

# A Fourier Analysis Based Attack against Physically Unclonable Functions

Fatemeh Ganji, Shahin Tajik, Jean-Pierre Seifert

Security in Telecommunications  
Technische Universität Berlin  
Germany

{fganji,stajik,jpseifert}@sec.t-labs.tu-berlin.de

**Abstract.** Electronic payment systems have leveraged the advantages offered by the RFID technology, whose security is promised to be improved by applying the notion of Physically Unclonable Functions (PUFs). Along with the evolution of PUFs, numerous successful attacks against PUFs have been proposed in the literature. Among these are machine learning (ML) attacks, ranging from heuristic approaches to provable algorithms, that have attracted great attention. Our paper pursues this line of research by introducing a Fourier analysis based attack against PUFs. More specifically, this paper focuses on two main aspects of ML attacks, namely being provable and noise tolerant. In this regard, we prove that our attack is naturally integrated into a provable Probably Approximately Correct (PAC) model. Moreover, we show that our attacks against known PUF families are effective and applicable even in the presence of noise. Our proof relies heavily on the intrinsic properties of these PUF families, namely arbiter, Ring Oscillator (RO), and Bistable Ring (BR) PUF families. We believe that our new style of ML algorithms, which take advantage of the Fourier analysis principle, can offer better measures of PUF security.

**Keywords:** Physically Unclonable Functions, Boolean Analysis, Noise Sensitivity, Low Degree Algorithm, Machine Learning, PAC Learning.

## 1 Introduction

Payment systems, including electronic payment and ticketing systems, provide one of the prominent examples of the diversified applications of RFID-tags. As an effective and cost-efficient security mechanism for these tags, PUFs have been introduced in the literature cf. [10, 48, 49]. The design of PUFs relies on inherent manufacturing process variations, being uncontrollable, but exploitable by a circuitry to generate either a source of randomness or an instance-specific fingerprint [18]. The growing need for using PUFs in several applications stems from two main issues. On the one hand, the ineffectiveness of traditional security measures, e.g., secure key generation/storage, has been widely accepted. On the other hand, the inevitable fact that overbuilt and counterfeit hardware primitives can be used in various important applications further contribute to this need for

robust security measures [26]. Since the notion of PUFs has been introduced to address the aforementioned issues, several studies have focused on the advantages and disadvantages of this concept. Designing such circuits and their respective security assessments, more particularly, cryptanalysis of PUFs are within two ends of the wide spectrum of these studies.

In addition to invasive and semi-invasive attacks, e.g., [20, 37, 45–47], a broad range of cryptanalysis of PUFs is covered by non-invasive attacks, for instance [42]. A great variety of these frameworks and numerous models have been developed around the principles of linear algebra [11], stochastic optimization [5], and machine learning (ML) [12, 14–16, 23, 42]. When launching the latter attacks, the adversary observes only a small subset of challenges and their corresponding responses (i.e., the inputs and the outputs of the PUF) in order to build a model of the challenge-response behavior of the PUF. Therefore, when compared with invasive and semi-invasive attacks, these attacks are cost-effective and non-destructive, and consequently, attractive for adversaries.

Applying empirical ML algorithms (e.g., [42]) in the assessment of the security of PUFs marked the beginning of an era, after which the well-established concepts and existing algorithms in the field of ML were applied to analyze the security of these primitives. Beyond the early heuristic methods, probably approximately correct (PAC) learning frameworks have been developed to *prove* vulnerabilities for the known families of intrinsic PUFs to ML attacks [12, 14–16, 19]. The results of these studies have been acknowledged, and form now a solid basis for the design of PUFs, c.f. [51]. Albeit being useful for this purpose, the question remains open whether practical aspects of the design of PUFs have been adequately reflected by the PAC learning frameworks. More specifically, except the proof provided for XOR-arbiter PUFs [15], PAC learning in the presence of noise has not been discussed in the literature so far.

This issue is of twofold importance. First, the term “noise” in the PUF-related literature refers to the observation that applying the same challenge may result in obtaining different responses due to the environmental changes, see, e.g. [30]. These noisy responses reveal some information about the challenge-response behavior of the PUF, in a similar way to side channel information, which can be beneficial to model the PUF [5, 8, 9]. Understanding the mechanisms of generating noisy responses is therefore essential for designing a PUF that is robust against such hybrid attacks. Secondly, the gap between the existing noise models in the ML- and PUF-related literature should be bridged primarily by a thorough understanding of differences and similarities between these models. Accordingly, a refined model of noisy PUFs should be established, which provides a firm basis for analyzing the security of these primitives against ML attacks. This paper aims to address these issues by providing the following contributions.

**Establishing a refined model of noisy PUFs that is in line with models widely accepted in ML theory.** In our model, we take into consideration the impact of noise on the final response of a PUF and as well at the inter-stage behavior of a PUF. We demonstrate that this model agrees with the noise models in ML theory, namely, attribute and classification noise.

**Introducing a new ML attack relying on the principles of Fourier analysis.** Thanks to the representation of PUFs as Boolean functions, we explore the properties of PUFs from the Fourier analysis perspective. We introduce the notion of noise sensitivity of Boolean functions representing PUFs as a powerful analysis tool. Moreover, for known and widely-used PUFs a so-called low degree algorithm approximating the Fourier coefficients of the corresponding Boolean functions is presented in this paper.

**Provability of our ML attack, even in the presence of attribute and classification noise.** Eventually, we prove that for known families of PUFs our attack can be launched to learn their challenge-response behavior, with prescribed levels of accuracy and confidence, even if the challenge-response pairs are noisy.

