2, \mu_i^3)$ on a message m_i, $\mathcal{R}$ will proceed as follows. First he checks the validity of the digital signature μ_i^3 on μ_i^2, in case it is invalid, he will output $\perp$, otherwise he will check the list $\mathcal{L}$, if he finds a record R_i having as first field the message m_i, he will proceed to the next step, namely, check whether the fourth field of R_i is equal to μ_i^1, if it is the case, $\mathcal{R}$ will issue a ZK proof of the equality of the decryption of μ_i^1 and the string used for the commitment μ_i^2. $\mathcal{R}$ can issue these proofs without the knowledge of $\Gamma.\mathsf{sk}$ using the rewinding technique (the proofs are ZK and thus simulatable) or by using the second field of R_i (randomness used to produce the encryption μ_i^1). Now, if R_i contains m_i in its first field, but its fourth field is different from μ_i^1, then $\mathcal{R}$ will check the next record R_j ($j > i$) having m_i in its first field and proceed in a similar fashion. Actually, if the message m_i is queried more than once, then it will occur in many records in $\mathcal{L}$. If $\mathcal{R}$ browses through all the records but none of them contains m_i and μ_i^1 in their first and fourth field resp, then for all the records R_i containing m_i in their first field, $\mathcal{A}$ will invoke his PCA oracle on the ciphertext μ_i^1 and the third fields of these records. If one of the queries yields "yes" as an answer, e.g., there exists a record $R_j = (m_i, t_j, r_j, e_j, c_j, \sigma_j)$ such that its third field r_j is a decryption of μ_i^1, then according to whether r_j is (is not) the opening value of the commitment μ_i^2 on m_i, $\mathcal{R}$ will issue a ZK proof of the equality (inequality) of the decryption of μ_i^1 and the string used for the commitment μ_i^2. Again such a proof is possible to issue using the rewinding technique (the value t_j cannot be used here because it was not used to encrypt r_j in μ_i^1). Finally, if no query to the PCA oracle yields the answer "yes", then $\mathcal{R}$ will issue the denial protocol, namely simulate a ZK proof, using the rewinding technique, of the inequality of the decryption of μ_i^1 and of the string used for the commitment μ_i^2.
Selective conversion is similarly carried out with the exception of issuing the decryption of μ_i^1 instead of the confirmation protocol and $\perp$ instead of the denial protocol.
The difference between the above simulation and the real execution of the algorithm is when the signature $\mu_i = (\mu_i^1, \mu_i^2, \mu_i^3)$ is valid, however, μ_i^1 is not an encryption of a string r_i already issued to $\mathcal{A}$ during a selective conversion query regarding the message m_i and a presumed signature on it. We distinguish two cases, either m_i was never queried for signature, in which case such a signature would correspond to an existential forgery on the construction and thus to an existential forgery on the underlying digital signature. Or, m_i was queried before for signature. Let $\mu_j = (\mu_j^1, \mu_j^2, \mu_j^3)$ be the output confirmer signature to such a query. Since μ_i^1 is encryption of some r_i which was never used to generate signatures on m_i, then with overwhelming probability $\mu_i^2 \neq \mu_j^2$ (both are commitment on m_i with different random strings). Thus, in this case (μ_i^2, μ_i^3) will correspond to an existential forgery on the underlying digital signature scheme. We conclude that the above simulation is indistinguishable from the real execution with probability at least $(1 - \epsilon')^{q_v + q_{sc}}$, as the digital signature scheme underlying the construction is (t, ϵ', q_s)-EUF-CMA secure by assumption.

At some point, $\mathcal{A}$ will output two messages m_0, m_1. The latter will then choose uniformly at random a bit $b \xleftarrow{R} \{0, 1\}$, and two different random strings r_0 and r_1 from the corresponding space. $\mathcal{R}$ will output

