

# Group Decryption<sup>\*</sup>

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**Abstract.** Anonymity is one of the main concerns in group-oriented cryptography. However, most efforts, for instance, group signatures and ring signatures, are only made to provide anonymity on the sender's point of view. There is only few work done to ensure anonymity in a cryptographic sense on the recipient's point of view in group-oriented communications. In this paper, we formalize the notion of *group decryptions*. It can be viewed as an analogous of group signatures in the context of public key encryptions. In this notion, a sender can encrypt a committed message intended to any member of a group, managed by a group manager, while *the recipient of the ciphertext remains anonymous*. The sender can convince a verifier about this fact *without* leaking the plaintext or the identity of the recipient. If required, the group manager can verifiably open the identity of the recipient. We propose an efficient group decryption scheme that is proven secure in the random oracle model. The overhead in both computation and communication is *independent of the group size*. A full ciphertext is about 0.2K bytes in a typical implementation and the scheme is practical to protect the recipient identity in privacy-sensitive group-oriented communications.

**Keywords.** group-oriented cryptography, group decryption, anonymity, bilinear pairing

## 1 Introduction

Anonymity is the main concern in group-oriented cryptography. It has attracted a lot of attentions in the context of digital signatures and extensively studied in the literature, such as group signatures, ring signatures, etc. However, these types of anonymous signatures only provide anonymity on the sender's viewpoint in the communication. There is only few work done to ensure anonymity on the recipient's viewpoint using cryptographic primitives. This paper concentrates on the *identity privacy of recipients* in group-oriented public key encryptions and proposes practical solutions.

Let us consider the following scenario. Alice wants to send a secret message to Bob who is one of the department managers in a company. For some security reasons and the purpose of protecting the managers from dealing with junk messages, the company gateway system does not allow the message in the network, unless it is directed for some department manager. However, Bob may not want to let the gateway know that he is the intended recipient of the

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<sup>\*</sup> We noted that Aggelos Kiayias and Yiannis Tsiounis and Moti Yung recently presented an independent paper Group Encryption which achieved the same goals with an different implementation at: <http://eprint.iacr.org/2007/015.pdf>. Their work was submitted on 12 Jan 2007 from database record but not publicly accessible until 19 Jan 2007 due to a reviewing process. We submitted this report to eprint on 21 Jan 2007.

message sent by Alice. By knowing only the public information of managers of the company, the gateway system has to determine whether the encrypted message is allowed to stay in the network or not. In other words, the gateway needs to test whether the message is indeed sent to one of the managers in the company, without knowing *who* is the intended recipient of the message. Furthermore, in the case of dispute, we may hope a trusted third party can reveal the identity of the recipient.

There may exist other applications where the recipients' anonymity is useful. For instance, in the privacy enhanced optimistic fair exchange scenario, the two parties exchanging the secrets may not want to reveal their identities to the third party. This notion may also be useful in identity escrow and transactions over the Internet. In [16], Kiayias *et al.* illustrate other examples of recipient anonymity such as anonymous trusted third party escrow, secure oblivious retriever storage and ad hoc access structure group signature.

There are some related notions on the anonymity of signers in the context of signatures. Group signatures, introduced by Chaum and van Heyst [9,5,8], provide signers' anonymity. Any group member can sign messages on behalf of the group, but the resulting signatures keep the identity of signer secret. In the standard definition, there is a third party who can open the signature, or undo its anonymity in the case of dispute. A ring signature, introduced in [22], is an alternative mean to achieve anonymity for ad-hoc groups without requiring any trusted manager. It is used to convince any third party that at least one member in an ad-hoc group has indeed issued the signature on behalf of the group. In contrast to the group signatures, the anonymity in ring signatures cannot be revoked.

In the context of public key based encryptions, recently, Bellare et al. [3] presented a notion of key privacy in public-key encryption schemes. However, the setting, goal and model in their notion are different from ours. They studied the setting of asymmetric encryptions to capture a security property for public-key-based encryption schemes that an attacker cannot determine the public keys that were used to generate the ciphertexts that it sees. Notice that the attacker cannot verify whether the ciphertexts are valid for some of the potential recipients, and no trusted party can trace the intended recipient. They use the classic chosen plaintext attack (CPA) and chosen ciphertext attack (CCA) to model the adversary in their notion. Their goal is to find public key encryption schemes with a special property referred to as recipient anonymity or key privacy. They managed to show that the existing well-known schemes such as the ElGamal encryption [12], the Cramer-Shoup encryption [11] and the RSA-OAEP [7,23] provide such a recipient anonymity with or without some slight modifications.

In [19,18], a similar notion of custodian-hiding verifiable encryption schemes was presented. In their notion, a sender can verifiably encrypt a message under a chosen public key from a public key list but the intended recipient is anonymous. Since there is no group manager to administer the potential recipients, their notion is designed for only ad-hoc applications and hence each ciphertext has to contain the public key list of potential recipients. Their instantiations suffer from a linear cost in both communication and computation in addition to the public key list in each ciphertext. In case of dispute, no group manager can revoke the anonymity of the receiver.

In this paper, we first contribute the formalization of group decryption. Then we realize an efficient scheme and prove its security in the random oracle model. To enable our realization, we also present several sub-protocols which may find independent applications.

- *Model and security definitions.* We formalize the notion of group decryptions. It can be viewed as an analogous of group signatures in the context of public key encryptions. In

this notion, a sender can encrypt a committed message to any intended group member managed by a group manager while *the recipient of the ciphertext remains anonymous*. The sender can convince a verifier about this fact without leaking the plaintext or the identity of the recipient. If required, *the group manager can verifiably open the identity of the recipient* (for instance, in case of dispute).

- *Efficient implementation.* We realize an efficient group decryption scheme from SXDH bilinear pairings. We prove that the scheme is secure in the random oracle model[6]. The overhead in both computation and communication is independent of the group size and the scheme is practical.
- *Commitment schemes and  $\Sigma$ -protocols of group element in pairing groups.* We also present several efficient sub-protocols such as commitment schemes to commit a group element in pairing groups and the corresponding zero-knowledge proof protocols. These sub-protocols are of independent interest and maybe useful for other applications.

The rest of the paper is organized as follows. The next section formalizes the notion of group decryptions. In Section 3, we review the underlying computational assumptions. Section 4 presents the building blocks. We propose our group decryption scheme in Section 5, followed by a conclusion in the last section.

## 2 Modeling Group Decryption

In this section, we formalize the notion of group decryption. It allows a sender to verifiably encrypt a committed message to any group member while the intended recipient remains anonymous. In case of dispute, the anonymity can be verifiably revoked by the group manager.

### 2.1 Group Decryption Algorithms

A group decryption (GE) scheme involves four parties, a group manager who administrate the group and trace the recipients when necessary, a group of registered members who anonymously receives message from senders, a sender who may be group members or outsiders and has secret message to be sent to the registered members, and a verifier who can verify the GE protocol without secret inputs. A GE scheme consists of the following procedures.

- **ParaGen:** It is a polynomial time algorithm which on input a security  $\lambda$ , outputs the system-wide parameters  $\pi$ .
- **GKeyGen:** It is a polynomial time algorithm which on input the system parameters  $\pi$ , outputs the group public and secret key pair  $(gpk, gsk)$ .
- **UKeyGen:** It is a polynomial time algorithm which on input the system parameters  $\pi$ , outputs a user's public and secret key pair  $(upk, usk)$ . It outputs independent key pairs in different runs with the same input  $\pi$ .
- **Join:** It is a polynomial time interactive algorithm between a user  $\mathcal{U}$  who wants to join a group and the group manager  $\mathcal{GM}$ . The user has input  $usk$  while the group manager has input  $gsk$ . The common input is  $(\pi, gpk, upk)$ . The user has output  $(mpk, msk)$  which is the public and secret key pair of  $\mathcal{U}$  as a legitimate group member. The group manager outputs an updated local database which includes a tracing trapdoor  $T_{\mathcal{U}}$  corresponding to group member  $\mathcal{U}$ . The tracing trapdoors forms a tracing list  $L_T$  secretly maintained by the group manager.  $T_{\mathcal{U}}$  and  $T_{\mathcal{U}'}$  may be identical or not for  $\mathcal{U} \neq \mathcal{U}'$ . All the legitimate group members' public keys  $mpk$  comprise of a public key list  $L_{pk}$ .