## 2 Notation and preliminaries

### 2.1 PUFs

First, we stress that our paper does not cover the topics of formalization and formal definitions of the PUFs. For more details on these topics see, e.g., [3, 4]. Note that hereafter the term “PUF” refers to the most popular, and known families of standalone PUFs: arbiter PUFs, Ring Oscillator (RO) PUFs, and Bistable Ring (BR) PUFs. Here, a standalone PUF means a PUF that is not composed of a combination of some PUFs (e.g., XOR arbiter PUFs) or other means. Generally speaking, PUFs are physical mappings from the inputs to the outputs, i.e., from the given *challenges* to the respective *responses*. These mappings are characterized by physical properties of the platform, on which the PUF is implemented. From among several security properties of PUFs, here we consider solely unclonability. Let the mapping  $f_{\text{PUF}} : \mathcal{C} \rightarrow \mathcal{Y}$ , where  $f_{\text{PUF}}(c) = y$ , describes a PUF. Ideally, for a given PUF  $f_{\text{PUF}}$  unclonability reflects the fact that creating a clone, i.e., a (physical) mapping  $g_{\text{PUF}} \neq f_{\text{PUF}}$ , is virtually impossible, where the challenge-response behavior of  $g_{\text{PUF}}$  is *similar* to  $f_{\text{PUF}}$  [3].

### 2.2 Boolean Functions as representations of PUFs

Similar to the most relevant studies on PUFs, we adopt the general definition of PUFs that is the physical mappings (see Sec. 2.1). This enables us to represent PUFs as Boolean functions over the finite field  $\mathbb{F}_2$ . To this end, consider  $V_n = \{c_1, c_2, \dots, c_n\}$  that is the set of Boolean attributes or variables, being either *true* or *false* denoted by “1” and “0”, respectively. Moreover, let  $C_n = \{0, 1\}^n$  be the set of all binary strings with  $n$  bits, and an *assignment* be a mapping from  $V_n$  to  $\{0, 1\}$ . Therefore, an assignment can be thought of as an  $n$ -bits string, where the  $i^{\text{th}}$  bit associated with the value of  $c_i$  (i.e., “0” or “1”).

A Boolean formula is a mapping that assigns values from the set  $\{0, 1\}$  to an assignment. In this regard, each Boolean attribute is also a formula, i.e.,  $c_i$  is a possible formula. If the Boolean formula assigns “1” to a Boolean assignment, it is a *positive example* of the concept, otherwise a *negative example*. Furthermore, a Boolean function  $f : C_n \rightarrow \{0, 1\}$  defines a Boolean formula accordingly.

In general, Boolean functions can be represented by several different classes of functions, e.g., juntas, Linear Threshold functions (LTFs), and Decision Lists (DLs), cf. [38, 41]. A  $k$ -junta is a Boolean function, whose output is determined solely by an unknown set of  $k$  variables. A list  $L$  containing  $r$  pairs  $(f_1, v_1), \dots, (f_r, v_r)$  is called a DL, where the Boolean formula  $f_i$  is a conjunction of Boolean attributes, and  $v_i \in \{0, 1\}$  with  $1 \leq i \leq r - 1$ . For  $i = r$ , we have  $v_r = 1$ . When representing a Boolean function by a decision list,  $L(c) = v_j$ , where  $c \in C_n$  and  $j$  is the smallest index in  $L$  so that  $f_j(c) = 1$ . Let  $k$ -DL denote the set of DLs, where each  $f_i$  is a conjunction of at most  $k$  Boolean attributes.

Before defining linear threshold functions, we define the encoding scheme  $\chi(0_{\mathbb{F}_2}) := +1$ , and  $\chi(1_{\mathbb{F}_2}) := -1$ . Hence, the Boolean function  $f$  can be defined as  $f : \{-1, +1\}^n \rightarrow \{-1, +1\}$ . Such a function is called a linear threshold function, if there are coefficients  $\omega_1, \omega_2, \dots, \omega_n \in \mathbb{R}$  and  $\theta \in \mathbb{R}$  such that  $f(c) = \text{sgn}((\sum_{i=1}^n \omega_i c_i) - \theta)$ . Without loss of generality, we assume that  $\sum_{i=1}^n \omega_i c_i \neq \theta$  for every  $c \in C_n$ .

**Noise Sensitivity of Boolean Functions** This term should not be mistaken as the notion of noise discussed in the PUF-related literature. The noise sensitivity of the Boolean function  $f : \{-1, +1\}^n \rightarrow \{-1, +1\}$  can be defined as follows (see Sec. 2.2 for more details on the encoding scheme required to define the noise sensitivity). Let  $c$  be a string chosen randomly and uniformly. By flipping each bit of this string independently with probability  $\varepsilon$  ( $0 \leq \varepsilon \leq 1$ ) we obtain the string  $c'$ . The noise sensitivity of  $f$  at  $\varepsilon$  is

$$NS_\varepsilon(f) := \Pr[f(c) \neq f(c')].$$

When studying the noise sensitivity of Boolean functions, applying methodologies developed for the spectral analysis of Boolean functions can provide a better understanding of this notion. The Fourier expansion of a Boolean function can be written as

$$f(c) = \sum_{S \subseteq [n]} \hat{f}(S) \chi_S(c),$$

where  $[n] := \{1, \dots, n\}$ ,  $\chi_S(c) := \prod_{i \in S} c_i$ , and  $\hat{f}(S) := \mathbf{E}_{c \in \mathcal{U}}[f(c) \chi_S(c)]$ . Note that  $\mathbf{E}_{c \in \mathcal{U}}[\cdot]$  indicates the expectation over examples chosen uniformly.