to his challenger the label $m_b \| \Sigma.\mathsf{pk}$ and the strings r_0, r_1. He receives then a ciphertext c, encryption of $r_{b'}$, for some $b' \xleftarrow{R} \{0,1\}$. To answer his challenger, $\mathcal{R}$ will compute a commitment c_b on the message m_b using the string $r_{b''}$ where $b'' \xleftarrow{R} \{0,1\}$. Then, $\mathcal{R}$ will output $\mu = (c, c_b, \Sigma.\mathsf{sign}_{\Sigma.\mathsf{sk}}(c_b))$ as a challenge signature to $\mathcal{A}$. Note that $\mathcal{A}$ can only exploit information leaked from c about the opening value of c_b because the commitment scheme is by assumption hiding.
Note that at this stage, $\mathcal{R}$ cannot request his PCA oracle on (c, r_i), $i \in \{0,1\}$ under the label $m_b \| \Sigma.\mathsf{pk}$. $\mathcal{R}$ would need to query his PCA oracle on such a quantity if he gets a verification (conversion) query on a signature $(c, c_b, -)$ and the message m_b. $\mathcal{R}$ will respond to such a query by simulating the denial protocol (output $\perp$). This simulation differs from the real algorithm when $(c, c_b, -)$ is valid on m_b. Again, such a scenario won't happen with probability at least $(1-\epsilon')^{q_v+q_{sc}}$, because the query would form a strong existential forgery on the digital signature scheme underlying the construction.
The rest of the proof follows in a straightforward way. Now, let $\mu = (c, c_b, \Sigma.\mathsf{sign}_{\Sigma.\mathsf{sk}}(c_b))$ be the challenge signature. In case, c is an encryption of $r_{b''}$ (that is if $b' = b''$), then μ corresponds to a valid confirmer signature on m_b. Otherwise, it is not a valid signature on neither m_b nor m_{1-b}. In fact, c_b is a commitment on m_b using a string different from the decryption of c under the label $m_b \| \Sigma.\mathsf{pk}$. Let b_a the bit output by $\mathcal{A}$. $\mathcal{R}$ will output b'' to his challenger in case $b = b_a$ and $1 - b''$ otherwise.

The advantage of $\mathcal{A}$ in such an attack is defined by

$$\epsilon = \mathsf{adv}(\mathcal{A}) = \Pr[b_a = b | b' = b''] - \frac{1}{2}$$

Whereas the advantage of $\mathcal{R}$ is given by

$$\begin{aligned}
\mathsf{adv}(\mathcal{R}) &= (1-\epsilon')^{q_v+q_{sc}} \left[\Pr[b = b_a, b' = b''] + \Pr[b \neq b_a, b' \neq b''] - \frac{1}{2}\right] \\
&= (1-\epsilon')^{q_v+q_{sc}} \left[\Pr[b = b_a | b' = b''] \Pr[b' = b''] + \Pr[b \neq b_a | b' \neq b''] \Pr[b' \neq b''] - \frac{1}{2}\right] \\
&= (1-\epsilon')^{q_v+q_{sc}} \left[\frac{1}{2}(\epsilon + \frac{1}{2}) + \frac{1}{2}\frac{1}{2} - \frac{1}{2}\right] \\
&= \frac{\epsilon}{2}(1-\epsilon')^{q_v+q_{sc}}
\end{aligned}$$

The last but one equation is due to the facts $\Pr[b' \neq b''] = \Pr[b' = b''] = \frac{1}{2}$ as $b'' \xleftarrow{R} \{0,1\}$, and to the fact that, in case $b' \neq b''$, the probability that $\mathcal{A}$ answers b is exactly $\frac{1}{2}$ since in that case the challenge signature is not valid on both messages.

□

E Security of the Modified "Signature of a Commitment" Paradigm

E.1 Proof of Theorem 4

Proof. (Sketch)

Let $\mathcal{A}$ be an EUF-CMA attacker against the construction. We construct an EUF-CMA attacker $\mathcal{R}$ against the underlying digital signature scheme as follows.

$\mathcal{R}$ gets the parameters of the digital signature from his attacker, and chooses a suitable encryption and commitment scheme. Simulation of the confirmedSign queries (on messages m_i) is done by first computing a commitment c_i on m_i using some random string r_i, then encrypting the string r_i in e_i and finally requesting the challenger for a digital signature σ_i on $e_i \| c_i$. The string (e_i, c_i, σ_i) is output to $\mathcal{A}$ along with a proof of the equality of the decryption of e_i and the opening value of c_i. Such a proof can be issued using the cryptosystem private key that $\mathcal{R}$ knows or the randomness used to encrypt r_i in e_i. Confirmation/denial and selective conversion queries can be perfectly simulated with the knowledge of the cryptosystem private key.

