

Provably Secure Identity-Based Authenticated Key Agreement Protocols Without Random Oracles

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Abstract. We present the *first* provably secure ID-based key agreement protocol, inspired by the ID-based encryption scheme of Gentry, in the standard (non-random-oracle) model. We show how this key agreement can be used in either escrowed or escrowless mode. We also give a protocol which enables users of separate private key generators to agree on a shared secret key. All our proposed protocols have comparable performance to all known protocols that are proven secure in the random oracle model.

Keywords: identity-based cryptography, authenticated key agreement, bilinear pairings, standard model

1 Introduction

Key agreement protocols are fundamental for establishing communications between two parties over an insecure network. If in a protocol one party is assured that no other party other than the designated party (or parties) may gain access to the particular established secret key, then the protocol is said to provide *implicit key authentication* (IKA). An authenticated key agreement (AK) protocol provides mutual IKA between (or among) parties. In addition, a key agreement protocol is said to provide key confirmation (of B to A) if A is assured that B actually possesses the session key. An AK protocol that provides mutual key confirmation is called an authenticated key agreement with key confirmation protocol (or an AKC protocol). Although key agreement protocols can employ either private or public-key cryptography, we consider two-party key agreement protocols in the public-key setting in this paper.

The idea of *identity(ID)-based cryptography* was first introduced by Shamir in 1984 [25]. The basic idea behind an ID-based cryptosystem is that end users can choose an arbitrary string (e.g., email addresses or other online identifiers) as their public keys. This eliminates much of the computational overhead associated with key management [20]. In 2001, Boneh and Franklin [1] gave the first fully functional solution for ID-based encryption (IBE) – an variant of the ElGamal [15] encryption scheme – using the pairing on elliptic curves. Since then, numerous ID-based AK protocols have been proposed (e.g., [26,27,13,24,12]).

Motivations.

1. The random oracle has been a popular technique in provable security since its formalization by Bellare and Rogaway [7]. Although some have argued that a proof in the random oracle model is more of a heuristic proof than a real one, existing provably-secure *ID-based* AK protocols are usually proven secure in the random oracle model (e.g., [13,20,11,28,5]). It is generally acknowledged that security in the random oracle model does not, however, imply security in the real world.

2. Most ID-based key agreement protocols have the inherent property of *session key escrow* (i.e., the private key generator (PKG) can recover all the session key agreed by its users). As noted in [20], this property may either be acceptable or unacceptable depending on individual situations. For example, key escrow is essential in situations where confidentiality and audit trail are legal requirements. There are, however, situations where key escrow is preferred to be “switched” off for privacy concerns [20].
3. Key agreement between different networks/domains may be desirable in some situation. For example, for the encrypted VoIP to work on a global scale, there must be compatibility between different networks. Therefore, key agreement between separate networks/domains (i.e., with different PKGs) is crucial.

Contributions. Based on the IBE system due to Gentry [17], we propose a new signature-/encryption-less ID-based AK protocol. We then prove the protocol secure in the standard model. This is, to the best of our knowledge, the *first* ID-based protocols to be proven secure in the standard model. Two other extensions to this basic protocol are also provided.

1. Efficient protocols that can be instantiated with or without session key escrow.
2. An efficient protocol that allows users registered with different private key generators (PKGs) to establish a shared session key.

Our design strategy. Signature-/encryption-less key agreement protocols have numerous advantages, and namely from an efficiency point of view. They are suitable for some constrained environments [19]. A common strategy of constructing a signature-/encryption-less *authenticated* key agreement protocol using the ElGamal [15] public key encryption scheme was first suggested by Matsumoto, Takashima and Imai. In [22], the authors designed several authenticated Diffie-Hellman key agreement protocols (i.e., to provide the original Diffie-Hellman protocol with key authentication), which is well-known as the MTI key agreement family. In particular, the MTI/A0 protocol is derived from the ElGamal encryption in such a way that we name it as a mutual “**Encryption and Decryption**” mechanism (refer to Appendix A for details). Similar but more recent proposals include the protocol of Goss [18], KEA [23] authenticated key agreement designed by NSA in 1994 (declassified in 1998) and Protocol 4 from Blake-Wilson et al. [3]. The design of our new ID-based AK protocol follows the idea behind the MTI/A0 protocol.

Paper Organization. The rest of the paper is structured as follows. In Section 2, we present the necessary background materials. Section 3 reviews Gentry’s ID-based encryption scheme. We then present our new protocols in section 4. In Section 5, we prove the security of our basic protocol without using random oracles. Section 6 concludes the paper. Finally, a high-level description of the common design strategy is given in Appendix A.

2 Technical Backgrounds

2.1 Security Attributes

Security attributes for AK(C) protocols have been identified in several previous work [4,21,6,13]. We briefly explain the security attributes as follows (refer to [4,21] for more detailed discussions):

- **Known-key secrecy.** Suppose an established session key between two entities is disclosed, the adversary is unable to learn other established session keys.
- **Perfect forward secrecy (PFS).** If both long-term secret keys of two entities (i.e. the protocol principals) are disclosed, the adversary is unable to derive old session keys established by that two entities.

- **PKG forward secrecy (PKG-FS)**. If in an ID-based key agreement protocol, the master key known only to the PKG is disclosed, the adversary is unable to derive old session keys established by that two entities. Note this attribute implies that the PKG is not able to passively escrow any session key of its users.
- **Key-compromise impersonation (K-CI) resilience**. Assume that entities A and B are two principals. Suppose A 's secret key is disclosed. Obviously, an adversary who knows this secret key can impersonate A to other entities (e.g. B). However, it is desired that this disclosure does not allow the adversary to impersonate other entities (e.g. B) to A .
- **Unknown key-share (UK-S) resilience**. Entity A cannot be coerced into sharing a key with entity B without A 's knowledge, i.e., when A believes that the key is shared with some entity $C \neq B$, and B (correctly) believes the key is shared with A .
- **No key control**. Neither the two protocol principals (A and B) can predetermine any portion of the shared session key being established between them.