### 2.3 PAC Learning Model [24]

Consider a PAC learner that is a learning algorithm, which is given access to a set of *examples* to generate an approximately correct hypothesis, with high probability. More formally, suppose that  $F = \cup_{n \geq 1} F_n$  denotes a target concept class, i.e., a set of Boolean functions over the instance space  $C_n = \{0, 1\}^n$ . In this paper, we are interested in a useful extension of the PAC model, in which each example is drawn from the instance space  $C_n$  with regard to the uniform distribution  $U$ . The hypothesis  $h \in F_n$  that is a Boolean function over  $C_n$  is an  $\varepsilon$ -approximator for  $f \in F_n$ , if

$$\Pr_{c \in_U C_n} [f(c) = h(c)] \geq 1 - \varepsilon.$$

The complexity of a target concept  $f \in F$  is assessed by measuring the size of that under a target representation. In order to define the size of a target concept  $f \in F$ ,  $size(f)$ , we define the mapping  $size : \{0,1\}^n \rightarrow \mathbb{N}$ , relating a natural number  $size(f)$  with a target concept  $f \in F$ . A polynomial-time algorithm  $A$ , i.e., our learner, is provided with labeled examples  $(c, f(c))$ , where  $f \in F_n$ , and  $c$  is chosen uniformly at random from  $C_n$ . Here we concentrate on the strong uniform PAC learning algorithms, defined as follows.

**Definition 1** *A strong uniform PAC learning algorithm  $A$  for the target concept class  $F$  is given a polynomial number of labeled examples to generate an  $\varepsilon$ -approximator for  $f$  under the uniform distribution  $U$ , with probability at least  $1 - \delta$ . In this regard, for any  $n \geq 1$ , any  $0 < \varepsilon, \delta < 1$ , and any  $f \in F_n$ , the running time of the algorithm  $A$  is  $poly(n, 1/\varepsilon, size(f), 1/\delta)$ , where  $poly(\cdot)$  denotes a polynomial function.*

### 3 Noise: its Origin and Models

In this section we aim to come up with a model for PUFs enabling us to understand, how the noise can affect the functionality of a PUF. As mentioned in Sec. 1, in the PUF-related literature the response to a challenge is noisy, if repeated evaluations of the PUF with the respective challenge results in different responses. This is due to environmental variations and their impact on the functionality of physical components forming the PUF, e.g., the stages in an arbiter PUF. Environmental variations cover a wide range of uncontrollable random noise, e.g., thermal noise, uncertainties in measurement, cross talk, and power supply noise [3, 50]. According to the lessons from performance specifications of circuits, these random variations can be conventionally modeled as random variables following Gaussian distributions [2, 36, 44].

#### 3.1 Impact of the Noise on a Single Stage

To provide a better understanding of the impact of the environmental variations on the internal functionality of a PUF, we focus on a single constitutive physical component of the PUF, hereafter called a stage. As carefully formulated in [29], although the consideration of low-level physical details is a tedious task, (measurable) physical processes can be approximated by “hidden variables”, namely the process variable and the noise variable. The latter variable corresponds to the effect of random noise on a single stage of the PUF. This variable follows a Gaussian distribution  $N_i$ , with realization  $n_i$  for each evaluation of the PUF. The definition of process variable determines the effect of manufacturing process variations on a single stage of the PUF [29]. As discussed before, this variable denoted by  $X_i$  follows a Gaussian distribution. Similarly and independently,  $X_1, \dots, X_n$  can be defined, where  $n$  denotes the number of stages. In this manner, the mean value of the respective distribution ( $\mu$ ) is reported by manufactures as the nominal value, and the standard deviation  $\sigma_i$  is the result of the process variations, cf. [8, 16, 34]. Two realizations of the random variable

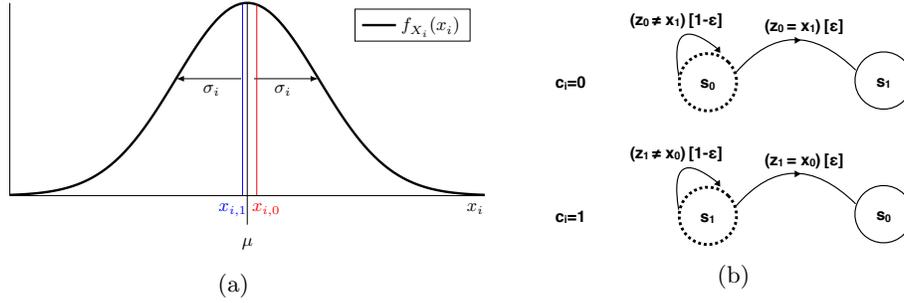


Fig. 1: (a) The Gaussian random variable  $X_i$  that corresponds to the  $i^{\text{th}}$  physical component of the PUF, and its two realizations  $x_{i,1}$  and  $x_{i,0}$ . In a meta-stable state these two realizations will be very close together. (b) Our simple Probabilistic (labeled) Transition Systems (PTS) describing how the noise can affect each stage in a PUF. The expressions included in parentheses denote the *labels*, whereas the information given in brackets refers to the probability of the transition between the states.