At some point, $\mathcal{A}$ will output a forgery $\mu^\star = (e^\star, c^\star, \sigma^\star)$ on some message $m^\star$, which has never been queried before. By definition, $\sigma^\star$ is a valid digital signature on $e^\star \| c^\star$. It will form an existential forgery on the digital signature scheme if $e^\star \| c^\star$ has never been queried before by $\mathcal{R}$ for a digital signature. Suppose there exists $1 \leq i \leq q_s$ such that $e^\star \| c^\star = e_i \| c_i$ where $\mu_i = (e_i, c_i, \sigma_i)$ was the output confirmer signature on the query m_i. Due to the special way the strings $e_i \| c_i$ are created, equality of the strings $e^\star \| c^\star$ and $e_i \| c_i$ implies equality of their suffixes (that start at the $(n+1)$-st position), namely, $c^\star$ and c_i. This equality implies the equality of m_i and $m^\star$ since the used commitment is binding by assumption. Thus $\mathcal{R}$ returns $(\sigma^\star, e^\star \| c^\star)$ as a valid existential forgery against the digital signature in question.

□

E.2 Proof of Theorem 5

Proof. Simulation of the key generation is similar to the proof of Theorem 3.

For a ConfirmedSign query on a message m_i, the reduction $\mathcal{R}$ (attacker against the cryptosystem) will proceed exactly as a real signer would do, with the exception of maintaining a list of records that contains the queried messages, the output confirmer signatures and the intermediate values used to produce these signatures, namely the random string used in the commitment and the randomness used to encrypt it. This list will be used later for the confirm/deny and selective conversion queries. In fact, for such queries, say (e_i, c_i, σ_i) on m_i, $\mathcal{R}$ will simulate the confirmation protocol (using the rewinding technique or the randomness used to encrypt the opening value of the commitment) if the encryption e_i appears in one record in the list (as an encryption of a string used for commitment), or simulate the denial protocol otherwise. Selective conversion of a confirmer signature whose first field appears in the list is done by revealing the opening value of the commitment, otherwise such a confirmer signature is converted to $\perp$.

The difference of this simulation with the real execution of the algorithm is when a queried signature, say (e_i, c_i, σ_i), is valid but e_i was never used to generate confirmer signatures. We distinguish two cases, either the underlying message m_i has been queried previously on not. In the latter case, such a signature would correspond to an existential forgery on the construction, thus, to an existential forgery on the underlying digital signature. In the former case, let (e_j, c_j, σ_j) be the output signature to $\mathcal{A}$ on the message m_i. Since e_i was never used before, then $c_i \neq c_j$ (both are commitments on m_i using different random strings). By the same argument used in the proof of Theorem 4 (for the analysis of the forger's output), we get that $e_i \| c_i \neq e_j \| c_j$. We conclude that the adversary would have to compute a digital signature on a string for which he never had obtained a signature. Thus, in the former case, the query would lead to an existential forgery on the underlying signature scheme. Since the latter is by assumption (t, ϵ', q_s)-EUF-CMA secure, the probability that the simulation differs from the real execution is at least $(1-\epsilon')^{q_v+q_{sc}}$.

Finally, in the challenge phase, the adversary outputs two challenging messages m_0, m_1. $\mathcal{R}$ will then produce two strings r_0, r_1 and hands them to his challenger. He gets as a response a challenge ciphertext e on r_b for some $b \in \{0,1\}$. $\mathcal{R}$ will choose a bit $b' \xleftarrow{R} \{0,1\}$ and produces a commitment c on a message $m_{b''}$, for some $b'' \xleftarrow{R} \{0,1\}$, using the string $r_{b'}$. Finally, he will produce a digital signature σ on $e\|c$. The challenge confirmer signature is (e, c, σ). Note, that if $b = b'$, the signature is valid on the message $m_{b''}$, otherwise, it is invalid on both messages. Note also that the adversary exploits only information leaked from the encryption e because the commitment scheme is hiding.