- **Encrypt**: It is a polynomial time algorithm which on input a secret message  $m$  in the structured message space, the intended group members’s public key  $mpk$  in the public key list and the system parameters  $\pi$ , outputs a ciphertext  $c$  in the ciphertext space.
- **EnVerify**: It is a polynomial time algorithm which on input a ciphertext  $c$ , the system parameters  $\pi$ , the group public key  $gpk$  and the public key list of the group members, outputs a bit 1 or 0 to represent that the ciphertext is valid or not.
- **Decryption**: It is a polynomial time algorithm which on input a valid ciphertext  $c$ , the system parameters  $\pi$ , the intended group member  $\mathcal{U}$ ’s public key  $mpk$  and secret key  $msk$ , outputs a message  $m$  in the message space.
- **Trace**: It is a polynomial time interactive algorithm between the group manager  $\mathcal{GM}$  and a verifier. The group manager has secret input  $(gsk, L_T)$  and a common input  $(\pi, gpk, L_{pk}, c)$  with the verifier, where  $c$  is a valid ciphertext. After the interactive algorithm is run, the manager  $\mathcal{GM}$  outputs  $mpk$  which represents the identity of the intended recipient of  $c$  and a zero-knowledge proof that the recipient has been corrected traced, while the verifier outputs a bit 1 or 0 to represent that the Trace procedure has been correctly run or not.

A group decryption scheme is said to be *correct* if all the parties follow the scheme honestly, the **EnVerify** algorithm outputs 1, the **Decryption** algorithm outputs the correct message and the verifier in the **Trace** procedure outputs 1.

## 2.2 Adversarial Model in Group Decryptions

We model the adversaries in group decryption schemes with the following oracles to which the adversaries can query. These oracles are maintained by a challenger.

- **UKeyGen Oracle**. For the  $i$ -th ( $i > 0$ ) query to this oracle, the adversary queries this oracle with an integer  $i$ . The challenger responds with the  $i$ -th user’s public key  $upk_i$  but keeps the corresponding secret key  $usk_i$ . The challenger maintains a counter  $n$  to records the query times and updates  $n = i$ .
- **Join Oracle**. The adversary queries this oracle with  $upk_i$  which is an output of the **UKeyGen**. The challenger runs the **Join** procedure for  $(upk_i, usk_i)$ . The transcript of this procedure and the corresponding group member public key  $mpk_i$  are sent to the adversary. The challenger updates the corresponding tracing list as the real scheme.
- **Corruption Oracle**. The adversary queries with  $mpk_i$  and obtains the corresponding secret key  $msk_i$  if  $mpk_i$  is in the group member public key list.
- **Encryption Oracle**. The adversary queries this oracle with  $(m, mpk_i)$ , where  $m$  is a message in the message space and  $mpk_i$  is in the group member public key list. The challenger responds the corresponding ciphertext  $c$ .
- **Decryption Oracle**. The adversary queries this oracle with a valid ciphertext for decryption. The challenger responds with the corresponding message.
- **Trace Oracle**. The adversary queries this oracle with a valid ciphertext. The challenger responds with the identity of the traced recipient and a proof to show that the ciphertext was sent to the traced recipient.

## 2.3 Security Definitions of Group Decryption

The security of group decryption schemes includes three aspects, i.e., the semantic security against chosen-ciphertext attacks, anonymity and traceability. We define these security notions in the standard two-stage games.

Let us first consider the semantic security against chosen-ciphertext attacks. It states that the attacker cannot get any information of the encrypted message by accessing all the above oracles, including the decryption procedure provided that secret key of the intended recipient and the target ciphertext were not queried. More formally, it is defined by the following game between a challenger  $\mathcal{CH}$  and an adversary  $\mathcal{A}$ .

**Setup:**  $\mathcal{CH}$  runs ParaGen and GkeyGen algorithms to generate the system parameters  $\pi$  and the group public and secret key pair  $(gpk, gsk)$ .  $(\pi, gpk)$  are sent to the attacker  $\mathcal{A}$ .  $\mathcal{CH}$  also initializes a counter and three lists  $L_U, L_M, L_T$  to recorder the users, the group members, and the tracing trapdoors.

**Phase 1:**  $\mathcal{A}$  can adaptively make all the oracles defined above.

**Challenge:**  $\mathcal{A}$  chooses a tuple  $(m_0, m_1, mpk_i)$ , where  $m_0, m_1$  are in the message space and  $mpk_i \in L_{pk}$  was never not queried to the Corruption oracle.  $\mathcal{CH}$  randomly selects a bit  $b \in \{0, 1\}$  and outputs the challenge ciphertext  $c^* = \text{Encrypt}(\pi, mpk_i, m_b)$ .  $\mathcal{CH}$  sends  $c^*$  to  $\mathcal{A}$ .

**Phase 2:**  $\mathcal{A}$  may make another sequence of queries as in Phase 1 with a constraint that the Corruption oracle cannot be queried on  $mpk_i$  and  $c^*$  cannot be queried to the Decrypt oracle.

**Output:** Finally  $\mathcal{A}$  outputs a guess bit  $b' \in \{0, 1\}$ .  $\mathcal{A}$  wins if  $b' = b$ . The advantage of  $\mathcal{A}$  is defined as  $|\Pr[b = b'] - \frac{1}{2}|$ .

**Definition 1.** *We say that a group decryption scheme is semantically secure against chosen ciphertext attacks if no polynomially bounded adversary has on-negligible advantage in the above game.*

The anonymity states that the attacker cannot determine the intended recipient of a ciphertext with a probability non-negligible greater than the probability of a random guess, provided that the attacker did not query the receivers' secret keys and the trace oracle. Note that the attacker can choose the message to be encrypted. Formally, the anonymity is defined by the following game between a challenger  $\mathcal{CH}$  and an adversary  $\mathcal{A}$ .

**Setup:** It is the same as the semantic security game.

**Phase 1:**  $\mathcal{A}$  can adaptively make all the oracles defined above.

**Challenge:**  $\mathcal{A}$  chooses a tuple  $(m, mpk_{i_0}, mpk_{i_1})$ , where  $mpk_{i_0}, mpk_{i_1} \in L_{pk}$  were never queried to the Corruption oracle and  $m$  is in the message space.  $\mathcal{CH}$  randomly selects a bit  $b \in \{0, 1\}$  and outputs the challenge ciphertext  $c^* = \text{Encrypt}(\pi, mpk_{i_b}, m)$ .  $\mathcal{CH}$  sends  $c^*$  to  $\mathcal{A}$ .

**Phase 2:**  $\mathcal{A}$  may make another sequence of queries as in Phase 1 except that the Corruption oracle cannot be queried on  $mpk_{i_0}, mpk_{i_1}$  and  $c^*$  cannot be queried to the Trace oracle.

**Output:** Finally  $\mathcal{A}$  outputs a guess bit  $b' \in \{0, 1\}$ .  $\mathcal{A}$  wins if  $b' = b$ . The advantage of  $\mathcal{A}$  is defined as  $|\Pr[b = b'] - \frac{1}{2}|$ .

**Definition 2.** *We say that a group decryption scheme is anonymous if no polynomially bounded adversary has a non-negligible advantage in the above game.*

A group decryption scheme should allow to revoke the identity of the recipient's identity in the case of dispute. The traceability of a group decryption scheme is defined by the following game between a challenger  $\mathcal{CH}$  and an adversary  $\mathcal{A}$ .