The main desirable *performance attributes* include low computational overheads, a minimal number of passes (the number of messages exchanged in a run of the protocol), and low communication overheads (total number of bits transmitted).

2.2 Security Model for ID-Based AK Protocols

We now review the formal security model for ID-based authenticated key agreement protocols due to Chen, Cheng and Smart [11]. Their model is an adapted version of the model of Blake-Wilson et al. [3] – an extension of the Bellare-Rogaway model [8] in the public key setting.

The model includes a set U of participants modeled by an oracle (e.g., $\Pi_{I,J}^n$ represents a participant I carrying out a protocol session in the belief that it is communicating with another participant J for the n -th time). Each participant has a long-term ID-based long-term public/private key pair, in which the public key is generated using her identity information and the private one is computed and issued secretly by a private key generator.

There is an active adversary (denoted by E) in the model modeled by a probabilistic polynomial time Turing Machine and has access to all the participants' oracles as well as the random oracles in the game. Participant oracles only respond to queries by the adversary and do not communicate directly among themselves and there exists at least a benign adversary who simply passes messages between participants faithfully.

Definition of security in the model depends on the notion of the *partner* oracles to any oracle being tested. In [11], partners have been defined by having the same session identifier (SID) which consists of a concatenation of the messages exchanged between the two. We define $SID(\Pi_{I,J}^n)$ as the concatenation of all messages that oracle $\Pi_{I,J}^n$ has sent and received.

Definition 1 (Partnership). *Two oracles $\Pi_{I,J}^n$ and $\Pi_{J,I}^{n'}$ are said to be partner oracles if they have accepted with the same SID.*

The security of a protocol is defined via a two-phase game between a challenger \mathcal{C} and the adversary E . In the first phase, the adversary E is allowed to issue the following queries in any order.

Send: E can send message M to $\Pi_{I,J}^n$. The oracle executes the protocol and responds with an outgoing message m or a decision to indicate accepting or rejecting the session. Any incoming and outgoing message is recorded on its transcript. If $M = \phi$ (denotes the null message), then the oracle initiates a protocol run.

Reveal: This query asks the oracle to reveal whatever session key it currently holds.

An oracle is called *revealed* if it has responded to a **Reveal** query.

Corrupt: This query asks a participant to reveal the long term private key. A participant is called corrupted if it has responded to a **Corrupt** query.

Test: At some point, E can make a **Test** query to some *fresh oracle* (see Definition 2 below). E receives either the session key or a random value from a particular oracle. Specifically, to answer the query the fresh oracle flips a fair coin $b \in \{0, 1\}$; if the answer is 0 it outputs the agreed session key, and if the answer is 1 it outputs a random element of \mathbb{G}_2 .

In the second phase, E can continue making **Send**, **Reveal** and **Corrupt** queries to the oracles, except that E is not allowed to reveal the target **Test** oracle or its partner oracle (if any), and E cannot corrupt participant J (assuming $\Pi_{I,J}^n$ is the **Test** oracle).

Output: Finally, E outputs a prediction (b') on b . E wins the game if $b' = b$, and we define E 's advantage (l is the security parameter) in winning the game as

$$\text{Advantage}^E(l) = |\Pr[b' = b] - 1/2|.$$

Definition 2 (Fresh Oracle). An oracle $\Pi_{I,J}^n$ is called *fresh* if it has accepted (and therefore holds a session key sk_i), it is not revealed, J has not been corrupted, and there is no revealed oracle $\Pi_{J,I}^{n'}$ which is a partner oracle of $\Pi_{I,J}^n$.

Remark 1. The above definition of fresh oracle is particularly defined to cover the security attribute of *key-compromise impersonation resilience* since it implies that the participant I could have been issued a **Corrupt** query [11].

Definition 3 (Secure AK Protocol [3]). A protocol is a secure AK protocol if:

1. In the presence of the benign adversary (who simply relays messages between parties without modification) on $\Pi_{I,J}^n$ and $\Pi_{J,I}^{n'}$, both oracles always accept holding the same session key, and this key is distributed uniformly on session key space; and if for every adversary E :
2. If uncorrupted oracles $\Pi_{I,J}^n$ and $\Pi_{J,I}^{n'}$ are partners in the sense of Definition 1 then both oracles accept and hold the same session key;
3. $\text{Advantage}^E(l)$ is negligible.

In the following, we briefly discuss the attributes that the above definitions of a secure AK achieves. Recall the security attributes which were mentioned above, and here we examine them one by one.

- *Known-key secrecy.* The property of known-key secrecy is implied by the above definitions of AK security. Since E is allowed to make **Reveal** queries to any oracles except for the target **Test** oracle $\Pi_{I,J}^n$ and its partner oracle $\Pi_{J,I}^{n'}$ to obtain any session keys. Even with the knowledge of many other session keys, E 's ability to distinguish between the session key held by $\Pi_{I,J}^n$ and a random number is still negligible. That is to say, the knowledge of any other session keys does not help E to deduce any information about the tested session key.
- *Perfect forward secrecy (PFS).* The definition does not imply the property of perfect forward secrecy. This is because the model does not allow the adversary to make queries of corrupted oracles and therefore does not model this type of attack.
- *Key-compromise impersonation (K-CI) resilience.* As mentioned above, the definition of fresh oracle imply the key compromise impersonation property.
- *Unknown key-share (UK-S) resilience.* The definition also imply the unknown key-share resilience property. If ID_I establishes a session key with ID_J though he believes that he is talking to ID_K , then there is an oracle $\Pi_{I,K}^n$ that holds this session key

sk_{IK} . At the same time, there is an oracle $\Pi_{J,I}^{n'}$ that holds this session key sk_{IK} , for some n' (normally $n' = n$). During an unknown key share attack, the user ID_K may not know this session key. Since $\Pi_{I,K}^n$ and $\Pi_{J,I}^{n'}$ are not partner oracles, the adversary can make a **Reveal** query to $\Pi_{J,I}^{n'}$ to learn this session key before asking a **Test** query to $\Pi_{I,K}^n$. Thus the adversary will succeed for this **Test** query challenge (i.e., the protocol is not secure) if the unknown key share attack is possible. By contradiction, a secure protocol in the model is resistant to the unknown key share attack.