The adversary will continue issuing his queries to $\mathcal{R}$, who will handle them as previously. Note that from now on during the verification/conversion queries, the adversary can ask a query $(e, c, -)$ on a message m_i, $i = 0, 1$. The probability that such a query is not valid is at least $(1-\epsilon')^{q_v+q_{sc}}$ since the digital signature scheme is (t, ϵ', q_s)-SEUF-CMA secure. At the end, the adversary outputs a bit b_a. Clearly the advantage of the adversary is $\epsilon = \Pr[b'' = b_a | b = b'] - \frac{1}{2}$. $\mathcal{R}$ will output b' in case $b'' = b_a$ and $1 - b'$ otherwise.

The advantage of $\mathcal{R}$ is clearly

$$\begin{aligned}
\mathsf{adv}(\mathcal{R}) &= (1-\epsilon')^{q_v+q_{sc}} \left[\Pr[b'' = b_a, b' = b] + \Pr[b'' \neq b_a, b' \neq b] - \frac{1}{2}\right] \\
&= (1-\epsilon')^{q_v+q_{sc}} \left[\Pr[b'' = b_a | b' = b] \Pr[b' = b] + \Pr[b'' \neq b_a | b' \neq b] \Pr[b' \neq b] - \frac{1}{2}\right] \\
&= (1-\epsilon')^{q_v+q_{sc}} \left[\frac{1}{2}(\epsilon + \frac{1}{2}) + \frac{1}{2}\frac{1}{2} - \frac{1}{2}\right] \\
&= \frac{\epsilon}{2}(1-\epsilon')^{q_v+q_{sc}}
\end{aligned}$$

□

F Efficient Instantiations using Certain Commitments and Cryptosystems

F.1 Proof of Theorem 6

We first remark that the function f used in the definition of the class $\mathbb{C}$ induces a group law in $\mathbb{H} = f(\mathbb{G})$ for the operation $\circ_s$. Moreover, we have $1_{\mathbb{H}} = f(1_{\mathbb{G}})$ and $\forall r \in \mathbb{G}$: $f(r)^{-1} = f(r^{-1})$.

Proof. For completeness, it is clear that if both parties follow the protocol, the prover will always be able to provide a proof that the verifier will accept.

For soundness, we show that the prover can cheat with probability at most 2^{-1} in one round if the verifier chooses b uniformly at random from $\{0, 1\}$. In fact, suppose that the prover can answer both challenges for the same commitment t_1. Let r_0 and r_1 be the responses of the prover to the challenges 0 and 1 respectively in Step 3. Since the verifier accepts the proof, we have, $t_1 = f(r_0) \circ_s I = f(r_1)$. Thus, $f(r_1) \circ_s f(r_0)^{-1} = f(r_1 * r_0^{-1}) = I$. Hence, the prover would know a preimage of I. We conclude that a cheating prover can cheat with at most $1/2$, provided f is one-way and the verifier is honest (chooses the bit b uniformly from $\{0, 1\}$). Repeating the protocol l times leads to a soundness error which is at most 2^{-l}.

To prove that the proof is ZK, we provide the following simulator.

1. Generate uniformly a random bit $b' \in_R \{0, 1\}$. If $b' = 0$, choose $r' \in_R \mathbb{G}$ and sends $t_1 = f(r') \circ_s I$, otherwise, choose $r'' \in_R \mathbb{G}$ and sends $t_1 = f(r'')$ to the verifier.
2. Get b from the verifier. If $b = b'$: if $b = 0$, the simulator sends back r', otherwise, it sends r''. If $b \neq b'$, it goes to Step 1.

The prover's first message is always the function f applied to a random value $r'' \in \mathbb{G}$, and so is the first message of the simulator. Since b' is chosen uniformly at random from $\{0, 1\}$, the probability that the simulator rewinds the verifier is:

$$1 - \Pr[b = b'] = 1 - (\Pr[b = 0, b' = 0] + \Pr[b = 1, b' = 1]) = 1 - (\frac{1}{2}p + \frac{1}{2}(1-p)) = 1 - \frac{1}{2} = \frac{1}{2}$$

where $p = \Pr[b = 0]$. Therefore, the expected number of rewinds is 2 and as a consequence, the simulator runs in expected linear time. Finally, the distribution of the answers of the prover and of the simulator is again the same. We conclude that the protocol is ZK. It also remains ZK if it is run l times in parallel, where l is either constant or logarithmic in the security parameter. In fact, the simulator of the parallel composition of the protocol will be the parallel composition of the above simulator. Thus, the expected running time of the new simulator is 2^l (probability of not rewinding the verifier is 2^{-l}), which is either constant or polynomial in the security parameter. □

