

Unique-Path Identity Based Encryption With Applications to Strongly Secure Messaging

Paul Rösler¹, Daniel Slamanig², and Christoph Striecks²

¹ FAU Erlangen-Nürnberg paul.roesler@fau.de

² AIT Austrian Institute of Technology {daniel.slamanig,christoph.striecks}@ait.ac.at

Abstract. *Hierarchical Identity Based Encryption* (HIBE) is a well studied, versatile tool used in many cryptographic protocols. Yet, since the performance of all known HIBE constructions is broadly considered prohibitive, some real-world applications avoid relying on HIBE at the expense of security. A prominent example for this is secure messaging: *Strongly secure* messaging protocols are provably equivalent to *Key-Updatable Key Encapsulation Mechanisms* (KU-KEMs; Balli et al., Asiacrypt 2020); so far, all KU-KEM constructions rely on *adaptive unbounded*-depth HIBE (Poettering and Rösler, Jaeger and Stepanovs, both CRYPTO 2018). By weakening security requirements for better efficiency, many messaging protocols dispense with using HIBE.

In this work, we aim to gain *better efficiency without sacrificing security*. For this, we observe that applications like messaging only need a restricted variant of HIBE for strong security. This variant, that we call *Unique-Path Identity Based Encryption* (UPIBE), restricts HIBE by requiring that each secret key can delegate at most one subordinate secret key. However, in contrast to fixed secret key delegation in Forward-Secure Public Key Encryption, the delegation in UPIBE, as in HIBE, is uniquely determined by variable identity strings from an exponentially large space. We investigate this mild but surprisingly effective restriction and show that it offers substantial complexity and performance advantages.

More concretely, we generically build *bounded-depth* UPIBE from only *bounded-collusion IBE* in the standard model; and we generically build *adaptive unbounded*-depth UPIBE from only *selective bounded*-depth HIBE in the random oracle model. These results significantly extend the range of underlying assumptions and efficient instantiations. We conclude with a rigorous performance evaluation of our UPIBE design. Beyond solving challenging open problems by reducing complexity and improving efficiency of KU-KEM and strongly secure messaging protocols, we offer a new definitional perspective on the bounded-collusion setting.

1 Introduction

Traditionally, Hierarchical Identity Based Encryption (HIBE) [HL02, GS02] is motivated by a real-world scenario in which a sender wants to securely encrypt a message to a receiver without knowing their individual public key. Using a global main *public* key as well as a string that identifies the receiver (e.g., their email address `bob@pc.2023.ec.iacr.org`), the sender can encrypt the message via (H)IBE. To decrypt a ciphertext, the receiver can obtain their individual secret key by requesting delegation from the global main *secret* key. The hierarchy in HIBE provides a fine grained, leveled delegation: the secret key of `bob@pc.2023.ec.iacr.org` is delegated from secret key of `pc.2023.ec.iacr.org` which proceeds up to delegation from secret key of `org`. Thereby, each secret key can only delegate secret keys of subordinate identities. For the specific case of Identity Based Encryption (IBE) [Sha84, BF01], only the global main secret key can delegate identity-specific secret keys, which reduces the level depth to 1.

HIBE AS A POWERFUL BUILDING BLOCK. Independent of this real-world use case, HIBE turns out to be a versatile, powerful tool in the design of larger cryptographic protocols. For example, HIBE is used as the main component in designs of Broadcast Encryption (BE) [DF02], Forward-Secure Public Key Encryption (FS-PKE) [CHK03], Puncturable FS-PKE [GM15], 0-RTT Key Exchange with Forward Secrecy [GHJL17, DJSS18], and Key-Updatable Key Encapsulation Mechanisms (KU-KEM) for Ratcheted Key Exchange (RKE) [PR18b]. In most of these cases, the reason for relying on HIBE is rather the strength of HIBE secret key delegation than the traditional motivation of encrypting messages to an identity whose individual public key is unknown.

Notably, not all of these constructions utilize the full power of standard HIBE. For instance, FS-PKE can be based on relaxed Binary-Tree Encryption (BTE) [CHK03, Kat04]. Furthermore, KU-KEM constructions [PR18b, JS18, JMM19b, BRV20] only delegate secret keys along a single path of identities.

INTRODUCING UNIQUE-PATH IBE. Motivated by such restricted delegations, we introduce the notion of *Unique-Path Identity Based Encryption* (UPIBE). As in HIBE, UPIBE allows a sender to encrypt messages to a receiver whose individual public key is unknown by using only a string that specifies the receiver’s identity as well as a global main public key. On the receiver side, UPIBE assumes that a secret key in one level delegates at most one secret key of the subjacent level. In contrast to FS-PKE, unique-path delegation in UPIBE still respects identity (sub-)strings from an exponential size string space on each level. Consequently, a receiver with email address `bob@pc.2023.ec.iacr.org` cannot decrypt ciphertexts encrypted to identity `charlie@pc.2023.ec.iacr.org`. Beyond the cryptographic utility, there are real-world examples for such a unique-path delegation behavior in linear vertical or horizontal hierarchies.³

One perspective on UPIBE could be that it lifts the bounded-collusion setting from IBE [DKXY02] to HIBE by restricting adversaries in corrupting at most one delegated secret key in the identity hierarchy. Instead, we view the characteristic of UPIBE complementary or even orthogonal to the bounded-collusion setting: While bounded collusion means that the overall number of corrupted secret keys is limited, UPIBE limits the number of delegations—and, hence, corruptions—structurally *per delegated secret key*. In the specific case of UPIBE, we permit one delegation per secret key, but this can be extended to two or more delegations per secret key. Indeed, one of our results motivates research on HIBE with *at most two delegations* per secret key (see Section 4), which we leave as a question for future work and concentrate on UPIBE here.

UPIBE AS AN ABSTRACTION OF KU-KEM. In the context of strongly secure messaging, many cryptographic protocols use a building block called Key-Updatable Key Encapsulation Mechanism (KU-KEM) [PR18b, JS18, JMM19b, BRV20]. This extended form of standard KEM provides an update mechanism with which public keys and secret keys can be updated independently with respect to arbitrary bit strings. In addition to the security guarantees of a standard KEM, updates in KU-KEM are required to achieve *forward-secrecy* and *effective divergence*. This means that an updated secret key cannot decrypt ciphertexts directed to prior versions of the secret key; and an incompatibly updated secret key cannot decrypt ciphertexts produced with a corresponding (incompatible) public key.

The only known construction of KU-KEM relies on black-box HIBE with *unbounded hierarchy depth* secure against *adaptive adversaries* [PR18b, JS18, JMM19b, BRV20]. This induces a significant performance penalty and limits the choice of underlying assumptions (e.g., no practical⁴ unbounded-depth HIBE from lattices is known). Intuitively, KU-KEM secret key updates are realized via sequential HIBE delegations. Replacing black-box HIBE in this construction by black-box UPIBE is trivial. Thus, using a black-box HIBE scheme to realize UPIBE is henceforth referred to as *trivial UPIBE construction*. By introducing UPIBE as a more general notion for KU-KEM, we are the first to lift this specific tool to a suitable abstraction and reduce the power of (underlying) HIBE to the essential. As we will see, this also allows for a substantial gain in efficiency.

DEFINITIONS AND CONSTRUCTIONS OF SECURE MESSAGING. KU-KEM was developed as a building block for constructions of secure messaging protocols.⁵ Interestingly, the impractical performance of prior KU-KEM constructions even affected security definitions in the messaging literature. These definitions can be divided into two categories: (1) those that require *full security* with respect to the modeled threats and (2) those that *relax the security requirements* by limiting adversarial power. Generally, relaxed definitions allow for more efficient constructions. Specifically, the majority of fully secure messaging protocols relies on KU-KEM [PR18b, JS18, JMM19b, BRV20], whereas the main motivation for relaxing security definitions was to analyze or develop practical protocols that can dispense with employing KU-KEM for better efficiency [JMM19a, ACD19, DV19]. To emphasize and substantiate this partition of the literature, Balli et al. [BRV20] proved that KU-KEM is equivalent to fully secure messaging under weak randomness. Concretely, implementing KU-KEM in a messenger allows for diverging the secrets of

³ E.g., the chronological succession of presidents in a particular state or a ranking list that results from a competition.

⁴ We stress that the construction of selective-secure HIBE with unbounded delegations from CDH [DG17b] or from any fully secure IBE [DG17a] is an impressive, yet rather theoretic result.

⁵ Note that, instead of building full messaging protocols, the literature usually focuses on ratcheting (aka. continuous key exchange) protocols that relate to the former in the same way as KEM relates to PKE: Reaching analogous security guarantees, the former transmits encrypted messages and the latter establishes symmetric keys.

communication partners Alice and Bob when an adversary impersonates Alice towards Bob. Hence, the price that practical messengers—including all protocols deployed in practice—pay is reduced resilience against such impersonations. We conclude that KU-KEM and, therefore, UPIBE play a central role in (strongly) secure messaging.

EFFICIENCY OF UPIBE AND KU-KEM. The inefficiency of the trivial KU-KEM construction from black-box HIBE leads to two questions that were posed as open problems in prior work [PR18b, JS18, BRV20] and which we will address via the UPIBE approach:

- (1) *Can we build (KU-KEM from) UPIBE based on weaker assumptions?*
- (2) *Can we build (KU-KEM from) UPIBE with better efficiency?*

We are the first to affirm both questions in three steps.⁶ But instead of only giving answers for the specific case of KU-KEM, we generalize it to the UPIBE setting which highlights the reasons for our improvements.

First, we consider bounded-depth UPIBE, which means that the maximal number of secret-key delegation levels is bounded a priori. Our generic construction of bounded-depth UPIBE is based on *bounded-collusion IBE*, for which we have practical instantiations from standard assumptions like DDH or QR in the standard model [DKXY02, GLW12, TW14].⁷ In a second step, we extend the design of our bounded-depth UPIBE construction to obtain an unbounded-depth UPIBE scheme. This unbounded-depth UPIBE construction with adaptive security can be based on *bounded-depth* HIBE with only *selective* security in the random oracle model. Finally, we prove that KU-KEM can be based on UPIBE, where the number of key updates in KU-KEM is proportionate to the number of key delegations in UPIBE.

Instantiating our unbounded-depth UPIBE construction with the bounded-depth HIBE by Boneh et al. [BBG05] reveals the strengths of our approach. We compare this instantiation to the best known instantiation of *trivial* unbounded-depth UPIBE via the unbounded-depth HIBE by Gong et al. [GCTC16]. This comparison shows that our construction is significantly more efficient by most relevant measures. In particular, it outperforms the trivial approach substantially in terms of performance, ciphertext sizes, and encryption key sizes.

A notable feature of our unbounded-depth UPIBE construction is that its efficiency can be dynamically configured via a parameter ε . Roughly, ε trades ciphertext size against secret key size. Depending on the performance priorities in a setting (bandwidth, algorithm runtime, etc.) and depending on the expected user behavior (average length of identity strings, average number of encryptions per identity, etc.), this parameter can optimize our construction for deployment under various conditions. Setting the parameter ε to infinity yields the known trivial UPIBE construction [PR18b, JS18]; consequently, there always exists an ε for which our new UPIBE construction is indeed the best known one.

CONTRIBUTIONS. Our first contribution is to abstract the tools in KU-KEM constructions to the more general field of Identity Based Encryption by, simultaneously, reducing the power of standard HIBE to the essential: Unique-Path IBE. Our definition from Section 2 shows that this new perspective on structurally limited delegation and collusion is seamlessly embedded in existing (H)IBE notions.

For comprehensibility, we start with building the simpler bounded-depth UPIBE construction in Section 3, which is secure in the standard model (StM):

$$\text{Adaptive Bounded-Collusion IBE} \implies_{\text{StM}} \text{Adaptive Bounded-Depth UPIBE}$$

This construction shows that UPIBE can be based on significantly reduced complexity assumptions with a practically⁴ relevant design. We also give a concrete instantiation with small ciphertexts (two group elements) and secret keys (six group elements and one symmetric key) from DDH that takes advantage of construction internals of a bounded-collusion IBE by Dodis et al. [DKXY02].

⁶ An independent, concurrent article accepted at PKC'22 claimed to present a new direct KU-KEM construction with reduced complexity, too. The authors of the article shared their final version after acceptance. We identified a fatal security weakness in their KU-KEM construction and reported this weakness to the authors who then withdrew the article:

Collins D., Vaudenay S.: Reducing the Complexity of Optimally Secure Ratcheted Key Exchange, <https://pkc.iacr.org/2022/acceptedpapers.php>.

⁷ An alternative approach from standard assumptions would be to rely on the fully secure IBE from CDH by Garg and Döttling [DG17b]. Unfortunately, this will not yield a practical instantiation.

By developing two powerful extensions on top of our first generic UPIBE construction, we are ultimately able to build unbounded-depth UPIBE:

$$\text{Adaptive Bounded-Depth HIBE} \implies_{\text{StM}} \text{Adaptive Unbounded-Depth UPIBE}$$

While conceptually inheriting core ideas of our *bounded*-depth UPIBE, this second *unbounded*-depth UPIBE construction in Section 4 unfolds the full strength of our approach. Its efficiency is dynamically configurable for different deployment settings and, instantiated with the most suitable bounded-depth selective HIBE [BBG05], it reaches the best performance results compared to existing work. Along the way, inspired by techniques that turn *selective* secure bounded-depth HIBEs *adaptive* secure [BB04, BBG05], we develop a guessing technique which allows for a significantly broader choice of underlying assumptions and more efficient instantiations in the random oracle model (ROM):

$$\text{Selective Bounded-Depth HIBE} \implies_{\text{ROM}} \text{Adaptive Unbounded-Depth UPIBE}$$

We note that when instantiating our construction with lattice HIBEs [ABB10, CHKP10], we obtain the first KU-KEM secure under conjectured post-quantum assumptions.

We systematically analyze the performance of our approach when being used to instantiate KU-KEM in Section 7. It is notable that all prior KU-KEM constructions are a trivial special case of our new techniques. This means that our new constructions always offer the best (known) performance. For clarity, we first present semantically secure constructions of UPIBE. Using techniques known from KEM combiners [GHP18], we show in Section 5 that our constructions can also be made secure against chosen-ciphertext attacks if the underlying (H)IBE schemes are.

FUTURE DIRECTIONS. We believe that generalizing our UPIBE approach by increasing the bound of delegations/corruptions per secret key is promising and relevant. For example, our unbounded-depth UPIBE relies on bounded-depth HIBE where delegations per secret key can be bounded to 2. Utilizing the delegation bound of 2 in order to enhance performance compared to black-box bounded-depth HIBE (with unbounded delegation per secret key) would be beneficial. Another direction for future work is to enhance our generic results by designing (more) direct constructions that carefully take advantage of internals of specific (H)IBE schemes for better efficiency.

1.1 Technical Overview

To understand the core idea of our UPIBE constructions, we briefly discuss the subtle difference between the security definitions of standard HIBE and UPIBE. Although these definitions are conceptually identical, the crucial limitation of UPIBE is that at most one delegation per secret key is permitted. This means that the large tree of delegated secret keys in HIBE is reduced to a unique delegation path in UPIBE. Consequently, adversaries will essentially expose at most one UPIBE secret key—all descendant UPIBE keys can be obtained via delegation by the adversary itself. Consider the identity string that corresponds to this exposed UPIBE secret key. In relation to this identity string, our natural security definition requires only two types of challenge ciphertexts to remain secure: (1) those that are encrypted to *true prefix* identity strings and (2) those that are encrypted to identity strings *branching off* the exposed key’s identity string. All remaining challenges can be solved trivially with the exposed secret key. Our UPIBE constructions exploit this fact to turn all prefix identity strings (case 1) into branched off identity strings (case 2) by adding a special suffix at the end of every UPIBE identity string.

COMBINED HIBE EXPOSURE. Having the definitional difference in mind, we will see that multiple colluding exposures in HIBE can be significantly more damaging than the single permitted exposure in UPIBE. More concretely, HIBE constructions have to make sure that challenge ciphertexts remain secure under *any combination* of (non-trivial) secret key exposures in the delegation hierarchy. Since the unique-path delegation in UPIBE permits at most *one exposure*, UPIBE constructions have to protect challenge ciphertexts only against the single most damaging secret key exposure. We illustrate this gap by considering the effect of a specific combination of HIBE secret key exposures.

For this we let two exposed HIBE secret keys have identities $id_{\text{ex},1} = (id'_1)$ and $id_{\text{ex},2} = (id_1, id'_2)$, and a single HIBE challenge have identity $id_{\text{ch}} = (id_1, id_2)$, such that $id_1, id'_1, id_2, id'_2 \in \{0, 1\}^\lambda$, where λ is the bit-length per delegated sub-identity string. This means, $id_{\text{ex},2}$ and id_{ch} branch in delegation level 2 with $id'_2 \neq id_2$, and $id_{\text{ex},1}$ branches off the former two identity strings in level 1 with $id'_1 \neq id_1$. Observe that the exposed key with identity $id_{\text{ex},1}$ still contains information for delegating subordinate keys to

the second level, e.g., to sub-identity id_2 which results in full identity $id^* = (id'_1, id_2)$. In contrast, the exposed key with identity $id_{ex,2}$ does not (need to) contain this information anymore as it is delegated to level 2 already. However, exposed key with identity $id_{ex,2}$ may contain information about its own delegation path along the first level with sub-identity string id_1 , which differs from the information contained in exposed key with identity $id_{ex,1} = (id'_1)$. One major difficulty for building HIBE is to make sure that the information about delegation along id_1 from exposed key $id_{ex,2}$ cannot be combined harmfully with the secrets available for delegation to level 2 from exposed key $id_{ex,1}$. In particular, this combination should not suffice to obtain a secret key for identity $(id_1, id_2) = id_{ch}$ because this would solve the challenge. Since the single permitted exposure in UPIBE prevents such combined exposures, we can simplify the design of our UPIBE constructions, which makes them more efficient. We stress that this difference between HIBE and UPIBE is an inevitable implication of our natural definition.

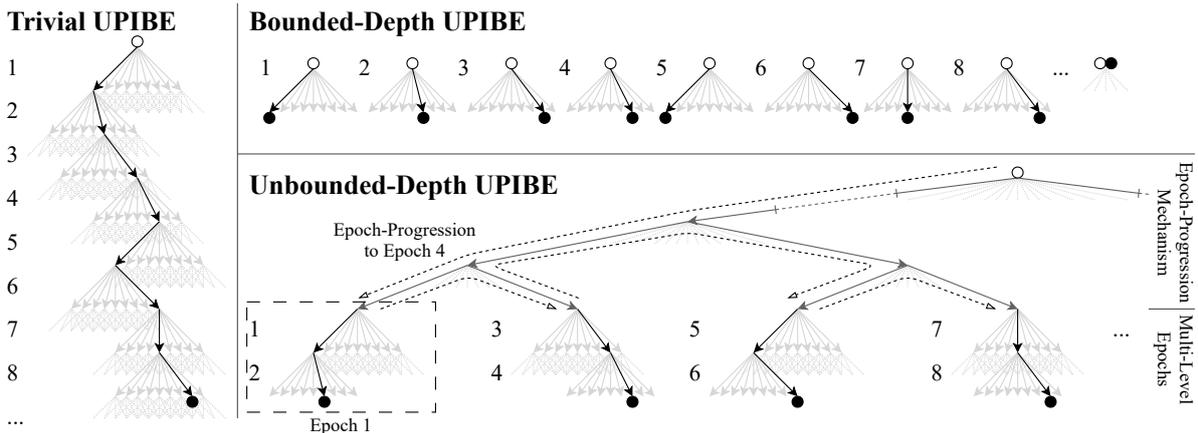


Fig. 1: Conceptual illustration of delegations in the trivial, bounded-depth, and unbounded-depth UPIBE constructions (here with $\varepsilon = 2$). The black (path of) arrows realize delegation of a UPIBE identity string with level depth 8. Light gray arrows indicate alternative and further delegations. White circles represent the (composed) main public key(s) and filled dots represent the (composed) delegated secret key(s).

BOUNDED-DEPTH UPIBE. One interpretation of the above observation is that our constructions can assume key material for lower level delegations to be per se harmless. Using this guarantee, our bounded-depth UPIBE construction implements each UPIBE delegation level with an individual IBE instance. Intuitively, this turns the vertical delegation path into a horizontal delegation sequence, as illustrated in Figure 1. Our construction’s UPIBE main public and secret key consist of all underlying IBE instances’ main public and secret keys, respectively. For encryption, the UPIBE identity string is split into multiple IBE sub-strings. The UPIBE ciphertext is then obtained by executing IBE encryption for each level’s sub-string and concatenating the resulting IBE ciphertexts. On UPIBE delegation, the respective level’s IBE main secret key is removed after delegating an identity-specific secret key for that level. To prove security of this construction, we use the fact that every challenge identity branches off the exposed key’s identity in one of its passed delegation levels. Our reduction embeds an underlying IBE challenge in this branching level, which turns a successful UPIBE adversary into a successful IBE adversary. The above description of our scheme is highly simplified and neglects subtle enhancements that lead to better performance. Although conceptually simple in the bounded-depth case, this construction does not extend (trivially) to the unbounded-depth setting.

UNBOUNDED-DEPTH UPIBE. Therefore, we develop two crucial extensions: First, we replace each delegation level by an ε -level *delegation epoch*. In every such epoch, ε many sequential delegations can be processed. (See Figure 1 where $\varepsilon = 2$.) This reduces the number of concatenated ciphertexts by a factor of $1/\varepsilon$. Then, we add an *epoch-progression mechanism* on top of our construction. With this mechanism, delegation from a fully-delegated epoch progresses dynamically to the next fresh epoch. This allows us to dispose of the static list of IBE instances from our bounded-depth construction. One can think of the epoch-progression mechanism as a Forward-Secure PKE scheme that generates at every step a fresh starting point for a multi-level epoch in which the actual UPIBE delegations are conducted. The

security proof for our unbounded-depth UPIBE follows the same idea as the one for our bounded-depth construction, only that it reduces to bounded-depth HIBE. To rely on only *selective* bounded-depth HIBE, we develop a special guessing technique that avoids the exponential loss factor induced by known techniques [BB04, BBG05] for turning selective HIBE adaptive secure. We believe that the solid design—in addition to its enhanced performance—makes our construction attractive for practical applications (such as secure messengers).