$X_i$ , namely  $x_{i,1}$  and  $x_{i,0}$ , are generated during manufacturing. Without loss of generality, suppose that the following holds. When  $c_i = 1$  is applied to the  $i^{\text{th}}$  stage, the realization  $x_{i,1}$  is chosen to be involved in generating the final response of the PUF, whereas  $x_{i,0}$  corresponds to  $c_i = 0$ . Moreover, suppose that the order relation between these realizations is  $x_{i,1} > x_{i,0}$ . Now, the *total impact of hidden variables on a stage* can be formulated as  $Z_i = X_i + N_i$  ( $1 \leq i \leq n$ ), where  $Z$  is clearly a Gaussian random variable. In addition, the realizations of this random variable are  $z_{i,1} = x_{i,1} + n_{i,1}$  and  $z_{i,0} = x_{i,0} + n_{i,0}$ , relating to the challenge bit applied to the PUF. Since the realizations  $z_{i,1}$  and  $z_{i,0}$  are related to two different evaluations of the PUF (with  $c_i = 1$  and  $c_i = 0$ , respectively), the noise realizations vary, as indicated by different indices. As defined in [29], the final response of the PUF is determined by these realizations. Obviously, the difference between  $z_{i,1}$  and  $z_{i,0}$  is the main factor contributing to the final response of the PUF. Now consider the  $i^{\text{th}}$  stage that is a meta-stable state, see Fig. 1(a). Here by meta-stable condition we refer to the fact that the realizations of the random variable  $X_i$ , i.e.,  $x_{i,1}$  and  $x_{i,0}$  can be very close together so that under the effect of environmental noise one realization can be equal to another:  $z_{i,0} = x_{i,1}$  or  $z_{i,1} = x_{i,0}$ , depending on the value of the challenge bit  $c_i$  [7]. To explain this, a simple Probabilistic (labeled) Transition Systems (PTS) can be defined as follows [40].

- There are two processes (i.e., sequences of events) corresponding to the value of the challenge bit applied to the  $i^{\text{th}}$  stage:  $c_i = 1$  and  $c_i = 0$ , see Fig. 1(b).
- In both processes, the set of states  $S$  contains two states denoted by  $s_0$  and  $s_1$ . The state  $s_0$  represented the case that the challenge bit  $c_i = 0$  is applied and in an ordinary condition (i.e., not meta-stable) we expect that  $x_{i,0}$  would

be involved in generating the final response of the PUF. Similarly, the state  $s_1$  can be defined.

- $s_{int} \in S$  is the initial state in each process, shown by dashed circles in each process. And the set of action labels is  $L$ .
- A transition probability function  $T : S \times L \times S \rightarrow [0,1]$  represents, under which circumstances and what degree of probability the system transits from one state to another. Clearly,  $\sum_{(l_i, s_j) \in L \times S} T(s_{int}, l_i, s_j) = 1$ .

Precisely defining our PTS, the tuple  $\mathcal{A}_i$  represents the process related to the case, when the challenge bit  $c_i$  is applied:  $\mathcal{A}_i = (S, L, T)$ .

In each of the processes, as illustrated in Fig. 1(b), the PTS may transit from one state to another with probability  $\varepsilon$ , otherwise it remains in its initial state. For instance, applying the challenge bit  $c_i = 0$ , the initial state  $s_0$  indicates that  $x_{i,0}$  would contribute to the final response of the PUF. However, if this stage (the  $i^{\text{th}}$  stage) is in a meta-stable state, i.e.,  $z_{i,0} = x_{i,1}$ , it is not possible to *differentiate* whether  $x_{i,1}$  or  $z_{i,0}$  would be involved to generate the final response of the PUF<sup>1</sup>. In other words, it can be thought that  $c_i = 1$  is applied and the final response is under the influence of  $x_{i,1}$ , i.e., the PTS is in  $s_1$  state. More precisely, we define a discrete random variable  $A$  corresponding to the event of a transition between  $s_0$  and  $s_1$ . Formally, let  $\Omega := \{\text{transition}, \text{stay}\}$  denote the sample space of the random variable  $A$  defined as  $A(\omega) = 1$  if  $\omega = \text{transition}$ , and otherwise,  $A(\omega) = 0$ . Obviously, this random variable follows a Bernoulli distribution with the success probability  $\varepsilon$ , i.e.,  $A \sim \text{Bern}(\varepsilon)$ .

Furthermore, as described above, we have translated the impact of noise on a single stage to a transition from one state to another state. Consequently, this change in the states can be seen as a probabilistic change of a challenge bit, e.g., the challenge bit  $c_i = 0$  is flipped to challenge bit  $c_i = 1$  or vice versa. To precisely summarize, with regard to our model, when applying the challenge  $c$  that is a Boolean string, the input to the PUF  $f_{\text{PUF}}$  can be written as  $c \oplus a$ , where  $\oplus$  denotes the bit-wise XOR operator and  $a$  is a random string composed of bits generated independently from the distribution  $\text{Bern}(\varepsilon)$ .

### 3.2 Impact of the Noise on the Measuring Element of the PUF

In the second phase, we should take into consideration the impact of uncertainty on the generation of the final response. In the literature this issue has been already explored, when discussing the precision of the measuring element (e.g., the arbiter in the case of arbiter PUFs), which makes a decision whether the response of the PUF to the respective challenge is “0” or “1” [8, 16, 32]. Clearly, being limited in the precision, such component may change the output of the PUF. For the purpose of this paper, we are not interested in the real-world distribution of this noise, but in how the effect of this noise on the responses can be precisely described. A useful interpretation of this effect is given in [15], namely that after generating the response of the PUF an unfair coin (Head with probability  $1 - \eta$ )

<sup>1</sup> Note that such transition does not always lead to a change in the response of the PUF.