F.2 Proof of Theorem 8

Proof. The confirmation protocol depicted in Figure 3 is a parallel composition of the proofs depicted in Figures 1 and 2. Therefore completeness and soundness follow as a direct consequence of the completeness and soundness of the underlying proofs (see [21]).

To prove that the protocol is ZK. We provide the following simulator (for one execution):

1. Generate $b' \in_R \{0,1\}$. If $b' = 0$, choose $r' \in_R \mathbb{G}$ and sends $t_1 = f(r') \circ_s I$ and $t_2 = \mathsf{encrypt}(r') \circ_e e$, otherwise, choose $r'' \in_R \mathbb{G}$ and sends $t_1 = f(r'')$ and $t_2 = \mathsf{encrypt}(r'')$ to the verifier.
2. Get b from the verifier. If $b = b'$: if $b = 0$, the simulator sends back r' and the randomness used to encrypt it in $\mathsf{encrypt}(r')$, otherwise, it sends r'' and simulates the proof of t_2 being an encryption of r'' (this proof is simulatable since it is by assumption ZK). If $b \neq b'$, it goes to Step 1.

The prover's first message is an encryption of a random value $r'' \in_R \mathbb{G}$, in addition to $f(r'')$, and so is the simulator's first message. Therefore the distributions of the prover and of the simulator outputs are the same in the first round of the proof. Moreover, the expected number of rewinds is 2 ($\Pr(b \neq b') = \frac{1}{2}$), making the simulator run in expected linear time. The distribution of the prover's messages in the third round is also similar to that of the simulator's messages. We conclude that the confirmation protocol is ZK. Parallel execution of the protocol will remain also ZK if the number of executions l is constant or logarithmic in the security parameter (see the above proof). $\square$

F.3 Proof of Theorem 9

Proof. With the standard techniques, we prove that the denial protocol depicted in Figure 3 is complete and sound with error probability 2^{-l} (l is the number of rounds) provided the verifier is honest and the cryptosystem is one way. Similarly, we provide the following simulator to prove the ZK property.

1. Generate $b' \in_R \{0,1\}$. If $b' = 0$, choose $r' \in_R \mathbb{G}$ and sends $t_1 = f(r') \circ_s I$ and $t_2 = \Gamma.\mathsf{encrypt}(r') \circ_e e$, otherwise, choose $r'' \in_R \mathbb{G}$ and a random $t_1 \in_R f(\mathbb{G})$ and $t_2 = \Gamma.\mathsf{encrypt}(r'')$.
2. Get b from the verifier. If $b = b'$: if $b = 0$, the simulator sends back r' and the randomness used to encrypt it in $\Gamma.\mathsf{encrypt}(r')$, otherwise, it sends r'' and simulates the proof of t_2 being an encryption of r'' (this proof is simulatable since it is by assumption zero knowledge). If $b \neq b'$, it goes to Step 1.

The prover's first message is an encryption of some random value r'', and the element $t_1 = f(r'' * r^{-1}) \circ_s I$. The simulator's first message is an encryption of a random value r'', and in case $b = 0$ the element $t_1 = f(r'' * r^{-1}) \circ_s I$, whereas in the case $b = 1$, it is the element $t_1 \in_R f(\mathbb{G})$ (independent of r''). Distinguishing these two cases it at least as hard as breaking the IND-CPA security of the underlying cryptosystem. In fact, if the verifier is able to distinguish these two cases, it can be easily used to break the cryptosystem in the IND-CPA sense. Therefore, under the assumption of the IND-CPA security of the cryptosystem, the simulator's and prover's first message distributions are indistinguishable. Moreover, the simulator runs in expected linear time, since the number of rewinds is 2. Moreover, the distribution of the prover's and the simulator's message in the last round are again, by the same argument, indistinguishable under the IND-CPA security of the cryptosystem. Finally, with same argument as above, parallel execution of the protocol remains also ZK if the number of executions is constant or logarithmic in the security parameter. $\square$