**Setup:** It is the same as the semantic security game.

**Probe Phase:**  $\mathcal{A}$  can adaptively make queries to all the oracles defined above.

**Output:**  $\mathcal{A}$  outputs a valid ciphertext  $c^*$ .  $\mathcal{A}$  wins if  $\mathcal{CH}$  cannot output the identity of the recipient in the member list or provide a valid zero-knowledge proof of the traced identity accepted by an honesty verifier.

**Definition 3.** We say that a group decryption scheme is traceable if no polynomially bounded adversary has negligible probability to win the above game.

### 3 Preliminaries

#### 3.1 Bilinear Pairings

We review some general concepts of pairing groups [4,15,24]. Let **PairingGen** be an algorithm that, on input a security parameter  $1^\lambda$ , outputs a tuple  $\mathcal{Y} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e)$ , where  $\mathbb{G}_1 = \langle g_1 \rangle$  and  $\mathbb{G}_2 = \langle g_2 \rangle$  have the same prime order  $p$ .  $e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_3$  is an efficient bilinear map if the following conditions hold:

1. **Non-degeneration:**  $e(g_1, g_2) \neq 1$ ;
2. **Bilinearity:** For all  $h_1 \in \mathbb{G}_1, h_2 \in \mathbb{G}_2$  and  $u, v \in \mathbb{Z}$ ,  $e(h_1^u, h_2^v) = e(h_1, h_2)^{uv}$

From [14], there are three types of pairing groups:

1.  $\mathbb{G}_2 = \mathbb{G}_1$ . Accordingly, we denote it  $\mathcal{Y} = (p, \mathbb{G}, \mathbb{G}_3, g, e) \leftarrow \text{PairingGen}(1^\lambda)$  for simplicity.
2.  $\mathbb{G}_2 \neq \mathbb{G}_1$  in which there is an *efficient distortion map*  $\psi : \mathbb{G}_2 \rightarrow \mathbb{G}_1$  but there is no *efficient distortion map*  $\varphi : \mathbb{G}_1 \rightarrow \mathbb{G}_2$ , where the distortion map satisfies  $\psi(g_2^u) = \psi(g_2)^u \in \mathbb{G}_1$  for any  $u \in \mathbb{Z}_p$ .
3.  $\mathbb{G}_2 \neq \mathbb{G}_1$  but there is no *efficient distortion map*  $\psi : \mathbb{G}_2 \rightarrow \mathbb{G}_1$  or  $\varphi : \mathbb{G}_1 \rightarrow \mathbb{G}_2$ .

If  $\mathbb{G}_2 \neq \mathbb{G}_1$  and there are efficient homomorphisms  $\psi : \mathbb{G}_2 \rightarrow \mathbb{G}_1$  and  $\varphi : \mathbb{G}_1 \rightarrow \mathbb{G}_2$ , it can be re-interpreted as Type 1. The Type 1 case is implemented using supersingular curves. The curves of Type 2 are ordinary and the homomorphism from  $\mathbb{G}_2 \rightarrow \mathbb{G}_1$  is the trace map. The curves of Type 3 are ordinary and  $\mathbb{G}_2$  is typically taken to be the kernel of the trace map.

#### 3.2 Computational Assumptions

Suppose that  $\mathcal{Y} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$  are SXDH pairing groups, where  $\mathbb{G}_1, \mathbb{G}_2$ , and  $\mathbb{G}_3$  are public. Our proposals are based on the following assumptions about pairing groups. We recall that these assumptions have been used by previous works in the literature [1,2,4,17].

**Assumption 1 (Inverse of Bilinear Pairing (IBP) Assumption)** Let  $\mathcal{Y} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$ . Given random value  $A \in \mathbb{G}_2$ , for any probabilistic polynomial time (PPT) adversary  $\mathcal{A}$ , the probability to compute  $X \in \mathbb{G}_1$  satisfying  $e(X, g_2) = A$  is negligible in  $\lambda$ .

The IBP assumption is weaker than the co-CDH assumption [4]. An adversary  $\mathcal{A}$  breaking the IBP assumption can be efficiently converted into an adversary  $\mathcal{B}$  to break the co-CDH assumption. The transformation is trivial: Given a co-CDH challenge  $(g_1, g_2, g_1^u, g_2^v)$ ,  $\mathcal{B}$  computes  $A = e(g_1^u, g_2^v) = e(g_1, g_2)^{uv}$  and queries  $\mathcal{A}$  with  $(A, g_1, g_2)$ .  $\mathcal{B}$  straightforwardly uses  $\mathcal{A}$ 's

reply  $X = g_1^{uv}$  to answer the co-CDH challenge. Similarly, if  $\mathbb{G}_1 = \mathbb{G}_2$ , the IBP assumption is implied by the classical CDH assumption in the case  $\mathbb{G} = \mathbb{G}_1 = \mathbb{G}_2$ .

The IBP assumption is an analog of the RSA assumption in the pairing group settings. We will use a strong version of the IBP assumption which can be viewed as an analog of the strong RSA assumption in the pairing group settings. This assumption holds only in the SXDH pairing groups (Type 3).

**Assumption 2 (Strong Inverse of Bilinear Pairing (SIBP) Assumption)** *Let  $\mathcal{Y} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$  be pairing groups of Type 3. Given random value  $h_2 \in \mathbb{G}_2$ , for any PPT adversary  $\mathcal{A}$ , the probability to compute a pair  $(X, Y) \in \mathbb{G}_1^2$  satisfying  $e(X, g_2) = e(Y, h_2)$  is negligible in  $\lambda$ .*

In pairing groups of type 3, the conventional DDH assumption holds in both  $\mathbb{G}_1$  and  $\mathbb{G}_2$ . Hence, such pairing groups are also called SXDH (symmetric external Diffie-Hellman) pairing groups [1]. In [1], Ateniese *et al.* exploited such pairing groups to built their practical group signatures without random oracles.

**Assumption 3 (Symmetric External Diffie-Hellman (SXDH) Assumption)** *Let  $\mathcal{Y} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$  be pairing groups of Type 3. The SXDH assumption states that the standard DDH assumption holds in both  $\mathbb{G}_1$  and  $\mathbb{G}_2$ .*

The LRSW assumption is a discrete-logarithm assumption originally introduced by Lysyanskaya *et al.* [17] and used in many subsequent works. Recently, a stronger form of the LRSW assumption, called Strong LRSW, was introduced by Ateniese *et al.* [2]. Strong LRSW only holds in SXDH pairing groups (Type 3).

**Assumption 4 (Strong LRSW Assumption)** *For SXDH pairing groups  $\mathcal{Y} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$ , Let  $X, Y \in \mathbb{G}_2$  be chosen at random, and  $O_{X,Y}(\cdot)$  be an oracle that takes as input a value  $v \in \mathbb{Z}_p^*$ , and outputs an LRSW-tuple  $(a, a^x, a^{y+vx})$  for a random  $a \in \mathbb{G}_1$ . Then for any PPT adversary  $\mathcal{A}^{(\cdot)}$  and all  $u \in \mathbb{Z}_p^*$ ,*

$$\Pr \left[ \begin{array}{l} x \leftarrow \mathbb{Z}_p, y \leftarrow \mathbb{Z}_p \\ X = g_2^x, Y = g_2^y \end{array} \middle| \begin{array}{l} (a_1, a_2, a_3, a_4, a_5) \leftarrow \mathcal{A}^{O_{X,Y}(\cdot)}(g_1, g_2, X, Y) \wedge a_1 \in \mathbb{G}_1 \\ \wedge a_2 = a_1^u \wedge a_3 = a_1^x \wedge a_4 = a_1^{ux} \wedge a_5 = a_1^{y+ux} \wedge u \notin \mathbb{Q} \end{array} \right] \leq \frac{1}{\text{poly}(\lambda)}$$

where  $\mathbb{Q}$  is the set of queries  $\mathcal{A}$  makes to  $O_{X,Y}(\cdot)$ .