CHOSEN-CIPHERTEXT SECURITY. We investigate the options to obtain CCA security for UPIBE. Unfortunately, the well known generic BCHK (often also called CHK) compiler for HIBEs [CHK04, BCHK07] is not applicable to UPIBE. While opting for a form of verification-by-re-encryption akin to the Fujisaki-Okamoto (FO) transform [FO99] is applicable, one introduces significant computational overhead as well as is bound to the ROM. Instead, we leverage chosen-ciphertext security of the underlying building blocks by effectively tying together the concatenated ciphertexts in every UPIBE ciphertext. For simplicity, we referred to UPIBE as a *Message Encryption* primitive so far, but all our results actually consider *Key Encapsulation*. Therefore, in the case of bounded-depth UPIBE we can make use of techniques developed in the context of KEM combiners [GHP18]. These versatile techniques only change the final computation of the encapsulated UPIBE key instead of explicitly authenticating the concatenated ciphertext. A similar idea, though in the ROM, can be applied in the case of unbounded-depth UPIBE where the underlying HIBE instances can be efficiently made CCA secure via the BCHK compiler. As a result, our chosen-ciphertext secure constructions are only minimally less efficient than our semantically secure ones.

2 UPIBE Definition

For clarity, we consider Identity Based *Key Encapsulation* primitives instead of Identity Based *Message Encryption* in this work. In line with this, we call public and secret keys *encapsulation* and *decapsulation* keys, respectively. Since Unique-Path IBE is a special case of Hierarchical IBE, we introduce all relevant IBE notions modularly at once.

Syntax. All of the considered *Identity Based Encapsulation* (IBE) schemes are quadruples $\text{IE} = (\text{IE.gen}, \text{IE.enc}, \text{IE.dec}, \text{IE.del})$ of algorithms with encapsulation and decapsulation key spaces \mathcal{EK} and \mathcal{DK} , respectively, symmetric key space \mathcal{K} , and ciphertext space \mathcal{C} .

We specify the considered types of IBE via parameters L , λ , and D . L fixes the maximal number of *sequential* delegations (i.e., the maximal number of levels aka. the *depth*), λ fixes the bit-length of identity strings for each delegation, and D fixes the maximal number of delegations *per* decapsulation key. That means, for unbounded-depth HIBE we have $(L, D) = (\infty, 2^\lambda)$, for bounded-depth HIBE we have $(L, D) = (L, 2^\lambda)$ for some fixed value L , for unbounded-depth UPIBE we have $(L, D) = (\infty, 1)$, and for bounded-depth UPIBE we have $(L, D) = (L, 1)$ for some fixed value L . (Bounds that are exponential in the security parameter are considered *practically unbounded*, too.) We treat bounded-collusion IBE as a bounded-depth HIBE with $L = 1$ such that the number of colluding users is encoded as the number of maximal delegations for the main decapsulation key $D = D$ for some constant D .

The four IBE algorithms' syntax is defined as follows:

- $\text{IE.gen} : \emptyset \rightarrow_{\S} \mathcal{EK} \times \mathcal{DK}$
- $\text{IE.enc} : \mathcal{EK} \times \{0, 1\}^{l \cdot \lambda} \rightarrow_{\S} \mathcal{C} \times \mathcal{K}$, where $0 < l \leq L$
- $\text{IE.dec} : \mathcal{DK} \times \mathcal{C} \rightarrow \mathcal{K}$
- $\text{IE.del} : \mathcal{DK} \times \{0, 1\}^\lambda \rightarrow_{\S} \mathcal{DK}$

For efficiency reasons, we add derivation algorithm $\text{IE.der} : \mathcal{EK} \times \{0, 1\}^\lambda \rightarrow_{\S} \mathcal{EK}$ that computes (compact) identity-specific encapsulation keys. This allows for reducing the combined size of a main encapsulation key ek and a multi-level identity string $id = (id_1, \dots, id_l)$, such that $\text{IE.enc}(ek, (id_1, \dots, id_l))$ can be turned into $\text{IE.enc}(\text{IE.der}(\dots \text{IE.der}(ek, id_1) \dots, id_l), \epsilon)$.

Correctness. For correctness of all considered types of IBE with parameters L , λ , and D , we require for all $(ek, dk_0) \leftarrow_{\S} \text{IE.gen}$, all $id = (id_1, \dots, id_l)$ with $id_i \in \{0, 1\}^\lambda, 0 < i \leq l \leq L$, all $dk_i \leftarrow_{\S} \text{IE.del}(dk_{i-1}, id_i)$, and all $(c, k) \leftarrow_{\S} \text{IE.enc}(ek, id)$, that $\text{IE.dec}(dk_l, c) = k$.

Security. We define experiment $\text{IND}_{\text{IE}}^b(\mathcal{A})$, $b \in \{0, 1\}$ that models multi-instance key indistinguishability. For all considered types of IBE schemes IE, this experiment provides the following oracles to adversary \mathcal{A} for which we provide a full pseudo-code specification in Figure 2:

- **Gen:** **Generates** a fresh main key pair $(ek, dk) \leftarrow_{\S} \text{IE.gen}$ and returns ek
- **Del(i, id, id^*):** **Delegates** decapsulation key $dk_{i,(id,id^*)} \leftarrow_{\S} \text{IE.del}(dk_{i,id}, id^*)$ from $dk_{i,id}$ with identity string $id^* \in \{0, 1\}^\lambda$, unless $dk_{i,id}$ results from L sequential delegations from a main decapsulation key, or D delegations from $dk_{i,id}$ were already queried
- **Chall(i, id):** Issues a **challenge** encapsulation $(c, k_0) \leftarrow_{\S} \text{IE.enc}(ek_i, id)$ to main encapsulation key ek_i and identity string $id \in \{0, 1\}^{l-\lambda}$, $0 < l \leq L$ and returns c as well as key k_b , where $k_1 \leftarrow_{\S} \mathcal{K}$, unless an exposed decapsulation key was delegated from ek_i 's main decapsulation key dk_i with an identity string that equals or is a prefix of id
- **Exp(i, id):** **Exposes** decapsulation key $dk_{i,id}$, generated or delegated from main decapsulation key dk_i and identity string id , unless a challenge encapsulation to ek_i and identity string id' was queried, such that (ek_i, dk_i) form a main key pair and id equals or is a prefix of id'

Eventually, the adversary terminates by outputting a guess b' and wins iff $b = b'$.

Game $\text{IND}_{\text{IE},L,\lambda,D}^b(\mathcal{A})$	Oracle $\text{Del}(i, id, id^*)$
00 $n \leftarrow 0$	16 Require $id \in \{0, 1\}^{l-\lambda}$, $0 \leq l < L$
01 $b' \leftarrow_{\S} \mathcal{A}$	17 Require $DK_i[id] \neq \perp$
02 Stop with b'	18 Require $D > \{id' DK_i[id id'] \neq \perp\} $
Oracle Gen	19 $dk \leftarrow_{\S} \text{IE.del}(DK_i[id], id^*)$
03 $(ek_n, dk_n) \leftarrow_{\S} \text{IE.gen}$	20 $DK_i[id id^*] \leftarrow dk$
04 $DK_n[\cdot] \leftarrow \perp$	21 Return
05 $DK_n[\epsilon] \leftarrow dk_n$	Oracle Exp(i, id)
06 $CH_n[\cdot] \leftarrow \emptyset$; $XP_n \leftarrow \emptyset$	22 Require $DK_i[id] \neq \perp$
07 $n \leftarrow n + 1$	23 $XP' \leftarrow \{id^* \in \{0, 1\}^* : id \preceq id^*\}$
08 Return ek_{n-1}	24 Require $\forall id^* \in XP_i \cup XP' : CH_i[id^*] = \perp$
Oracle Chall(i, id)	25 $XP_i \stackrel{\perp}{\leftarrow} XP'$
09 Require $0 \leq i < n$	26 Return $DK_i[id]$
10 Require $id \in \{0, 1\}^{l-\lambda}$, $0 < l \leq L$	Oracle Dec(i, id, c)
11 Require $id \notin XP_i$	27 Require $DK_i[id] \neq \perp$
12 $(c, k_0) \leftarrow_{\S} \text{IE.enc}(ek_i, id)$	28 Require $c \notin CH_i[id]$
13 $CH_i[id] \stackrel{\perp}{\leftarrow} \{c\}$	29 $k \leftarrow \text{IE.dec}(DK_i[id], c)$
14 $k_1 \leftarrow_{\S} \mathcal{K}$	30 Return k
15 Return (c, k_b)	

Fig. 2: Game IND for defining security of (un-)bounded-depth (Bounded-Collision/Hierarchical/Unique-Path) IBE. By allowing queries to oracle Dec, we model chosen-ciphertext attacks (instead of only chosen-plaintext attacks). $\{0, 1\}^*$ is the set of arbitrary bit-strings, and $x \preceq y$ tests if bit-string x equals or is a prefix of bit-string y .

If adversary \mathcal{A} specifies the challenge(s) at the beginning of the game without adaptively seeing the return values of other queries, we call \mathcal{A} **selective** and otherwise **adaptive**.

With the above adversarial oracles, we capture **chosen-plaintext** attacks. Selective chosen-plaintext attacks is a rather weak adversary model that helps us focusing on the core of our novel ideas when presenting our constructions. Yet, we also present adaptive **chosen-ciphertext** secure constructions. An adversary, attacking such constructions, can additionally query the following oracle:

- **Dec(i, id, c):** **Decapsulates** $k \leftarrow \text{IE.dec}(dk_{i,id}, c)$ of ciphertext c under $dk_{i,id}$ and returns k , unless c was given to \mathcal{A} as a challenge encapsulation to ek_i and id , $dk_{i,id}$ was (sequentially) delegated from dk_i with respect to id , and (ek_i, dk_i) form a main key pair

Definition 1. The advantage of adversary \mathcal{A} in winning IND_{IE}^b is

$$\text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{A}) := |\Pr[\text{IND}_{\text{IE}}^0(\mathcal{A}) = 1] - \Pr[\text{IND}_{\text{IE}}^1(\mathcal{A}) = 1]|.$$

Compared to standard (bounded-depth) (H)IBE security experiments, the only difference is our restriction to at most D delegation queries per decapsulation key. Yet, challenges can be queried without limiting the choice of identity strings, even for UPIBE.

3 Bounded-Depth UPIBE from Bounded-Collusion IBE

We present our bounded-depth UPIBE construction in Figure 3 by explaining its components one after another, starting with decapsulation keys and ciphertexts.

Structure of Keys and Ciphertexts. The core idea behind our UPIBE constructions is that delegations along the unique ‘vertical’ path of identity levels are realized ‘horizontally’. That means, for each delegation level in our UPIBE construction from Figure 3 with bounded-depth L , we use a separate bounded-collusion IBE instance. Think of these IBE instances being placed horizontally next to one another from left to right as shown in Figure 1.

To understand this idea, we describe the structure of UPIBE decapsulation keys. A UPIBE decapsulation key delegated to level l contains three different types of keys, two of which are IBE decapsulation keys: (1) One *ordinary delegated* IBE decapsulation key for *each* of the first l levels, (2) an additional *special delegated* IBE decapsulation key for *only* level l , and (3) a symmetric *forwarding* key from which (*un-delegated*) IBE main decapsulation keys for all remaining $L - l$ levels are computed. See Figure 3 lines 02-06 for UPIBE key generation that consists of generating all IBE main encapsulation keys and sampling the initial symmetric forwarding key.

Proc IE.gen	Proc IE.enc(ek, id)
00 $E[\cdot] \leftarrow \perp; D[\cdot] \leftarrow \perp$	21 Require $id \in \{0, 1\}^{l-\lambda}, 0 < l \leq L$
01 $fk_0 \leftarrow_{\S} \{0, 1\}^{\lambda}$	22 $E \leftarrow ek$
02 For $l = 0$ to $L - 1$:	23 $id_0 \parallel \dots \parallel id_{l-1} \leftarrow id$ with $id_j \in \{0, 1\}^{\lambda}$
03 $(fk_{l+1}, s) \leftarrow G(fk_l)$	24 For $j = 0$ to $l - 2$:
04 $(ek', dk') \leftarrow \text{IE.gen}'(s)$	25 $(c'_j, k'_j) \leftarrow_{\S} \text{IE.enc}'(E[j], id_j \parallel 1)$
05 $E[l] \leftarrow ek'$	26 $(c'_{l-1}, k'_{l-1}) \leftarrow_{\S} \text{IE.enc}'(E[l - 1], id_{l-1} \parallel 0)$
06 $ek \leftarrow E; dk \leftarrow (0, \perp, fk_0)$	27 $C \leftarrow c'_0 \parallel \dots \parallel c'_{l-1}$
07 Return (ek, dk)	28 $K \leftarrow W(k'_0, \dots, k'_{l-1}, C)$
	29 Return (C, K)
Proc IE.del(dk, id)	Proc IE.dec(dk, C)
08 Require $id \in \{0, 1\}^{\lambda}$	30 $(l, D, fk) \leftarrow dk$
09 $(l, D, fk) \leftarrow dk$	31 $c_0 \parallel \dots \parallel c_{l-1} \leftarrow C$ with $c_j \in \mathcal{C}$
10 Require $l < L$	32 Require $l = l'$
11 If $l > 0$:	33 For $j = 0$ to $l - 2$:
12 $(dk'_0, dk'_1) \leftarrow D[l - 1]$	34 $k'_j \leftarrow_{\S} \text{IE.dec}'(D[j], c_j)$
13 $D[l - 1] \leftarrow dk'_1$	35 $(dk'_0, dk'_1) \leftarrow D[l - 1]$
14 $(fk', s) \leftarrow G(fk)$	36 $k'_{l-1} \leftarrow_{\S} \text{IE.dec}'(dk'_0, c_{l-1})$
15 $(ek', dk') \leftarrow \text{IE.gen}'(s)$	37 $K \leftarrow W(k'_0, \dots, k'_{l-1}, C)$
16 $dk'_0 \leftarrow_{\S} \text{IE.del}'(dk', id \parallel 0)$	38 Return K
17 $dk'_1 \leftarrow_{\S} \text{IE.del}'(dk', id \parallel 1)$	
18 $D[l] \leftarrow (dk'_0, dk'_1)$	
19 $dk \leftarrow (l + 1, D, fk')$	
20 Return dk	

Fig. 3: Construction of bounded-depth UPIBE IE with parameters $(L, \lambda, D = 1)$ from PRG G and bounded-collusion IBE scheme IE' with parameters $(L', \lambda', D') = (1, \lambda + 1, 2)$ and ciphertext space \mathcal{C} . Core function W is realized as XOR-sum $\bigoplus_{j=0}^{l-1} k'_j$ and ignores input C . In our chosen-ciphertext secure instantiation, we additionally generate a dummy ciphertext $\hat{c} \leftarrow_{\S} \mathcal{C}$ and key $\hat{k} \leftarrow_{\S} \mathcal{K}$ in IE.gen , which is included into ek and W to pad all unused indices $i \leq L$ with \hat{c} and \hat{k} respectively.

A UPIBE ciphertext, encapsulated to level l (i.e., to identity $id \in \{0, 1\}^{l-\lambda}$), consists of one IBE ciphertext for each of the first $l - 1$ levels encoded with suffix 1 (lines 23-25) and one additional IBE ciphertext that targets the special delegated IBE decapsulation key at level l encoded with suffix 0 (line 26). To decapsulate the former $l - 1$ ciphertexts (lines 33-34), the receiver needs to be in possession of the first $l - 1$ ordinary delegated IBE decapsulation keys. Hence, successful decapsulation shows that the receiver holds a UPIBE decapsulation key that was correctly delegated along the first $l - 1$ levels of the identity path. By also being able to decapsulate the special l th IBE ciphertext (lines 35-36), the receiver additionally shows that it holds the full UPIBE decapsulation key that was delegated along all l levels—and particularly not a UPIBE decapsulation key that was delegated along an *extended* identity path.

While a UPIBE ciphertext is a concatenation of all l IBE ciphertexts, the encapsulated UPIBE key is an XOR-sum of all l encapsulated IBE keys (lines 27-28). We generalize the computation of

the encapsulated key via core function W to simplify the description of our chosen-ciphertext secure construction in Section 5.

Delegation of a UPIBE decapsulation key is in line with the above ideas by conducting four steps: (a) Removing the special IBE decapsulation key at current level l , yet keeping all ordinary IBE decapsulation keys until level l (lines 11-13), (b) computing the next forwarding key as well as a seed by evaluating a PRG on the current forwarding key (line 14), (c) generating the main IBE decapsulation key at level $l + 1$ from the obtained seed (line 15), and delegating both the special delegated IBE decapsulation key for level $l + 1$ (line 16) as well as the ordinary delegated IBE decapsulation key for level $l + 1$ (line 17) from this new main IBE decapsulation key, and, lastly, (d) removing the just obtained main IBE decapsulation key at level $l + 1$ as well as the old forwarding key.

Intuition for Security. The security argument for this construction uses the fact that adversaries can expose at most one UPIBE decapsulation key per instance during the security experiment.⁸ This single exposure reveals precisely one special delegated IBE decapsulation key—the current one—the chain of ordinary IBE decapsulation keys that were delegated along the exposed UPIBE key’s identity path, and the current symmetric forwarding key from which future levels’ IBE main decapsulation keys can be obtained. After such an exposure, two types of UPIBE ciphertexts must remain secure: Those that target *true prefixes* of the exposed key’s identity string, and those that target identity strings *branching off* the exposed key’s identity string.⁹ Ciphertexts targeting a true prefix identity string, indeed, remain secure because their decapsulation requires the use of a higher level special delegated IBE decapsulation key. Such prior level special IBE keys were removed before the exposure and are, therefore, not contained in the exposed UPIBE key. Similarly, the decapsulation of ciphertexts that target a branched off identity string require the use of an inaccessible IBE decapsulation key—namely, an ordinary IBE decapsulation key that was delegated along this branch. Consequently, exposed UPIBE decapsulation keys do not affect ciphertexts that are required to remain secure. Finally, we note that at most two delegated decapsulation keys per IBE instance are leaked at an exposure of a UPIBE decapsulation key. Thus, relying on bounded-collusion IBE suffices, where the number of colluding users is at most 2.

Performance. Bounded-depth UPIBE (and bounded-depth KU-KEM) actually often suffice for secure messaging protocols.¹⁰ So far, the only known instantiation of bounded-depth UPIBE is trivially derived from bounded-depth HIBE. With our bounded-depth UPIBE construction we demonstrate a significant reduction in complexity of the underlying hardness assumption: bounded-collusion IBE instead of bounded-depth HIBE. Furthermore, we use this construction to make the reader familiar with the core ideas of our unbounded-depth UPIBE construction in Section 4.

Without any additional assumptions on the underlying bounded-collusion IBE, the size of UPIBE encapsulation keys in our construction is linear in the maximal level depth L , UPIBE decapsulation keys grow with the number of conducted delegations, and UPIBE ciphertexts grow in the bit-length of their corresponding identity string.

When instantiating our construction with the DDH-based bounded-collusion IBE by Dodis et al. [DKXY02], we can take advantage of the group structure to aggregate and shrink encapsulation keys, decapsulation keys, and ciphertexts. We give the concrete instantiation in Appendix E in which a UPIBE decapsulation key consists of 6 exponents and 1 symmetric key, a UPIBE ciphertext consists of 2 group elements, and a UPIBE encapsulation key consists of $2 + 3(L - l)$ group elements, where l is the level for which the current encapsulation key is derived via algorithm `IE.der`. This is highly efficient for settings in which distribution and storage of large encapsulation keys is cheap.¹¹ Enhancing this construction to also obtain a compact, constant size encapsulation key remains an interesting open problem.

⁸ With the exposed UPIBE decapsulation key, the adversary can compute all subsequent delegations and decapsulations itself, so further exposures are meaningless.

⁹ Branching here means that for two identity strings id, id^* with $\ell^* = \min(|id|, |id^*|)$, strings id and id^* differ in at least one of the first ℓ^* bits.

¹⁰ E.g., the number of conducted key delegations in the bidirectional messaging protocol in [PR18a, see page 22] is upper-bounded by the maximal number of ciphertexts that cross the wire during a round-trip time (i.e., at most a few dozens).

¹¹ Consider asymmetric communication for which ciphertexts should be small and encapsulation keys can be large: E.g., sending large encapsulation keys on hardware memory from time to time via resupply flights to the International Space Station, and sending ciphertexts over the air back to earth.

Security. For clarity, we first consider chosen-plaintext security IND_{IE}^b of our UPIBE construction:

Theorem 1. *Bounded-depth UPIBE protocol IE from Figure 3 offers adaptive key indistinguishability in the standard model. More precisely, for every adaptive chosen-plaintext adversary \mathcal{A} attacking protocol IE in games IND_{IE}^b according to Definition 1 with parameters $(L, \lambda, D = 1)$, there exists an adversary \mathcal{B}_{G} attacking PRG G according to Definition 3 and an adaptive chosen-plaintext adversary $\mathcal{B}_{\text{IE}'}$ attacking bounded-collusion IBE IE' in games $\text{IND}_{\text{IE}'}$ according to Definition 1 with parameters $(L', \lambda', D') = (1, \lambda + 1, 2)$ such that $\text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{A}) \leq q_{\text{Gen}} \cdot L^2 \cdot \text{Adv}_{\text{G}}^{\text{ind}}(\mathcal{B}_{\text{G}}) + q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot L \cdot \text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B}_{\text{IE}'})$, where q_{Gen} and q_{Chall} are the number of queries to oracles Gen and Chall by adversary \mathcal{A} , respectively, and the running time of \mathcal{B}_{G} and $\mathcal{B}_{\text{IE}'}$ is about that of \mathcal{A} .*

Security Proof Overview. For clarity in notation, we refer to oracles in game IND_X^b by adding the scheme's identifier X as a subscript to the oracle names (i.e., Gen_X , Chall_X , etc). Also, we first sketch our proof by focusing on a reduction from single-instance security of UPIBE to multi-instance security of IBE.

Using the PRG, we begin with a hybrid argument that replaces all unexposed symmetric forwarding keys and IBE main key pairs with independently sampled ones. Our reduction $\mathcal{B}_{\text{IE}'}$ then almost directly passes oracle queries from adversary \mathcal{A} against our UPIBE construction IE in game IND_{IE} to oracles of game $\text{IND}_{\text{IE}'}$ against the underlying bounded-collusion IBE scheme IE'. The responses of oracles in game $\text{IND}_{\text{IE}'}$ can then be used almost directly to answer adversary \mathcal{A} 's oracle queries in game IND_{IE} . That means, \mathcal{A} 's queries to oracle Gen_{IE} can be answered by using responses of simple queries to oracle $\text{Gen}_{\text{IE}'}$; the same holds for queries to oracle Del_{IE} .