- *No key control.* The above definition of AK security does not imply resilience to key control attacks that are launched by one of the protocol participants. However, key control attacks launched by an outside adversary are captured by the model. In the model, all participants are assumed to be honest participants. If the protocol is not attacked (i.e., can be proven secure in the model), then we can be assure that the session key established is distributed uniformly at random in the session key space. Otherwise, the adversary E must have a non-negligible ability to distinguish between the session key held by $\Pi_{I,J}^n$ and a random number.

2.3 Bilinear Pairings

We briefly review the necessary facts about bilinear pairings [1]. Let \mathbb{G}_1 be a cyclic multiplicative group generated by an element g , whose order is a prime p , and \mathbb{G}_2 be a cyclic multiplicative group of the same prime order p . We assume that the discrete logarithm problem (DLP) in both \mathbb{G}_1 and \mathbb{G}_2 are hard.

Definition 4. *An admissible pairing e is a bilinear map $e : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$, which satisfies the following three properties:*

1. Bilinear: If $u, v \in \mathbb{G}_1$ and $a, b \in \mathbb{Z}_p^*$, then $e(u^a, v^b) = e(u, v)^{ab}$;
2. Non-degenerate: $e(g, g) \neq 1$;
3. Computable: If $u, v \in \mathbb{G}_1$, one can compute $e(u, v) \in \mathbb{G}_2$ in polynomial time.

2.4 Complexity Assumptions

The security of Gentry’s IBE system [17] is based on a complexity assumption that they call *the truncated augmented bilinear Diffie-Hellman exponent assumption* (the truncated q -ABDHE). We recall the truncated decision q -ABDHE problem as follows (refer to [17] for detailed description).

Truncated Decision q -ABDHE Problem. Given a vector of $q + 3$ elements

$$(g', g'^{\alpha^{q+2}}, g, g^\alpha, g^{\alpha^2}, \dots, g^{\alpha^q}) \in \mathbb{G}_1^{q+3}$$

as input (here $\alpha \in \mathbb{Z}_p$), an algorithm \mathcal{B} that outputs $b \in \{0, 1\}$ has advantage ϵ in solving the truncated decision q -ABDHE if

$$\begin{aligned} & | \Pr[\mathcal{B}(g', g'^{\alpha^{q+2}}, g, g^\alpha, g^{\alpha^2}, \dots, g^{\alpha^q}, e(g^{\alpha^{q+1}}, g')) = 0 \\ & - \Pr[\mathcal{B}(g', g'^{\alpha^{q+2}}, g, g^\alpha, g^{\alpha^2}, \dots, g^{\alpha^q}, Z) = 0] | \geq \epsilon \end{aligned}$$

where the probability is over the random choice of generators $g, g' \in \mathbb{G}_1$, the random choice of $\alpha \in \mathbb{Z}_p$, the random choice of $Z \in \mathbb{G}_2$, and the random bits consumed by \mathcal{B} .

The security of our escrowless version of the key agreement protocol is based on the following variant of the truncated q -ABDHE problem, with two additional elements g and g^α being given as input. We note that this modified problem is believed to be as hard as the original one.

Modified Truncated q -ABDHE Problem. Given a vector of $q + 5$ elements

$$(g', g'^{\alpha^{q+2}}, g, g^\alpha, t, t^\alpha, t^{\alpha^2}, \dots, t^{\alpha^q}) \in \mathbb{G}_1^{q+5}$$

as input, an algorithm \mathcal{B} that outputs $b \in \{0, 1\}$ has advantage ϵ in solving the *modified truncated decision q -ABDHE* if

$$\begin{aligned} & |\Pr[\mathcal{B}(g', g'^{\alpha^{q+2}}, g, g^\alpha, t, t^\alpha, t^{\alpha^2}, \dots, t^{\alpha^q}, e(t^{\alpha^{q+1}}, g')) = 0 \\ & - \Pr[\mathcal{B}(g', g'^{\alpha^{q+2}}, g, g^\alpha, t, t^\alpha, t^{\alpha^2}, \dots, t^{\alpha^q}, Z) = 0] | \geq \epsilon \end{aligned}$$

where the probability is over the random choice of generators $g, g', t \in \mathbb{G}_1$, the random choice of $\alpha \in \mathbb{Z}_p$, the random choice of $Z \in \mathbb{G}_2$, and the random bits consumed by \mathcal{B} .

Definition 5 ((*Modified*) Truncated Decision (t, ϵ, q) -ABDHE Assumption). We say that the truncated decision (t, ϵ, q) -ABDHE assumption (resp. the modified truncated decision (t, ϵ, q) -ABDHE assumption) holds in \mathbb{G}_1 if no t -time algorithm has advantage at least ϵ in solving the truncated decision q -ABDHE problem (resp. the modified truncated decision q -ABDHE problem) in \mathbb{G}_1 .

3 Review of Gentry's IBE Scheme

In this section, we review the first construction of Gentry from [17], which is a chosen-plaintext secure ElGamal-type IBE scheme (proven in the standard model).

Let \mathbb{G}_1 and \mathbb{G}_2 be groups of prime order p , and let $e : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$ be the bilinear pairing. The IBE system works as follows.

Setup: The PKG chooses two random generators $g, h \in \mathbb{G}_1$ and a random $\alpha \in \mathbb{Z}_p$, calculates $g_1 = g^\alpha \in \mathbb{G}_1$. It sets the public *params* as $\langle g, g_1, h \rangle$ and the *master-key* as α .

Key Generation: To generate a private key for identity $ID \in \mathbb{Z}_p$, the PKG generates a random $r_{ID} \in \mathbb{Z}_p$, and outputs the private key as $d_{ID} = \langle r_{ID}, h_{ID} \rangle$, where $h_{ID} = (hg^{-r_{ID}})^{1/(\alpha-ID)}$. (The PKG ensures that $ID \neq \alpha$ and it always assigns identical r_{ID} for a given identity ID .)