## 4 Building Blocks

In this section, as building blocks of our group decryptions, we present new commitment schemes to commit group element in pairings. The commitment schemes works similarly to the well-known discrete logarithm and Pedersen commitments [20]. Then we propose zero-knowledge proofs of the committed values and show that these sub-protocols are  $\Sigma$ -Protocols.

### 4.1 Commitment

A commitment scheme consists of four efficient algorithms:  $\mathcal{C} = (\text{ParaGen}, \text{Com}, \text{Open}, \text{Ver})$ . The generation algorithm  $\text{ParaGen}(1^k)$ , where  $k$  is the security parameter, outputs a public commitment key  $pk$  (possibly empty, but usually consisting of public parameters for the commitment scheme). Given a message  $m$  from the associated message space  $\mathcal{M}$ ,  $\text{Com}_{pk}(m; r)$  produces a commitment string  $c$  for the message  $m$ . The opening algorithm  $\text{Open}_{pk}(m; r)$  produces a decommitment value  $d$  for  $c$ . Finally, the verification algorithm  $\text{Ver}_{pk}(m, c, d)$

accepts (i.e., outputs 1) if the pair  $(c, d)$  is a valid commitment/decommitment pair for  $m$ . We require that for all  $m \in \mathcal{M}$ ,  $\text{Ver}_{pk}(m, \text{Com}_{pk}(m; r), \text{Open}_{pk}(m; r)) = 1$  holds with all but negligible probability. Without loss of generality, we assume that the opening algorithm simply outputs its randomness  $r$  as the decommitment, and the verification algorithm simply checks if  $c = \text{Com}_{pk}(m; r)$ .

- **Hiding:** No PPT adversary (who knows  $pk$ ) can extract  $m$  from  $\text{Com}_{pk}(m; r)$ . Sometimes, we may need a stronger hiding property where no PPT adversary can distinguish the commitments to any two message of its choice:  $\text{Com}_{pk}(m_1, r_1), \text{Com}_{pk}(m_2, r_2)$ . That is,  $\text{Com}_{pk}(m, r)$  reveals “no information” about  $m$  even allowing the adversary to choose the message space.
- **Binding:** Having the knowledge of  $pk$ , it is computationally hard for any PPT adversary  $\mathcal{A}$  to come up with  $c, m, d, m', d'$  such that  $(c, d)$  and  $(c, d')$  are valid commitment pairs for  $m$  and  $m'$ , but  $m \neq m'$  (such a tuple is said to be a collision). That is,  $\mathcal{A}$  cannot find a value  $c$  which it can open in two different ways.

## 4.2 $\Sigma$ -Protocols

Let  $\mathcal{R} = (x, w)$  be some NP-relation (i.e., it is efficiently testable to see if  $(x, w) \in \mathcal{R}$  and  $|w| \leq \text{poly}(|x|)$ ). We usually call  $x$  the input, and  $w$  the witness (for  $x$ ). Consider a three move protocol run between a PPT prover  $\mathbf{P}$ , with input  $(x, w) \in \mathcal{R}$ , and a PPT verifier  $\mathbf{V}$  with input  $x$ .  $\mathbf{P}$  chooses a random string  $r_p$ , computes  $a = \text{Start}(x, w; r_p)$ , and sends  $a$  to  $\mathbf{V}$ .  $\mathbf{V}$  then chooses a random string  $c$  (called “challenge”) from some appropriate domain  $E$  and sends it to  $\mathbf{P}$ . Finally,  $\mathbf{P}$  responds with  $z = \text{Finish}(x, w, c; r_p)$ . The verifier  $\mathbf{V}$  then computes and returns a bit  $b = \text{Check}(x, a, c, z)$ . We require that  $\text{Start}$ ,  $\text{Finish}$ , and  $\text{Check}$  be polynomial-time algorithms, and that  $|c| \leq \text{poly}(|x|)$ . Such a protocol (given by procedures  $\text{Start}$ ,  $\text{Finish}$ , and  $\text{Check}$ ) is called a  $\Sigma$ -Protocol for  $\mathcal{R}$  if it satisfies the following properties, called completeness, special soundness, and special honest-verifier zero-knowledge:

- **Completeness:** If  $(x, w) \in \mathcal{R}$  then the verifier outputs  $b = 1$  (with all but negligible probability).
- **Special Soundness:** There exists a PPT algorithm  $\text{Extract}$ , called the (knowledge) extractor, such that it is computationally infeasible to produce an input tuple  $(x, a, c, z, c', z')$  such that  $c \neq c'$  both lie in the proper “challenge” domain,  $\text{Check}(x, a, c, z) = \text{Check}(x, a, c', z') = 1$ , and yet  $\text{Extract}(x, a, c, z, c', z')$  fails to output a witness  $w$  such that  $(x, w) \in \mathcal{R}$ . Intuitively, if some prover can correctly respond to two different challenges  $c$  and  $c'$  on the same first flow  $a$ , then the prover must “know” a correct witness  $w$  for  $x$  (in particular,  $x$  has a witness).
- **Special HVZK:** There exists a PPT algorithm  $\text{Sim}$ , called the simulator, such that for any  $(x, w) \in \mathcal{R}$  and for any fixed challenge  $c$ , the following two distributions are computationally indistinguishable. The first distribution  $(x, a, c, z)$  is obtained by running an honest prover  $\mathbf{P}$  (with some fresh randomness  $r_p$ ) against a verifier whose challenge is fixed to  $c$ . The second distribution  $(x, a', c, z')$  is obtained by computing the output  $(a', z') \leftarrow \text{Sim}(x, c)$  (with fresh randomness  $r_s$ ). Intuitively, this says that for any  $a$ -priori fixed challenge  $c$ , it is possible to produce a protocol transcript computationally indistinguishable from an actual run with the prover (who knows  $w$ ).

With the well-known Fiat-Shamir transformation [13], the HVZK protocols can be converted into digital signatures. They can be proven secure in the random oracle model due to the fork lemma[21].

### 4.3 Knowledge Proof of Committed Element in Pairings Groups

We present a commitment scheme to commit to elements in pairing groups then show how to prove the knowledge of the committed values. This commitment scheme is similar to the discrete logarithm commitment scheme.

Let  $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$ . The public commitment key is  $pk = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e)$ . To commit a group element  $x \in \mathbb{G}_1$ , one computes the commitment

$$A = e(x, g_2).$$

To open the commitment  $A$ , the committer shows  $m \in \mathbb{G}_1$  to the verifier. The verifier checks that  $x \stackrel{?}{\in} \mathbb{G}_1$  and  $e(x, g_2) \stackrel{?}{=} A$ . The verifier outputs 1 if both verifications hold; otherwise it outputs 0. Clearly, the commitment scheme is computationally hiding and binding based on the IBP assumption.

Similarly to the knowledge proof of discrete logarithm, we present a knowledge proof of the knowledge of the committed  $m$  in  $A$ , and denote the protocol by

$$PK\{x|A = e(x, g_2)\}.$$

The 3-move protocol is as follows.

**Step 1** The prover (i.e., the committer) randomly selects  $r \in \mathbb{G}_1$  and sends  $B = e(r, g_2)$  to the prover.

**Step 2** The verifier challenges the prover with a random  $c \in \mathbb{Z}_p^*$ .

**Step 3** The prover responds with  $s = rx^c$ .

**Step 4** The verifier checks that  $s \stackrel{?}{\in} \mathbb{G}_1$  and  $e(s, g_2) \stackrel{?}{=} BA^c$ . The verifier outputs 1 if both checks hold; otherwise it outputs 0.