However, embedding challenges from game $\text{IND}_{\text{IE}'}$ in challenges of game IND_{IE} is non-trivial. To understand this, we observe that the hardness of a challenge in game IND_{IE} depends on the delegation path of the first (and w.l.o.g. only) exposed UPIBE decapsulation key in game IND_{IE} . More precisely, let id^* be the identity string that corresponds to the delegation path of the first exposure via oracle Exp_{IE} . A challenge directed to identity string id is only considered hard if id is a true prefix of id^* , or if id and id^* differ in at least one of their first ℓ^* bits, where $\ell^* = \min(|id|, |id^*|)$. On a query to oracle Chall_{IE} with identity string id , our reduction $\mathcal{B}_{\text{IE}'}$ splits id into its λ -long sub-strings and then identifies in which of these sub-strings the first difference between id and id^* occurs. For this branching sub-string, reduction $\mathcal{B}_{\text{IE}'}$ queries an IBE challenge via oracle $\text{Chall}_{\text{IE}'}$. The resulting IBE challenge-ciphertext and IBE challenge-key are then embedded in the corresponding UPIBE challenge-ciphertext and UPIBE challenge-key output of oracle Chall_{IE} . However, reduction $\mathcal{B}_{\text{IE}'}$ learns string id^* only as soon as adversary \mathcal{A} calls oracle Exp_{IE} . Hence, for each challenge issued before this first exposure query, reduction $\mathcal{B}_{\text{IE}'}$ has to guess in which sub-string the identities branch. Embedding this guessing step in a hybrid argument introduces a loss factor of at most $q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot L$, where q_{Gen} and q_{Chall} are the numbers of queries to oracles Gen_{IE} and Chall_{IE} by adversary \mathcal{A} , resp., and L is the maximal number of delegation levels for our UPIBE construction. We provide our formal proof for multi-instance security in Appendix B.1.

4 Unbounded-Depth UPIBE from Bounded-Depth HIBE

Our unbounded-depth UPIBE construction extends our bounded-depth construction from Section 3 twofold: Horizontally, it replaces each level—realized by an IBE instance in our bounded-depth construction—by a *multi-level epoch*. Each epoch can internally handle up to ε sub-identity levels/delegations. The second extension replaces the static list of IBE main keys at the top of our bounded-depth UPIBE construction by a dynamic *epoch-progression* mechanism. This mechanism realizes a dynamic progression from one epoch to another and, thereby, eliminates the a-priori bounded number of sub-identity levels/delegations; see Figure 1 for a schematic illustration.

The only component used to build our unbounded-depth UPIBE construction is a single bounded-depth HIBE scheme. To understand how the (unbounded number of) UPIBE delegations are processed by this bounded-depth HIBE, we invite the reader to look at the tree of identities/delegations in this HIBE that is indicated by gray (dotted) lines and arrows in Figure 1.

Epoch-Progression via Forward-Secure PKE Technique. In the top α levels of the HIBE tree, we implement the epoch-progression mechanism, where $\alpha = \lceil \log(2^\kappa/\varepsilon) \rceil$ and κ is the security parameter. Of these α top HIBE delegation levels, we only make use of a binary delegation (sub-)tree. Each path in

this binary tree part of the HIBE tree is the binary-encoding of an epoch number, where first epoch 0 is encoded as the left-most path and last epoch $2^{\kappa}/\varepsilon - 1$ is encoded as the right-most path. The lowest nodes in this top binary tree part (i.e., nodes in level α) represent *epoch starting nodes*. The first epoch starts at the left-most node which corresponds to the identity string that binary-encodes 0 (i.e., $0^{\alpha \cdot \lambda'}$, where λ' is the bit-length of HIBE identity sub-strings per level/delegation). We defer the explanation of how UPIBE delegations are realized *within* epochs to the next paragraph. As soon as an epoch is completed, the next epoch starts at the adjacent binary-tree node to the right in level α . (That is, starting nodes of epochs 2 and 3 correspond to identity strings $0^{\alpha \cdot \lambda' - 1} \| 1$ and $0^{(\alpha-1) \cdot \lambda' - 1} \| 1 \| 0^{\lambda'}$, respectively, where each level's identity sub-string contains a $(\lambda' - 1)$ -long 0-bit padding prefix.)

Progression from one epoch starting node to the next one follows the well known idea of Forward-Secure PKE from Binary Tree Encryption [CHK03].¹² Roughly, the epoch-progression mechanism iteratively delegates HIBE decapsulation keys along the α -long path from the root to the current epoch starting node. During this path delegation, also decapsulation keys of (binary-tree) siblings along this path are delegated. After each delegation on this path, the respective parent node's key from which the two sibling keys were delegated is deleted. Only the first epoch progression starts at the root of the HIBE tree. All following epoch progressions start from the lowest level for which a delegated sibling key exists. This mechanism ensures that only starting nodes of future epochs but not of previous epochs are accessible.

Multi-Level Epochs. Our UPIBE construction splits identity strings of length $l \cdot \lambda$ into $\varepsilon \cdot \lambda$ -long *epoch sub-strings*. Each individual epoch sub-string is delegated in ε steps vertically in the HIBE tree under its epoch starting node (i.e., each epoch contains ε delegation levels). Hence, every epoch sub-string in the HIBE tree looks exactly the same as its UPIBE identity sub-string counterpart (see Figure 1). However, instead of being concatenated vertically in the HIBE tree, one can think of the vertical epoch sub-strings hanging next to one another from left to right under their epoch starting nodes in level α .

Structure of Keys and Ciphertexts. Despite these two crucial extensions, the overall idea of our unbounded-depth UPIBE construction is very close to its bounded-depth counterpart from Section 3. This becomes evident when looking at the structure of UPIBE decapsulation keys and ciphertexts.

A UPIBE decapsulation key at delegation level l contains three types of delegated HIBE decapsulation keys: (1) up to α *epoch-progression* decapsulation keys, (2) one *ordinary* decapsulation key for each of the previous $\lceil l/\varepsilon \rceil - 1$ epochs and, potentially, one *ordinary* decapsulation key for the current epoch, and (3) a *special* decapsulation key for the current epoch. The epoch-progression decapsulation keys replace the single symmetric forwarding key from our bounded-depth construction. This allows for efficient delegation of *future* epochs' initial decapsulation keys, yet preventing access to *previous* epochs' initial decapsulation keys. Ordinary and special decapsulation keys are used for the actual decapsulation of UPIBE ciphertexts (almost) as in our bounded-depth construction.

The concrete components of a UPIBE decapsulation key are as follows. One *ordinary* HIBE decapsulation key, delegated to the lowest HIBE tree level $\alpha + \varepsilon$, is stored for each finished epoch. All remaining HIBE decapsulation keys, ever delegated in these prior epochs, are removed from the (delegated) UPIBE decapsulation key. For the current epoch, a *special* decapsulation key delegated to HIBE level $\alpha + (l \bmod \varepsilon)$ in that epoch is stored in the UPIBE decapsulation key, where l is the overall number of UPIBE delegations so far. When delegating the UPIBE decapsulation key, this *special* HIBE decapsulation key is replaced by a new one for the next level. Only in the last level $\alpha + \varepsilon$ of the current epoch where $(l = -1 \bmod \varepsilon)$, the UPIBE decapsulation key contains two HIBE keys: a *special* and an *ordinary* HIBE decapsulation key.

A UPIBE ciphertext for level l consists of one HIBE ciphertext per existing epoch, where $\lceil l/\varepsilon \rceil$ is the number of existing epochs. Each of the first $\lceil l/\varepsilon \rceil - 1$ ciphertexts is directed to its epoch's ordinary decapsulation key, and the last ciphertext is directed to the current epoch's special decapsulation key.

All UPIBE delegations within an epoch delegate a new special HIBE decapsulation key from the previous level's special HIBE decapsulation key. After each delegation, this previous special HIBE decapsulation key is removed. In the lowest level of an epoch—in HIBE tree level $\alpha + \varepsilon$ —an additional

¹² For clarity in our explanation, we slightly deviate from the original BTE-to-FS-PKE idea by Canetti et al. [CHK03]: We do not use all nodes in the BTE tree as epoch starting points but only nodes in the lowest level of this BTE component.

ordinary HIBE decapsulation key is delegated the from previous level's special HIBE decapsulation key. This ordinary HIBE decapsulation key is never removed from the UPIBE decapsulation key.

Intuition for Security. The intuitive security argument for this construction resembles the one from Section 3. Recall that, on exposure of a UPIBE decapsulation key, only those UPIBE encapsulations must remain secure whose targeted identity string either is a *true prefix* of the exposed key's identity string or *branches off* the exposed key's identity string.⁹ Encapsulations to true prefix identity strings have their last HIBE encapsulation directed to an *earlier* special HIBE decapsulation key. This special key is not stored in the exposed UPIBE decapsulation key anymore, since the latter only contains the *current* level's special HIBE decapsulation key. Encapsulations to branched off identity strings have the HIBE encapsulation of the branching epoch directed to an ordinary HIBE decapsulation key that was never stored in the exposed UPIBE decapsulation key. Finally, all exposed decapsulation keys of the epoch-progression mechanism only reveal parts of the HIBE tree from which future epochs can be delegated. Thus, UPIBE encapsulations of our unbounded-depth construction remain secure under non-trivial exposures of UPIBE decapsulation keys.

Construction. We specify our unbounded-depth UPIBE construction formally in Figure 4. This construction uses a bounded-depth HIBE with maximal level depth $L = \alpha + \varepsilon = \lceil \log(2^\kappa/\varepsilon) \rceil + \varepsilon$, where κ is the security parameter.

<pre> Proc IE.gen 00 $E[\cdot] \leftarrow \perp; D_{ep}[\cdot] \leftarrow \perp; D_{fs}[\cdot] \leftarrow \perp$ 01 $(ek', dk'_0) \leftarrow_{\S} \text{IE.gen}'$ 02 For $j = 0$ to $\alpha - 1$: 03 $dk''_0 \leftarrow_{\S} \text{IE.del}'(dk'_0, 0^{\lambda+1})$ 04 $dk''_1 \leftarrow_{\S} \text{IE.del}'(dk'_0, 0^\lambda \ 1)$ 05 $dk'_0 \leftarrow dk''_0; D_{fs}[j] \leftarrow dk''_1$ 06 $D_{ep}[0] \leftarrow dk'_0$ 07 $ek \leftarrow ek'; dk \leftarrow (0, D_{fs}, D_{ep})$ 08 Return (ek, dk) Proc IE.enc(ek, id) 09 Require $id \in \{0, 1\}^{l-\lambda}, l \in \mathbb{N}^+$ 10 $id_0 \ \dots \ id_{l-1} \leftarrow id$ with $id_j \in \{0, 1\}^\lambda$ 11 $d \leftarrow l \bmod \varepsilon; e \leftarrow \lceil l/\varepsilon \rceil$ 12 For $e' = 0$ to $e - 2$: 13 $id' \leftarrow e$ 14 $(e'_0, \dots, e'_{\alpha-1}) \leftarrow e'$ with $e'_j \in \{0, 1\}$ 15 For $j = 0$ to $\alpha - 1$: 16 $id' \leftarrow_{\S} 0^\lambda \ e'_j$ 17 For $d' = 0$ to $\varepsilon - 2$: 18 $id' \leftarrow_{\S} id_{e'-\varepsilon+d'} \ 1$ 19 $id' \leftarrow_{\S} id_{e'-\varepsilon+\varepsilon-1} \ 0$ 20 $(c'_{e'}, k'_{e'}) \leftarrow \text{IE.enc}'(ek, id')$ 21 $id' \leftarrow e$ 22 $(e'_0, \dots, e'_{\alpha-1}) \leftarrow e - 1$ with $e'_j \in \{0, 1\}$ 23 For $j = 0$ to $\alpha - 1$: 24 $id' \leftarrow_{\S} 0^\lambda \ e'_j$ 25 For $d' = 0$ to $d - 1$: 26 $id' \leftarrow_{\S} id_{(e-1)-\varepsilon+d'} \ 1$ 27 $(c'_{e-1}, k'_{e-1}) \leftarrow \text{IE.enc}'(ek, id')$ 28 $C \leftarrow c'_0 \ \dots \ c'_{e-1}$ 29 $K \leftarrow W(k'_0, \dots, k'_{e-1}, C)$ 30 Return (C, K) </pre>	<pre> Proc IE.dec(dk, C) 31 $(l, D_{fs}, D_{ep}) \leftarrow dk$ 32 $d \leftarrow l \bmod \varepsilon; e \leftarrow \lceil l/\varepsilon \rceil$ 33 $c_0 \ \dots \ c_{e-1} \leftarrow C$ with $c_j \in \mathcal{C}$ 34 Require $e = e'$ 35 For $j = 0$ to $e - 2$ 36 $k'_j \leftarrow_{\S} \text{IE.dec}'(D_{ep}[j], c_j)$ 37 If $d \neq \varepsilon - 1$: $dk'_1 \leftarrow D_{ep}[e - 1]$ 38 Else: $(dk'_0, dk'_1) \leftarrow D_{ep}[e - 1]$ 39 $k'_{e-1} \leftarrow_{\S} \text{IE.dec}'(dk'_1, c_{e-1})$ 40 $K \leftarrow W(k'_0, \dots, k'_{e-1}, C)$ 41 Return K Proc IE.del(dk, id) 42 Require $id \in \{0, 1\}^\lambda$ 43 $(l, D_{fs}, D_{ep}) \leftarrow dk$ 44 $d \leftarrow l \bmod \varepsilon; e \leftarrow \lceil l/\varepsilon \rceil$ 45 If $d = 0 \wedge e > 0$: 46 $(dk'_0, dk'_1) \leftarrow D_{ep}[e - 1]$ 47 $D_{ep}[e - 1] \leftarrow dk'_0$ 48 $j \leftarrow \text{msdb}(e - 1, e)$ 49 $dk'_0 \leftarrow D_{fs}[j]; D_{fs}[j] \leftarrow \perp$ 50 For j to $\alpha - 1$: 51 $dk''_0 \leftarrow_{\S} \text{IE.del}'(dk'_0, 0^{\lambda+1})$ 52 $dk''_1 \leftarrow_{\S} \text{IE.del}'(dk'_0, 0^\lambda \ 1)$ 53 $dk'_0 \leftarrow dk''_0; D_{fs}[j] \leftarrow dk''_1$ 54 $D_{ep}[e] \leftarrow dk'_0$ 55 If $d \neq \varepsilon - 1$: 56 $D_{ep}[e] \leftarrow_{\S} \text{IE.del}'(D_{ep}[e], id \ 1)$ 57 Else: 58 $dk'_0 \leftarrow_{\S} \text{IE.del}'(D_{ep}[e], id \ 0)$ 59 $dk'_1 \leftarrow_{\S} \text{IE.del}'(D_{ep}[e], id \ 1)$ 60 $D_{ep}[e] \leftarrow (dk'_0, dk'_1)$ 61 $dk \leftarrow (l + 1, D_{fs}, D_{ep})$ 62 Return dk </pre>
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Fig. 4: Generic construction of unbounded-depth UPIBE IE with parameters $(L = \infty, \lambda, D = 1)$ from bounded-depth HIBE scheme IE' with ciphertext space \mathcal{C} with parameters $(L', \lambda', D') = (\alpha + \varepsilon, \lambda + 1, 2)$, where $\alpha = \lceil \log(2^\kappa/\varepsilon) \rceil$. Additional parameter ε fixes the epoch depth in the lower path component and κ is the security parameter. Function $\text{msdb}(x, y)$ computes the most significant bit in which the bit-representations of x and y differ and core function W is realized as XOR-sum $\bigoplus_{j=0}^{e-1} k'_j$ and ignores input C . In our chosen-ciphertext secure instantiation we instantiate W with random oracle H^* .

The UPIBE encapsulation key consists solely of the main HIBE encapsulation key. The initial UPIBE decapsulation key is generated by executing the epoch-progression mechanism with the main HIBE

decapsulation key to derive the first epoch’s starting decapsulation key (Figure 4, lines 02-06). More concretely, this mechanism delegates one ephemeral and one stored decapsulation key in each of the first α HIBE levels (lines 03-04). Ephemeral key dk'_0 is replaced after delegating the two decapsulation keys of the next level. Stored key dk'_1 will be used for future epoch progressions. In level α , ephemeral key dk'_0 is set as the first epoch’s starting decapsulation key. We explain the specific encoding- and padding-scheme for identity strings at the end of this paragraph.

UPIBE encapsulation splits the targeted identity string id into $\varepsilon \cdot \lambda$ -long *epoch sub-strings*. Our pseudo-code separates the processing of the first $e - 1$ epoch sub-strings (lines 12-20) from the last epoch’s sub-string (lines 21-27). Roughly, each epoch sub-string (composed in lines 17-19 resp. 25-26) is prepended with a binary encoding of the corresponding epoch number (lines 14-16 resp. 22-24). The binary encoding prefix represents the epoch-progression path to the epoch’s starting node. For every epoch, an HIBE encapsulation directed to the concatenated string of binary-encoded *epoch number* and *epoch identity sub-string* is executed (line 20 resp. 27). The final UPIBE ciphertext is a simple concatenation of all epoch HIBE ciphertexts; the output UPIBE key is an XOR-sum of all encapsulated epoch HIBE keys.

On UPIBE decapsulation, the input ciphertext is decomposed, and each of the resulting HIBE ciphertexts is decapsulated. For all previous epochs, the stored lowest level ordinary decapsulation key is used for decapsulation (lines 35-36). In the current epoch, the special decapsulation key is used for this (line 39). Depending on whether the current epoch reached its lowest level or not, the special decapsulation key is stored solitarily (line 37) or together with the ordinary decapsulation key (line 38).

In most cases, UPIBE delegation simply uses the current epoch’s special HIBE decapsulation key together with input identity string id to delegate a new special HIBE decapsulation key that replaces the prior one (lines 56). Only if the lowest level of the current epoch is reached, an additional ordinary HIBE decapsulation key is delegated and stored (line 58-60). A subsequent delegation starts a new epoch and, therefore, uses the epoch-progression mechanism (lines 45-54). This mechanism starts by deleting the previous epoch’s special decapsulation key (lines 46-47). Then, it identifies the lowest existing decapsulation key in the underlying binary-tree structure (line 48) with which the next epoch starting node is delegated (lines 50-54). This subsequent starting node—basically the immediate neighbor node in the binary tree—is used as the new epoch’s initial decapsulation key.

We elaborate on some implementation details. To realize a binary tree in the epoch-progression mechanism, the binary encoding of epoch numbers is padded with $(\lambda' - 1 = \lambda)$ leading 0-bits in every level (lines 03-04, 16, 24, 51-52). For the composition of epoch sub-strings, each *level’s* identity *sub-string* is appended with a 1-bit (lines 18, 26) except for the last level in any previous epoch; previous epochs’ last level sub-strings have an appended 0-bit (lines 19). This corresponds to the use and delegation of special and ordinary decapsulation keys (lines 56, 58-59).

Depth of Epoch-Progression Mechanism. To allow for sufficiently many epoch progressions, the depth of the upper binary tree within the HIBE tree is $\alpha = \lceil \log_2(2^\lambda/\varepsilon) \rceil$. One can reduce α by 1 if not only leafs in this binary tree represent epoch starting decapsulation keys, but also all inner nodes in this tree. This would resemble the original Forward-Secure PKE idea by Canetti et al. [CHK03]. Furthermore, one can reduce α by increasing the number of delegations per decapsulation key in the tree (e.g., from binary to ternary tree). Although this decreases the HIBE depth, it would increase the size of UPIBE decapsulation keys.

Depth of Multi-Level Epochs. Our unbounded-depth UPIBE construction is parameterized by variable ε that determines the number of delegations per epoch. We note that for $\varepsilon = \infty$, our UPIBE construction reduces to the known trivial delegation design via *unbounded*-depth HIBE [PR18b, JS18, BRV20]. Thus, there is *always* an ε for which our construction is *at least as efficient* as the previous approach. Beyond that, using the flexibility of parameter ε , our construction’s performance can be adapted to different use cases. For example, depending on whether ciphertexts or decapsulation keys should be small, and depending on the expected number of delegations in a setting, an optimal value ε can be configured. Our full evaluation is in Section 7.

2-Delegation HIBE. We want to note that each HIBE decapsulation key in our construction from Figure 4 delegates at most two child decapsulation keys. Thus, while reducing the level depth parameter L substantially from infinity in UPIBE to a bounded value in the underlying HIBE, parameter D only grows

from 1 delegation per secret key in UPIBE to 2 in the underlying HIBE. With our definition framework from Section 2 and our new perspective on delegation-restricted HIBE, we lay the foundation for future work that may investigate whether bounded-depth HIBE with limited delegation of $D = 2$ can be built more efficiently than general bounded-depth HIBE.

Security. To support comprehensibility and avoid idealized assumptions, we first reduce adaptive chosen-plaintext security IND_{IE}^b of our UPIBE construction to adaptive security of the underlying HIBE in the standard model. In Section 4.1, we augment our reduction with a new guessing technique that allows us to trade the strength of the underlying HIBE (only selective security instead of adaptive security) against idealized assumptions (random oracle model instead of standard model). Relying only on selective secure HIBEs for adaptive security of our UPIBE significantly extends the class of available HIBE constructions from the literature. For full security against chosen-ciphertext attacks, we consider different generic and direct techniques in Section 5.

Theorem 2. *Unbounded-depth UPIBE protocol IE from Figure 4 offers adaptive key indistinguishability in the standard model. More precisely, for every adaptive chosen-plaintext adversary \mathcal{A} attacking protocol IE in games IND_{IE}^b according to Definition 1 with parameters $(L = \infty, \lambda, D = 1)$, there exists an adaptive chosen-plaintext adversary \mathcal{B} attacking bounded-depth HIBE IE' in games $\text{IND}_{\text{IE}'}^b$ according to Definition 1 with parameters $(L', \lambda', D') = (\lceil \log(2^\kappa/\varepsilon) \rceil + \varepsilon, \lambda + 1, 2)$ such that $\text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{A}) \leq q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot \lceil l_{\text{long}}/\varepsilon \rceil \cdot \text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B})$, where κ is the security parameter, ε is the construction's epoch-depth parameter, q_{Gen} and q_{Chall} are the numbers of queries to oracles Gen and Chall by adversary \mathcal{A} , respectively, l_{long} is the level-depth of the longest identity string queried to oracle Chall by adversary \mathcal{A} , and the running time of \mathcal{B} is about that of \mathcal{A} .*

Security Proof Overview. Our security proof for Theorem 2 is very similar to the one for Theorem 1. The major technical difference is that here the security of each UPIBE instance is reduced to only one bounded-depth HIBE instance's security. Reduction \mathcal{B} , again, simulates all oracle queries of adversary \mathcal{A} in game IND_{IE} via queries to oracles in game $\text{IND}_{\text{IE}'}$. As in our proof from Section 3, for certain UPIBE challenge queries to oracle Chall_{IE} , the reduction has to guess where to embed underlying HIBE challenges of game $\text{IND}_{\text{IE}'}$. A hybrid argument that implements these guesses cause the loss factor in our advantage bound. The general strategy for embedding challenges is to determine where the identity string input of oracle Chall_{IE} branches off the delegation path of (potentially) exposed UPIBE decapsulation keys. In contrast to our proof of Theorem 1, reduction \mathcal{B} here only needs to guess the *epoch of the sub-string* in which the identity strings of challenge and exposure branch lie. We provide our formal proof in Appendix B.2.