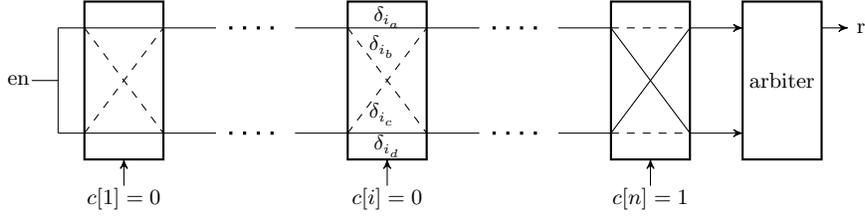


Fig. 2: Schematic of an arbiter PUF composed of  $n$  stages and an arbiter terminating the chain. When applying a challenge bit to a respective stage, either the realizations  $x_{i,1}$  or the realization  $x_{i,0}$  is chosen to be involved in generating the response of the PUF. For instance, we have  $x_{i,1} = \delta_{i_a} - \delta_{i_d}$  and  $x_{i,0} = \delta_{i_b} - \delta_{i_c}$ .

is flipped. Depending on the outcome, the final response is determined: when the outcome is Head, the response generated by the PUF remains unchanged, or otherwise, the response of the PUF is flipped. Here we follow the same principle to model the uncertainty with regard to the final response. Let a random variable  $B$  represent the impact of a limited precision of the physical component that makes the decision about the final answer. As explained above, this random variable also follows a Bernoulli distribution with the success probability  $1 - \eta$ , i.e.,  $B \sim \text{Bern}(1 - \eta)$ . For the sake of readability hereafter we denote  $\text{Bern}(1 - \eta)$  by  $R$ , and  $\text{Bern}(\varepsilon)$  by  $D$ . We have already defined the random string  $a$  containing independent random bits drawn according to the distribution  $D$  (see Sec. 3.1), and the random bit  $b$  drawn from the distribution  $R$ . Hence, the final response of the PUF can be formalized as  $y = f_{\text{PUF}}(c \oplus a) \oplus b$ .

### 3.3 Practical Implications of the Noise Model

In this section, we explain, how a relation between the parameters introduced in Sec. 3.1-3.2) and real-world PUFs can be established.

**Arbiter PUFs:** First we consider the impact of the noise on a single stage of an arbiter PUF. For the arbiter PUF family, the realizations  $x_{i,0}$  and  $x_{i,1}$  are associated with the difference between the delays of crossed and straight signal paths, namely,  $\delta_{i_a} - \delta_{i_d} = x_{i,1}$  and  $\delta_{i_b} - \delta_{i_c} = x_{i,0}$ , see Fig. 2. When the difference between these variables is small and the challenge bit  $c_i$  is applied to the stage, in the presence of the noise it is not possible to make a decision whether  $x_{i,0}$  or  $x_{i,1}$  has impacted the final response of the PUF.

Moreover, the impact of the noise on the response of the PUF can be explained by considering the limited precision of the arbiter terminating the chain, see Fig. 2 [16]. In this case, if after the final stage the delay difference between the upper and the lower paths is smaller than the precision of the arbiter, the arbiter could enter a metastable state and thus generate a wrong response.

**Ring Oscillator (RO) PUFs:** The response of this PUF is generated according to the difference between the frequencies of two ROs selected by the

challenge. In other words, the challenge determines a pair of ROs that contributes to the final response of the PUF. When the frequency differences of ROs in two pairs vary insignificantly, under noisy conditions one of those RO pairs can mimic another one. Therefore, it can be thought that some of the bits of the challenge applied to the PUF are flipped so that another RO pair makes impact on the final response of the PUF.

Furthermore, the limited precision of the counters measuring the frequencies of the ROs can affect the response of the PUF. More precisely, if the difference in the oscillation frequencies of a selected RO pair is not significant, the counters cannot measure the frequencies with high precision. Comparison of uncertain frequency measurements can lead to the generation of a wrong response.

**Bistable Ring (BR) PUFs:** Although a precise analytical model of the BR PUF is missing in the literature [12], we can still describe the impact of the noise on individual stages. For a given challenge in the BR PUF,  $n$  inverters are selected, and upon setting the reset signal to low, the created inverter ring starts to oscillate until it settles down to a valid logical state. In this case, the process variables can be intrinsic differences in the propagation delays and electrical gains of each inverter. Therefore, based on the environmental conditions, the noise can be added to the realization of the process variables. However, in contrast to the arbiter and RO PUFs, there is no *explicit* measuring element in this PUF architecture. But the required additional measurement element which introduces noise for this type of PUF is explicitly discussed in [12, 21, 22].

### 3.4 Modeling the Noise from the Perspective of Machine Learning

With regard to the discussion from the previous sections 3.1- 3.3, PUFs can be thought of as Boolean functions, whose input-output behavior is determined by random process variations as well as the inevitable impact of random noise. In line with this view, a model of PUFs as illustrated in Fig. 3 can be established. This model can be seen as an extension of a model introduced and evaluated practically in [29]. Our model is composed of two components: random and deterministic components. The random component represents the random environmental noise, whereas the deterministic component accounts for the deterministic Boolean function realized in the chip. In other words, in the absence of noise, the response of the PUF to a challenge applied repeatedly remains the same. The building blocks as well as the parameters related to the model have been introduced previously, although their interpretations in the machine learning context have not yet been considered. As the next step in our framework, this section elaborates on how the functionality of a noisy PUF, as shown in Fig. 3, can be described from the point of view of machine learning. To this end, the following Lemma plays an important role, cf. [6].

**Lemma 1** *Consider  $U$ ,  $D$  and  $R$  that are a uniform, and two arbitrary distributions<sup>2</sup> over the space  $\{0, 1\}$ , respectively. Moreover, let the function  $f : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$*

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<sup>2</sup> Regarding the physical properties of noisy PUFs we have defined the distributions  $D$  and  $R$  precisely, but in general these distribution can be arbitrary.