G The "Signature of an Encryption" Paradigm

G.1 Efficiency analysis

Confirmation/denial protocols. We showed that the confirmation (and also the confirmedSign) protocol, in confirmer signatures from the "Signature of an Encryption" paradigm, amounts to proving that a ciphertext encrypts a given plaintext. This is in general easy since in most encryption schemes, one can define, given a ciphertext c and its underlying plaintext m, two homomorphic one way functions f and g, and two quantities I and J such that $f(r) = I$ and $g(\mathsf{sk}) = J$, where r is the randomness used to encrypt m in c and sk is the private key of the cryptosystem in question. Examples of such encryptions are [18], the cryptosystem defined in [4] which uses the linear Diffie-Hellman KEM, Paillier [33], and also Cramer-Shoup [13] and [9]. The confirmation (confirmedSign) protocol in this case will be reduced to a proof of knowledge of a preimage of J (I) by the function g (f), for which we provided an efficient proof in Figure 1.

Concerning the denial protocol, it is not always straightforward. In most discrete-logarithm-based encryption, this protocol amounts to a proof of inequality of discrete logarithms as in [18, 4, 13]. In case the cryptosystem belongs to the class $\mathbb{E}$ defined in Definition 1, Figure 4 provides an efficient proof that c encrypts some $\tilde{m} \neq m$. In the protocol provided in this figure, f denotes an arbitrary *deterministic homomorphic injective one way function*:

$$f(m \star m') = f(m) \circ_s f(m')$$

1. The prover chooses $m' \xleftarrow{R} \mathbb{G}$, computes and sends $t_1 = f(m') \circ_s f(m)$ and $t_2 = \Gamma.\mathsf{encrypt}(m') \circ_e c$ to the verifier.
2. The verifier chooses $b \xleftarrow{R} \{0, 1\}$ and sends it to the prover.
3. If $b = 0$, the prover sends m' and the randomness used to encrypt it in $\Gamma.\mathsf{encrypt}(m')$.
   Otherwise, he sends $m' * \tilde{m}$ and proves that t_2 is an encryption of $m' * \tilde{m}$. 1
4. If $b = 0$, the verifier checks that t_1 and t_2 are computed as in Step 1.
   Otherwise, he checks the proof of decryption of t_2:
   It it fails, he rejects the proof.
   Otherwise, he accepts the proof if $f(m' * \tilde{m}) \neq t_1$.

Fig. 4. Proof system for membership (non membership) to the language $\{(m, c) \colon \exists \tilde{m} : \tilde{m} = \Gamma.\mathsf{decrypt}(c) \wedge \tilde{m} \neq m\}$ Common input: $(m, c, \Gamma.\mathsf{pk})$ and Private input: $\Gamma.\mathsf{sk}$ or randomness encrypting $\tilde{m}$ in c

With the standard tools, the above denial protocol can be shown to be a Σ protocol with computational ZK, if the cryptosystem Γ is IND-CPA secure, and is (the protocol) ZK close under parallel composition if the number of rounds in logarithmically bounded.

Selective Conversion. The selective conversion in confirmer signatures from the "Signature of an Encryption" paradigm consists of a non interactive proof of the confirmation protocol. As mentioned earlier in this document, there has been recently an important progress in this area. We note in this paragraph two solutions.