4.1 Relaxing Assumptions: Adaptive UPIBE from Selective HIBE

The above outlined standard model proof for our unbounded-depth UPIBE construction from Figure 4 relies on *adaptive* secure bounded-depth HIBE. Yet, the most suitable bounded-depth HIBEs (e.g., [BBG05]) are only *selective* secure. Generic techniques for turning selective secure schemes adaptive secure, as done in [BB04, BBG05, ABB10, CHKP10], rely on the random oracle model and induce an exponential loss factor in the HIBE's maximal level depth L . The simple idea of these techniques is to replace each identity sub-string in the construction by the output of a random oracle evaluated on this identity sub-string (i.e., $id_0 \parallel \dots \parallel id_l$ is replaced by $H(id_0) \parallel \dots \parallel H(id_l)$). The reduction then embeds sub-strings of the selective challenge identity in randomly chosen random-oracle-outputs. A reduction succeeds if it embeds the *selective challenge* sub-strings in those random-oracle-outputs whose input identity sub-strings form the *adaptive challenge*. This induces an exponential loss in the maximal number of identity sub-strings per adaptive challenge. This is problematic because our UPIBE construction relies on an adaptive secure bounded-depth HIBE with parameter $L = \alpha + \varepsilon = \lceil \log(2^\kappa/\varepsilon) \rceil + \varepsilon$, which is linear in the security parameter κ . Thus, the loss factor would be exponential in κ when following the generic approach of turning the underlying HIBE adaptive secure [BB04, BBG05, ABB10, CHKP10] before using this HIBE to instantiate our unbounded-depth UPIBE construction.

Solution: Guessing Essentials Only. Due to the way our construction makes use of the underlying bounded-depth HIBE, we can carefully change the generic approach from [BB04, BBG05] in order to

relax the assumption on the HIBE from adaptive to selective security. Thus, we significantly extend the set of bounded-depth HIBE schemes from the literature with which we can instantiate our unbounded-depth UPIBE construction. Our main observation is that the two (virtual) components in our UPIBE construction—*epoch-progression mechanism* and *multi-level epochs*—encode information of different density. For this, consider an HIBE identity string to which our UPIBE encapsulation internally issues an HIBE encapsulation. The first part of such an HIBE identity string encodes an integer that represents the epoch number in the upper epoch-progression mechanism. The second part encodes a sub-string of the actual UPIBE identity string (i.e., the identity sub-string for one epoch).

In order to embed a selective HIBE challenge in the adaptive UPIBE challenge, our reduction has to predict the branching epoch’s full HIBE identity string in advance. In this epoch, the UPIBE challenge identity branches off the delegated identity of the corresponding (exposed) UPIBE decapsulation key. To predict this epoch’s full HIBE identity string, we treat the two parts—epoch number and sub-string of UPIBE identity—differently. The branching *epoch number* can simply be guessed with high probability. The reason is that polynomially bounded users (and adversaries) only issue UPIBE identity strings of polynomial length. Thus, also the number of epochs used to represent a UPIBE identity string is polynomially bounded. To predict the second part of the HIBE identity string—the branching epoch’s sub-string of the actual UPIBE identity string—we employ the generic technique [BB04, BGG05] based on the random oracle model. Since the depth of each multi-level epoch is bounded by constant parameter ε , the loss induced by this technique is only polynomial (not exponential) in κ .

Concrete Adjustments. We interpose a random oracle H in the following lines of our construction in Figure 4: 18: $id' \stackrel{u}{\leftarrow} H(id_{e' \cdot \varepsilon + d'} \| 1)$; 19: $id' \stackrel{u}{\leftarrow} H(id_{e' \cdot \varepsilon + \varepsilon - 1} \| 0)$; 26: $id' \stackrel{u}{\leftarrow} H(id_{(e-1) \cdot \varepsilon + d'} \| 1)$; 56: $D_{ep}[e] \leftarrow_{\$} \text{IE.del}'(D_{ep}[e], H(id \| 1))$; 58: $dk'_0 \leftarrow_{\$} \text{IE.del}'(D_{ep}[e], H(id \| 0))$; 59: $dk'_1 \leftarrow_{\$} \text{IE.del}'(D_{ep}[e], H(id \| 1))$. However, we leave the identity sub-strings of the upper epoch-progression mechanism untouched. Thus, lines 03-04, 16, 24, and 51-52 remain the same. A full proof of following Theorem 3 is given in Appendix B.3.

Theorem 3. *Adjusting unbounded-depth UPIBE protocol IE from Figure 4 offers adaptive key indistinguishability in the random oracle model. More precisely, let H be a random oracle, then for every adaptive chosen-plaintext adversary \mathcal{A} attacking protocol IE in games IND_{IE}^b according to Definition 1 with parameters $(L = \infty, \lambda, D = 1)$, there exists a selective chosen-plaintext adversary \mathcal{B} attacking bounded-depth HIBE IE' in games $\text{IND}_{\text{IE}'}^b$ according to Definition 1 with parameters $(L', \lambda', D') = (\lceil \log(2^\kappa / \varepsilon) \rceil + \varepsilon, \lambda + 1, 2)$ such that $\text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{A}) \leq q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot ((l_{\text{long}})^2 \cdot (q_H)^\varepsilon) \cdot \text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B})$, where κ is the security parameter, ε is the construction’s epoch-depth parameter, q_{Gen} , q_{Chall} , and q_H are the number of queries to oracles Gen, Chall and the random oracle by adversary \mathcal{A} , respectively, l_{long} is the level-depth of the longest identity string queried to oracle Chall by adversary \mathcal{A} , and the running time of \mathcal{B} is about that of \mathcal{A} .*

5 CCA Secure UPIBE

Now we turn our focus on the task of achieving chosen-ciphertext security for bounded- and unbounded-depth UPIBE. While it might be tempting to think that similar to HIBEs one could generically convert CPA-secure UPIBE into CCA-secure ones using the BCHK (often also called CHK) compiler [CHK04, BCHK07], this unfortunately does not work: BCHK needs one delegation *per* decapsulation from the same decapsulation key, but UPIBE only offers one delegation for each decapsulation key *in total*. Thus, we need to adopt different strategies for constructing CCA-secure UPIBE.

5.1 Bounded-depth UPIBE

FO-Transform. Having in mind that we construct bounded-depth UPIBE from (bounded-collision) IBE, a natural choice is to apply the Fujisaki-Okamoto (FO) transform [FO99] and in particular one of its modular variants [HHK17]. FO typically considers single instances, but in our construction of UPIBE one has to deal with multiple parallel IBE ciphertexts and this requires some care. Recently, Cini et al. in [CRSS20] considered this issue of parallel ciphertexts in FO for reducing decryption errors as well as constructing Bloom-Filter KEMs (BFKEMs) from IBE. Though [CRSS20] relies on a single IBE instance,

it is quite straightforward to adapt their approach to UPIBE.¹³ Unfortunately, using FO in this way, besides being bound to the random oracle model (ROM), requires an overhead of l encryptions of the underlying IBE during decapsulation, which can be significant.

Split-Key PRF. An alternative, more efficient, and more flexible approach is made possible when we view our UPIBE construction in Section 3 as parallel bounded-collision IBE and take inspiration from Giacon et al. [GHP18]. In particular, recall that our overall ciphertext $C = c'_0 \parallel \dots \parallel c'_{l-1}$ is the concatenation of l ciphertexts of independent IBEs and the encapsulation key is computed as $K \leftarrow W(k'_0, \dots, k'_{l-1}, C)$, where W represents what is called a core function by Giacon et al. [GHP18]. We note that [GHP18] focuses on parallel KEM combiners, and show that if W is a split-key pseudorandom function (skPRF), it yields a CCA-secure KEM if at least one of the l KEMs is CCA secure. Various instantiations of skPRFs in the ROM and standard model with different types of trade-offs are discussed in [GHP18]. For instance the PRF-then-XOR composition $W(k'_0, \dots, k'_{l-1}, C) := \bigoplus_{i=0}^{l-1} F_i(k'_i, C)$, where F_i 's are PRFs, is a skPRF in the standard model. Our focus now is not on combiners and as the use of our instances is dynamic (i.e., the depth can vary), this does not work for UPIBE. Here we need to require that *all* instances are CCA secure. Nevertheless, as we discuss below, the use of an skPRF still gives advantages when it comes to standard model constructions.

Achieving CCA-secure IBE. While CCA security can be easily achieved in the ROM by starting from a CPA-secure (bounded-collision) IBE and applying the FO transform, the overall overhead due to the FO is identical when directly applying FO (as discussed above). However, we can obtain CCA-secure bounded-depth UPIBE in the standard model when relying on an IBE scheme that directly provides CCA security in the standard model (e.g., [Gen06] or the CCA-secure version of the bounded-collision IBE in [DKXY02]). Alternatively, if one accepts that the IBEs are replaced by CPA-secure depth 2 HIBEs, one can simply use the BCHK compiler [CHK04, BCHK07].

Now, we will show that the bounded-depth UPIBE protocol from Figure 3 is CCA-secure when the underlying bounded-collision IBE IE' is CCA-secure (e.g., [DKXY02]) and the core function W is based on a split-key pseudorandom function F with $n = L$ (cf. Appendix A for the definition). For reasons that we will discuss below, we include a special KEM key \hat{k} and a special ciphertext \hat{c} into ek of the UPIBE protocol IE in order to “pad” calls to W to always take L inputs (for all cases where depth $l < L$).

Theorem 4. *Bounded-depth UPIBE protocol IE from Figure 3 offers adaptive key indistinguishability under chosen-ciphertext attacks in the standard model. More precisely, for every adaptive chosen-ciphertext adversary \mathcal{A} attacking protocol IE in games IND_{IE}^b according to Definition 1 with parameters $(L, \lambda, D = 1)$, there exists an adversary \mathcal{B}_G attacking PRG G according to Definition 3, an adversary \mathcal{B}_W against the split-key pseudorandomness of W according to Definition 2, and an adaptive chosen-ciphertext adversary $\mathcal{B}_{IE'}$ attacking bounded-collision IBE IE' in games $IND_{IE'}^b$ according to Definition 1 with parameters $(L', \lambda', D') = (1, \lambda + 1, 2)$ such that $\text{Adv}_{IE}^{\text{ind}}(\mathcal{A}) \leq q_{\text{Gen}} L^2 \cdot \left(q_{\text{Gen}} q_{\text{Chall}} L \cdot \text{Adv}_G^{\text{ind}}(\mathcal{B}_G) + 1 \right) + 2q_{\text{Gen}} q_{\text{Chall}} L \cdot \left(q_{\text{Chall}} \cdot \text{Adv}_{F_i}^{\text{pr}}(\mathcal{B}_W) + \text{Adv}_{IE'}^{\text{ind}}(\mathcal{B}_{IE'}) \right)$, where q_{Chall} and q_{Gen} are the number of queries to oracle Chall and Gen by adversary \mathcal{A} , and the running times of \mathcal{B}_G , \mathcal{B}_W , and $\mathcal{B}_{IE'}$ is about that of \mathcal{A} .*

Security Proof Overview. The strategy for the proof is analogous to that of Theorem 1, but we will proceed in a sequence of Games moving from the game IND_{IE}^0 to IND_{IE}^1 , which allows us to follow the strategy by Giacon et al. [GHP18]. In contrast to their proof, in our case all instances are required to be CCA secure. This is since we require CCA security of the underlying IBE IE' at the branching positions of identities that are asked to the challenge oracle, which can be placed at any of the L positions adaptively. We need to take some care when using the pseudorandomness of the split-key pseudorandom function for W , as we use $n = L$ but the number of required inputs vary with the actual depth of the identities l . Therefore, we always use L inputs for calls to W where for the $L - l$ rightmost inputs we simply use a fixed key \hat{k} and ciphertext \hat{c} (we will not make this fact explicit in the proof). We provide a formal proof in Appendix C.1.

¹³ We would sample a random key k and derive $(r_0, \dots, r_{l-1}, k') = G(k)$ from a random oracle G and encapsulate k_i with randomness r_i for the i 'th instance such that $K = k_0 \oplus \dots \oplus k_{l-1}$ and then use k' as the overall encapsulation key.

5.2 Unbounded-depth UPIBE

For the same reasons as discussed in Section 5.1 we prefer to avoid a generic use of the FO transform for proving CCA security of our unbounded-depth UPIBE. Unfortunately, the generic skPRF approach pursued in Section 5.1 requires an a priori bound on the depth, which is not the case for unbounded-depth UPIBE.

Consequently, although we follow the same overall idea, as already mentioned in Figure 4, we instantiate the core function W directly by a random oracle H^* , i.e., derive the overall key as $K \leftarrow H^*(k_1, \dots, k_l, c_1, \dots, c_l)$ where (k_i, c_i) are the encapsulation outputs of the chosen-ciphertext secure bounded HIBE. Since our focus is on efficiency, and the strategy to prove Theorem 3 already requires the ROM, this seems to be a meaningful choice. For CCA security of the single ciphertexts of the underlying bounded-depth HIBE, the most efficient approach is a use of the BCHK compiler [CHK04, BCHK07]. This yields a very flexible approach as due to the choice of the required strongly secure signature scheme there are many performance and bandwidth trade-offs available (see also Section 7). Using this strategy we can show the following for our unbounded-depth UPIBE.

Theorem 5. *Adjusting unbounded-depth UPIBE protocol IE from Figure 4 as described in Section 4.1 offers adaptive key indistinguishability under chosen-ciphertext attacks in the random oracle model. More precisely, let H and H^* be random oracles, then for every adaptive chosen-ciphertext adversary \mathcal{A} attacking protocol IE in games IND_{IE}^b according to Definition 1 with parameters $(L = \infty, \lambda, D = 1)$, there exists a selective chosen-ciphertext adversary \mathcal{B} attacking bounded-depth HIBE IE' in games $\text{IND}_{\text{IE}'}^b$ according to Definition 1 with parameters $(L', \lambda', D') = (\lceil \log(2^\kappa/\varepsilon) \rceil + \varepsilon, \lambda + 1, 2)$ such that $\text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{A}) \leq q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot ((l_{\text{long}})^2 \cdot (q_{\text{H}})^\varepsilon) \cdot \left(\text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B}) + \frac{q_{\text{Chall}} \cdot q_{\text{H}^*}}{|\mathcal{K}|} \right)$ where κ is the security parameter, ε is the construction's epoch-depth parameter, q_{Chall} , q_{Gen} , q_{H} and q_{H^*} are queries to oracles Chall, Gen and random oracles H and H^* by adversary \mathcal{A} , respectively, l_{long} is the level-depth of the longest identity string queried to oracle Chall by adversary \mathcal{A} , and the running time of \mathcal{B} is about that of \mathcal{A} .*

Security Proof Overview. The proof of Theorem 5 is very similar to that of Theorem 3 and the main difference to the CCA secure bounded-depth UPIBE, we do not use the split-key pseudorandomness of W , but directly model it as a random oracle. This simplifies the proof significantly, as we can replace the keys obtained from calls to the challenge oracle simply via a modification of the random oracle simulation and then bounding the probability of detecting this departure. We provide a formal proof in Appendix C.2.

6 Key-Updatable KEM from UPIBE

A Key-Updatable Key Encapsulation Mechanism (KU-KEM) [JS18, PR18b] is a KEM $K = (K.\text{gen}, K.\text{enc}, K.\text{dec}, K.\text{up})$ with additional update algorithms $K.\text{up}$ for encapsulation keys and decapsulation keys. The computation of each update $ek' \leftarrow_{\S} K.\text{up}(ek, ad)$ resp. $dk' \leftarrow_{\S} K.\text{up}(dk, ad)$ is determined by a bit string ad that is arbitrarily chosen by the user. One can think of these update bit strings as new information (aka. *associated data*) that is added to the context of the ongoing session. Updates of encapsulation keys and decapsulation keys can be conducted independently without information being transmitted between holders of encapsulation and decapsulation key. The feature of *independent* updates *with respect to bit strings* constitutes the crucial difference to significantly weaker notions like Updatable PKE [JMM19a, DKW21] that offer more efficient instantiations. We refer the interested reader to a discussion by Balli et al. [BRV20] who elaborate on the shortcomings of Updatable PKE in the context of *strongly* secure messaging.

As long as both components of a KU-KEM key pair are updated with respect to the same bit strings—meaning, their context is updated compatibly—the key pair remains compatible. More precisely, a generated pair consisting of encapsulation key and decapsulation key remains compatible if the list of bit strings for updates applied on the encapsulation key equals the list of bit strings for updates applied on the decapsulation key. We follow the slightly stronger variant of KU-KEM by Balli et al. [BRV20] that furthermore requires for compatibility of a key pair that the list of bit strings for updates together with the list of sent and received encapsulation ciphertexts equals on both sides.

For security of KU-KEM, two goals beyond pure key-indistinguishability are required: (1) *Forward-secrecy*, meaning that an updated future version of the current decapsulation key can be exposed to an

adversary without harming confidentiality of ciphertexts produced with a current or previous (compatible) version of the corresponding encapsulation key—in short, old ciphertexts remain secure if future decapsulation keys are exposed; (2) *Effective divergence*, meaning that an incompatible decapsulation key can be exposed to an adversary without harming confidentiality of ciphertexts produced with the corresponding (incompatible) encapsulation key—in short, any difference in update bit strings makes encapsulation key and decapsulation key fully independent.

KU-KEM is a special form of UPIBE where KU-KEM update bit strings are implemented via UPIBE identity sub-strings, KU-KEM decapsulation key updates are realized via UPIBE delegations, and KU-KEM encapsulation key updates are realized via UPIBE derivations. The construction of KU-KEM from UPIBE is, therefore, straight forward: $K.\text{gen} := \text{IE.gen}$; $K.\text{up}(ek, ad) := \text{IE.der}(ek, ad = id)$ resp. $K.\text{up}(dk, ad) := \text{IE.del}(dk, ad = id)$; $K.\text{enc}(ek)$ executes $\text{IE.enc}(ek, \epsilon)$ and updates ek via $\text{IE.der}(ek, ad = c)$; $K.\text{dec}(dk, c)$ executes $\text{IE.dec}(dk, c)$ and updates dk via $\text{IE.del}(dk, ad = c)$. (Pseudo-code is given in Figure 5.) This construction was first proposed by Poettering and Rösler [PR18b] and slightly adapted in other works [JS18, BRV20]. Yet, we are the first to reduce the underlying assumption from general unbounded-depth HIBE to unbounded-depth UPIBE.

Proc K.gen	Proc K.up(ek, ad)	Proc K.up(dk, ad)
00 ($ek_{\text{IE}}, dk_{\text{IE}}$) $\leftarrow_{\$}$ IE.gen	02 $ek \leftarrow \text{IE.der}(ek_{\text{IE}}, ad)$	07 $dk \leftarrow_{\$}$ IE.del(dk, ad)
01 Return ($ek_{\text{IE}}, dk_{\text{IE}}$)	03 Return ek	08 Return dk
	Proc K.enc(ek)	Proc K.dec(dk, c)
	04 (c, k) $\leftarrow_{\$}$ IE.enc(ek_{IE}, ϵ)	09 $k \leftarrow \text{IE.dec}(dk, c)$
	05 $ek \leftarrow \text{IE.der}(ek_{\text{IE}}, c)$	10 $dk \leftarrow_{\$}$ IE.del(dk, c)
	06 Return (ek, c, k)	11 Return (dk, k)

Fig. 5: Generic construction of KU-KEM K from a UPIBE scheme IE. This construction (almost) equally appears in [PR18b, JS18, BRV20] but these variants assume full HIBE functionality and security for IE instead.

We defer the formal definition of KU-KEM by Balli et al. [BRV20] as well as our proof of Theorem 6 to Appendix D. This proof tightly reduces the security of the KU-KEM construction to adaptive chosen-ciphertext security of the underlying unbounded-depth UPIBE scheme.

Theorem 6. *KU-KEM protocol K from Figure 5 offers one-wayness of encapsulated keys. More precisely, for every adaptive chosen-ciphertext adversary \mathcal{A} attacking protocol K in game KUOW_K from Figure 9, there exists an adaptive chosen-ciphertext adversary \mathcal{B} attacking unbounded-depth HIBE IE in games IND_{IE}^b according to Definition 1 with parameters $(L', \lambda', D') = (\infty, \lambda, 1)$ such that $\text{Adv}_K^{\text{kuow}}(\mathcal{A}) \leq \text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{B})$, where the running time of \mathcal{B} is about that of \mathcal{A} .*

Necessity of UPIBE for KU-KEM. Despite a subtle syntactical difference between these two primitives, the chosen KU-KEM notion is (likely) equivalent to UPIBE. The difference is that UPIBE encapsulation and decapsulation are stateless but their KU-KEM counterparts are stateful. More concretely, KU-KEM en- and decapsulation output and, thereby, may change the input en- resp. decapsulation key. In contrast, UPIBE encapsulation and decapsulation only take these keys as inputs (i.e., without changing them).

To prove that KU-KEM implies UPIBE, a direct simulation of UPIBE en- resp. decapsulation based on KU-KEM en- resp. decapsulation would, therefore, need to ignore changes applied to the KU-KEM keys. This works well for a simulating a single encapsulation (aka. challenge) query and a single decapsulation query, resp. Moreover, using a standard hybrid argument, one can amplify the single permitted challenge query for the multi-challenge setting. However, lifting the limitation of only a single decapsulation query seems more complicated. Thus, the described approach would (only) prove that KU-KEM implies 1-CCA secure UPIBE. We leave a full proof as an open problem for future work.