Encryption: The sender picks randomly a $s \in \mathbb{Z}_p$, using the receiver's identity ID , sets the ciphertext to be (to encrypt message $m \in \mathbb{G}_2$)

$$C = (g_1^s g^{-s \cdot ID}, e(g, g)^s, m \cdot e(g, h)^{-s}).$$

Decryption: To decrypt ciphertext $C = (u, v, w)$, the decrypter of the identity ID computes

$$m = w \cdot e(u, h_{ID}) \cdot v^{r_{ID}}.$$

Consistence: The recipient can correctly decrypt C to get m since

$$\begin{aligned} & e(u, h_{ID}) \cdot v^{r_{ID}} \\ &= e(g^{s(\alpha-ID)}, h^{1/(\alpha-ID)} g^{-r_{ID}/(\alpha-ID)}) \cdot e(g, g)^{sr_{ID}} \\ &= e(g, h)^s. \end{aligned}$$

4 Proposed ID-Based Key Agreement Protocols

In this section, we propose three new ID-based authenticated key agreement protocols based on Gentry's IBE scheme (refer to Section 3).

4.1 An ID-Based Key Agreement Protocol with Escrow

Our first protocol (named as Protocol I) does not provide PKG forward secrecy (or master-key forward secrecy), i.e., when the master key α of the PKG is compromised, an adversary who gets it can recover all the users' past session keys. Equivalently, this means that the PKG can *escrow* all the session keys (refer to [13,20] for more details). Clearly, for any ID-based key agreement protocol, perfect forward secrecy (PFS) is implied by the PKG forward secrecy. Whereas, our second protocol (named as Protocol II) provides the PKG forward secrecy. Both of our protocols are two-pass protocol with *mutual* implicit key authentication (IKA). We note that it is readily to extend our protocols into 3-pass AKC protocols, using the common method given in [3,13].

As with all the other ID-based AK protocols we assume the existence of a PKG that is responsible for the creation and secure distribution of users' private keys.

Protocol I consists of three stages, i.e. **Setup**, **Key Generation** and **Key Agreement**. The **Setup** and **Key Generation** stages are identical to that of Gentry's IBE scheme [17].

Suppose two principals Alice and Bob are about to agree on a session key (we denote their identity *Ident* as A and B , respectively), we follow previous notations and hereafter, let $g_{Ident} = g_1 g^{-ID_{Ident}}$ and $g_T = e(g, g)$, where $Ident \in \{A, B\}$. We denote the participant's private key as $\langle r_{ID}, h_{ID} \rangle$. The **Key Agreement** stage is as follows.

Key Agreement. To establish a shared session key, Alice and Bob each firstly generates an ephemeral private key (say x and $y \in \mathbb{Z}_p$), and compute the corresponding ephemeral public keys $T_{11} = g_B^x$, $T_{12} = g_T^x$ and $T_{21} = g_A^y$, $T_{22} = g_T^y$. They then exchange $T_1 = T_{11}||T_{12}$ and $T_2 = T_{21}||T_{22}$ as described in Figure 1 (where the symbol "||" denotes concatenation).

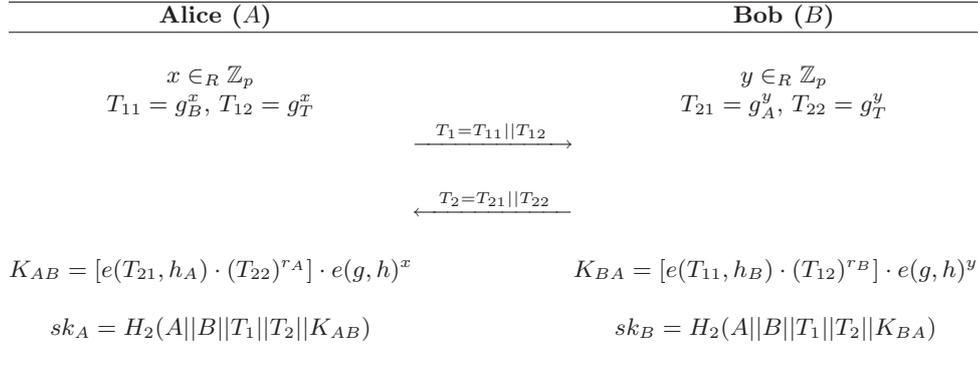


Fig. 1. Protocol I

1. Alice computes the shared secret K_{AB} as follows:

$$K_{AB} = [e(T_{21}, h_A) \cdot (T_{22})^{r_A}] \cdot e(g, h)^x.$$

2. Bob computes the shared secret K_{BA} as follows:

$$K_{BA} = [e(T_{11}, h_B) \cdot (T_{12})^{r_B}] \cdot e(g, h)^y.$$

Protocol Correctness: By the bilinearity of the pairing, we can easily get the following equations:

$$\begin{aligned}
K_{AB} &= e(T_{21}, h_A) \cdot (T_{22})^{r_A} \cdot e(g, h)^x \\
&= e(g_A^y, (g^{-r_A} h)^{1/(\alpha-ID_A)}) \cdot (g_T^y)^{r_A} \cdot e(g, h)^x \\
&= e(g^{y(\alpha-ID_A)}, (g^{-r_A} h)^{1/(\alpha-ID_A)}) \cdot (g_T^y)^{r_A} \cdot e(g, h)^x \\
&= e(g^y, g^{-r_A} h) \cdot (g_T^y)^{r_A} \cdot e(g, h)^x \\
&= e(g^y, g^{-r_A} h) \cdot e(g, g)^{y r_A} \cdot e(g, h)^x \\
&= e(g^y, g^{-r_A} h) \cdot e(g^y, g^{r_A}) \cdot e(g, h)^x \\
&= e(g^y, g^{-r_A} h g^{r_A}) \cdot e(g, h)^x \\
&= e(g^y, h) \cdot e(g, h)^x \\
&= e(g, h)^{x+y}.
\end{aligned}$$

$$\begin{aligned}
K_{BA} &= e(T_{11}, h_B) \cdot (T_{12})^{r_B} \cdot e(g, h)^y \\
&= e(g_B^x, (g^{-r_B} h)^{1/(\alpha-ID_B)}) \cdot (g_T^x)^{r_B} \cdot e(g, h)^y \\
&= e(g^{x(\alpha-ID_B)}, (g^{-r_B} h)^{1/(\alpha-ID_B)}) \cdot (g_T^x)^{r_B} \cdot e(g, h)^y \\
&= e(g^x, g^{-r_B} h) \cdot (g_T^x)^{r_B} \cdot e(g, h)^y \\
&= e(g^x, g^{-r_B} h) \cdot e(g, g)^{x r_B} \cdot e(g, h)^y \\
&= e(g^x, g^{-r_B} h) \cdot e(g^x, g^{r_B}) \cdot e(g, h)^y \\
&= e(g^x, g^{-r_B} h g^{r_B}) \cdot e(g, h)^y \\
&= e(g^x, h) \cdot e(g, h)^y \\
&= e(g, h)^{x+y} \\
&= K_{AB}.
\end{aligned}$$