Damgård et al. [16]'s solution. This solution transforms a 3-move interactive ZK protocol P with linear answer to a non interactive ZK one (NIZK) using a homomorphic cryptosystem in a registrated key model, i.e., in a model where the verifier registers his key. More precisely, let a be the first message computed by the prover in P, $c \in \mathbb{N}$ be the challenge sent by the verifier, and finally let $z = u + cv$ be the answer computed by the prover in the third step, where $u, v \in \mathbb{N}$. Let further Γ denote a homomorphic encryption scheme such that $\Gamma.\mathsf{encrypt}(m+m') = \Gamma.\mathsf{encrypt}(m) \cdot \Gamma.\mathsf{encrypt}(m')$, where m and m' are integer values in a suitable range. If the verifier chooses a key pair $(\Gamma.\mathsf{pk}, \Gamma.\mathsf{sk})$ and publishes an encryption e of the challenge c, then the prover can compute a as usual and $\Gamma.\mathsf{encrypt}(z) = \Gamma.\mathsf{encrypt}(u)e^v$ and sends these quantities to the verifier in one pass. The verifier decrypts $\Gamma.\mathsf{encrypt}(z)$ to obtain z and checks whether (a, c, z) is an accepting transcript. The authors in [16] proposed an efficient illustration using Paillier's encryption and the proof of equality of two discrete logarithms. We conclude that with such a technique, the "signature of an encryption" approach accepts an efficient instantiation if the considered cryptosystem allows proving the correctness of a decryption using a proof of equality of two discrete logarithms, e.g., [18, 4, 13].

Groth and Sahai [25]'s solution. The authors in this work provide an efficient NIZK for the language:

$$\mathsf{PoK} = \{(a, b)\colon c_1 = u^a \wedge c_2 = v^b \wedge c_3 = g^{a+b}\}$$

The common input is $g, c_1, c_2, u, v \in (\mathbb{G}, \cdot)$ where $(\mathbb{G}, \cdot)$ is a bilinear group. The private input is either (a, b) or $(\mathsf{DL}_g(u), \mathsf{DL}_g(v))$, where $\mathsf{DL}_g(u)$ denotes the discrete logarithm of u in base g. We conclude then that the "signature of an encryption" approach accepts an efficient instantiation if the considered cryptosystem is the one defined in [4], since the a proof of the above language can be used to prove that a given ciphertext decrypts to a given message.

G.2 Security analysis

Theorem 10. *The above construction is (t, ϵ, q_s)-EUF-CMA secure if the underlying digital signature is also (t, ϵ, q_s)-EUF-CMA secure.*

Proof. The adversary $\mathcal{R}$ against the signature underlying the construction will get the parameters of the digital signature he is trying to attack from his challenger. Then, he will choose a suitable cryptosystem. Simulation of signatures is simple; on a query m_i, $\mathcal{R}$ will first compute an encryption c_i of m_i, then request his challenger for a signature on c_i. Let σ_i be the answer of such a query. $\mathcal{R}$ will then output (c_i, σ_i) and produces a ZK proof that c_i decrypts in m_i. Such a proof, in addition to all the proofs involved in the verification/conversion queries is possible for $\mathcal{R}$ to give with the knowledge of the cryptosystem private key.

At some time, the adversary $\mathcal{A}$ against the construction will output a forgery $(c^\star, \sigma^\star)$ on a message $m^\star$, that has never been queried before. $\sigma^\star$ is by definition a digital signature on $c^\star$. The former has never been queried by $\mathcal{R}$ for digital signature, since otherwise $m^\star$ would have been queried before. We conclude that $(c^\star, \sigma^\star)$ is also a valid forgery on the signature scheme. □

Theorem 11. *The above construction is $(t, \epsilon, q_s, q_v, q_{sc})$-INV1-CMA secure if it uses a (t, ϵ', q_s)-SEUF-CMA secure digital signature and a $(t + q_s(q_v + q_{sc}), \epsilon(1 - \epsilon')^{q_v + q_{sc}})$-IND-CPA secure cryptosystem.*

Proof. Let $\mathcal{A}$ be the invisibility adversary against the construction, we construct an IND-CPA adversary $\mathcal{R}$ against the underlying cryptosystem as follows.

$\mathcal{R}$ gets the parameters of the target cryptosystem from his challenger, and chooses a suitable digital signature scheme. For a confirmedSign query on m_i, $\mathcal{R}$ will proceed as in the real algorithm, with the exception of maintaining a list $\mathcal{L}$ of records that consists of the query, its encryption, the randomness used to produce the encryption, and finally the digital signature on the encryption. $\mathcal{R}$ can produce digital signatures on any encryption with the knowledge of the signature scheme private key. Moreover, he can confirm any signature he has just generated with the knowledge of the randomness used in the encryption.