7 Evaluation

Our evaluation considers (asymptotic and concrete) parameter sizes of one-way CCA (formally, KUOW) secure KU-KEMs built *trivially* from unbounded-depth HIBEs on the one side and KU-KEMs based on our UPIBE construction that relies on bounded-depth HIBEs from Section 5.2 on the other side. A compact summary of how UPIBE variable sizes relate generically to the underlying HIBE variable sizes is

given in Appendix D.1. Before starting the concrete analysis, we note that CCA security of (un)bounded-depth HIBEs can be generically achieved efficiently via the BCHK transform [CHK04, BCHK07] using a strongly secure one-time-signature scheme.¹⁴

Since we have applicability and performance in mind for our application towards optimally secure messaging protocols, we include bounded-depth HIBE schemes that are secure in the random-oracle model (ROM). Moreover, we looked at all applicable unbounded-depth HIBEs and selected three constructions [Lew12, LP20, GCTC16] that suit the application we have in mind best. Depending on the concrete bounded-depth HIBE scheme, it is a common technique to reduce public parameter sizes in the ROM [BBG05]. This, however, does not work generically. Particularly, in the HIBE scheme by Gong et al. (GCTC) [GCTC16], the underlying encapsulation key structure seemingly prevents this form of parameter compression. The same seems to be the case for Langrehr-Pan (LP) [LP20], while Lewko (L) [Lew12] already has compact encapsulation keys (however, with a large constant).

For our KU-KEM construction via the UPIBE paradigm (where we only require a selectively secure HIBE with polynomially bounded depth), the strongest candidate is the Boneh-Boyen-Goh (BBG) HIBE [BBG05]. Here, encapsulation key size is only two group elements using the ROM. However, we cannot utilize the ROM to reduce the size of BBG decapsulation keys since these keys require a certain structure. Hence, the BBG HIBE has linear-size decapsulation keys, but enjoys constant-size encapsulation keys *and* ciphertexts (all in the maximal depth).

By considering the most efficient (un)bounded-depth HIBE schemes, we conduct a fair comparison between KU-KEMs from trivial UPIBE via unbounded-depth HIBE and KU-KEMs from our novel UPIBE construction. In Table 1, we list CCA secure KU-KEMs from CCA secure (un)bounded-depth HIBEs with relevant size and performance parameters.

UPIBE	Via HIBE	Encapsulation key size	Ciphertext size	Decapsulation key size	Model
Triv.	L [Lew12]	$60 G_1 + 2 G_T + l\lambda$	$(10l + 12) G_1 $	$(10l + 60) G_2 $	StM
Triv.	LP [LP20]	$(2\gamma + 4) G_1 + (2\gamma + 6) G_2 + l\lambda$	$(7l + 11) G_1 $	$(7l + 2) G_2 $	StM
Triv.	GCTC [GCTC16]	$(3n + 9 + \lceil l/n \rceil) G_1 + 3 G_T $	$(9\lceil (l+1)/n \rceil + 2) G_1 $	$((9 + 3n)\lceil l/n \rceil + 3n + 9 - 3l) G_2 $	StM
Ours	BBG [BBG05]	$(1 + \lceil l/\varepsilon \rceil) G_1 + 1 G_2 $	$(3\lceil l/\varepsilon \rceil) G_1 $	$(\mathcal{O}(\alpha \cdot (\alpha + \varepsilon)) + \lceil l/\varepsilon \rceil + \alpha) G_2 $	ROM

UPIBE	Via HIBE	Key generation (# exp.)	Encapsulation (# exp.)	Decapsulation (# exp., # pairings)	Ass.
Triv.	L [Lew12]	$60 (G_1), 80 (G_2), 2 (G_T)$	$60l + 62 (G_1), 2 (G_T)$	$(61l (G_2), 10l + 1)$	DLIN
Triv.	LP [LP20]	$(2\gamma + 4) (G_1), (2\gamma + 6) (G_2)$	$(7l + 11) (G_2), 2 (G_T)$	$((7(l+1) + 2) (G_2), (7l + 2) + 1)$	SXDH
Triv.	GCTC [GCTC16]	$6(n + 3) (G_2), 1 (G_T)$	$(15\lceil l/n \rceil + 3l) (G_1), 3 (G_T)$	$(15\lceil l/n \rceil + 3l (G_2), 9\lceil l/n \rceil + 1)$	SXDH
Ours	BBG [BBG05]	$1 (G_1), 1 (G_2)$	$((\lceil l/\varepsilon \rceil + 5) (G_1), 1 (G_T))$	$(\varepsilon + \alpha/\varepsilon + 2) (G_2), 2\lceil l/\varepsilon \rceil)$	BDHE

Table 1: Comparison of CCA secure KU-KEMs with parameter sizes and performance instantiated from the standard-model unbounded-depth HIBEs L [Lew12], LP [LP20], and GCTC [GCTC16] (trivially) and the bounded-depth HIBE BBG [BBG05] (via our KU-KEM-from-UIBE approach from Section 6). Here, $\alpha + \varepsilon$ is the maximum level (and α can be considered linear in the security parameter), l is the current number of key updates, γ is the output bit length of a collision-resistant hash function, and ε is the epoch-depth in our UPIBE. $n \geq 1$ is the performance parameter of GCTC [GCTC16]. We use the type-3 pairing setting with $e : G_1 \times G_2 \rightarrow G_T$ for prime-order groups G_1, G_2 , and G_T . Here, we do not consider the tightness of the reductions to the underlying assumptions.

We see that all but one known trivial KU-KEM instantiations via [Lew12, GCTC16, LP20] have ciphertext and decapsulation-key sizes that scale linearly in the number of delegations (which corresponds to KU-KEM key updates). Only GCTC [GCTC16] has a trade-off for ciphertext and key sizes via their performance parameter n . With our non-trivial UPIBE approach from bounded-depth HIBEs, taking the BBG scheme [BBG05] as instantiation, we obtain ciphertext sizes that only scale linearly in the number of *epochs*, which can be adjusted by the depth-parameter ε as described in Section 4. Moreover, our KU-KEM approach via BBG enjoys very short encapsulation keys. This yields a significant reduction in encapsulation key and ciphertext sizes for KU-KEMs compared to other approaches (see Table 1).

¹⁴ In our concrete setting, for standard-model HIBEs, we use Groth’s pairing-free signature scheme [Gro06] while for HIBEs in the ROM, we use Schnorr signatures [Sch90].

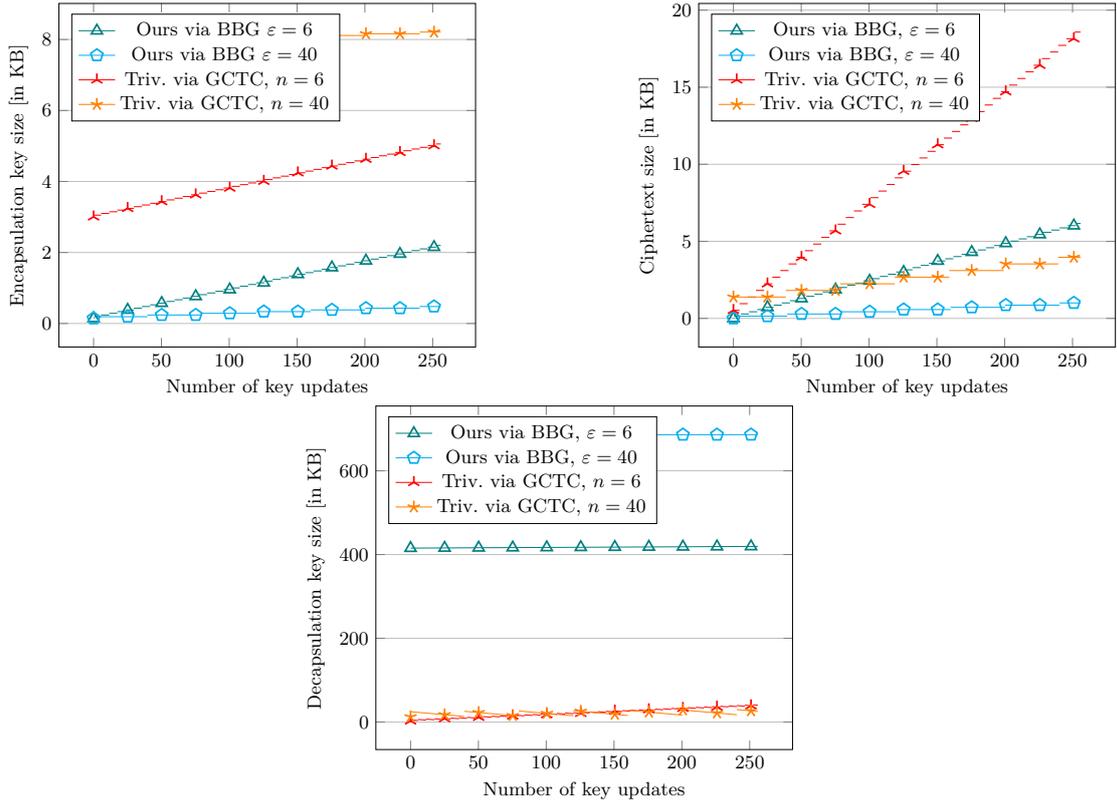


Fig. 6: Comparison of CCA secure KU-KEM encapsulation and decapsulation key as well as ciphertext sizes in kilobytes (KB) from (un)bounded-depth HIBEs. For the pairing group, we chose BLS12-381 (which gives around 128 bit security); this means per element in G_1 , G_2 , and G_T , we have 382, 764, 4572 bits.

Detailed Analysis. For our following analysis concerning parameter sizes and performance, from the three trivial standard-model KU-KEMs based on unbounded-depth HIBEs [Lew12, GCTC16, LP20], we chose GCTC [GCTC16] which outperforms the other two—particularly because of their scalability parameter n that allows to trade-off ciphertext and encapsulation/decapsulation key sizes.¹⁵ Hence, the GCTC scheme is the best suitable reference instantiation of KU-KEM via the *trivial* UPIBE construction for a concrete comparison regarding the applications we have in mind.

Application Requirements. Our focus is on short ciphertexts and encapsulations keys (for bandwidth reasons) while on the sender and the receiver sides, we want fast encapsulation and fast decapsulation, respectively. As we argue now, our non-trivial UPIBE approach with BBG outperforms the trivial KU-KEM construction with GCTC in all of the metrics mentioned above. We recall that our KU-KEM decapsulation is based on the actual ciphertext decapsulation and an additional key delegation of the underlying HIBE. Moreover, we can compress the identity string via algorithm IE.der to compute an identity-specific encapsulation key for BBG and GCTC. We currently do not see how to perform this compression for [Lew12, LP20].

Bandwidth Comparison. We observe that the performance parameter n in GCTC plays a similar role as our depth parameter ϵ in UPIBE; hence, we compare it at the same level. As illustrative examples, we choose $\epsilon = n = 6$ and $\epsilon = n = 40$. From the graphs in Figures 6 and 7, we see that the encapsulation key for the BBG-based KU-KEM is very short. The ciphertext size of all KU-KEMs scales with ϵ and n . Our BBG-based approach has the shortest ciphertext sizes of all. For decapsulation key sizes, the GCTC approach is more efficient; however, as we argued with the application of secure messaging in mind, this is tolerable. Hence, concerning parameter sizes, we conclude that the BBG approach has shorter

¹⁵ Essentially, GCTC [GCTC16] improves Lewko [Lew12] towards shorter ciphertext sizes and LP [LP20] deals with tightness of the Lewko scheme [Lew12], at the expense of rather large encapsulation keys (see γ -factor).

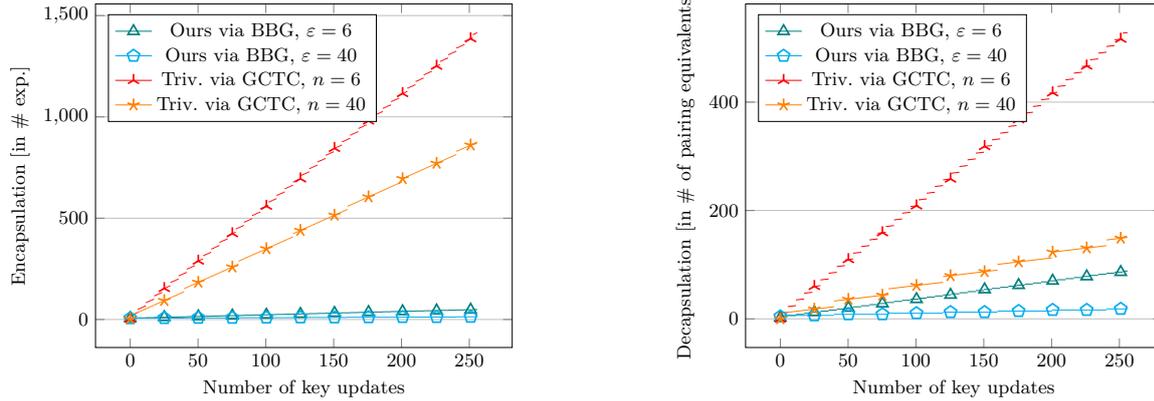


Fig. 7: Comparison of CCA secure KU-KEM key generation, encapsulation, and decapsulation performance from un-/bounded-depth HIBEs. We estimate that a G_1 exponentiation is 10 times more efficient than a pairing.

ciphertexts and smaller encapsulation key at the expense of slightly larger decapsulation keys compared to the trivial GCTC-based KU-KEM approach.

Computation Comparison. In terms of computation complexity (Figure 7), we see that the BBG approach significantly outperforms the GCTC-based approach for encapsulation and decapsulation. The (initial) key generation for the BBG-based and for GCTC-based approaches are comparable efficiency-wise and constant in the number of key updates; our approach needs α many exponentiations while GCTC’s number of exponentiations scales linearly in their performance parameter n . For encapsulation and decapsulation (where latter uses key delegation *and* decryption of the underlying HIBE), the BBG-based KU-KEM is more efficient; particularly, in situations when a large number of key updates is needed. See that the larger ε , the more efficient is the decapsulation of the BBG-based KU-KEM approach. The reason is that the BBG HIBE ciphertexts are of constant size and need only a constant number of pairings per ciphertext for decryption.

Summary. In conclusion, a KU-KEM via our unbounded-depth UPIBE construction, instantiated with the BBG HIBE, has shorter ciphertext and encapsulation-key sizes compared to the GCTC-based solution with analogous parameter choices (being the most efficient unbounded-depth HIBE known for trivial UPIBE) at the expense of a slightly larger decapsulation key. Additionally, the decapsulation and, particularly, the encapsulation of the BBG-based KU-KEM are significantly more efficient compared to the GCTC-based trivial KU-KEM. Hence, for our envisioned application of strongly secure messaging, we can tolerate slightly larger decapsulation keys while achieving more efficient decapsulation and encapsulation as those operations happen rather often in KU-KEMs.

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A Basic Definitions

Split-Key Pseudorandom Functions. A split-key pseudorandom function [GHP18] is a function that behaves like a random function if at least one component of its key is picked uniformly at random (while the other components may be known or even chosen by the adversary). In the following let $\mathcal{K} = \mathcal{K}_1 \times \dots \times \mathcal{K}_n$ be finite key, \mathcal{X} a finite input and \mathcal{Y} a finite output space. We consider a function $F : \mathcal{K} \times \mathcal{X} \rightarrow \mathcal{Y}$ and for each $i \in [n]$ associate to each adversary \mathcal{A} its advantage $\text{Adv}_{F,i}^{\text{pr}}(\mathcal{A})$.

Definition 2 ([GHP18]). *We say that F is a split-key pseudorandom function (skPRF) if the advantages $\text{Adv}_{F,i}^{\text{pr}}(\mathcal{A}) := |\Pr[\text{PR}_i^0(\mathcal{A}) = 1] - \Pr[\text{PR}_i^1(\mathcal{A}) = 1]|$ are negligible for all PPT adversaries, where the experiment is given in Figure 8.*

Game $\text{PR}_i^b(\mathcal{A})$	Oracle $\text{Eval}(k', x)$
00 $X \leftarrow \emptyset$	04 If $x \in X$: Abort
01 $k_i \leftarrow_{\S} \mathcal{K}_i$	05 $X \leftarrow X \cup \{x\}$
02 $b' \leftarrow_{\S} \mathcal{A}^{\text{Eval}}$	06 $k_1 \dots k_{i-1} k_{i+1} \dots k_n \leftarrow k'$
03 Stop with b'	07 $y \leftarrow F(k_1 \dots k_i \dots k_n, x)$
	08 $y^0 \leftarrow y; y^1 \leftarrow_{\S} \mathcal{Y}$
	09 Return y^b

Fig. 8: Security experiment PR_i^b , $1 \leq i \leq n$, modeling the split-key pseudorandomness of function F .

Pseudo-Random Generator. We use the following standard security notion for PRGs.

Definition 3. *The advantage of an adversary \mathcal{A} in breaking the security of a pseudo-random generator $G : \{0, 1\}^n \rightarrow \{0, 1\}^m$ with l outputs is defined as $\text{Adv}_G^{\text{ind}} := |\Pr[\mathcal{A}_G(G(s)) = 1 : s \leftarrow_{\S} \{0, 1\}^n] - \Pr[\mathcal{A}_G(r) = 1 : r \leftarrow_{\S} \{0, 1\}^m]|$.*

B Full Proofs for UPIBE Constructions

Our proofs of theorems 1, 2, and 3 are sketched on a high level in sections 3, 4, and 4.1 already. These proofs are conceptually very similar but differ both in the guessing step that induces the loss factor and in the reduction to multi-instance vs. single-instance assumptions. In particular, the selective-to-adaptive security proof in Appendix B.3 provides an evolved guessing technique.

B.1 Bounded-Depth UPIBE

We proceed with our formal proof of Theorem 1 for multi-instance security of our bounded-depth UPIBE from Figure 3 in three game-hops.

Hybrid 1: Replacing PRG Outputs. Game 1 implements a hybrid argument of q_{Gen} steps, that internally consist of up to L sub-steps each, where q_{Gen} is the number of queries to oracle Gen_{IE} by adversary \mathcal{A} . In every step $\iota \in [q_{\text{Gen}}]$, oracle Exp_{IE} is changed as follows: At the beginning of each step, a variable $l \in [L]$ is randomly sampled. If the instance generated with the ι th query to oracle Gen_{IE} is ever exposed, then oracle Exp_{IE} aborts on the following condition: The number of delegations previously applied to this instance's exposed UPIBE decapsulation key differs from l . If oracle Exp_{IE} does not abort, step ι internally proceeds in l sub-steps in which oracles Gen_{IE} and Del_{IE} are changed as follows: For the first l delegation steps of this instance, instead of computing forwarding key fk and seed s as the PRG output, both values are sampled uniformly at random. An adversary that successfully distinguishes between two subsequent sub-steps is turned directly into an adversary \mathcal{B}_G that breaks PRG G , which leads to a loss bounded by $\text{Adv}_G^{\text{ind}}(\mathcal{B}_G)$. The number of sub-steps is l , which is at most L , leading to a loss of L . Furthermore, the loss, induced by guessing the correct number of delegations that are conducted before an exposure, is bounded by L . Having q_{Gen} steps in total for this hybrid leads to full loss of:

$$q_{\text{Gen}} \cdot L^2 \cdot \text{Adv}_G^{\text{ind}}(\mathcal{B}_G).$$

At the end of this hybrid, all IBE *main key pairs*, whose IBE main decapsulation keys are never exposed, are freshly generated. The remaining IBE main key pairs, in particular those that are exposed, are generated as in the original construction.

Hybrid 2: Guessing Branched Sub-String. Game 2 implements a hybrid argument of q_{Chall} steps, where q_{Chall} is the number of queries to oracle Chall_{IE} by adversary \mathcal{A} . In every sub-step $\iota \in [q_{\text{Chall}}]$, the ι th query to oracle Chall_{IE} and a corresponding query to oracle Exp_{IE} are changed. Queries to oracles Gen_{IE} and Del_{IE} are, in contrast, processed as at the end of the previous hybrid.

At the beginning of each hybrid step, integer i^* is sampled uniformly at random from space $[q_{\text{Gen}}]$, where q_{Gen} is the overall number of queries to oracle Gen_{IE} . If the ι th query to oracle Chall_{IE} has inputs (i, id) such that $i \neq i^*$, then Game 2 in sub-step ι aborts.

Furthermore, if the ι th query to oracle Chall_{IE} with inputs (i, id) is issued as long as $XP_i = \emptyset$, oracle Chall_{IE} changes its behavior for this query as follows: Oracle Chall_{IE} samples an integer l^* uniformly at random from space $[L]$ and stores (i, id, l^*) as a guess. On the first corresponding subsequent successful

query to oracle Exp_{IE} (i.e., when set XP_i initially becomes non-empty) with inputs (i, id') , Game 2 in sub-step ι aborts if this prior guess was wrong. That is, if a guess of the form (i, id, l^*) was stored, such that id and id' equal in the l^* th identity sub-string, Game 2 in sub-step ι aborts, unless id is a true prefix of id' . More precisely, let $id = (id_0, \dots, id_{|id|/\lambda-1})$ and $id' = (id'_0, \dots, id'_{|id'|/\lambda-1})$ be concatenations of sub-strings, such that all sub-strings id_j and id'_j are in set $\{0, 1\}^\lambda$. Then, Game 2 in sub-step ι aborts if $id_{l^*} = id'_{l^*}$ unless for all $j \in [|id|/\lambda]$ it holds that $id_j = id'_j$.

We observe that all valid challenges that target identity string id must differ from the exposed decapsulation key's identity string id' in some sub-string, unless id is a true prefix of id' . Hence, the guess in the ι th query to oracle Chall_{IE} is correct with probability at least $1/L$, leading to a loss factor of L per step in this hybrid argument. Furthermore, the guess at the beginning of a hybrid sub-step about the instance targeted in the ι th query to oracle Chall_{IE} is correct with probability at least $1/q_{\text{Gen}}$, leading to a loss factor of q_{Gen} per step in this hybrid argument. Therefore, the entire hybrid argument induces a loss factor of at most:

$$q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot L,$$

where q_{Gen} and q_{Chall} are the numbers of queries to oracles Gen_{IE} and Chall_{IE} by adversary \mathcal{A} , respectively.

Enhanced Hybrid 2: Only Guessing Necessary Information. By sampling variable l^* from set $[|id|/\lambda]$, one can more precisely guess the index of the branching sub-string, where id is the challenge identity string for instance i in the ι th query. This reduces the loss factor from $q_{\text{Chall}} \cdot L$ to $q_{\text{Chall}} \cdot l_{\text{long}}$, where l_{long} is the level-depth of the longest identity string id_{long} queried to oracle Chall_{IE} by adversary \mathcal{A} (i.e., $l_{\text{long}} = |id_{\text{long}}|/\lambda$). This enhancement becomes (more) important when proving security of our unbounded-depth UPIBE in Appendix B.2.