The final shared secret session key is then $sk = H_2(A||B||T_1||T_2||K)$, where $H_2 : \{0, 1\}^* \rightarrow \{0, 1\}^k$ is a *key derivation function* (in which $k = |sk|$). Note here we include the transcript (T_1 and T_2) in the key derivation function to resist the potential *key replicating attack* (whereby an adversary is somehow able to manipulate the shared secret K using his own contributions while he still does not know the value of K) [10].

Efficiency. Protocol I is *role symmetric*, which means that each party performing the same operations. Protocol I has comparable computational efficiency to those protocols from [13,20] (which are only proven secure in the random oracle model). In Protocol I, each participant has to generate a random number, perform one exponentiations in \mathbb{G}_1 , two exponentiations in \mathbb{G}_2 , and compute two pairings (which are the most expensive operations in the protocol and one of these pairings can be precomputed). We leave out multiplication in \mathbb{G}_1 and pairing multiplication in \mathbb{G}_2 , and hashing as they are fast to compute compared to the other principle operations.

Compared with the Chen-Kudla protocol [13] and the McCullagh-Barreto protocol [20], Protocol I is less efficient on the message bandwidth since two message blocks (as opposed to only one) need to be distributed by each user.

Escrow. The escrow property derives from the PKG's ability to recover the shared session key, since with the knowledge of all the two users' private keys, the PKG is also able to calculate $e(g, h)^x$ and $e(g, h)^y$ using the publicly transmitted data.

4.2 An ID-Based Key Agreement Protocol Without Escrow

In this subsection, we show how to turn off the session key escrow property of the above protocol by making a slight modification to the **Setup** and **Key Generation** algorithms to Gentry's IBE system. Similar to the idea used in [13], we calculate an extra *Diffie-Hellman* shared key from the two participants' *ephemeral* contributions. However, unlike

the protocols in [13], Protocol II does not bring additional communication cost. We now introduce Protocol II, which has PKG forward secrecy (or, master-key forward secrecy).

Setup: Compared with the original **Setup** algorithm, the new public parameter has one more generator (denoted as t) of group \mathbb{G}_1 . Now the PKG chooses three random generators $g, h, t \in \mathbb{G}_1$ and a random $\alpha \in \mathbb{Z}_p$, calculates $g_1 = g^\alpha \in \mathbb{G}_1$. It sets the public *params* as $\langle g, g_1, h, t \rangle$ and the *master-key* as α .

Key Generation: To generate a private key for identity $ID \in \mathbb{Z}_p$, the PKG generates a random $r_{ID} \in \mathbb{Z}_p$, and outputs the private key as $d_{ID} = \langle r_{ID}, h_{ID} \rangle$, where $h_{ID} = (ht^{-r_{ID}})^{1/(\alpha-ID)}$. (Again, the PKG ensures that $ID \neq \alpha$ and it always assigns identical r_{ID} for a given identity ID .)

Suppose that Alice and Bob are about to agree on a session key (we denote their identity *Ident* as A and B , respectively), we let $g_{Ident} = g_1 g^{-ID_{Ident}}$ and $t_T = e(g, t) \in \mathbb{G}_2$ ¹. The **Key Agreement** stage is as follows.

Key Agreement. To establish a shared session key, Alice and Bob each firstly generates an ephemeral private key (say x and $y \in \mathbb{Z}_p$), and compute the corresponding ephemeral public keys $T_{11} = g_B^x$, $T_{12} = t_T^x$ and $T_{21} = g_A^y$, $T_{22} = t_T^y$. They then exchange $T_1 = T_{11}||T_{12}$ and $T_2 = T_{21}||T_{22}$ and compute the session key as described in Figure 2, with extra operations (in contrast to Protocol I) underlined.

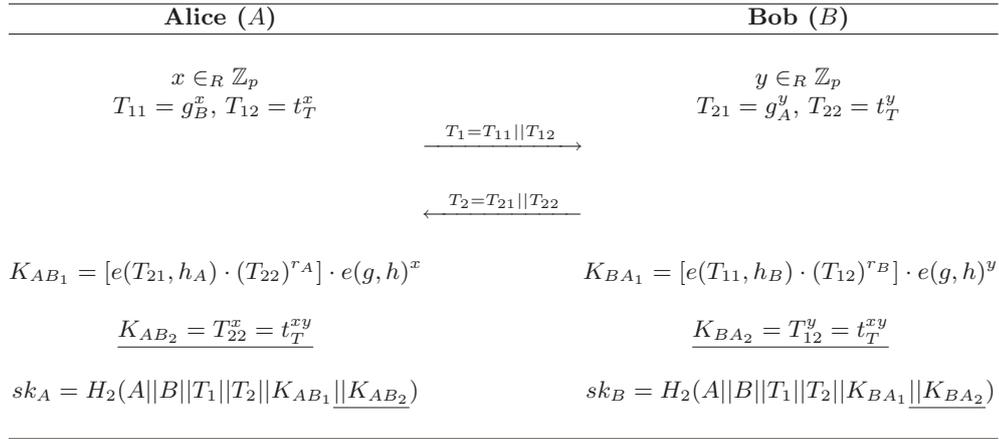


Fig. 2. Protocol II

Protocol Correctness: Both Alice and Bob will compute the same session key since we have the following equations:

