

# Remarks on One Arbitrated Quantum-signature Scheme

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**Abstract** Zeng and Keitel proposed an arbitrated quantum signature scheme In 2002. Recently, Curty and Lütkenhaus pointed out that the protocol is not operationally specified. In a reply, Zeng gave more details of the scheme. The author also claimed that the scheme is suitable for unknown messages. In this letter, we remark that the invented scenario in the original scheme is artificial. This is because its security entirely depends on the presence of a trustworthy arbitrator. Moreover, the claim that the original scheme is suitable for unknown messages is not sound.

**Keywords** quantum digital signature, blind signature, arbitrator.

## 1 Introduction

A digital signature of a message is a number dependent on some secret known only to the signer, and, additionally, on the content of the message being signed [6]. Signatures must be verifiable; if a dispute arises as to whether a party signed a document (caused by either a lying signer trying to repudiate a signature it did create, or a fraudulent claimant), an unbiased third party should be able to resolve the matter equitably, without requiring access to the signers secret information (private key). The importance of digital signatures to modern electronic commerce has become overwhelming such that Rivest [7] has written “[they] may prove to be one of the most fundamental and useful inventions of modern cryptography.”

The security of all public key digital signature schemes presently depends on the inability of a forger to solve certain difficult mathematical problems, such as factoring large numbers [8]. Regretfully, with a quantum computer factoring becomes tractable [9], thus allowing signatures to be forged. In 2001, Gottesman and Chuang [4] proposed a quantum digital signature scheme whose security is based on fundamental principles of quantum physics. It allows a sender (Alice) to sign a message in such a way that the signature can be validated by a number of different people, and all will agree either that the message came from Alice or that it has been tampered with. The public

keys in the scheme can only be used once, unlike more sophisticated digital signature schemes. So this simple protocol can serve as a model for a quantum scheme.

In 2002, Zeng and Keitel [10] proposed an arbitrated quantum-signature scheme (AQSS for short). The suggested algorithm is implemented by a symmetrical quantum key cryptosystem and Greenberger-Horne-Zeilinger (GHZ) [5] triplet states. Its security relies on the availability of an arbitrator. In 2008, Curty and Lütkenhaus [3] pointed out that the protocol is not clearly operationally defined and several steps are ambiguous. Moreover, they argue that the security statements are incorrect. In the reply [11], the author gave more detailed presentations and proofs of the scheme. He also claimed that the scheme is suitable for unknown messages.

In this letter, we revisit the scheme using a general technique and show that the invented scenario in [11] is artificial. The claim that the original scheme is suitable for unknown messages is not sound. This is because it is unreasonable that in a signature scheme the final verifier can not know the content of the signed message.

## 2 Review of the AQSS

We now briefly review the arbitrated quantum signature scheme [10]. The following description follows that in [11].

[Step-I1] Obtaining keys  $K_a$  and  $K_b$ . The lengths of these keys depend on the chosen cryptographic algorithms in the signing and verifying phases.

[Step-I2] Distributing GHZ triplet states  $\psi$ .

[Step-S1] Alice presents a message state  $|P\rangle = \{|p_1\rangle, |p_2\rangle, \dots, |p_n\rangle\}$  with  $|p_i\rangle = \alpha_i|0\rangle + \beta_i|1\rangle$ .

[Step-S2] Alice generates  $|R\rangle = \{|r_1\rangle, |r_2\rangle, \dots, |r_n\rangle\}$ .

[Step-S3] Alice obtains a four-particle state  $|\phi\rangle_i$  via entangling the message state  $|p_i\rangle$  and the GHZ state  $|\psi\rangle$  according to the Eq. (8) defined in Ref. [10].

[Step-S4] Alice executes a Bell measurement on  $|\phi\rangle_i$  and obtains the results  $\mathcal{M}_a$  expressed in Eq. (9) in the Ref. [10].

[Step-S5] Alice creates the signature  $|S\rangle$  of the message  $|P\rangle$  via encrypting the Bell measurement results  $\mathcal{M}_a$  and the generated  $|R\rangle$  using a quantum symmetrical key cryptosystem, e.g., the quantum one-time pad algorithm.

[Step-S6] Alice sends  $|P\rangle$  followed by the signature  $|S\rangle$  to Bob.

[Step-V1] Bob measures his GHZ particles and obtains the results  $\mathcal{M}_b$ , then he encrypts  $\mathcal{M}_b$ ,  $|S\rangle$ , and  $|P\rangle$  with his key  $K_b$  to obtain  $y_b$ . After that Bob sends  $y_b$  to the arbitrator.

[Step-V2] The arbitrator generates a verification parameter  $\gamma$  according to Eq. (13) in Ref. [10].