Game 3 in Hybrid 2. After guessing the instance and the branching level for the challenge query targeted by current hybrid step ι , we randomize the challenge keys in each of the first ι queries to oracle Chall_{IE} in Game 3. More concretely, in Game 3 the first $\iota - 1$ queries to oracle Chall_{IE} output a randomly sampled challenge key. In addition to that, the ι th query to oracle Chall_{IE} replaces the l^* th component of the challenge key with a randomly sampled key component. That is, if l^* was sampled in prior Game 2, then this is the branching level in which Game 3 embeds a random key component of the ι th challenge. If, otherwise, l^* was not sampled for the ι th query in Game 2 because a prior exposure of the corresponding decapsulation key already fixed the branching level, then this fixed branching level determines the component of the challenge key that is randomly sampled for the output of this ι th challenge query. All subsequent queries to oracle Chall_{IE} remain unchanged in the ι th step of Hybrid 2 in Game 3.

Reduction. Our reduction $\mathcal{B}_{\text{IE}'}$ uses an adversary that successfully distinguishes between hybrid steps $\iota - 1$ and ι in Game 3 to succeed in game $\text{IND}_{\text{IE}'}$ against the underlying bounded-collusion IBE. This reduction simulates the l^* th execution of algorithm $\text{IE.gen}'$ in the i^* th query to oracle Gen_{IE} via a query to oracle $\text{Gen}_{\text{IE}'}$; all remaining key pairs (of other instances as well as other levels of instance i^*) are generated honestly by the reduction itself. The reduction simulates both potential executions of algorithm $\text{IE.del}'$ in oracle Del_{IE} for instance i^* 's level l^* via queries to oracle $\text{Del}_{\text{IE}'}$. Furthermore, it obtains a potentially exposed delegated IBE decapsulation key for level l^* in a query to oracle Exp_{IE} with input instance i^* via a query to oracle $\text{Exp}_{\text{IE}'}$.

For the ι th query to oracle Chall_{IE} with inputs (i^*, id) , reduction $\mathcal{B}_{\text{IE}'}$ either uses guess (i^*, id, l^*) from Game 2 or, if $XP_{i^*} \neq \emptyset$, identifies branching level l^* directly (by comparing input id and the exposed identity string) to assemble the challenge ciphertext. Let $id = (id_0, \dots, id_{|id|/\lambda-1})$ be a concatenation of sub-strings, such that all sub-strings id_j are in set $\{0, 1\}^\lambda$. Reduction $\mathcal{B}_{\text{IE}'}$ issues a query to oracle $\text{Chall}_{\text{IE}'}$ with inputs $(1, id_{l^*} \parallel \text{suf})$, where $\text{suf} = 0$ if $l^* = |id|/\lambda - 1$, and $\text{suf} = 1$ otherwise. Oracle $\text{Chall}_{\text{IE}'}$ responds with a ciphertext-key pair (c^*, k^*) . Ciphertext c^* replaces c'_{l^*} in the concatenated UPIBE challenge ciphertext and key k^* replaces k'_{l^*} in the XORed UPIBE challenge key for the simulation of the l^* th execution of algorithm $\text{IE.enc}'$ in oracle Chall_{IE} . (All remaining IBE ciphertexts and keys in a challenge query are computed honestly by $\mathcal{B}_{\text{IE}'}$ itself.) Thus, only IBE ciphertext and key of branching level l^* are replaced in the composed UPIBE ciphertext and key, respectively. All challenge keys for the first $\iota - 1$ queries to oracle Chall_{IE} were replaced by random keys in hybrid step $\iota - 1$ already.

Finally, we observe that the only difference between sub-steps $\iota - 1$ and ι is that the ι th query to oracle Chall_{IE} returns a key with a single randomized key component. This randomization matches the behavior of the embedded IBE challenge. We now argue that this embedded IBE challenge remains valid if the ι th query to oracle Chall_{IE} remains valid, too. This means that a successful distinguisher between sub-steps $\iota - 1$ and ι indeed successfully solves the embedded IBE challenge.

Note that UPIBE challenges are enforced to remain valid by security experiment IND_{IE} . This enforcement forbids the exposure of a UPIBE decapsulation key with delegation identity string id' if there exists a challenge with identity string id such that id' is a prefix of id . Our reduction uses this mechanism by identifying the (IBE) identity sub-string in which id' and id differ. This difference is either based on a real branch in UPIBE identity strings id' and id , or based on a *special* branch in the underlying IBE identity sub-string. (The latter is induced if id is a prefix of id' , such that the last IBE identity sub-string of UPIBE string id has an attached *special* suffix 0-bit.) On exposure of UPIBE decapsulation key with $id' = (id'_0, \dots, id'_{|id'|/\lambda-1})$, only those IBE decapsulation keys are revealed that were delegated to (encoded variants of) sub-strings of id' . That is, for UPIBE instance i^* , IBE decapsulation keys of levels j delegated to identity sub-string $id'_j || \text{suf}$ are exposed, where $\text{suf} = 1$ if $j^* \neq |id'|/\lambda - 1$, and $\text{suf} = 0$ otherwise. That means, IBE challenges that were directed to different identity sub-strings (on the branching level aka. of the branching IBE instance) remain valid because the corresponding IBE decapsulation keys were not revealed. Thus, the embedded IBE challenge remains valid.

Consequently, a successful distinguisher between two subsequent hybrid sub-steps is reduced to the security of the underlying IBE. After all hybrid sub-steps, all queried challenges have random keys such that output bit b' of adversary \mathcal{A} is independent of challenge bit b , which concludes our proof with:

$$\text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{A}) \leq q_{\text{Gen}} \cdot \mathsf{L}^2 \cdot \text{Adv}_{\text{G}}^{\text{ind}}(\mathcal{B}_{\text{G}}) + q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot \mathsf{L} \cdot \text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B}_{\text{IE}'})$$

□

B.2 Unbounded-Depth UPIBE

We proceed with our formal proof of Theorem 2 for multi-instance security of our unbounded-depth UPIBE from Figure 4 in two game-hops.

Hybrid: Guessing Branched Sub-String. Game 1 implements a hybrid argument of q_{Chall} steps, where q_{Chall} is the number of queries to oracle Chall_{IE} by adversary \mathcal{A} . In every sub-step $\iota \in [q_{\text{Chall}}]$, the ι th query to oracle Chall_{IE} and a corresponding query to oracle Exp_{IE} are changed. Queries to oracles Gen_{IE} and Del_{IE} are, in contrast, processed exactly as in the original Game IND_{IE} .

At the beginning of each hybrid step, integer i^* is sampled uniformly at random from space $[q_{\text{Gen}}]$, where q_{Gen} is the overall number of queries to oracle Gen_{IE} . If the ι th query to oracle Chall_{IE} has inputs (i, id) such that $i \neq i^*$, then Game 1 in sub-step ι aborts.

Furthermore, if the ι th query to oracle Chall_{IE} with inputs (i, id) is issued as long as $XP_i = \emptyset$, oracle Chall_{IE} changes its behavior for this query as follows: Oracle Chall_{IE} samples an integer e^* uniformly at random from space $[\lceil |id|/(\varepsilon \cdot \lambda) \rceil]$ and stores (i, id, e^*) as a guess. On the first corresponding subsequent successful query to oracle Exp_{IE} (i.e., when set XP_i initially becomes non-empty) with inputs (i, id') , Game 1 in sub-step ι aborts if this prior guess was wrong. That is, if a guess of the form (i, id, e^*) was stored, such that id and id' equal in the e^* th epoch sub-string, Game 1 in sub-step ι aborts, unless id is a true prefix of id' . More precisely, let $id = (id_0, \dots, id_{\lceil |id|/(\varepsilon \cdot \lambda) \rceil - 1})$ and $id' = (id'_0, \dots, id'_{\lceil |id'|/(\varepsilon \cdot \lambda) \rceil - 1})$ be concatenations of epoch sub-strings, such that all sub-strings id_j and id'_j except for the respective last ones are in set $\{0, 1\}^{\varepsilon \cdot \lambda}$; The last epoch sub-strings $id_{\lceil |id|/(\varepsilon \cdot \lambda) \rceil - 1}$ and $id'_{\lceil |id'|/(\varepsilon \cdot \lambda) \rceil - 1}$ are in set $\{0, 1\}^{l \cdot \lambda}$ and $\{0, 1\}^{l^* \cdot \lambda}$, respectively, where $l \leq \varepsilon$ and $l^* \leq \varepsilon$. Then, Game 1 in sub-step ι aborts if $id_{e^*} = id'_{e^*}$ unless for all $j \in [\lceil |id|/(\varepsilon \cdot \lambda) \rceil]$ it holds that $id_j = id'_j$.

We observe that all valid challenges that target identity string id must differ from the exposed decapsulation key's identity string id' in some epoch sub-string, unless id is a true prefix of id' . Hence, the guess in the ι th query to oracle Chall_{IE} is correct with probability at least $1/\lceil \varepsilon/l_{\text{long}} \rceil$, where l_{long} is the level-depth of the longest identity string queried to oracle Chall_{IE} by adversary \mathcal{A} (i.e., $l_{\text{long}} = \lceil |id_{\text{long}}|/\lambda \rceil$). Thus, the loss factor induced per step in this hybrid argument is $\lceil \varepsilon/l_{\text{long}} \rceil$. Furthermore, the guess at the beginning of a hybrid sub-step about the instance targeted in the ι th query to oracle Chall_{IE} is

correct with probability at least $1/q_{\text{Gen}}$, leading to a loss factor of q_{Gen} per step in this hybrid argument. Therefore, the entire hybrid argument induces a loss factor of at most:

$$q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot \lceil \varepsilon / l_{\text{long}} \rceil,$$

where q_{Gen} and q_{Chall} are the numbers of queries to oracles Gen_{IE} and Chall_{IE} by adversary \mathcal{A} , respectively.

Game 2 in Hybrid. After guessing the instance and the branching epoch for the challenge query targeted by current hybrid step ι , we randomize the challenge keys in each of the first ι queries to oracle Chall_{IE} in Game 2. More concretely, in Game 2 the first $\iota - 1$ queries to oracle Chall_{IE} output a randomly sampled challenge key. In addition to that, the ι th query to oracle Chall_{IE} replaces the e^* th epoch component of the challenge key with a randomly sampled key component. That is, if e^* was sampled in prior Game 1, then this is the branching epoch in which Game 2 embeds a random key component of the ι th challenge. If, otherwise, e^* was not sampled for the ι th query in Game 1 because a prior exposure of the corresponding decapsulation key already fixed the branching epoch, then this fixed branching epoch determines the component of the challenge key that is randomly sampled for the output of this ι th challenge query. All subsequent queries to oracle Chall_{IE} remain unchanged in the ι th step of Hybrid 1 in Game 2.

Reduction. Our reduction \mathcal{B} uses an adversary that successfully distinguishes between hybrid steps $\iota - 1$ and ι in Game 2 to succeed in game $\text{IND}_{\text{IE}'}$ against the underlying bounded-depth HIBE. This reduction simulates the execution of algorithm $\text{IE.gen}'$ in the i^* th query to oracle Gen_{IE} via a query to oracle $\text{Gen}_{\text{IE}'}$; all remaining key pairs (of other UPIBE instances) are generated honestly by the reduction itself. (Note that, for the i^* th UPIBE instance in Game IND_{IE} , reduction \mathcal{B} maintains a single HIBE instance in Game $\text{IND}_{\text{IE}'}$.) The reduction simulates all executions of algorithm $\text{IE.del}'$ in oracle Del_{IE} for instance i^* via queries to oracle $\text{Del}_{\text{IE}'}$. Furthermore, it obtains exposed delegated HIBE decapsulation keys for instance i^* in a query to oracle Exp_{IE} via a query to oracle $\text{Exp}_{\text{IE}'}$.

For the ι th query to oracle Chall_{IE} with inputs (i^*, id) , reduction \mathcal{B} either uses guess (i^*, id, e^*) from Game 1 or, if $XP_{i^*} \neq \emptyset$, identifies branching epoch e^* directly (by comparing input id and the exposed identity string) to assemble the challenge ciphertext. Let $id = (id_0, \dots, id_{\lceil |id|/(\varepsilon \cdot \lambda) \rceil - 1})$ be a concatenation of epoch sub-strings, such that all sub-strings id_j except for the last one are in set $\{0, 1\}^{\varepsilon \cdot \lambda}$. Reduction \mathcal{B} , modifies epoch sub-string id_{e^*} by appending a 1-bit or 0-bit to each level's sub-string, depending on whether it is the last level in that epoch, and depending on whether it is the last epoch of the entire string (see Figure 4 lines 17-19 resp. 25-26). Furthermore, modified epoch sub-string id_{e^*} is prepended by a binary encoding of e^* according to the epoch-progression mechanism (see Figure 4 lines 14-16). Finally, a query to oracle $\text{Chall}_{\text{IE}'}$ with inputs (i, id_{e^*}) is issued, where id_{e^*} is the modified epoch sub-string. Oracle $\text{Chall}_{\text{IE}'}$ responds with a ciphertext-key pair (c^*, k^*) . Ciphertext c^* replaces c'_{e^*} in the concatenated UPIBE challenge ciphertext and key k^* replaces k'_{e^*} in the XORed UPIBE challenge key for the simulation of algorithm IE.enc in oracle Chall_{IE} . (All remaining HIBE ciphertexts and keys in this challenge query are computed honestly by \mathcal{B} itself.) Thus, only HIBE ciphertext and key of branching epoch e^* are replaced in the composed UPIBE ciphertext and key, respectively. All challenge keys for the first $\iota - 1$ queries to oracle Chall_{IE} were replaced by random keys in hybrid step $\iota - 1$ already.

Finally, we observe that the only difference between sub-steps $\iota - 1$ and ι is that the ι th query to oracle Chall_{IE} returns a key with a single randomized key component. This randomization matches the behavior of the embedded HIBE challenge. We now argue that this embedded HIBE challenge remains valid if the ι th query to oracle Chall_{IE} remains valid, too. That means that a successful distinguisher between sub-steps $\iota - 1$ and ι indeed successfully solves the embedded HIBE challenge.

Note that UPIBE challenges are enforced to remain valid by security experiment IND_{IE} . This enforcement forbids the exposure of a UPIBE decapsulation key with delegation identity string id' if there exists a challenge with identity string id such that id' is a prefix of id . Our reduction uses this mechanism by identifying the (HIBE) epoch sub-string in which id' and id differ. This difference is either based on a real branch in UPIBE identity strings id' and id , or based on a *special* branch in the underlying HIBE identity sub-string. (The latter is induced if id is a prefix of id' , such that the last HIBE identity sub-string of UPIBE string id has an attached *special* suffix 0-bit.)

On exposure of UPIBE decapsulation key with $id' = (id'_0, \dots, id'_{\lceil |id'|/(\varepsilon \cdot \lambda) \rceil - 1})$, the following HIBE decapsulation keys are revealed: (1) epoch-progression keys for future epochs and (2) multiple ordinary

keys and one special key that were all delegated to (encoded variants of) epoch sub-strings of id' . We observe that any issued HIBE challenge must be embedded in the current or in one of the previous epochs—with respect to a branch of an exposed UPIBE decapsulation key. This means that no HIBE challenge is directed to decapsulation keys of descendants of revealed epoch-progression keys. Thus, neither of the revealed epoch-progression keys invalidates an issued HIBE challenge. Similarly, every HIBE challenge is directed to an epoch identity sub-string that differs from a revealed HIBE decapsulation key's epoch identity sub-string in this epoch (i.e., in the branching epoch). Thus, a HIBE challenge is neither invalidated due to exposed ordinary or special HIBE decapsulation keys. Hence, the embedded HIBE challenge remains valid.

Consequently, a successful distinguisher between two subsequent hybrid sub-steps is reduced to the security of the underlying HIBE. After all hybrid sub-steps, all queried challenges have random keys such that output bit b' of adversary \mathcal{A} is independent of challenge bit b , which concludes our proof with:

$$\text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{A}) \leq q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot \lceil l_{\text{long}}/\varepsilon \rceil \cdot \text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B}).$$

□

B.3 Unbounded-Depth UPIBE from Selective Bounded-Depth HIBE

Our proof of Theorem 2 in prior Section B.2 assumes adaptive security of the employed bounded-depth HIBE. We now relax this assumption to selective security only. Thus, we significantly extend the set of bounded-depth HIBE schemes from the literature with which we can instantiate our unbounded-depth UPIBE construction.

Shortcoming of Generic Selective-to-Adaptive-Proof. In order to relax the assumption from adaptive to selective security, we extend a known technique from the literature [BB04, BBG05]. This technique generically lifts bounded-depth HIBEs from selective to adaptive security by using a programmable random oracle: Intuitively, every identity (sub-)string id is hashed by the random oracle and the hash output $H(id)$ is used as the actual input to the encapsulation and delegation algorithms, respectively. In the reduction proof, the *selective challenge identity* sub-strings are programmed as random oracle outputs. The probability that these outputs are programmed on inputs that form the *adaptive challenge identity* is exponential in the depth of the HIBE. Since our UPIBE construction uses a bounded-depth HIBE where the depth is linear in the security parameter, applying this technique directly leads to a meaningless result.

Solution: Guess Branch-Epoch First. Due to the way our construction makes use of the underlying bounded-depth HIBE, we can carefully change the approach from [BB04, BBG05] for relying only on a selectively secure HIBE. For this purpose, we observe that our proof from Section B.2 always only relies on the security guarantees of the so called *branching epoch*. In this epoch, the respective challenge identity branches off the identity that is delegated with a corresponding (exposed) decapsulation key. Following our proof strategy from Section B.2, this branching epoch will fix the epoch in which the underlying (selective) HIBE challenge is embedded.

Epoch-Delegations via Random Oracle. We recall that the generic approach to turn selectively-secure HIBE into adaptive-secure HIBE simply hashes every identity sub-string id via a random oracle $H(id)$ before this hash is given as input to HIBE algorithms $\text{IE.enc}'$ and $\text{IE.del}'$ (instead of having the plain sub-strings id as inputs). We apply the same change to our unbounded-depth UPIBE construction for all identity sub-strings processed in the *lower multi-level epoch component*. That is, we implement random oracle H in the following lines of our construction from Figure 4:

```

18:  $id' \stackrel{\$}{\leftarrow} H(id_{e' \cdot \varepsilon + d'} \| 1)$ 
19:  $id' \stackrel{\$}{\leftarrow} H(id_{e' \cdot \varepsilon + \varepsilon - 1} \| 0)$ 
26:  $id' \stackrel{\$}{\leftarrow} H(id_{(e-1) \cdot \varepsilon + d'} \| 1)$ 
56:  $D_{ep}[e] \stackrel{\$}{\leftarrow} \text{IE.del}'(D_{ep}[e], H(id \| 1))$ 
58:  $dk'_0 \stackrel{\$}{\leftarrow} \text{IE.del}'(D_{ep}[e], H(id \| 0))$ 
59:  $dk'_1 \stackrel{\$}{\leftarrow} \text{IE.del}'(D_{ep}[e], H(id \| 1))$ 

```

However, we leave the identity sub-strings of the upper epoch-progression mechanism untouched. Thus, lines 03-04, 16, 24, and 51-52 remain the same.

The idea behind these changes is as follows: In each hybrid step that covers a single UPIBE challenge, we guess one epoch at the beginning of Game IND_{IE} . For this targeted epoch, we guess the entire HIBE identity sub-string. The special property of this guessed epoch is that the UPIBE identity string branches off the corresponding (potentially) exposed UPIBE decapsulation key's identity string. By implementing these guesses in a way that increases their success probability, we can commit to a *selective* bounded-depth HIBE challenge identity string at the beginning of underlying HIBE Game IND . Thus, we turn the adaptive adversary against the UPIBE scheme into a selective adversary. The guess for a UPIBE challenge consists of two components: (1) the *epoch number* of the epoch in which the UPIBE challenge identity string branches of the identity string on decapsulation side and (2) the *actual identity sub-strings* in the lower multi-level epoch part for this branching epoch.

Clever Guessing. We start with guessing the UPIBE epoch number in which the challenge identity branches off its counterpart on the decapsulation side. This guess fixes the branched epoch's identity path in the *upper epoch-progression mechanism* of our UPIBE construction, which induces a polynomial loss factor per challenge.

Next, we would guess the actual identity sub-strings in the *lower multi-level epoch part* of our UPIBE construction for the branching epoch. Instead of guessing these (up to) ε identity sub-strings in plain, we use the random oracle to increase the probability of guessing correctly. That is, we guess for each of the ε levels in the branching epoch when their identity sub-string is queried to the random oracle for the first time. By programming the a priori committed (selective) HIBE identity sub-strings as random oracle outputs into these ε random oracle queries, we obtain a selective HIBE adversary. The probability of guessing these random oracle queries correctly for one UPIBE challenge is bounded by $1/(q_{\text{H}})^{\varepsilon}$, where q_{H} is the total number of random oracle queries. Using this approach in a hybrid argument to cover all UPIBE challenges, we obtain a loss factor that is polynomial (and not exponential!) in the security parameter.

We proceed with our formal proof of Theorem 3 for multi-instance security of our unbounded-depth UPIBE from Figure 4.

Hybrid 1: Instances. Game 1 implements a hybrid argument of q_{Gen} steps, where q_{Gen} is the number of queries to oracle Gen_{IE} by adversary \mathcal{A} . In every step $\rho \in [q_{\text{Gen}}]$, queries to oracles Chall_{IE} and Exp_{IE} are changed when being directed to instance $i = \rho$. All remaining queries to these oracles as well as queries to oracles Gen_{IE} and Del_{IE} are processed exactly as in the original Game IND_{IE} . The changed behavior of oracles Chall_{IE} and Exp_{IE} is implemented in the following hybrid.

Hybrid 2: Challenge Epoch Branch & Oracle Queries. Game 2 implements a hybrid argument of $q_{\text{Chall},\rho}$ steps, where $q_{\text{Chall},\rho}$ is the number of queries to oracle Chall_{IE} with inputs (ρ, \cdot) by adversary \mathcal{A} . Note that $q_{\text{Chall},\rho} \leq q_{\text{Chall}}$, where q_{Chall} is the total number of queries to oracle Chall_{IE} by adversary \mathcal{A} .