¹ Note that in Protocol I, the corresponding component is $g_T = e(g, g)$ instead.

$$\begin{aligned}
K_{AB_1} &= e(T_{21}, h_A) \cdot (T_{22})^{r_A} \cdot e(g, h)^x \\
&= e(g_A^y, (ht^{-r_A})^{1/(\alpha-ID_A)}) \cdot (t_T^y)^{r_A} \cdot e(g, h)^x \\
&= e(g^{y(\alpha-ID_A)}, (ht^{-r_A})^{1/(\alpha-ID_A)}) \cdot (t_T^y)^{r_A} \cdot e(g, h)^x \\
&= e(g^y, ht^{-r_A}) \cdot (g_T^y)^{r_A} \cdot e(g, h)^x \\
&= e(g^y, ht^{-r_A}) \cdot e(g, t)^{y r_A} \cdot e(g, h)^x \\
&= e(g^y, ht^{-r_A}) \cdot e(g^y, t^{r_A}) \cdot e(g, h)^x \\
&= e(g^y, ht^{-r_A} t^{r_A}) \cdot e(g, h)^x \\
&= e(g^y, h) \cdot e(g, h)^x \\
&= e(g, h)^{x+y} \\
&= K_{BA_1},
\end{aligned}$$

$$K_{AB_2} = T_{22}^x = e(g, t)^{xy} = K_{BA_2}.$$

Efficiency. Protocol II is also role symmetric. Obviously, compared with Protocol I, Protocol II only requires one more exponentiations in \mathbb{G}_2 for each participant.

Protocol II has exactly the same communication efficiency as Protocol I.

Escrowless. Notice that with α , the PKG is able to calculate g^x (resp. g^y) from $T_{11} = g^{x(\alpha-ID_B)}$ (resp. $T_{21} = g^{y(\alpha-ID_A)}$). Protocol II avoids session key escrow since the PKG is still *not* able to compute $t_T^{xy} = e(g, t)^{xy}$. We note that calculating the keying term t_T^{xy} from t_T^x and t_T^y involves solving the Computational Diffie-Hellman Problem (CDHP) over the group \mathbb{G}_2 .

4.3 Key Agreement Between Users of Separate PKGs

We now look at ID-based key agreement protocols between users of separate PKGs. The first such protocol was suggested by Chen and Kudla in [13], and [20] also gives an efficient construction. Based on our new protocol given above, we propose another ID-based key agreement protocol across separate PKGs. Again, we note that this protocol can be instantiated in escrowed or escrowless mode.

For key agreement to be feasible between users of distinct PKGs, it is required that the PKGs use the globally agreed domain parameters. Since elliptic curves, suitable group generator points (i.e., g and h) and other cryptographic tools such as hash functions, have been standardised for security applications, for example in the NIST FIPS standards, it is therefore reasonable to assume the availability of standard pairing-friendly curves as well [20]. Once these group generator points and curves have been agreed upon, each PKG can generate their own random master secret key (i.e. α).

Suppose that Alice is a user of the private key generator PKG_A (with a master secret α_A), and Bob is a user of the private key generator PKG_B (with a master secret α_B). Therefore, Alice's private key is generated by PKG_A with α_A and Bob's private key is generated by PKG_B with α_B . Assume that Alice (resp. Bob) has obtained an authentic copy of the public key of PKG_B (resp. PKG_A). Alice and Bob now perform the authenticated key agreement depicted in Figure 3.

On Escrow(less). Correctness of Protocol III can be easily checked. In this protocol, neither PKG_A nor PKG_B can escrow the established session keys. But if the two PKGs work together (or collude) they still can passively escrow all their users' session keys via $K = [e(T_{21}, h_A) \cdot (T_{22})^{r_A}] \cdot [e(T_{11}, h_B) \cdot (T_{12})^{r_B}]$, since the two users' private keys $\langle r_A, h_A \rangle$ and $\langle r_B, h_B \rangle$ are known to their PKGs.

Note that it is straightforward to turn off the session key escrow property of Protocol III, simply in a parallel way to the construction of Protocol II from Protocol I.

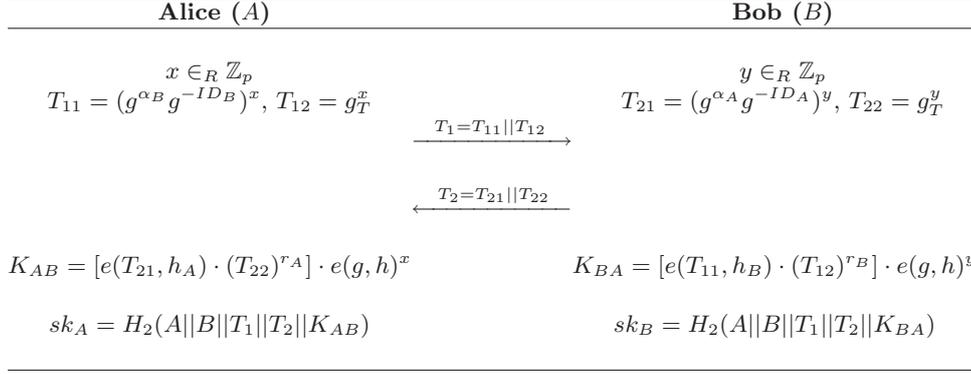


Fig. 3. Protocol III

Efficiency. Protocol III has identical computational and communication efficiencies with Protocol I.

5 Security Proof

Since the above three protocols have similar structures, it is sufficient to consider only the security of the basic protocol, Protocol I. We postpone the proofs of the security of Protocol II and Protocol III to the full version of this paper. Here we suggest that Protocol II can be proven secure using the *modified* truncated decision q -ABDHE assumption (see Section 2.4).

With the description of the security model in Section 2.2, we now state:

Theorem 1. *If the truncated decision q -ABDHE assumption holds for the pair of groups \mathbb{G}_1 and \mathbb{G}_2 , then Protocol I is a secure AK protocol.*

Proof. Our proof is closely based on the security proof of Gentry’s first IBE scheme [17].