**Theorem 1.** *The above knowledge proof protocol  $PK\{x|A = e(m, g_2)\}$  is  $\Sigma$ -protocol.*

*Proof.* The completeness of the above protocol is obvious. Now we prove the soundness and zero-knowledge. We first show the *special soundness* by construction of an efficient knowledge extractor if the malicious committer can respond to different challenges  $c \neq c'$  on the same first flow. Let  $s \neq s' \in \mathbb{G}_1$  be the two different responses. From the verification equations, it holds that  $e(s, g_2) = BA^c$  and  $e(s', g_2) = BA^{c'}$ . Assume that  $r \in \mathbb{G}_1$  satisfies that  $e(r, g_2) = B$ . Then we have that  $s = rx^c$  and  $s' = rx^{c'}$ . It follows that  $sx^{-c} = s'x^{-c'}$ . Hence,  $x^{c-c'} = s/s'$ . Since  $c - c' \neq 0$ ,  $x = (s/s')^{\frac{1}{c-c'}}$ . The knowledge is extracted.

Then we prove the *special HVZK*, i.e., to construct an efficient simulator  $\text{sim}$ , which on input public parameters  $pk$  and  $A$ , outputs a simulated transcript  $(B', c', s')$  indistinguishable from the output  $(B, c, s)$  of a real run of the protocol. The simulation works as follows. Randomly select  $s' \leftarrow \mathbb{G}_1, c' \leftarrow \mathbb{Z}_p^*$ . Compute  $B' = e(s', g_2)/A^{c'}$ . Clearly,  $e(s', g_2) = B'A^{c'}$ , and  $(B', c', s')$  has identical distribution as the output  $(B, c, s)$  of a real run of the protocol. This completes the proof.  $\square$

### 4.4 Equality Proof of Committed Elements in Pairing Groups

Let the prover committed two values  $A = e(x, g_2)$  and  $B = e(x, h_2)$ , where  $h_2$  is an independent generator of  $\mathbb{G}_2$ . We present a zero-knowledge proof of the equality of committed group elements and denote the protocol by

$$PK\{x|A = e(x, g_2) \wedge B = e(x, h_2)\}.$$

The protocol works as follows.

- **Step 1** The prover randomly selects  $r \leftarrow \mathbb{G}_1$  and sends  $D = e(r, g_2)$  and  $E = e(r, h_2)$  to the verifier.
- **Step 2** The verifier challenges the prover with  $c \leftarrow \mathbb{Z}_p^*$ .
- **Step 3** The prover responds with  $s = rx^c$ .
- **Step 4** The verifier checks that  $s \stackrel{?}{\in} \mathbb{G}_1$ ,  $e(s, g_2) \stackrel{?}{=} DA^c$  and  $e(s, h_2) \stackrel{?}{=} EB^c$ . The verifier outputs 1 if all checks hold; otherwise, output 0.

Similarly to Theorem 1, we have the following claim.

**Theorem 2.** *The above protocol  $PK\{x|A = e(x, g_2) \wedge B = e(x, h_2)\}$  is a  $\Sigma$ -protocol.* □

#### 4.5 Knowledge Proof of Pedersen Commitment in Pairing Groups

The Pedersen commitment [20] of discrete logarithms is a widely used commitment scheme. In this subsection, we present a Pedersen commitment of a group element in SXDH pairing groups. Let  $\mathcal{Y} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_2, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$  be pairing groups of Type 3. Given random values  $h_2 \in \mathbb{G}_2$ , the Pedersen commitment of a secret element  $X \in \mathbb{G}_1$  is

$$A = e(X, g_2)e(R, h_2)$$

where  $r$  is randomly chosen from  $\mathbb{G}_1$ .

Similarly to the classical Pedersen commitment, we argue that this commitment is unconditionally hiding and computationally binding. On the one hand, given  $A \in \mathbb{G}_3$  and  $g_2, h_2$  as independent generators of  $\mathbb{G}_2$ , for any  $X' \in \mathbb{G}_1$ , there exists an  $R' \in \mathbb{G}_1$  such that  $e(R', h_2) = A/e(X', g_2)$ . Hence, it is unconditionally hiding. On the other hand, if the committer can output two pairings  $(X, R) \neq (X', R')$  such that  $A = e(X, g_2)e(R, h_2)$  and  $A = e(X', g_2)e(R', h_2)$ , then we have  $e(x, g_2)/e(x', g_2) = e(r', h_2)/e(r, h_2)$ . Hence, we can output a pair  $(x/x', r'/r)$  satisfying  $e(x/x', g_2) = e(r'/r, h_2)$ . However, this is infeasible in SXDH pairing groups under the SIBP assumption. Hence the above commitment is computationally binding.

We provide a knowledge proof of the Pedersen commitment in SXDH pairing groups. We denote the protocol by

$$PK\{X, R|A = e(X, g_2)e(R, h_2)\}.$$

The protocol runs as follows.

- **Step 1** The prover randomly selects  $V, W \leftarrow \mathbb{G}_1$  and sends  $D = e(V, g_2)e(W, h_2)$  to the verifier.
- **Step 2** The verifier challenges the prover with  $c \leftarrow \mathbb{Z}_p^*$ .
- **Step 3** The prover responds with  $S = VX^c, Z = WR^c$ .
- **Step 4** The verifier checks that  $S, Z \stackrel{?}{\in} \mathbb{G}_1$  and  $e(S, g_2)e(Z, h_2) \stackrel{?}{=} DA^c$ . The verifier outputs 1 if both checks hold; otherwise, output 0.

The completeness of the protocol is straightforward. For the security, we have the following result. We omit it as it is very similar to Theorem 1.

**Theorem 3.** *The above protocol  $PK\{X, R|A = e(X, g_2)e(R, h_2)\}$  is a  $\Sigma$ -protocol.* □

#### 4.6 Knowledge Proof of Discrete Logarithm of Pedersen Commitment in Pairing Groups

In our GE scheme, we need a further zero-knowledge proof that the prover knows the discrete logarithm of the committed element to a given base. We denote the protocol by

$$PK\{X, R, x | A = e(X, g_2)e(R, h_2) \wedge X = g_1^x\},$$

where a prover proves the knowledge of  $X, R \in \mathbb{G}_1$  and  $x \in \mathbb{Z}_p$  satisfying  $A = e(X, g_2)e(R, h_2)$  and  $X = g_1^x$  without leaking  $X, R$  or  $x$ . The protocol is as follows.

- **Step 1** The prover randomly selects  $\gamma \leftarrow \mathbb{Z}_p, W \leftarrow \mathbb{G}_1$  and sends  $D = e(g_1^\gamma, g_2)e(W, h_2)$  to the verifier.
- **Step 2** The verifier challenges the prover with  $c \leftarrow \mathbb{Z}_p^*$ .
- **Step 3** The prover responds with  $\sigma = \gamma + cx, Z = WR^c$ .
- **Step 4** The verifier finally checks that  $\sigma \stackrel{?}{\in} \mathbb{Z}_p, Z \stackrel{?}{\in} \mathbb{G}_1, e(g_1^\sigma, g_2)e(Z, h_2) \stackrel{?}{=} DA^c$ . The verifier outputs 1 if all checks hold; otherwise, output 0.

**Theorem 4.** *The above protocol  $PK\{X, R, x | A = e(X, g_2)e(R, h_2) \wedge X = g_1^x\}$  is a  $\Sigma$ -protocol.*

One can easily follow Theorem to prove the above theorem and we omit the proof. If we view  $X$  as the identity or public key of the prover and  $x$  its private key, the above protocol allows a HVZK proof of the secret key corresponding to a public key hidden in a commitment and may be useful for anonymous systems.

## 5 Proposed Group Decryption Scheme

In this section, we propose a group decryption scheme following the definition. We notice that, currently and independently, Kiayias, Tsiounis and Yung [16] presented a primitive referred to as *group encryption* and an efficient instantiation to achieve the same security goals as ours. We refer to this primitive as group decryption to stress the anonymity on the receiver's viewpoint. We briefly compare our proposal with theirs here.