In every step $\iota \in [q_{\text{Chall},\rho}]$, we change the behavior of the ι th query to oracle $\text{Chall}_{\text{IE}}(\rho, id)$ as well as the behavior of a corresponding query to oracle $\text{Exp}_{\text{IE}}(\rho, \cdot)$. For clarity, we split each step ι into three sub-steps:

- a) In the first sub-step, we guess the length l_{id} of challenge identity string id that is input to the ι th query of instance ρ to oracle $\text{Chall}_{\text{IE}}(\rho, id)$. If this guess is wrong, the game aborts. The guess is correct with probability at least $1/l_{\text{long}}$, where l_{long} is the level-depth of the longest identity string queried to oracle Chall_{IE} by adversary \mathcal{A} (i.e., $l_{\text{long}} = |id_{\text{long}}|/\lambda$).
- b) Consider the full identity string id' according to which the decapsulation key of instance ρ is delegated via oracle Del_{IE} throughout the entire game. In the second sub-step, we guess the first position β_{id} in identity string id' in which it branches off the challenge identity string id that is input to the ι th query of instance ρ to oracle $\text{Chall}_{\text{IE}}(\rho, id)$. If this guess is wrong, the game aborts. This guess is also correct with probability at least $1/l_{\text{long}}$ as β_{id} is guessed on a sub-string (and not on a bit) granularity.
- c) In the third sub-step, we consider the (up to) ε identity sub-strings of id in the epoch that contains the β_{id} th sub-string of id (i.e., the branching epoch), where id is the challenge identity to the ι th query to instance's ρ oracle $\text{Chall}_{\text{IE}}(\rho, id)$. For each sub-string in this epoch, we guess when it is queried to the random oracle for the first time. If this guess is wrong, the game aborts. The guess is

correct with probability at least $(q_H)^\varepsilon$, where q_H is the total number of queries to random oracle H by adversary \mathcal{A} .

ι.a) String Length. Consider queries to oracle Chall_{IE} with inputs (i, id) . Overall, the behavior of the original oracle Chall_{IE} is changed for all of these queries that are directed to instances $i < \rho$ as well as for the first ι queries that are directed to instance $i = \rho$. We call these queries the *relevant queries* in step ι subsequently. All remaining irrelevant queries to oracle Chall_{IE} remain unchanged in this hybrid step.

At the beginning of the game, we sample an integer l_{id} for the most recent relevant challenge query (i.e., the ι th challenge query for instance ρ). This integer l_{id} is sampled uniformly at random from space $[l_{\text{long}}]$, where l_{long} is the level-depth of the longest identity string queried to oracle Chall by adversary \mathcal{A} (i.e., $l_{\text{long}} = \lfloor id_{\text{long}}/\lambda \rfloor$).

We then check for this integer if it correctly predicts the respective challenge identity's length. That means, the most recent relevant query (i, id) to challenge oracle Chall_{IE} aborts the game if $l_{id} \neq \lfloor id/\lambda \rfloor$, where l_{id} is the respective prediction for this query.

Each prediction l_{id} is correct with probability at least $1/l_{\text{long}}$, where l_{long} is the level-depth of the longest identity string queried to oracle Chall_{IE} by adversary \mathcal{A} (i.e., $l_{\text{long}} = \lfloor id_{\text{long}}/\lambda \rfloor$). Thus, the loss factor induced per step $\iota.a$ in this hybrid argument is

$$1/l_{\text{long}}.$$

ι.b) Identity String Branch. After sampling the prediction of string length, we sample another integer β_{id} for the most recent relevant query (i, id) at the beginning of the game. This integer β_{id} is also sampled uniformly at random from space $[l_{\text{long}}]$.

We then check for this integer β_{id} if it correctly predicts the respective branch between challenge identity string id and delegation identity string id' . For this, consider most recent relevant query (i, id) and the full identity string id' with which the decapsulation key of instance i is delegated throughout the game. To check if id branches off id' in position β_{id} , we have to distinguish two cases: (A) The first exposure of instance i is queried before the respective most recent relevant challenge query with inputs (i, id) . In this case, challenge oracle Chall_{IE} checks if the branch prediction was correct. (B) The respective most recent relevant challenge (i, id) is queried before instance i is exposed for the first time. In this case, the expose oracle Exp_{IE} checks if the branch prediction was correct.

In either case, the game aborts if the branch prediction was not correct. That is, if integer β_{id} was sampled for the most recent relevant challenge query (i, id) such that id and id' differ in a sub-string $\beta' < \beta_{id}$ or equal in the β_{id} th sub-string, hybrid step $\iota.b$ aborts, unless id is a true prefix of id' . More precisely, let $id = (id_0, \dots, id_{l_{id}-1})$ and $id' = (id'_0, \dots, id'_{\lfloor id'/\lambda \rfloor - 1})$ be concatenations of sub-strings, such that all sub-strings id_j and id'_j are in set $\{0, 1\}^\lambda$. Then, hybrid step $\iota.b$ aborts if $\exists \beta' < \beta_{id} : \perp \neq id_{\beta'} \neq id'_{\beta'} \neq \perp$ or if $\perp \neq id_{\beta_{id}} = id'_{\beta_{id}} \neq \perp$, unless $\beta_{id} = l_{id}$ and for all $j \in [l_{id}]$ it holds that $id_j = id'_j \neq \perp$.

We observe that all valid challenges of instance i that target identity string id must differ from the exposed corresponding decapsulation key's identity string id' in some epoch sub-string, unless id is a true prefix of id' . Hence, the respective prediction of β is correct with probability at least $1/l_{\text{long}}$, where l_{long} is the level-depth of the longest identity string queried to oracle Chall_{IE} by adversary \mathcal{A} (i.e., $l_{\text{long}} = \lfloor id_{\text{long}}/\lambda \rfloor$). Thus, the loss factor induced per step $\iota.b$ in this hybrid argument is

$$1/l_{\text{long}}.$$

ι.c) Random Oracle Queries. For the most recent relevant challenge (i, id) , we now look at the up to ε sub-strings of id that are contained in the branching epoch that also contains the β_{id} th sub-string. That means, we look at all identity sub-strings contained in the branching epoch of this most recent relevant challenge. For all these sub-strings, we identify the respective random oracle queries that take them as input.

To prepare this step, we first calculate the number of actual identity sub-strings contained in this epoch. Consider the most recent relevant query (i, id) with (predicted) length l_{id} and (predicted) branch position β_{id} . If $\lfloor l_{id}/\varepsilon \rfloor > \lfloor \beta_{id}/\varepsilon \rfloor$, then epoch $\lfloor \beta_{id}/\varepsilon \rfloor - 1$ is filled with $r = \varepsilon$ sub-strings. Otherwise, epoch $\lfloor l_{id}/\varepsilon \rfloor - 1 = \lfloor \beta_{id}/\varepsilon \rfloor - 1$ is filled with only $r = (l_{id} \bmod \varepsilon)$ sub-strings.

For each of these r identity sub-strings processed in epoch $\lfloor \beta_{id}/\varepsilon \rfloor - 1$, we guess when it is queried as input to the random oracle for the first time. That is, we sample integers ϕ_1, \dots, ϕ_r uniformly at

random from set $[q_H]$, where q_H is the total number of queries to the random oracle throughout the game. Then we consider the r identity sub-strings processed in the actual lower multi-level epoch part of epoch $\lceil \beta_{id}/\varepsilon \rceil - 1$ in our UPIBE construction. We abort if the ℓ th of these identity sub-string processed in epoch $\lceil \beta_{id}/\varepsilon \rceil - 1$ was not input to random oracle query ϕ_ℓ .

Each of these guesses is correct with probability at least $1/q_H$ and all guesses together are correct with probability at least

$$1/(q_H)^\varepsilon.$$

Loss Factor of Hybrids. The two hybrid arguments induce a loss factor of at most

$$q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot (l_{\text{long}})^2 \cdot (q_H)^\varepsilon,$$

where q_{Gen} and q_{Chall} are the numbers of queries to oracle Gen_{IE} and Chall_{IE} by adversary \mathcal{A} , respectively.

Selective Adversary. We maintain the perspective of the hybrid argument from above for the actual reduction to selective security of the underlying bounded-depth HIBE. Thus, our proof reduces the security of one relevant UPIBE challenge to the security of an underlying HIBE challenge after another.

Consider one relevant challenge (i, id) with (predicted) length l_{id} , (predicted) branch position β_{id} , and (predicted) random oracle queries ϕ_1, \dots, ϕ_r . In order to embed a selective HIBE challenge in this (relevant) UPIBE challenge, we commit at the beginning of the game to the following HIBE identity string. This identity string begins with a padded binary encoding of epoch number $\lceil \beta_{id}/\varepsilon \rceil - 1$, where padding and encoding are conducted according to the epoch-progression mechanism of our unbounded-depth UPIBE construction. For the appended UPIBE epoch identity sub-string, we pre-sample r random oracle output strings of length λ' each. These r strings are concatenated and then appended to the padded binary encoding of the epoch number. This combined identity string constitutes the selective HIBE challenge that is embedded in one relevant adaptive UPIBE challenge. To maintain a sound simulation, random oracle query ϕ_ℓ outputs the ℓ th of the initially r pre-sampled strings.

Reduction. Since this reduction recycles many parts of the proof of Theorem 2, we keep its description brief. We now concentrate on the relevant challenge (ρ, id) added in step ι of the hybrid argument with length l_{id} and branch β_{id} . Note that the end of the corresponding ι th query to oracle Chall_{IE} of instance ρ , UPIBE challenge-key from HIBE keys $K = (k_0, \dots, k_{l_{id}-1})$ and UPIBE challenge-ciphertext from HIBE ciphertexts $C = (c_0, \dots, c_{l_{id}-1})$ are assembled. The reduction for this hybrid step ι replaces the $(\lceil \beta_{id}/\varepsilon \rceil - 1)$ th components of K and C with the corresponding selective HIBE challenge-key k^* and challenge-ciphertext c^* , respectively. Thus, the actual UPIBE challenge-key and ciphertext query is computed as $(K^*, C^*) = (k_0, \dots, k_{\lceil \beta_{id}/\varepsilon \rceil - 2}, k^*, k_{\lceil \beta_{id}/\varepsilon \rceil}, \dots, k_{l_{id}-1}, c_0, \dots, c_{\lceil \beta_{id}/\varepsilon \rceil - 2}, c^*, c_{\lceil \beta_{id}/\varepsilon \rceil}, \dots, c_{l_{id}-1})$.

Finally, we see that a UPIBE adversary winning in step ι of instance ρ directly breaks IND security of the underlying selective secure bounded-depth HIBE scheme IE' . As described above, our reduction $\mathcal{B}_{\rho, \iota}$ uses the committed HIBE identity string at the beginning of Game IND to fix the selective HIBE challenge, which is embedded in UPIBE epoch $\lceil \beta_{id}/\varepsilon \rceil - 1$ of UPIBE challenge ι for instance ρ . We omit repeating the arguments from the proof of Theorem 2 for why HIBE challenges remain valid based on the validity of UPIBE challenges.

Proof Result. After combining all game hops of our hybrid arguments, an adversary in final step $\rho = q_{\text{Gen}}, \iota = q_{\text{Chall}}$ can only guess the challenge bit randomly. Thus, combining the loss factors from all game hops, we obtain

$$\text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{A}) \leq q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot (l_{\text{long}})^2 \cdot (q_H)^\varepsilon \cdot \text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B}).$$

□

C CCA Security Proofs for UPIBE

C.1 Bounded-Depth UPIBE

We proceed with our formal proof via game-hops and for simplicity we focus on a reduction from single-instance security of UPIBE IE to multi-instance security of IBE IE' and use the simple guessing strategy from Theorem 1. For clarity in notation, we refer to oracles in game IND_X^b by adding the scheme's identifier X as a subscript to the oracle names (i.e., $\text{Gen}_X, \text{Chall}_X, \text{Del}_X, \text{Exp}_X, \text{Dec}_X$).

Game 0. This game represents the original game IND_{IE}^0 , and thus $\text{Adv}^{G_0} = \Pr[\text{IND}_{\text{IE}}^0(\mathcal{A}) = 1]$.

Game 1. Game 1 implements a hybrid argument via q_{Gen} steps exactly as done in the proof of Theorem 1 to make all of the IBE main keys independent.

Transition Game 0-1. An adversary that successfully distinguishes between two sub-steps can be turned directly into an adversary \mathcal{B}_G that breaks PRG G , which leads to a loss bounded by $\text{Adv}_G^{\text{ind}}(\mathcal{B}_G)$. The number of sub-steps is l , which is at most L , leading to a loss of L . Furthermore, the loss, induced by guessing the correct number of delegations that are conducted before an exposure, is bounded by L . Having q_{Gen} steps we obtain that $\text{Adv}^{G_0} \leq \text{Adv}^{G_1} + q_{\text{Gen}} \cdot L^2 \cdot \text{Adv}_G^{\text{ind}}(\mathcal{B}_G)$.

Game 2. Game 2 implements a hybrid argument of q_{Chall} steps, where q_{Chall} is the number of queries to oracle Chall_{IE} by adversary \mathcal{A} . In every sub-step $\iota \in [q_{\text{Chall}}]$, the ι th query to oracle Chall_{IE} and a corresponding query to oracle Exp_{IE} are changed. Queries to oracles Gen_{IE} and Del_{IE} are, in contrast, processed as at the end of the previous hybrid.

At the beginning of each hybrid step, integer i^* is sampled uniformly at random from space $[q_{\text{Gen}}]$, where q_{Gen} is the overall number of queries to oracle Gen_{IE} . If the ι th query to oracle Chall_{IE} has inputs (i, id) such that $i \neq i^*$, then Game 2 in sub-step ι aborts.

Furthermore, if the ι th query to oracle Chall_{IE} with inputs (i, id) is issued as long as $XP_i = \emptyset$, oracle Chall_{IE} changes its behavior for this query as follows: Oracle Chall_{IE} samples an integer l^* uniformly at random from space $[L]$ and stores (i, id, l^*) as a guess. On the first corresponding subsequent successful query to oracle Exp_{IE} (i.e., when set XP_i initially becomes non-empty) with inputs (i, id') , Game 2 in sub-step ι aborts if this prior guess was wrong. That is, if a guess of the form (i, id, l^*) was stored, such that id and id' equal in the l^* th identity sub-string, Game 2 in sub-step ι aborts, unless id is a true prefix of id' . More precisely, let $id = (id_0, \dots, id_{|id|/\lambda-1})$ and $id' = (id'_0, \dots, id'_{|id'|/\lambda-1})$ be concatenations of sub-strings, such that all sub-strings id_j and id'_j are in set $\{0, 1\}^\lambda$. Then, Game 2 in sub-step ι aborts if $id_{l^*} = id'_{l^*}$ unless for all $j \in [|id|/\lambda]$ it holds that $id_j = id'_j$.

We observe that all valid challenges that target identity string id must differ from the exposed decapsulation key's identity string id' in some sub-string, unless id is a true prefix of id' . Hence, the guess in the ι th query to oracle Chall_{IE} is correct with probability at least $1/L$, leading to a loss factor of L per step in this hybrid argument. Furthermore, the guess at the beginning of a hybrid sub-step about the instance targeted in the ι th query to oracle Chall_{IE} is correct with probability at least $1/q_{\text{Gen}}$.

Transition Game 1-2. We observe that all valid challenges that target identity string id must differ from the exposed decapsulation key's identity string id' in some sub-string, unless id is a true prefix of id' . Hence, the ι th guess in oracle Chall_{IE} is correct with probability at least $1/L$, leading to a loss factor of L per step in this hybrid argument. Furthermore, the guess at the beginning of a hybrid sub-step about the instance targeted in the ι th query to oracle Chall_{IE} is correct with probability at least $1/q_{\text{Gen}}$. Therefore, the entire hybrid argument induces a loss factor of at most $q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot L$, where q_{Gen} and q_{Chall} are the number of queries to oracles Gen_{IE} and Chall_{IE} by adversary \mathcal{A} . Consequently, $\text{Adv}^{G_1} = q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot L \cdot \text{Adv}^{G_2}$.

Game 3. In this Game 3 we change how the challenge oracle Chall_{IE} works and for that maintain a list L_{C^*} . In particular, for a challenge query to oracle Chall_{IE} with input id , either use the guess (id, l^*) from game 2 or, if $XP \neq \emptyset$, identify the branching level l^* directly. For each such (id, l^*) sent to Chall_{IE} it runs $(c_{l^*}, k_{l^*}) \leftarrow \text{IE}'.\text{enc}(ek_{l^*}, id \| \text{suf})$, where $\text{suf} = 0$ if $l^* = |id|/\lambda - 1$, and $\text{suf} = 1$ otherwise, then samples $k \leftarrow_{\S} \mathcal{K}_{l^*}$ and sets $(c_{l^*}, k_{l^*}) \leftarrow (c_{l^*}, k)$, i.e., replaces the key of the l^* th instance with a uniform random key, and then assembles the challenge ciphertext c^* and key k^* . We set $L_{C^*}[c_{l^*}] \leftarrow k$, so that on every decapsulation query that contains this ciphertext component, we use the respective key consistently.

Transition Game 2-3. We can argue this via the chosen-ciphertext security of IE' . Reduction $\mathcal{B}_{\text{IE}'}$ issues a query to oracle $\text{Chall}_{\text{IE}'}$ with inputs $(l^*, id_{l^*} \| \text{suf})$, where $\text{suf} = 0$ if $l^* = |id|/\lambda - 1$, and $\text{suf} = 1$ otherwise. Oracle $\text{Chall}_{\text{IE}'}$ responds with a ciphertext-key pair (c_{l^*}, k_{l^*}) and the final concatenated UPIBE challenge ciphertext c^* contains c_{l^*} and for the final key k^* is obtained by using k_{l^*} in W . (All remaining IBE IE' ciphertexts and keys in a challenge query are computed honestly by $\mathcal{B}_{\text{IE}'}$ itself.) Thus, only IBE IE' ciphertext and key of branching level l^* are replaced in the composed UPIBE ciphertext and

key, respectively. Depending on the challenge bit of $\mathcal{B}_{\text{IE}'}$'s challenger in the experiment $\text{IND}_{\text{IE}'}^b$, we either simulate Game 2 or Game 3 and thus as a result we have that $\text{Adv}^{G_2} \leq \text{Adv}^{G_3} + \text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B}_{\text{IE}'})$.

We now construct a hybrid over q_{Chall} steps in order to replace the key k^* output by calls to Chall_{IE} with uniformly random keys. However, since this change will add an artifact to the decapsulation procedure, i.e., queries that contain a ciphertext c_{l^*} corresponding to the respective branching identity id_{l^*} are not answered with W but with uniform keys k , we need to add another q_{Chall} hybrids to remove this artifact again. We will use two consecutive hybrids to do (the odd ones $i = 1, 3, \dots$) and undo (the even ones $j = 2, 4, \dots$) these changes.

Game 4.i. Initialize variables \mathbf{C} , \mathbf{C}_i and a list L . On the i 'th query to Chall_{IE} with branching identity id on position l^* we replace the key k^* obtained from W by a uniformly random key $k^* \leftarrow_{\S} \mathcal{K}$ (and note that we already sample k_{l^*} uniformly at random). Let c_{l^*} be the corresponding IBE IE' ciphertext in the overall ciphertext C^* of IE, then we keep track of the l^* 'th ciphertext $\mathbf{C}_i \leftarrow c_{l^*}$ and $\mathbf{C} \leftarrow C^*$. On each query to IE.dec with ciphertext C we do the following (if $C = \mathbf{C}$ the query is not allowed and we return \perp): If we already have the ciphertext in the list, i.e., $L[c] \neq \perp$, we return $L[c]$. Otherwise if the l^* 'th ciphertext component of C equals \mathbf{C}_i , we use k_{l^*} as the corresponding key, evaluate W and save its output to $L[c]$. Now, if $c_{l^*} = \mathbf{C}_i$, then we overwrite $L[c]$ with a random key $k \leftarrow_{\S} \mathcal{K}$ and finally we return $L[c]$.

Game 4.j. Here, we undo the changes from the $j - 1$ 'th Chall_{IE} query and in particular, if $c_{l^*} = \mathbf{C}_j$, we no longer overwrite $L[c]$ with a uniformly random key $k \leftarrow_{\S} \mathcal{K}$ but use the output of W again.

Transition Game 3-4.1/4.2-4.2 $q_{\text{Chall}} - 1$. We can argue this via the split-key pseudorandomness of W. In particular within the i 'th call to oracle Chall_{IE} we compute all the ciphertexts and thus the overall ciphertext C^* using the single IBE IE' instances (padded accordingly with the ciphertext \hat{c}). Except for the l^* 'th position of the branching identity id , reduction \mathcal{B}_W issues a call to $\text{Eval}(k', C^*)$, where k' is the padded key that omits the key at position l^* . Similarly for every call to IE.dec which contains c_{l^*} , i.e., $c_{l^*} = \mathbf{C}_i$, \mathcal{B}_W equivalently makes a call to Eval . Depending on the challenge bit of \mathcal{B}_W 's challenger in the experiment PR_i^b we either simulate Game 3/4.j (where in case of PR_i^0 we implicitly set k_{l^*}) or Game 4.i and thus as a result we have that $\text{Adv}^{G_3} \leq \text{Adv}^{G_{4.2q_{\text{Chall}}}} + q_{\text{Chall}} \text{Adv}_{F_i}^{\text{pr}}(\mathcal{B}_W)$.

Transition Game 4.1-4.2 q_{Chall} . We can argue this via the split-key pseudorandomness of W and the reduction works analogous to above with the only difference that for l^* 'th position of the branching identity id in the Chall_{IE} , the reduction no longer needs to call $\text{Eval}(k', C^*)$, but again choose a uniform key $k^* \leftarrow_{\S} \mathcal{K}$. As a result, we have $\text{Adv}^{G_{4.2q_{\text{Chall}}}} \leq \text{Adv}^{G_{4.2q_{\text{Chall}}}} + q_{\text{Chall}} \text{Adv}_{F_i}^{\text{pr}}(\mathcal{B}_W)$.

Game 5. In this Game we revert the change from Game 3 and in particular for each branching identity id on position l^* send to Chall_{IE} we no longer use the randomly sampled key $k \leftarrow_{\S} \mathcal{K}_{l^*}$ to replace the key k_{l^*} obtained via $(c_{l^*}, k_{l^*}) \leftarrow \text{IE}'.\text{enc}(ek_{l^*}, id \parallel \text{suf})$, where $\text{suf} = 0$ if $l^* = |id|/\lambda - 1$, and $\text{suf} = 1$ otherwise. Consequently, we again use the real output of the l^* 'th IBE IE' instance and then assemble the challenge ciphertext. Note that keys k^* output by Chall_{IE} are still chosen uniformly at random from \mathcal{K} .