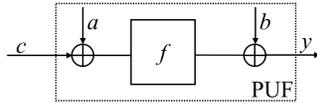


Fig. 3: Our model composed of blocks representing a noisy PUF. The random string  $a$  contains independent random bits drawn according to the distribution  $D$  (see Sec. 3.1), and the random bit  $b$  is drawn from the distribution  $R$  (see Sec. 3.2). From the machine learning point of view, they refer to attribute noise and classification noise, respectively.

be an arbitrary Boolean function. Let  $C \in_U \mathbb{F}_2^n$ ,  $A \in_D \mathbb{F}_2^n$  and  $B \in_R \mathbb{F}_2$  be independent random strings and a random variable, respectively. The random variables  $(C, f(C \oplus A) \oplus B)$  and  $(C \oplus A, f(C) \oplus B)$  follow identical distribution.

For the proof of this Lemma, we refer to [6]. The conclusions drawn from it are of great importance for us. First, from the machine learning point of view, the noise represented by the random variable  $A$  is called the “attribute noise”, whereas the random variable  $B$  corresponds to the “classification noise”. The issue of attribute and classification noise learnability has been well addressed in the relevant literature, see, e.g., [6, 52]. Secondly, thanks to the seminal paper published by Bshouty et al. [6], a relationship between machine learning under noisy conditions and the noise sensitivity of Boolean functions has been established. The consequence of this relation is that efficient algorithms developed to estimate the Fourier coefficients of an unknown function can be applied to learn the respective function *even* under noisy conditions.

## 4 Fourier Analysis Based Attacks against PUFs

To mount our attack, we apply an algorithm proposed by Linial, Mansour, and Nisan [28] to estimate the Fourier coefficients of an unknown function (i.e., so-called LMN-style algorithm). The rationale behind the LMN-style algorithm, originally called “low degree” algorithm [35], is that some classes of Boolean functions can be approximated by taking into account solely a small number of their Fourier coefficients (called “low” coefficients), corresponding to small subsets of  $[n]$  (see Sec. 2.2).

**Theorem 1. (Low degree algorithm)** [28, 35, 39] *Assume that an algorithm can determine a set  $\mathcal{S} \subseteq 2^{[n]}$  containing subsets of  $[n]$  so that  $\sum_{S \in \mathcal{S}} \hat{f}(S)^2 \geq 1 - \varepsilon$ . The algorithm is given a pre-defined confidence level  $\delta$  and access to a polynomial number of input-output pairs of the Boolean function  $f$  that are chosen uniformly at random. With probability  $1 - \delta$  the algorithm delivers a Boolean function  $h$  that is an  $\varepsilon$ -approximator of the Boolean function  $f$  such that*

$$\sum_{S \subseteq [n]} \left( \hat{f}(S) - \hat{h}(S) \right)^2 \leq \varepsilon.$$

The running time of the algorithm is  $\text{poly}(|\mathcal{S}|, n, 1/\varepsilon, \log_2(1/\delta))$ .

For the proof of this theorem, we refer the reader to [28, 35], in which the mechanism for determining the set  $\mathcal{S}$ , and the lower bound on the number of input-output pairs required by the algorithm has been discussed extensively.

#### 4.1 An LMN-style Algorithm for RO PUFs

Although the security of these PUFs can be easily broken by simply reading out all CRPs, launching a machine learning attack in the specific circumstance of having limited access to the CRPs (e.g., eavesdropping them) has been addressed in the literature, see for instance [14, 42]. In the present case, learning of noisy RO PUFs has not been discussed. Our proof of the existence of an LMN-style algorithm for RO PUFs relies on the fact that these PUFs can be represented by  $k$ -DLs [14].

**Theorem 2.** [27, 35] *An LMN-style algorithm can be employed that with probability  $1 - \delta$  delivers a Boolean function  $h$  approximating a decision list  $L$ , which represents an RO PUF. The running time of this algorithm is  $\text{poly}(n, \log_2(1/\varepsilon), \log_2(1/\delta))$ .*

The proof sketch can be summarized as follows. According to results presented in [14], an RO PUF can be represented by a DL. Furthermore, Mansour proved that a DL could be approximated by a Boolean function  $h$ , whose Fourier coefficients concentrate only on a small set of variables, namely,  $\log_2(1/\varepsilon)$  variables [35]<sup>3</sup>. To find this set of variables, the low degree algorithm can be applied to deliver  $h$  and the running time of that is  $\text{poly}(n, \log_2(1/\varepsilon), \log_2(1/\delta))$ .

#### 4.2 An LMN-style Algorithm for Arbiter PUFs

To prove the existence of an LMN-style algorithm for arbiter PUFs we argue as follows. It is known that if a Boolean function exhibits a bounded, small noise sensitivity, its Fourier coefficients are mainly low coefficients. More precisely, the following Corollary can be proved (for the proof see Corollary 2.3.3 in [39]) that forms the basis for proof.

**Corollary 1** [39] *Consider  $\alpha : [0, 1/2] \rightarrow [0, 1]$  that is a strictly increasing continuous function so that  $NS_\varepsilon(f) \leq \alpha(\varepsilon)$ . We have  $\sum_{|S| \geq m} \hat{f}(S)^2 \leq \varepsilon$ , where  $m = 1/\alpha^{-1}(\varepsilon/2.32)$  and  $\alpha^{-1}(\cdot)$  denotes the inverse of the function  $\alpha(\cdot)$ .*

Now Corollary 2 states how Corollary 1 can be applied to prove the existence of an LMN-style algorithm for arbiter PUFs.