Their general idea is to let the sender first commit to the message to be sent. Then the sender encrypts the message using the intended receiver's public key. The sender also encrypts the receiver's public key as well as the associated certificate from the group manager using the open manager's public key. Finally, the sender proves to a verifier it has behaved honestly in a zero-knowledge manner. For a practical implementation, proper underlying encryption schemes have to be found to enable an easy zero-knowledge proof protocol. They realize their scheme with a cramer-shoup variation of the Paillier cryptosystem and obtain a CCA-2 secure scheme without using random oracles. The zero-knowledge proof protocol is interactive. It can be converted into a non-interactive one using the Fiat-Shamir transformation but the security now relies on the random oracle model. Without considering the transcripts introduced by the zero-knowledge proof to show the correctness of the encryption, their requires about 5K bytes.

We also first let the sender commit to the message to be sent. However, before encrypting the message, the sender rerandomizes the receiver's public key and the corresponding certificate, such that the rerandomized public key corresponds to the same secret key as the original one and any one can verify that the rerandomized certificate is still a signature of

the re-randomized public key, but no one can link them with the original public key and certificate except the intended receiver and the group manager. Then the sender encrypts the message using the rerandomized public key of some group member. Finally, the sender just prove that the last encryption operation takes the committed message as input because the intended receiver can use its original secret key to decrypt it. For a practical implementation, we have to find proper encryption schemes and methods to generate the receiver's public keys and their certificates allowing rerandomization. We realize our scheme with the original ElGamal encryption in the context of pairing groups and the  $\text{CL}^+$ -signature [1,2] to generate the certificates of the group members' public keys. We obtain CCA-2 security only in the random oracle model but our scheme is non-interactive. With an interactive zero-knowledge proof, our scheme can also achieve CCA-2 security without random oracles if the Cramer-Shoup encryption in the context of pairing groups is adopted. The full ciphertext including the transcript of zero-knowledge proofs in our scheme is about 0.2K bytes and an order shorter than that (5K bytes) in [16].

- **ParaGen:**  $\Upsilon = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$ .  $\mathcal{H}(\cdot) : \{0, 1\}^* \rightarrow \mathbb{Z}_p$  is cryptographic hash function. Let  $h_2$  be an independent generator of  $\mathbb{G}_2$ . The globe parameters are  $\pi = \{\Upsilon, \mathcal{H}, h_2\}$ .
- **GKeyGen:** Randomly select  $x, y \leftarrow \mathbb{Z}_p^*$ . Compute  $X = g_2^x, Y = g_2^y$ . The public and secret keys of the group manager are

$$gpk = (X, Y), gsk = (x, y).$$

- **UKeyGen:** Choose at random  $u \leftarrow \mathbb{Z}_p^*$ . Compute  $U = e(g_1, g_2)^u$ . The public and secret keys of the user are

$$upk = U, usk = u.$$

- **Join:** A user  $\mathcal{U}$  can join a group and become a group member via the following protocol with the group manager  $\mathcal{GM}$ .
  1.  $\mathcal{U}$  sends  $E = g_1^u, T = g_2^u$  to  $\mathcal{GM}$  via a (an Ind-CCA2) *confidential* channel and proves the knowledge of decryption key:  $\varrho = PK\{u | E = g_1^u\}$ .
  2.  $\mathcal{GM}$  checks the validity of  $\varrho$  and  $e(E, g_2) = e(g_1, T) = U$ . If any check fails or  $T$  has been in its local database,  $\mathcal{GM}$  aborts the Join protocol. Else,  $\mathcal{GM}$  blindly generates a  $\text{CL}^+$ -signature  $S = (a_1, a_2, a_3, a_4, a_5)$  on the user's secret key  $u$  corresponding to the public key  $U = e(g_1, g_2)^u$ , where

$$a_1 = g_1^\gamma, a_2 = E^\gamma, a_3 = a_1^x, a_4 = a_2^x, a_5 = (a_1 a_4)^y$$

for a randomly chosen value  $\gamma \leftarrow \mathbb{Z}_p^*$ .  $\mathcal{GM}$  sends  $S = (a_1, a_2, a_3, a_4, a_5)$  to  $\mathcal{U}$  as its group certificate corresponding to  $U$ , where the secret key is hidden in the form  $a_2 = a_1^u$ .  $\mathcal{GM}$  adds  $(T, U)$  to its local database.

3. The user checks that the group certificate  $S = (a_1, a_2, a_3, a_4, a_5)$  is a valid  $\text{CL}^+$ -signature [2]:

$$e(a_1, X) = e(a_3, g_2), e(a_2, X) = e(a_4, g_2), e(a_1 a_4, Y) = e(a_5, g_2).$$

If any equation does not hold, the Join protocol fails. Else, the user computes a knowledge signature (converted from the corresponding  $\Sigma$ -protocol with the Fiat-Shamir transformation)

$$\sigma = KS\{u, T | e(a_1, T) = e(a_2, g_2) \wedge e(g_1, T) = U \wedge a_1^u = a_2\}(gpk || upk || S)$$

on a message of the group public key, the user's own public key and the corresponding certificate. Here, a knowledge signature  $\sigma = KS\{x|y = f(x)\}(m)$  denotes a signature  $\sigma$  of message  $m$  to show the knowledge of  $x$  such that  $y = f(x)$ . The user  $\mathcal{U}$  who has become a group member obtains its member public key and secret key:

$$mpk = \{S, U, \sigma\}, msk = u.$$

– **Encryption:** Let a sender want to send a committed message  $m \in G_1$  to a group member  $\mathcal{U}$ . It can verifiably send it to  $\mathcal{U}$  without leaking the identity of  $\mathcal{U}$  as follows.

1. **Membership check:** The sender verifies the validity of  $S$  and  $\sigma$ . If any check fails, the sender aborts.
2. **Message commitment:** For  $m \in G_1$ , commit the secret message  $m$  as follows:

$$\delta \leftarrow G_1, c_0 = e(m, g_2)e(\delta, h_2).$$

3. **Key Re-randomization:** Randomly select  $r \leftarrow \mathbb{Z}_p^*$  and re-randomize the group certificate of  $U$  by computing:

$$c_1 = a_1^r, c_2 = a_2^r, c_3 = a_3^r, c_4 = a_4^r, c_5 = a_5^r.$$

4. **Message encryption:** Randomly choose  $s \leftarrow \mathbb{Z}_p^*$ , compute

$$c_6 = a_1^s, c_7 = m^{-1}a_2^s.$$

5. **Encryption proof:** Prove that  $(c_0, c_6, c_7)$  has been correctly generated by compute the knowledge signature

$$c_8 = KS\{M, s|e(c_7, g_2)c_0 = e(M, g_2)e(\delta, h_2) \wedge c_6 = a_1^s \wedge M = a_2^s\}(c_0||c_1||\cdots||c_7),$$

which is equivalent to the following knowledge signature:

$$c'_8 = KS\{m, s|c_0 = e(m, g_2)e(\delta, h_2) \wedge c_6 = a_1^s \wedge c_7 = m^{-1}a_2^s\}(c_0||c_1||\cdots||c_7).$$

Output  $c = (c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8)$  as the resulting ciphertext of message  $m$  to the anonymous group member  $\mathcal{U}$ .

– **Encryption Verification:** Any verifier can verify the validity of the ciphertext as follows:

1. Check that

$$e(c_1, X) = e(c_3, g_2), e(c_2, X) = e(c_4, g_2), e(c_1c_4, Y) = e(c_5, g_2).$$

2. Check that  $c_8$  is a valid knowledge signature as defined.

If any check fails, the ciphertext is rejected. Else it is accepted.

– **Decryption:** The group member  $\mathcal{U}$  with a secret key  $u$  decrypts a ciphertext  $c$  as follows:

1. Check that  $c_2 = c_1^u$  to decide whether the ciphertext is intended to  $\mathcal{U}$ .
2. Check that validity of the  $CL^+$ -signature by the equalities:

$$e(c_3, g_2) = e(c_1, X), e(c_4, g_2) = e(c_2, X), e(c_5, g_2) = e(c_1c_4, Y).$$

This check validates that the intended recipient is a registered group member.

3. Check that  $c_8$  is a valid knowledge signature to validate that the ciphertext is computed from the committed message.