Condition 1 follows from the assumption that the two oracles follow the protocol and E is benign. In this case, both oracles accept (since they both receive correctly formatted messages from the other oracle) holding the same key sk (since $K_{AB} = K_{BA}$ by the bilinearity of the pairing and the partnership).

Condition 2 follows from the fact that if the two oracles are uncorrupted, then they cannot be impersonated, and if they are partners then each has received properly formatted messages from the other. So they will both accept holding the same session key sk . In the following, we show that the Condition 3 is also satisfied.

For a contradiction, assume that there is an adversary E against Protocol I that has a non-negligible advantage ϵ in guessing correctly whether the response to a Test query is real or random (i.e., winning the attacking game defined in Section 2.2). Out of this adversary, we show how to construct a simulator S that solves the truncated decision q -ABDHE problem with non-negligible advantage ϵ' . Given input of the two groups $\mathbb{G}_1, \mathbb{G}_2$, the bilinear map e , and a random truncated decision q -ABDHE challenge $(g', g'^{\alpha^{q+2}}, g, g^\alpha, g^{\alpha^2}, \dots, g^{\alpha^q}, Z)$, S 's task is to distinguish whether Z is $e(g^{\alpha^{q+1}}, g')$ or a random element of \mathbb{G}_2 .

We assume that the game between S and E involves at most $q - 1$ parties. We shall slightly abuse the notation $\Pi_{A,B}^n$ to refer to the n -th one among all the oracles in the game, instead of the n -th instance of participant A . As n is only used to help identify oracles, this notation change will not affect the soundness of the model. Let N_s be an upper bound on the number of oracles invoked by E in the game and assume that S guesses that the oracle $\Pi_{i,J}^s$ ($s \in \{1, \dots, N_s\}$) is to be asked the Test query by the adversary E . S works by interacting with E as follows:

Setup: This stage is identical to that of [17]. S generates a random polynomial $f(x) \in \mathbb{Z}_p[x]$ of degree q . It sets $h = g^{f(\alpha)}$, computing h from $(g, g^\alpha, g^{\alpha^2}, \dots, g^{\alpha^q})$. It sets the public key as $(g, g_1 = g^\alpha, h)$ and sends it to E . Clearly, this public key has a distribution identical to that in the actual construction. Note that the master key is α , which is unknown to S .

Corrupt queries: S simulates the **Corrupt** query on input ID_i as follows. If $g^{ID_i} = g_1$, then we have $ID_i = \alpha$, S can use α to solve the truncated decision q -ABDHE problem immediately. Else, let $F_{ID}(x)$ denote the $(q-1)$ -degree polynomial $(f(x) - f(ID))/(x - ID)$. S sets the private key $\langle r_{ID_i}, h_{ID_i} \rangle$ to be $(f(ID_i), g^{F_{ID_i}(\alpha)})$. This is a valid private key for ID_i , since $g^{F_{ID_i}(\alpha)} = g^{(f(\alpha) - f(ID_i))/(\alpha - ID_i)} = (hg^{-f(ID_i)})^{1/(\alpha - ID_i)}$, as required. In order to keep the secrecy of the polynomial $f(x)$, we require that the total number of the **Corrupt** query is less than q (the degree of $f(x)$). Since $f(x)$ is a uniformly random polynomial, this private key appears to E to be correctly distributed.

Send queries: Let $f_2(x) = x^{q+2}$ and $F_{2, ID_J}(x) = (f_2(x) - f_2(ID_J))/(x - ID_J)$, which is a polynomial of degree $q+1$.

S answers all **Send** queries as specified for a normal oracle, i.e., for the only **Send** query to an oracle, S takes a random value in \mathbb{Z}_p to form its contribution.

When E asks a **Send** query to the oracle $\Pi_{I,J}^s$, S computes protocol messages T_{I1} and T_{I2} for $\Pi_{I,J}^s$ as follows:

$$T_{I1} = g^{f_2(\alpha) - f_2(ID_J)},$$

$$T_{I2} = Z \cdot e(g', \prod_{l=0}^q (g^{\alpha^l})^{F_{2, ID_J, l}}),$$

where $F_{2, ID_J, l}$ is the coefficient of x^l in $F_{2, ID_J}(x)$.

We write the protocol messages that the oracle $\Pi_{I,J}^s$ obtained as T_{J1} and T_{J2} . S computes the session secret K_{IJ} for $\Pi_{I,J}^s$ as follows:

$$K_{IJ} = [e(T_{I1}, h_J) \cdot T_{I2}^{r_J}] \cdot [e(T_{J1}, h_I) \cdot T_{J2}^{r_I}].$$

S computes the final session key sk_I of $\Pi_{I,J}^s$ as

$$sk_I = H_2(I || J || T_{I1} || T_{I2} || T_{J1} || T_{J2} || K_{IJ}).$$

Let $\lambda = (\log_g g') F_{2, ID_J}(\alpha)$. If $Z = e(g^{\alpha^{q+1}}, g')$, then $T_{I1} = g^{\lambda(\alpha - ID_J)}$, $T_{I2} = e(g, g)^\lambda$. As a result, $e(T_{I1}, h_J) \cdot T_{I2}^{r_J} = e(g, h)^\lambda$ and $K_{IJ} = e(g, h)^\lambda \cdot [e(T_{J1}, h_I) \cdot T_{J2}^{r_I}]$ is a valid and appropriately-distributed session secret for the oracle $\Pi_{I,J}^s$. Therefore, the output sk_I is a valid session key for oracle $\Pi_{I,J}^s$ under randomness of λ . Since $\log_g g'$ is uniformly random, λ is uniformly random, and so sk_I is a valid, appropriately-distributed challenge for E .

Reveal queries: If the query is directed at the oracle $\Pi_{I,J}^s$ or its partner oracle (if it exists), S aborts with failure (**Event 1**). Otherwise, it gives the session key hold by the oracle to E .

Test query: At some point in the simulation, E will ask a **Test** query of some oracle. If E does not choose the guessed oracles $\Pi_{I,J}^s$ to ask the **Test** query, then S aborts with failure (**Event 2**). However if E does pick $\Pi_{I,J}^s$ for the **Test** query, S provides E with sk_I as the response.