**Corollary 2** *Representing an arbiter PUF by an LTF, a Boolean function  $h$  approximating this LTF can be delivered by an LMN-style algorithm, whose running time is polynomial in  $n$ ,  $1/\varepsilon^2$ , and  $\log_2(1/\delta)$ .*

**Proof:** Thanks to the results reported in [18, 33, 42], LTFs are appropriate representations of arbiter PUFs. For any LTF  $f$  its noise sensitivity is a bounded, small value depending only on  $\varepsilon$ , namely we have  $NS_\varepsilon(f) \leq 8.54\sqrt{\varepsilon}$  [25]. Now fix  $\alpha(\varepsilon) = \sqrt{\varepsilon}$ . According to Corollary 1, the running time of the LMN-style algorithm is polynomial in  $O(n^m)$ , where  $m = 1/\alpha^{-1}(\varepsilon/2.32)$ . ■

<sup>3</sup> Here we do not discuss the details of the proof. For the proof cf. [35].

### 4.3 An LMN-style Algorithm for BR PUFs

Similar to the proof of the existence of an LMN-style algorithm for arbiter PUFs, we take advantage of the properties of the Boolean functions representing BR PUFs. More specifically, we rely on the fact that a BR PUF can be represented by  $k$ -junta, where  $k$  is a (relatively) small constant value for practical values of  $n$ , as demonstrated in [12]. Moreover, the noise sensitivity of a  $k$ -junta function is a bounded, small value:  $NS_\varepsilon(f) \leq k\varepsilon/2$ , see, e.g., [17]. Now the following corollary can be formulated to prove the existence of an LMN-style algorithm for BR PUFs.

**Corollary 3** *An LMN-style algorithm can be applied to deliver an  $\varepsilon$ -approximator for a  $k$ -junta representing a BR PUF. The running time is polynomial in  $n$ ,  $1/\varepsilon$ , and  $\log_2(1/\delta)$ .*

### 4.4 Provability in the Sense of PAC Model

The low degree algorithm mainly aims to provide an approximator of a Boolean function with a given probability, when it is given a polynomial number of input-output pairs of the Boolean function that are chosen uniformly at random. However, its existence has a serious consequence. More specifically, if the set  $\mathcal{S}$  is composed of all the subsets of low degree, Theorem 1 introduces a PAC learning algorithm under the uniform distribution [39]. Before formulating this precisely, we first shift our focus to the issue of dealing with noise.

As explored in Sec. 3.4, we take the attribute and the classification noise into account. The question is how these processes affect the functionality and the efficiency of an LMN-style algorithm. This issue is well addressed by Bshouty et al., [6], and here we briefly summarize their results. They have shown that the attribute and the classification noise attenuate the Fourier coefficients, which the LMN-style algorithm aims to estimate from the uniformly random examples. To be exact, assume that an LMN-style algorithm attempts to estimate the Fourier coefficient  $\hat{f}(S)$ . Under the noisy conditions, it delivers  $\hat{f}(S)(1 - 2\eta)\alpha_S$ , where  $(1 - 2\eta)$  and  $\alpha_S$  are attenuation factors corresponding to the classification and attribute noise, respectively. While the former factor is known, see e.g., [15], the latter requires more attention. The attenuation factor  $\alpha_S$  is the defined as  $\alpha_S := \mathbf{E}_{a \in D}[\chi_S(a)]$ , where  $\mathbf{E}_{a \in D}[\cdot]$  denotes the expectation over random examples drawn from the known distribution  $D$ . As discussed in Sec. 3.4, here we consider  $a$  that is a random string, whose bits are independently generated following a Bernoulli distribution  $\text{Bern}(2\varepsilon)$  with  $\varepsilon \in (0, 1/2]$ . Hence,  $|\alpha_S| = \prod_{i \in S} (1 - 2\varepsilon) = (1 - 2\varepsilon)^{|S|}$ . Note that the practical implication of the attenuation factors  $((1 - 2\eta)$  and  $\alpha_S$ ) is that after running the LMN-style algorithm each Fourier coefficient delivered by the algorithm should be multiplied by  $(1 - 2\eta)^{-1}$  and  $\alpha_S^{-1}$  to eliminate the impact of the noise.

Now we can summarize the above discussion and the results presented in Section 4.1-4.3 in a more formal manner, as stated in Corollary 4.

**Corollary 4** *Consider a given PUF that is represented by a Boolean function  $f_{PUF}$  and can be learned by applying an LMN-style learning algorithm. Then*

The number of ROs $N$	# CRPs in training set	# CRPs in test set	$ \mathcal{S} $	Accuracy	$\eta$	$\varepsilon$
256	5000	65536	1491	99.53	Noiseless	
	5000			97.45	0.1	0
	5500			98.64		
	5000			93.22	0.2	0
	7500			98.56		
	5000			89.06	0	0.1
	12000			98.73		
	5000			85.45	0	0.2
	20000			98.66		
512	7500	262144	1681	99.26	Noiseless	
	7500			93.29	0.1	0
	15000			98.66		
	7500			93.08	0.2	0
	18000			98.72		
	7500			86.03	0	0.1
	22000			99.01		
	7500			82.98	0	0.2
	30000			98.67		

Table 1: Results for learning RO PUFs with  $N$  rings.