If any check fails, the group member  $\mathcal{U}$  aborts the Decryption procedure. Else, it outputs

$$m = c_6^u / c_7.$$

- **Receiver Tracing:** The group manager can trace the recipient as follows. It checks whether there exists  $(T, \mathcal{U})$  in its local database such that

$$e(c_1, T) = e(c_2, g_2).$$

If so, output  $U$ . Else output an error message. The group manager can prove to a verifier that it has correctly traced the recipient with the following zero-knowledge proof:

$$PK\{T | e(c_1, T) = e(c_2, g_2) \wedge e(g_1, T) = U\}.$$

The correctness of the scheme follows from a straightforward verification. For the security, we have the following claims.

**Theorem 5.** *The proposed group decryption scheme is semantically secure against chosen ciphertext attacks in the random oracle model under the DDH assumption and the Strong LRSW assumption in SXDH pairing groups.*

*Proof.* We prove that a successful attacker in the semantical security game of our scheme can be used as a subroutine to break the DDH assumption in  $\mathbb{G}_1$ .

Assume that we are given a DDH challenge  $(g_1, g_1^\alpha, g_1^\beta, g_1^\delta) \in \mathbb{G}_1^4$ , where  $\mathbb{G}_1$  is from SXDH pairing groups  $\mathcal{Y} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$ . We are required to answer whether  $\delta = \alpha\beta$  or not. We first use this DDH challenge to simulate the oracles that the attacker may query and then use the attacker's reply to answer the DDH challenge.

**Setup.** We randomly choose  $h_2 \in \mathbb{G}_2$ . The globe parameters are  $\pi = \{\mathcal{Y}, \mathcal{H}, h_2\}$ , where the hash function  $\mathcal{H}(\cdot) : \{0, 1\}^* \rightarrow \mathbb{Z}_p$  is modeled as a random oracle. It is simulated in the standard way. That is, we maintain an  $\mathcal{H}$ -list and for any query  $\xi$ , if  $\xi$  is not in the list, we reply with a random number  $c \in \mathbb{Z}_p$  and add  $(\xi, c)$  to the  $\mathcal{H}$ -list. If the  $\xi$  has been in the list, we forward the precious reply to maintain consistent. We also randomly select  $x, y \leftarrow \mathbb{Z}_p^*$  and compute  $X = g_2^x, Y = g_2^y$ . The public and secret key pair of the group manager is  $gpk = (X, Y), gsk = (x, y)$ .  $\pi$  and  $gpk$  are sent to the attacker.

**UkeyGen Oracle.** Let the system contain at most  $n = \text{poly}(\lambda)$  users. We randomly choose an integer  $1 \leq i_0 \leq n$ . For the attacker's query  $i \neq i_0$ , we behave as the real scheme and send the corresponding public key to the attacker. If  $i = i_0$ , we send  $e(g_1^\alpha, g_2)$  as the  $i_0$ -th user's public key.

**Join Oracle.** For the attacker's query  $upk \neq e(g_1^\alpha, g_2)$  in the user public key list  $L_U$ , we behave as the real scheme. For query  $upk = e(g_1^\alpha, g_2)$ , we randomly choose a string in the ciphertext space to respectively simulate the ciphertext of  $(g_1^\alpha, g_2^\alpha)$ . Since the channel here is confidential and Ind-CCA2 secure (See the Join protocol in Section 5), the attacker cannot distinguish the simulation from the real scheme with non-negligible probability. We randomly select  $(c, s) \in \mathbb{Z}_p \times \mathbb{Z}_p^*$  to simulate the (non-interactive) knowledge proof  $PK\{\alpha | E = g_1^\alpha\}$ . Add  $(g_1^s E^c, c)$  to the  $\mathcal{H}$ -list. Use  $(x, y)$ , we can generate the group certificate  $S = (a_1, a_2, a_3, a_4, a_5)$  for  $E = g_1^\alpha$  as the real scheme. We can also similarly simulate the knowledge signature  $\sigma = KS\{\alpha, T | e(a_1, T) = e(a_2, g_2) \wedge e(g_1, T) = U \wedge a_1^\alpha = a_2\}(gpk || E || S)$  in the random oracle model without knowing  $\alpha, T$ .

**Corruption Oracle.** Whenever the attacker queries a group member's secret key  $msk$ , if  $mpk$  corresponds to  $g^\alpha$ , we declare failure and aborts the protocol, denoted by a bad event *Failure 1*. Else we can correctly answer with the corresponding  $msk$  since we have generated such group member's public and secret key pair as the real scheme. Here, *Failure 1* happens with probability  $\varepsilon_1 = \frac{n_c}{n}$ , where  $n_c$  is the number of corrupted members.

**Encryption Oracle.** We do as the real scheme but keep a list to record all the ciphertext we have produced.

**Decryption Oracle.** When the attacker queries this oracle with  $c = (c_1, \dots, c_8)$ , if it is not corresponding to the user public key  $g_1^\alpha$ , we can reply with the corresponding message  $m$  as the real scheme. If it is corresponding to the user public key  $g_1^\alpha$  but  $(c_1, \dots, c_7, c'_8)$  is in the ciphertext list but  $c_8 \neq c'_8$ , we reply with the plaintext  $m$  corresponding to  $(c_1, \dots, c_7, c'_8)$  if and only if the encryption verification on  $c$  holds. If  $(c_1, \dots, c_7)$  is not in the ciphertext list but the encryption verification on  $c$  holds, we run the attacker two times using the standard rewinding technique [21] to extract  $M, s$ . We reply with  $Mc_7^{-1}$  as the corresponding plaintext.

**Trace Oracle.** When the attacker queries this oracle with a valid ciphertext  $c = (c_1, \dots, c_8)$ , we trace it with the tracing trapdoors in the tracing list. If  $mpk$  does not correspond to  $g_1^\alpha$ , we reply as the real scheme. If  $mpk$  corresponds to  $g_1^\alpha$ , we use the standard simulation of the zero-knowledge proof in the random oracle model to reply the attacker. If all the trapdoors fail to trace the recipient, we output the group member corresponding to  $g_1^\alpha$  as the recipient. Else we trace the recipient as the real scheme. This simulation fails if the ciphertext is not generated for any recipient in the group member list which implies that the attacker has successfully forge a group member certificate results into a solution to the Strong LRSW assumption. Hence, this *Failure 2* happens with negligible probability  $\varepsilon_2$  assuming the Strong LRSW assumption.

In the challenge phase, the attacker queries with a tuple  $(m_0, m_1, mpk)$ , where  $m_0, m_1$  are in the message space and  $mpk = (a_1, a_2, a_3, a_4, a_5, upk, \sigma)$  is a valid group member's public key and has never been queried to the **Corruption Oracle**. If  $upk \neq e(g_1^\alpha, g_2)$ , we declare failure and denoted it by a bad event *Failure 3* which happens with probability  $1 - \frac{1}{n}$ . Else we randomly choose a bit  $b \in \{0, 1\}$  and do the following.

- Commit the chosen message  $m_b$  with  $c_0^* = e(m_b, g_2)e(\delta, h_2)$  for a random  $\delta \leftarrow \mathbb{G}_1$ .
- Compute  $c_1^* = g_1^\gamma, c_2^* = (g_1^\alpha)^\gamma, c_3^* = a_1^x, c_4^* = a_2^x, c_5^* = (c_1^*c_4^*)^y$  for a randomly chosen value  $\gamma \leftarrow \mathbb{Z}_p^*$ .
- Randomly choose  $s \leftarrow \mathbb{Z}_p^*$ , compute  $c_6^* = (g_1^\beta)^\gamma, c_7^* = m_b^{-1}(g_1^\delta)^\gamma$ .
- Simulate the knowledge signature in the random oracle model:  $c_8^* = KS\{M, s|e(c_7^*, g_2)c_0^* = e(M, g_2)e(\delta, h_2) \wedge c_6^* = (c_1^*)^s \wedge M = (c_2^*)^s\}(c_0^*||c_1^*||c_2^*||c_3^*||c_4^*||c_5^*||c_6^*||c_7^*)$ .
- Output  $c^* = (c_0^*, c_1^*, c_2^*, c_3^*, c_4^*, c_5^*, c_6^*, c_7^*, c_8^*)$  as the challenge ciphertext of message  $m_b$  to the group member  $mpk$ .