[Step-V3] The arbitrator sends his GHZ particles and the encrypted result  $y_{tb} = K_b(\mathcal{M}_a, \mathcal{M}_b, \gamma, |S\rangle)$

to Bob.

[Step-V4] Bob obtains the arbitrator's GHZ particles. In addition, Bob obtains  $\mathcal{M}_a, \mathcal{M}_b, |S\rangle$ , and  $\gamma$  via decrypting the received  $y_{tb}$ .

[Step-V5] Bob performs the initial verification via the parameter  $\gamma$ .

[Step-V6] Bob performs the further verification via comparing  $|P\rangle$  and  $|P'\rangle$ , where  $|P'\rangle$  is obtained according to the correlation of the GHZ triplet state.

In 2008, Curty and Lütkenhaus [3] pointed out it is unclear what are the real advantages of this protocol if all the parties know the state  $|P\rangle$ . In the reply [11], the author acknowledged

The AQSS works for known messages even though it is not very useful or efficient, which was never claimed. The main aims of the AQSS are to present another application of the entanglement in cryptology and to prove the possibility of a quantum-signature. Based on the AQSS we expected some further investigation of the quantum-signature.

Finally, the author stresses that the arbitrated quantum-signature scheme is, in principle, also suitable for the unknown message. He explains

The unknown message signature scheme is always called the "blind signature" in classic cryptology. The blind signature considers the cases where Alice or Bob, or even both Alice and Bob, do not know the content of the message to be signed and verified.

### 3 Analysis of the AQSS

We now revisit the arbitrated quantum signature scheme by a general technique.

[Step-I1'] Obtaining keys  $K_a$  and  $K_b$ .

[Step-S1'] Alice presents a message state  $|P\rangle$ .

[Step-S2'] Alice creates  $|S\rangle = K_a(|P\rangle)$  and sends the signature  $|S\rangle$  to Bob.

[Step-V1'] Bob creates  $y_b = K_b(|S\rangle)$  and sends  $y_b$  to the arbitrator.

[Step-V2'] The arbitrator decrypts  $y_b$  with the key  $K_b$  to obtain  $|S'\rangle$ . He then decrypts  $|S'\rangle$  with the key  $K_a$  to obtain  $|P'\rangle$ . If  $|P'\rangle = |P\rangle$ , he sets  $\gamma = 1$ . Otherwise,  $\gamma = 0$ . The arbitrator then creates  $y_{tb} = K_b(|P'\rangle, \gamma)$  and sends  $y_{tb}$  to Bob.

[Step-V3'] Bob decrypts  $y_{tb}$  with the key  $K_b$  to obtain  $|P''\rangle, \gamma'$ . He then checks  $\gamma' = 0$ ? If  $\gamma' = 0$ , he rejects it. If  $\gamma' = 1$ , he performs the further verification via checking  $|P\rangle = |P''\rangle$ ? If  $|P\rangle = |P''\rangle$ , he accepts it. Otherwise, he rejects it.

By the simplified protocol, we find the requirement for the expensive GHZ triplet-particle can be removed. Why can the simple protocol work well? This is because the arbitrator knows all private keys of the involved users. Actually, in the presence of an *absolutely* trustworthy arbitrator, almost

cryptographic primitives become easy to achieve. The authors [10] *misunderstand* the meaning of the term “arbitrator” in cryptology. It leads them to a peculiar protocol (the arbitrator shares the key  $K_a, K_b$  with Alice and Bob, respectively). As for the role of an arbitrator in cryptographic protocols, we refer to [2]:

An arbitrator is a disinterested third party trusted to complete a protocol. Trusted means that all people involved in the protocol accept what he says as true, what he does as correct, and that he will complete his part of the protocol. Arbitrators can help complete protocols between two mutually distrustful parties.

Notice that an arbitrated protocol does not entail that the arbitrator knows all private keys of the involved users.

In the reply [11], the author claimed that the original scheme is suitable for unknown messages. We now argue that the claim is false. First, we claim it is unreasonable that the final verifier can not know the content of the signed message. In fact, the ultimate motive of a signature is to assure the authorship (or at least agreement with the contents) of the signed message to the final verifier. Second, the author [11] also misunderstands the scenario for a classical blind signature. In cryptography, a blind signature, as introduced by D. Chaum [1], is a form of digital signature in which the content of a message is disguised (blinded) before it is signed. The resulting blind signature can be publicly verified against the original, unblinded message, in the manner of a regular digital signature. Therefore, we stress that the final verifier must know the content of the signed message.

## 4 Conclusion

In this letter, we remark that Zeng-Keitel arbitrated quantum signature scheme is artificial. We also clarify that in a blind signature scheme the final verifier knows the content of the signed message.

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