For a verification query (c_i, σ_i) on m_i, $\mathcal{R}$ will check $\mathcal{L}$ (after checking of course the validity of σ_i on m_i), if the record $R_i = (m_i, c_i, -, -)$ appears in the list, then he will issue a proof that c_i decrypts in m_i using the third component of the record. Otherwise, he will simulate a proof of the inequality of the decryption of c_i and m_i using the rewinding technique.

For a conversion query, $\mathcal{R}$ will proceed as in a verification query with the exception of providing the non-interactive variant of the proof he would issue if the signature is valid, and the symbol $\perp$ otherwise. This simulation differs from the real one when the queried signature (c_i, σ_i) is valid on m_i however c_i does not appear in the list (as first field of the output confirmer signatures). We distinguish two cases, either the message in question m_i has not been queried before for signature, in which case such a query would correspond to a valid existential forgery on the construction, and thus on the underling signature scheme. Or, the queried signature is on a message that has been queried before, which corresponds to an existential forgery on the underlying signature scheme. Since the signature scheme underlying the construction is (t, ϵ', q_s)-EUF-CMA secure, this scenario does not happen with probability at least $(1-\epsilon')^{q_v+q_{sc}}$.

At some point, $\mathcal{A}$ produces two messages m_0, m_1. $\mathcal{R}$ will forward the same messages to his challenger and obtain a ciphertext c, encryption of m_b for some $b \xleftarrow{R} \{0,1\}$. $\mathcal{R}$ will produce a digital signature on c and give the result in addition to c to $\mathcal{A}$ as a challenge confirmer signature. It easy to see that $\mathcal{A}$'s answer is sufficient for $\mathcal{R}$ to conclude. Note that after the challenge phase, $\mathcal{A}$ is allowed to issue confirmedSign, verification and conversion queries and $\mathcal{R}$ can handle them as previously. Namely the probability that the adversary does not issue a verification/conversion query of the type $(c, -)$ is at least $(1-\epsilon')^{q_v+q_{sc}}$ since the signature scheme underlying the construction is (t, ϵ', q_s)-SEUF-CMA secure.

□

Remark 5. Note that the IND-CPA requirement on the cryptosystem is also necessary. In fact, deterministic schemes, e.g., RSA (which is OW-CPA secure) are not allowed in the design, since an invisibility adversary will compute the encryptions of the two challenge messages and check whether one of them is the first field of the signature.

H A stronger security model

In [14], the author presented an elaborate security model. We discuss in this paragraph how one can extend the constructions seen so far to this model.

Security against malicious confirmers. The first difference between our model and the one in [14] is the unforgeability against *malicious* confirmers which is satisfied in the latter but not considered in the former. This property requires the construction to remain EUF-CMA secure even if the EUF-CMA adversary is allowed to choose the confirmer public key. One can easily see that the constructions presented in this document meet this property as the confirmer public key does not play any role in the unforgeability proofs.

Correctness of the conversion. Another difference lies in requesting the confirmer to provide a proof of the correctness of the conversion. This is vital, because in all constructions that realize the "signature of a commitment" paradigm, the confirmer, with the help of the signer, can convert *invalid* signatures; he can release the "real" opening value of the commitment, which does not have to be the decryption of the first field of the confirmer signature (the signer might keep a list of the issued signatures and the random strings used to produce the commitments). A way to overcome this, is to provide, along with the opening value of the commitment, a proof that it is the correct decryption of the first field of the alleged signature. In [14], the author suggested to use a protocol, i.e., an interactive proof, proving the correctness of the conversion. We propose to use non interactive proofs to get transferability, i.e., anybody can check the correctness of a converted confirmer signature. As mentioned in the previous subsection, there exists efficient ways to obtain non-interactive proofs of knowledge without using Fiat-Shamir heuristics. Again constructions shown before meet this stronger property (the reduction in the invisibility proofs can issue such proofs using the randomness used to produce the encryption of the commitment opening value). Finally, our constructions allow also the confirmer to convert *invalid* signatures although it is not his responsibility to convert ill-formed signatures. The confirmer can do so by issuing simply the decryption of the first field of signature (in case it is a well-formed ciphertext) along with a non-interactive proof of the correctness of the decryption. Anybody can then check that the released string does not open the commitment (second field of the confirmer signature).