Guess: After the **Test** query, the algorithm E outputs its guess $b' \in \{0, 1\}$.

We claim that if S does not abort during the simulation then E 's view is identical to its view in the real attack. Now we evaluate the probability that the simulation does not

abort. If the adversary indeed has chosen the s -th oracle (i.e. $\Pi_{I,J}^s$) as the test oracle, then by the rules of the game **Event 1** and **2** would not happen. We have

$$\Pr[S \text{ does not abort}] \geq \frac{1}{N_s}.$$

Solving the truncated decision q -ABDHE problem: S simply forward the output b' of E to its challenger of the truncated decision q -ABDHE problem.

Probability Analysis: If $Z = e(g^{\alpha^{q+1}}, g')$, then the simulation is perfect, and E will win the game with probability $\epsilon + 1/2$. Else, Z is uniformly random, and thus T_{I1} and T_{I2} are uniformly random and independent element of \mathbb{G}_1 and \mathbb{G}_2 , respectively. Since by the rules of the game that J is uncorrupted, we know that K_{IJ} and the response to the Test query (i.e. sk_I) are also uniformly random and independent from E 's view. In this case, E has no advantage in winning the game, i.e., E will guess correctly whether sk_I is real or random with probability $1/2$.

We are now ready to calculate the S 's advantage ϵ' in solving the truncated decision q -ABDHE problem:

$$\begin{aligned} & | \Pr[\mathcal{B}(g', g'^{\alpha^{q+2}}, g, g^\alpha, g^{\alpha^2}, \dots, g^{\alpha^q}, e(g^{\alpha^{q+1}}, g'))] = 0 \\ & - \Pr[\mathcal{B}(g', g'^{\alpha^{q+2}}, g, g^\alpha, g^{\alpha^2}, \dots, g^{\alpha^q}, Z) = 0] | \geq 1/2 + \epsilon - 1/2 = \epsilon. \end{aligned}$$

As mentioned above, the probability that S does not abort is at least $1/N_s$. Combining these two results, we have

$$\epsilon' = \epsilon/N_s.$$

The analysis of time-complexity is identical to that of the proof in [17]. In the simulation, S 's operation is dominated by computing $g^{F_{ID}(\alpha)}$ in response to E 's **Corrupt** query on ID , where $F_{ID}(x)$ is a polynomial of degree $q - 1$. Each such computation requires $O(q)$ exponentiations in \mathbb{G}_1 . The time-complexity of S is therefore $t + O(t_{exp} \cdot q^2)$. \square

6 Conclusion

We presented a new identity-based authenticated key agreement protocol inspired by Gentry's IBE system. We showed that the new protocol can be instantiated in either escrowed or escrowless mode, and also give a protocol which is suitable for circumstances where there are separate private key generators. Finally, we proved that the security of the proposed protocol (in its basic form) is relative to the decision truncated Augmented Bilinear Diffie-Hellman Exponent (the decision truncated q -ABDHE) problem without using random oracles.

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A A Design Strategy for ID-Based AK Protocols

Here we describe in detail the design strategy we used in this paper, i.e. a generic method for deriving ID-based authenticated key agreement protocols from any ElGamal-type ID-based encryption system.

To illustrate the ideas behind the strategy, we first recall the so-called *ElGamal one-pass unilateral key agreement protocol* that was first given in Chap. 12 of [21]. We quote it as follows.

ElGamal key agreement is a Diffie-Hellman variant providing a one-pass protocol with unilateral key authentication (of the intended recipient to the originator), provided the public key of the recipient is known to the originator a priori. While related to ElGamal encryption, the protocol is more simply Diffie-Hellman key agreement wherein the public exponential of the recipient is fixed and has verifiable authenticity (e.g., is embedded in a certificate).

Informally, the protocol proceeds as follows. The sender A forms a *shared* secret using her random input r_A in combination with B 's long-term public key Y_B by computing $K = Y_B^{r_A}$. On receipt of the ephemeral public key $T_A = r_A P$, the receiver B is able to reconstruct the session key $K = T_A^{x_B}$, where x_B is the corresponding private key to Y_B .

The two-pass MTI/A0 protocol can be seen as a parallel execution of ElGamal one-pass key agreement protocol. It yields session keys with *mutual* (implicit) key authentication against passive attacks. As in ElGamal one-pass key agreement, A sends to B a single message, resulting in the shared key K . B independently initiates an analogous protocol with A , resulting in the shared key K' . A and B then output the KK' as their agreed session key.

Note that although the original MTI/A0 protocol is of certain security weaknesses, e.g., it doesn't provide perfect forward secrecy and is vulnerable to some active attacks such as unknown key-share attacks [6], triangle attacks [9], the elegant idea behind it is very useful in Diffie-Hellman authenticated key agreement protocol design.

The above idea can be applied to the design of ID-based AK protocols. In fact, Smart's protocol [26] is the first such example.

So far, we are ready to define a general framework, named as *the generic MTI/A0 protocol* (GMP), for the design of ID-based AK protocols based on an ElGamal-type IBE scheme.

Definition 6 (Generic MTI/A0 Protocol (GMP)). *Suppose we have an ElGamal-type IBE scheme and two users (Alice and Bob) want to agree on a shared session key. They does the following:*

1. Alice (Bob) firstly generates her (his) ephemeral private key, then computes her (his) ephemeral public key PK_{eph} using the public parameters of the system, finally she (he) sends PK_{eph} to Bob (Alice).
2. Alice (Bob) uses the ephemeral private key generated in Step 1 to compute *the ElGamal encryption session key* K_{En} .
3. After receiving the ephemeral public key, they each calculate *the ElGamal decryption session key* K_{De} , using their own long-term private keys.

4. Alice (Bob) computes her (his) final session key sk as

$$sk = K_{En} * K_{De},$$

where the symbol “ $*$ ” stands for a commutative operation, e.g., multiplication, addition, or bitwise XOR.

Remark 2. The two users are able to successfully establish a shared session key after the above GMP.