Transition Game 4.2 $q_{\text{Chall}} - 5$. This is identical to the transition from Game 3 to Game 4 and follows from the chosen-ciphertext security of IE' . As a result, we have $\text{Adv}^{G_{4.2q_{\text{Chall}}}} \leq \text{Adv}^{G_5} + \text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B}_{\text{IE}'})$.

Game 6. In this Game we revert the change from Game 2. Consequently, we have that $\text{Adv}^{G_5} \leq \text{Adv}^{G_6} + q_{\text{Gen}} \cdot \mathbb{L}^2 \cdot \text{Adv}_{\text{G}}^{\text{ind}}(\mathcal{B}_{\text{G}})$.

Game 7. This Game represents the game IND_{IE}^1 and thus $\text{Adv}^{G_7} = \Pr[\text{IND}_{\text{IE}}^1(\mathcal{A}) = 1]$.

Note that Game 6 and Game 7 are identical up to the fact that we no longer need to guess any branch identities for Chall_{IE} , i.e., $\text{Adv}^{G_6} = \frac{1}{q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot \mathbb{L}} \cdot \text{Adv}^{G_7}$.

Proof Result. Overall we obtain $\text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{A}) \leq$

$$q_{\text{Gen}} \cdot \mathbb{L}^2 \cdot \left(q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot \mathbb{L} \cdot \text{Adv}_{\text{G}}^{\text{ind}}(\mathcal{B}_{\text{G}}) + 1 \right) + 2q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot \mathbb{L} \cdot \left(q_{\text{Chall}} \cdot \text{Adv}_{F_i}^{\text{pr}}(\mathcal{B}_W) + \text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B}_{\text{IE}'}) \right)$$

which concludes the proof. \square

C.2 Unbounded-Depth UPIBE

To prove Theorem 5, we can follow exactly the same strategy as in the proof of Theorem 3, but before the final reduction step, we introduce an additional hybrid to consider the core function W modeled as a random oracle H^* .

Randomize Challenge Key. We now use a second random oracle H^* that implements the core function W . As in our proof of Theorem 3, we use this random oracle to replace the challenge key of ι th relevant challenge query of instance ρ by a random key.

We now concentrate on the relevant challenge (ρ, id) added in step ι of the hybrid argument with length l_{id} and branch β_{id} . Note that the end of the corresponding ι th query to oracle Chall_{IE} of instance ρ , a query to random oracle H^* with inputs $(K, C) = (k_0, \dots, k_{l_{id}-1}, c_0, \dots, c_{l_{id}-1})$ is issued. Before issuing this random oracle query, in hybrid step ι one replaces the $(\lceil \beta_{id}/\varepsilon \rceil - 1)$ th key component of K with a randomly sampled key. Thus, the actual issued random oracle query at the end of this challenge query has input $(K^*, C) = (k_0, \dots, k_{\lceil \beta_{id}/\varepsilon \rceil - 2}, k^*, k_{\lceil \beta_{id}/\varepsilon \rceil}, \dots, k_{l_{id}-1}, c_0, \dots, c_{l_{id}-1})$. If the adversary ‘externally’ issued a query to the random oracle with the same inputs (K^*, C) , too, the hybrid in step ι of instance ρ aborts. Note that the total number of challenge keys is bounded by q_{Chall} and the total number of guesses by the adversary \mathcal{A} is bound by q_{H^*} . Consequently, the probability to abort in this hybrid is $\frac{q_{\text{Chall}} \cdot q_{H^*}}{|\mathcal{K}|}$.

Reduction. Finally, we see that distinguishing between hybrids in step ι of instance ρ can be reduced to breaking IND security of the underlying chosen-ciphertext selectively secure bounded-depth HIBE scheme IE' . This is true because our reduction $\mathcal{B}_{\rho, \iota}$ can use the committed HIBE identity string at the beginning of Game IND to fix the selective HIBE challenge. This challenge is embedded in UPIBE epoch $\lceil \beta_{id}/\varepsilon \rceil - 1$ of UPIBE challenge ι for instance ρ . Note that for every call to Dec_{IE} with id and C , if C is the output of a call to Chall_{IE} we return \perp . Otherwise, we run the required decapsulations to obtain K (for the replaced component of K we can use the challenge key) and return $H^*(K, C)$. The only way adversary \mathcal{A} can distinguish the hybrids is by querying the random oracle H^* on this challenged key, which solves the HIBE challenge. We omit repeating the arguments from the proofs of Theorem 3 for why HIBE challenges remain valid based on the validity of UPIBE challenges.

Proof Result. After combining all game hops of our hybrid arguments, an adversary in final step $\rho = q_{\text{Gen}}, \iota = q_{\text{Chall}}$ can only guess the challenge bit randomly. Thus, combining the loss factors from all game hops, we obtain

$$\text{Adv}_{\text{IE}}^{\text{ind}}(\mathcal{A}) \leq q_{\text{Gen}} \cdot q_{\text{Chall}} \cdot ((l_{\text{long}})^2 \cdot (q_H)^\varepsilon) \cdot \left(\text{Adv}_{\text{IE}'}^{\text{ind}}(\mathcal{B}) + \frac{q_{\text{Chall}} \cdot q_{H^*}}{|\mathcal{K}|} \right).$$

□

D Full Details on KU-KEM from UPIBE

We first formally define KU-KEM in line with prior definitions by [PR18b, BRV20] and then provide a proof of Theorem 6 from Section 6.

Syntax. A KU-KEM scheme is a quadruple $K = (K.\text{gen}, K.\text{enc}, K.\text{dec}, K.\text{up})$ of algorithms whose syntax is defined as follows¹⁶:

- $K.\text{gen} : \emptyset \rightarrow_{\mathfrak{s}} \mathcal{EK} \times \mathcal{DK}$
- $K.\text{enc} : \mathcal{EK} \rightarrow_{\mathfrak{s}} \mathcal{EK} \times \mathcal{C} \times \mathcal{K}$
- $K.\text{dec} : \mathcal{DK} \times \mathcal{C} \rightarrow \mathcal{DK} \times \mathcal{K}$
- $K.\text{up} : \mathcal{EK} \times \{0, 1\}^\lambda \rightarrow \mathcal{EK}$ resp. $K.\text{up} : \mathcal{DK} \times \{0, 1\}^\lambda \rightarrow_{\mathfrak{s}} \mathcal{DK}$

¹⁶ In contrast to prior work [PR18b, BRV20], we define the KU-KEM decapsulation algorithm as well as the KU-KEM update algorithm for decapsulation keys probabilistic. The reason for this is that, in our KU-KEM construction, these two algorithms use the probabilistic UPIBE delegation algorithm. Using a PRF to de-randomize the UPIBE delegation algorithm (and thereby the two probabilistic UPIBE algorithms) is immediate as already noted in [PR18b, BRV20].

Correctness. For correctness of KU-KEM, we require for all $(ek_0, dk_0) \leftarrow_{\S} \text{K.gen}$, all $ad = (ad_1, \dots, ad_l)$ with $ad_i \in \{0, 1\}^\lambda \cup \{\perp\}$, $0 < i \leq l$, all $ek_i \leftarrow \text{K.up}(ek_{i-1}, ad_i)$ and all $dk_i \leftarrow_{\S} \text{K.up}(dk_{i-1}, ad_i)$ if $ad_i \neq \perp$, all $(ek_i, c_i, k_i) \leftarrow_{\S} \text{K.enc}(ek_{i-1})$ and all $(dk_i, k'_i) \leftarrow_{\S} \text{K.dec}(dk_{i-1}, c_i)$ otherwise, that $k_i = k'_i$ for all $0 < i \leq l$.

Security. As it suffices both for the equivalence result in [BRV20] and to instantiate the ratcheted key exchange definitions in [PR18b], we use the one-way notion KUOW_K from [BRV20] to define security of a KU-KEM scheme K . This notion is formalized in Figure 9.

Definition 4. *The advantage of \mathcal{A} in winning KUOW_K is $\text{Adv}_K^{\text{kuow}}(\mathcal{A}) := \Pr[\text{KUOW}_K(\mathcal{A}) = 1]$.*

Game $\text{KUOW}_K(\mathcal{A})$	Oracle $\text{Solve}(i, tr, k)$
00 $n \leftarrow 0$	22 Require $1 \leq i \leq n$
01 Invoke \mathcal{A}	23 Require $tr \notin XP_i$
02 Stop with 0	24 Require $CK_i[tr] \neq \perp$
Oracle Gen	25 Reward $k = CK_i[tr]$
03 $n \leftarrow n + 1$	26 Return
04 $(ek_n, dk_n) \leftarrow_{\S} \text{K.gen}$	Oracle $\text{Up}_D(i, ad)$
05 $CK_n[\cdot] \leftarrow \perp$; $XP_n \leftarrow \emptyset$	27 Require $1 \leq i \leq n \wedge ad \in \mathcal{AD}$
06 $trs_n \leftarrow \epsilon$; $trr_n \leftarrow \epsilon$	28 $dk_i \leftarrow_{\S} \text{K.up}(dk_{i-1}, ad)$
07 $DK_n[\cdot] \leftarrow \perp$	29 $trr_i \leftarrow^u ad$
08 $DK_n[trr_n] \leftarrow dk_n$	30 $DK_i[trr_i] \leftarrow dk_i$
09 Return ek_n	31 Return
Oracle $\text{Up}_E(i, ad)$	Oracle $\text{Dec}(i, c)$
10 Require $1 \leq i \leq n \wedge ad \in \mathcal{AD}$	32 Require $1 \leq i \leq n \wedge c \in \mathcal{C}$
11 $ek_i \leftarrow \text{K.up}(ek_{i-1}, ad)$	33 $(dk_i, k) \leftarrow_{\S} \text{K.dec}(dk_{i-1}, c)$
12 $trs_i \leftarrow^u ad$	34 $trr_i \leftarrow^u c$
13 Return ek_i	35 $DK_i[trr_i] \leftarrow dk_i$
Oracle $\text{Enc}(i, r)$	36 If $CK_i[trr_i] \neq \perp$:
14 Require $1 \leq i \leq n$	37 Return
15 Require $r \in \mathcal{R} \cup \{\epsilon\}$	38 Return k
16 If $r = \epsilon$: $mr \leftarrow \text{fal}$; $r \leftarrow_{\S} \mathcal{R}$	Oracle $\text{Exp}(i, tr)$
17 Else: $mr \leftarrow \text{tru}$	39 Require $1 \leq i \leq n$
18 $(ek_i, c, k) \leftarrow \text{K.enc}(ek_{i-1}; r)$	40 Require $DK_i[tr] \in \mathcal{DK}$
19 $trs_i \leftarrow^u c$	41 $XP_i \leftarrow^u \{tr^* \in (\mathcal{AD} \cup \mathcal{C})^* :$
20 If $mr = \text{fal}$: $CK_i[trs_i] \leftarrow k$	$tr \prec tr^*\}$
21 Return (ek_i, c)	42 Return $DK_i[tr]$

Fig. 9: Security experiment KUOW from [BRV20], modeling one-way security of key-updatable KEM in a multi-instance/multi-challenge setting under randomness manipulation. Line 41 is a shortcut notion that can be implemented efficiently. CK : challenge keys, XP : exposed decapsulation keys, trs , trr : transcripts.

Construction. The KU-KEM construction by [BRV20] in Figure 5 is a variant of the one presented in [PR18b]. It realizes encapsulation key updates by concatenating the input associated data strings. Corresponding decapsulation key updates are based on UPIBE (or HIBE in [PR18b, BRV20]) delegations. Both KU-KEM encapsulation and decapsulation use the underlying UPIBE encapsulation and decapsulation algorithm, respectively, and then perform an update of the respective key pair component with respect to the en-/decapsulated ciphertext.

Proof. The reduction proof for Theorem 6 is as straight forward as the construction: Each KU-KEM instance i is simulated by, and reduced to, a UPIBE instance i . For this, all UPIBE algorithms in KU-KEM K are simulated via oracles from game IND. (K.gen in Gen_K with Gen_{IE} , K.up in Up_E directly, K.up in Up_D with Del_{IE} , K.dec in Dec_K with Dec_{IE} ; Exp_K with Exp_{IE} , Enc_K and Solve_K are explained blow.) UPIBE challenges are embedded in all queries to oracle Enc for which $r = \epsilon$ holds. The respective UPIBE challenge keys are stored just as normal UPIBE keys in array CK_i . In queries to oracle Enc for which we have $r \neq \epsilon$, algorithm K.enc is honestly executed by the reduction. If a query to oracle Solve is issued with a key that matches the corresponding UPIBE challenge key stored in

array CH_i , reduction \mathcal{B} immediately terminates and returns 0; if this never happens, reduction \mathcal{B} waits until adversary \mathcal{A} terminates and then terminates itself with return value 1. We observe that a successful adversary \mathcal{A} must provide a correct key during the execution of game KUOW. If no key provided to oracle Solve is correct, then the UPIBE challenge was a random key. In contrast, if a key provided to oracle Solve is correct, then the UPIBE challenge was a real key. We conclude our reduction by observing that the conditions under which challenges in games KUOW and IND remain valid are the same. \square

D.1 Generic Performance Summary

Table 10 depicts the asymptotic key and ciphertext sizes depending on the number of (en- resp. decapsulation) key updates l and the depth per epoch ε . See that the KU-KEMs in [PR18b, BRV20] rely on unbounded-depth HIBEs while our KU-KEM can be built from a bounded-depth HIBE.

Schemes	ek size	dk size	c size	# IE.enc	Depth of HIBE
Trivial [PR18b, JS18, BRV20]	1	1	1	1	$\mathcal{O}(l)$
Ours	$\mathcal{O}(l/\varepsilon)$	$\mathcal{O}(l/\varepsilon)$	$\mathcal{O}(l/\varepsilon)$	$\mathcal{O}(l/\varepsilon)$	$\alpha + \varepsilon$ $= \lceil \log(2^\kappa/\varepsilon) \rceil + \varepsilon$

Fig. 10: Comparison between KU-KEM constructions: Maximal size factor of KU-KEM en- resp. decapsulation keys and ciphertexts in relation to their HIBE equivalents; maximal number of HIBE encapsulations per KU-KEM encapsulation; and maximal depth of used HIBE, if l is the number of en- resp. decapsulation key updates, and ε is the depth per epoch in our KU-KEM construction from Figure 5.

More concretely, our unbounded-depth UPIBE is parameterized by the identity string length per delegation λ , security parameter κ , and epoch depth ε . It can be built from a bounded-depth HIBE with depth bound $L = \alpha + \varepsilon = \lceil \log(2^\kappa/\varepsilon) \rceil + \varepsilon$ and identity string length $\lambda + 1$. The UPIBE encapsulation key consists of one HIBE main encapsulation key, the UPIBE decapsulation key delegated l times consists of $\alpha + \lceil l/\varepsilon \rceil$ delegated HIBE decapsulation keys, where the first α decapsulation keys are delegated to levels between 1 and α (more precisely, one decapsulation key per level), the next $\lceil l/\varepsilon \rceil + 1$ decapsulation keys are delegated to level $\alpha + \varepsilon$, and the last decapsulation key is delegated to a level between α and $\alpha + \varepsilon$. A UPIBE ciphertext for an identity string of length $\lambda \cdot l$ consist of $\lceil l/\varepsilon \rceil$ HIBE ciphertexts, where each of the first $\lceil l/\varepsilon \rceil - 1$ ciphertexts targets an identity string of length $\lambda \cdot (\alpha + \varepsilon)$, and the last ciphertext targets an identity string of length between $\lambda \cdot \alpha$ and $\lambda \cdot (\alpha + \varepsilon)$.

E Bounded-Depth UPIBE From DDH

To demonstrate the practicality of our bounded-depth UPIBE construction from Figure 3, we instantiate it with an adjusted variant of the bounded-collusion IBE by Dodis et al. [DKXY02] in Figure 11. For comprehensibility, we only consider the simpler chosen-plaintext secure variant from [DKXY02]. Our adjustments equally apply to their slightly more complicated chosen-ciphertext secure variant.

Without any adjustments, the direct instantiation of our UPIBE with this IBE results in the following variable sizes: a UPIBE encapsulation key consists of $3 \cdot L$ group elements; a UPIBE decapsulation key consists of $2 + l \cdot 2$ exponents and one symmetric key, where l is the number of applied delegations; a UPIBE ciphertext consists of $l \cdot 2$ group elements, where l is the number of applied derivations, resp., the number of sub-string components of the input identity string.

We take advantage of the group structure to aggregate encapsulation keys and decapsulation keys, and we eliminate unnecessary redundancy in the ciphertext. More concretely, we multiply the identity-specific *ordinary* IBE encapsulation keys of all prior levels to one aggregated group element in Figure 11, line 30. Correspondingly, we add the identity-specific *ordinary* IBE decapsulation keys of all prior levels to one aggregated exponent in line 43. The ciphertext of the original IBE consists of a random twin-DH share. Instead of generating an independent share for each used IBE instance, our adjusted ciphertext contains only one such share (see line 14).

Since our security proof uses a hybrid argument that considers only one IBE instance at each step, the remaining instances can be seen as independent group elements, resp., exponents during this step that re-randomize the keys and ciphertext of the considered single instance. Hence, our adjustments do not affect the security proof. The same holds for the elegant proof by Dodis et al. [DKXY02].

<pre> Proc IE.gen 00 $ek'_1 \leftarrow 1; EK[\cdot] \leftarrow \perp$ 01 $dk'_1 \leftarrow (0, 0); D \leftarrow (0, 0)$ 02 $fk_0 \leftarrow_{\S} \{0, 1\}^{\kappa}$ 03 For $l = 0$ to $L - 1$: 04 $(fk_{l+1}, S) \leftarrow G(fk_l)$ 05 For $t : 0 \leq t \leq 2$: 06 $(x_t^*, y_t^*) \leftarrow S_t$ 07 $z_t^* \leftarrow g^{x_t^*} h^{y_t^*}$ 08 $EK[l] \leftarrow (z_t^*)_{0 \leq t \leq 2}$ 09 $ek \leftarrow (E, ek'_0, ek'_1, EK)$ 10 $dk \leftarrow (0, D, \perp, dk'_1, fk_0)$ 11 Return (ek, dk) Proc IE.enc(ek, id) 12 Require $id \in \{0, 1\}^{l \cdot \lambda} \cup \{\epsilon\}, 0 < l \leq L$ 13 $r \leftarrow_{\S} \mathbb{Z}_p$ 14 $c \leftarrow (g^r, h^r)$ 15 If $id \neq \epsilon$: 16 $id_0 \parallel \dots \parallel id_{l-1} \leftarrow id$ with $id_j \in \{0, 1\}^{\lambda}$ 17 For $j : 0 \leq j < l$: 18 $ek \leftarrow_{\S} \text{IE.der}(ek, id_j)$ 19 $(E, ek'_0, ek'_1, EK) \leftarrow ek$ 20 $k \leftarrow (E \cdot ek'_0)^r$ 21 Return (c, k) Proc IE.dec(dk, c) 22 $(l, D, dk'_0, dk'_1, fk) \leftarrow dk$ 23 $(u, v) \leftarrow c$ 24 $(X, Y) \leftarrow D$ 25 $(x_0, y_0) \leftarrow dk'_0$ 26 $k \leftarrow u^{X+x_0} v^{Y+y_0}$ 27 Return k </pre>	<pre> Proc IE.der(ek, id) 28 Require $id \in \{0, 1\}^{\lambda}$ 29 $(E, ek'_0, ek'_1, EK) \leftarrow ek$ 30 $E \leftarrow E \cdot ek'_1$ 31 $l \leftarrow \min(l : EK[l] \neq \perp)$ 32 $(z_t^*)_{0 \leq t \leq 2} \leftarrow EK[l]$ 33 $ek'_0 \leftarrow \prod_{t=0}^2 (z_t^*)^{(id \parallel 0)^t}$ 34 $ek'_1 \leftarrow \prod_{t=0}^2 (z_t^*)^{(id \parallel 1)^t}$ 35 $EK[l] \leftarrow_{\S} \perp$ 36 $ek \leftarrow (E, ek'_0, ek'_1, EK)$ 37 Return ek Proc IE.del(dk, id) 38 Require $id \in \{0, 1\}^{\lambda}$ 39 $(l, D, dk'_0, dk'_1, fk) \leftarrow dk$ 40 Require $l < L$ 41 $(X, Y) \leftarrow D$ 42 $(x_1, y_1) \leftarrow dk'_1$ 43 $D \leftarrow (X + x_1, Y + y_1)$ 44 $(fk', S) \leftarrow G(fk)$ 45 For $t : 0 \leq t \leq 2$: 46 $(x_t^*, y_t^*) \leftarrow S_t$ 47 $x_0 \leftarrow \sum_{t=0}^2 (id \parallel 0)^t x_t^*$ 48 $y_0 \leftarrow \sum_{t=0}^2 (id \parallel 0)^t y_t^*$ 49 $x_1 \leftarrow \sum_{t=0}^2 (id \parallel 1)^t x_t^*$ 50 $y_1 \leftarrow \sum_{t=0}^2 (id \parallel 1)^t y_t^*$ 51 $dk'_0 \leftarrow (x_0, y_0)$ 52 $dk'_1 \leftarrow (x_1, y_1)$ 53 $dk \leftarrow (l + 1, D, dk'_0, dk'_1, fk')$ 54 Return dk </pre>
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Fig. 11: Adjusted instantiation of our bounded-depth UPIBE from Figure 3 with DDH-based bounded-collision IBE by Dodis et al. [DKXY02]. Making use of the group structure, we aggregate encapsulation keys, ciphertexts, and decapsulation keys.

Due to our adjustments, the enhanced instantiation of our UPIBE from Figure 11 has the following variable sizes: a UPIBE encapsulation key consists of $2 + 3 \cdot (L - l)$ group elements, where l is the number of applied derivations; a UPIBE decapsulation key consists of $2 + 2 + 2$ exponents and one symmetric key; a UPIBE ciphertext consists of 2 group elements.

Corollary 1. *Bounded-depth UPIBE protocol IE from Figure 11 offers adaptive key indistinguishability in the standard model. More precisely, for every adaptive chosen-plaintext adversary \mathcal{A} attacking protocol IE in games IND_{IE}^b according to Definition 1 with parameters $(L, \lambda, D = 1)$ and non-negligible advantage, there exists an adversary \mathcal{B}_{G} breaking the PRG G according to Definition 3 and an adversary \mathcal{B}_{DDH} breaking the DDH problem.*