After receiving the challenge ciphertext  $c^*$ , the attacker can still query above oracles but  $mpk$  cannot be queried to the **Corruption Oracle** and  $c^*$  cannot be queried to the **Decryption Oracle**. We answer these queries as above.

Finally, the attacker will output its guess bit  $b'$ . We conclude that  $(g_1, g_1^\alpha, g_1^\beta, g_1^\delta)$  is a DDH tuple in  $\mathbb{G}_1$  if and only if  $b' = b$ . Note that  $c^*$  is valid ciphertext of  $m_b$  under the group member public key  $mpk$  if and only if  $(g_1, g_1^\alpha, g_1^\beta, g_1^\delta)$  is a DDH tuple. We answer successfully whenever the attacker has a correct guess. Let the attacker win this semantical security game with probability  $\varepsilon$ . Hence, we win the DDH challenge with probability  $(1 - \varepsilon_1)(1 - \varepsilon_2)(1 - \varepsilon_3)\varepsilon \approx (1 - \frac{n_c}{n})\frac{1}{n}\varepsilon$ . This completes the proof.  $\square$

**Theorem 6.** *The proposed group decryption scheme is anonymous in the random oracle model under the DDH assumption and the Strong LRSW assumption in SXDH pairing groups.*

*Proof.* We prove that a successful attacker in the anonymity game of our scheme can be used as a subroutine to break the DDH assumption in  $\mathbb{G}_1$ .

Assume that we are given a DDH challenge  $(g_1, g_1^\alpha, g_1^\beta, g_1^\delta) \in \mathbb{G}_1^4$ , where  $\mathbb{G}_1$  is from SXDH pairing groups  $\mathcal{Y} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$ . We are required to answer whether  $\delta = \alpha\beta$  or not. We first use a DDH challenge to simulate the oracles that the attacker may query and then use the attacker's reply to answer the DDH challenge.

The simulation is the same as the previous proof in the semantical security. In the challenge phase, the attacker queries with a tuple  $(m, mpk_{i_0}, mpk_{i_1})$ , where  $m$  is in the message space and  $mpk_{i_b} = (a_{1,b}, a_{2,b}, a_{3,b}, upk_{i_b}, \sigma_{i_b})$  are valid group member's public keys for  $b = 0, 1$  and have never been queried to the Corruption Oracle. If  $upk_{i_b} \neq e(g_1^\alpha, g_2)$  for  $b = 0, 1$ , we declare failure and denoted it by a bad event which happens with probability  $1 - \frac{2}{n}$ . Else  $upk_{i_b} = e(g_1^\alpha, g_2)$ . We compute the challenge ciphertext as follows.

- Commit the chosen message  $m$  with  $c_0^* = e(m, g_2)e(\delta, h_2)$  for a random  $\delta \leftarrow \mathbb{G}_1$ .
- Compute  $c_1^* = g_1^\beta, c_2^* = g_1^\delta, c_3^* = c_1^x, c_4^* = c_2^x, c_5^* = (c_1 c_4)^y$  for a randomly chosen value  $\gamma \leftarrow \mathbb{Z}_p^*$ .
- Randomly choose  $s \leftarrow \mathbb{Z}_p^*$ , compute  $c_6^* = (c_1^*)^s, c_7^* = m^{-1}(c_2^*)^s$ .
- Simulate the knowledge signature in the random oracle model using the standard simulating technique:  $c_8^* = KS\{M, s | e(c_7^*, g_2)c_0^* = e(M, g_2)e(\delta, h_2) \wedge c_6^* = (c_1^*)^s \wedge M = (c_2^*)^s\} (c_0^* || c_1^* || c_2^* || c_3^* || c_4^* || c_5^* || c_6^* || c_7^*)$ .
- Output  $c^* = (c_0^*, c_1^*, c_2^*, c_3^*, c_4^*, c_5^*, c_6^*, c_7^*, c_8^*)$  as the challenge ciphertext of message  $m_b$  to the group member  $mpk$ .

After receiving the challenge ciphertext  $c^*$ , the attacker can still query above oracles but  $mpk$  cannot be queried to the Corruption Oracle and  $c^*$  cannot be queried to the Trace Oracle. We answer these queries as above.

Finally, the attacker will output its guess bit  $b'$ . We conclude that  $(g_1, g_1^\alpha, g_1^\beta, g_1^\delta)$  is a DDH tuple in  $\mathbb{G}_1$  if and only if  $b' = b$ . Note that  $c^*$  is valid ciphertext of  $m$  under the group member public key  $mpk_{i_b}$  if and only if  $(g_1, g_1^\alpha, g_1^\beta, g_1^\delta)$  is a DDH tuple. We answer successfully whenever the attacker has a correct guess. Let the attacker win this semantical security game with probability  $\varepsilon$ . Similarly, we win the DDH challenge with probability  $(1 - \frac{2\varepsilon}{n})\frac{2}{n}\varepsilon$ . This completes the proof.  $\square$

**Theorem 7.** *The proposed group decryption scheme is traceable in the random oracle model under the Strong LRSW assumption in SXDH pairing groups.*

*Proof.* Assume that we are given a Strong LRSW challenge  $(g_1, X = g_2^x, Y = g_2^y) \in \mathbb{G}_1 \times \mathbb{G}_2^2$ , where  $\mathbb{G}_1, \mathbb{G}_2$  are from SXDH pairing groups  $\mathcal{Y} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, g_1, g_2, e) \leftarrow \text{PairingGen}(1^\lambda)$ . We are required to output a tuple  $(a_1 \in \mathbb{G}_1, a_2 = a_1^x, a_3 = a_1^y, a_4 = a_1^{ux}, a_5 = a_1^{y+uxy})$  for a value  $u \in \mathbb{Z}_p^*$  which has never been queried to the LRSW oracle. We first use the LRSW challenge and the LRSW oracle to simulate the oracles that the attacker may query and then use the attacker's reply to answer the Strong LRSW challenge.

The Setup is the same as the previous proof. We generate the users' key pairing as the real scheme. When attacker requires to add these users into the group, we use the secret user keys  $u \in \mathbb{Z}_p^*$  to ask the strong LRSW oracle. It return us with  $(a, a^x, a^{y+uxy})$  for a

random  $a \in \mathbb{G}_1$ . Then we obtain  $(a^r, (a^x)^r, (a^u)^r, (a^x)^{ur}, (a^{y+uxy})^r)$  which the corresponding group certificates and compute the rest parts as the real scheme. All the other actions can be perfectly simulated as we know the group members' secret keys. Finally, the attacker will output a valid ciphertext  $c' = (c'_1 \cdots, c'_8)$  which we cannot traced. Note that the condition 1 in the Encryption Verification procedure guarantees that  $(c'_1 \cdots, c'_5)$  is a strong LRSW tuple [2]. Hence,  $\log_{c'_1} c'_2 = u'$  is not the secret key of any group member and hence has never been queried to the LRSW oracle. Therefore,  $(c'_1 \cdots, c'_5)$  can be used to successfully answer the strong LRSW challenge. This contradicts to the strong LRSW assumption in SXDH pairing groups and completes the proof.  $\square$

## 6 Conclusion

In this paper, we formalized the notion of group decryptions. It allows a sender to verifiably encrypt a committed message intended to any member of a group, managed by a group manager, while the recipient of the ciphertext remains anonymous. In case of dispute, the group manager can verifiably open the identity of the recipient. We proposed the first group decryption scheme from pairing groups secure in the random oracle model. Our scheme has *constant complexity* in both computation and the communication. To achieve our scheme, we presented several sub-protocols. These sub-protocols are efficient and of independent interest.

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