*the PUF is PAC learnable under the uniform challenge distribution, even in the presence of attribute and classification noise. The running time of the PAC learner is poly( $|\mathcal{S}|, n, 1/\varepsilon, \log_2(1/\delta), (1/1 - 2\eta)$ ).*

## 5 Results and Discussion

The effectiveness of our proposed attack is evaluated by conducting simulations on data collected from PUFs that are implemented on FPGAs. The PUF simulators, as well as LMN algorithm, are implemented in Matlab [1]. To simulate the challenge-response behaviors of the PUFs, we have taken the real physical properties of the PUFs into account. For instance, the maximum delay deviation of each inverter and the precision of the arbiter used in our arbiter PUF chain are equal to 9 ps and 2.5 ps, respectively, as reported for a Xilinx Virtex-5 FPGA (65 nm technology) [31, 32]. The delays of the stages are generated with respect to a Gaussian distribution with the above-mentioned maximum deviation. By applying a random challenge chosen uniformly, the response of the arbiter PUF is generated and stored in our data set. As for RO PUFs, similar to the approach introduced in [14], the publicly accessible measurement results from a dataset [43] have been taken into account. These results contain 100 samples of the frequency of each and every ring-oscillators, which comprise RO PUFs with 512 rings implemented on 193 Xilinx Spartan-3 FPGAs (90 nm technology). These frequencies are the inputs of our RO PUF simulator that mimics the challenge-response behavior of RO PUFs with 256 and 512 ring-oscillators. The RO PUF simulator randomly selects  $N$  ( $N = 256, 512$ ) frequencies corresponding to  $N$  different ring-oscillators. Feeding the simulator with random challenges a pair of rings is chosen and their frequency are compared to generate the response.

Moreover, our BR PUF simulator relies on the results presented in [12], where BR PUFs have been implemented on Altera Cyclone IV FPGAs (60 nm

The number of stages $n$	# CRPs in training set	# CRPs in test set	$ \mathcal{S} $	Accuracy	$\eta$	$\varepsilon$
64	2000	100000	1078	99.19	Noiseless	
	2000			97.30	0.1	0
	2250			99.02		
	2000			97.06	0.2	0
	2350			98.86		
	2000			97.09	0	0.1
	2300			99.15		
	2000			97.97	0	0.2
	2300			99.28		
	128			2100	100000	1078
2100		98.81	0.1	0		
2300		99.06				
2100		96.94	0.2	0		
2500		99.29				
2100		97.23	0	0.1		
2500		99.23				
2100		98.04	0	0.2		
2500		99.45				

Table 2: Results for learning arbiter PUFs with  $n$  stages.

technology). The internal functionality of these PUFs has been simulated by taking  $k$ -juntas into account. This is valid since in a follow-up work [13], the authors of [12] have demonstrated that a BR PUF (with practical values of  $n$ , e.g., 32 and 64) belongs to the class of  $k$ -junta functions. Hence, to simulate the challenge-response behavior of BR PUFs, the value  $k$  and the conjunctive rule presented in above studies are considered.

For all PUFs, the procedure of adding classification and attribute noise is as discussed in Sec. 3. In our experiments, we have  $\delta = 0.01$ , and various levels of noises:  $\eta = 0, 0.1, 0.2$  and  $\varepsilon = 0.1, 0.2$ . All simulators and the LMN algorithms are implemented on a MacBook Pro with 2.6 GHz Intel Core i5 processor and 8 GB of RAM. The key difference between our approach and the methodology usually employed in ML attack scenario is that an adversary applying LMN algorithm needs to simply write a script (e.g., in Matlab), which computes a small set of Fourier coefficients. To this end, according to Theorem 1, the total number of relevant coefficients is  $|\mathcal{S}|$  and the Fourier coefficients can be computed in a straightforward manner as shown in Sec. 2.2.

Our results are presented in Table 1-3. The accuracy of the final model, i.e., the approximated the function  $f_{PUF}$  generated by using the low degree Fourier coefficients, is reported in these tables. For each experiment, the accuracy reported in the table is the minimum accuracy over 5 repetitions of the experiments that our algorithm achieves. First, as a reference, we conduct experiments on noiseless CRPs collected from PUFs. By adding the noise, the accuracy of the model decreases for the number of CRPs applied in the case of the noiseless PUF. Afterwards, we increase the number of CRPs in the training set to achieve virtually the same accuracy (with the maximum  $\pm 1\%$  difference) in both cases. The results demonstrate that as promised by Corollary 4, the increase in the number of CRPs needed in the presence of noise is polynomial in noise levels.

The number of stages $n$	# CRPs in training set	# CRPs in test set	$ \mathcal{S} $	Accuracy	$\eta$	$\varepsilon$
32	500	100000	59	99.72	Noiseless	
	500			95.91	0.1	0
	950			99.45		
	500			95.02	0.2	0
	950			99.63		
	500			94.39	0	0.1
	1000			99.55		
	500			94.72	0	0.2
	1200			99.40		
	64			500	100000	165
500		97.3	0.1	0		
900		99.19				
500		92.65	0.2	0		
950		98.93				
500		97.56	0	0.1		
950		99.23				
500		95.63	0	0.2		
950		99.39				

Table 3: Results for learning BR PUFs with  $n$  stages.

## 6 Conclusion and Remarks

Our paper presents the first study on the feasibility and the applicability of a new attack, i.e., the LMN algorithm against PUFs. This algorithm has not been applied in the context of PUFs, although being known to the ML community. Thus, similar to other ML attacks against PUFs discussed in the literature, the novelty of our approach is the introduction of a new attack against PUFs, even applicable in the case of noisy CRPs. The proposed attack mainly relies on approximating the low degree Fourier coefficients by applying a so-called low degree algorithm developed in ML theory.

Moreover, our paper is the first to introduce the notion of noise sensitivity to assess the security of PUFs. This notion not only reflects the physical properties of a PUF (discussed in Section 3), but also it is closely related to the resilience of a PUF against LMN attacks. In this respect, the implication of Corollaries 3 and 4 (related to the existence of an LMN algorithm for PUFs) is that since the noise sensitivity of the Boolean functions representing the PUFs is a small, bounded value, an attacker can launch the LMN attack. Moreover, in the case of noisy PUFs, the attenuation factors can affect the efficiency of the LMN algorithm. In other words, if the noise sensitivity is well adjusted by the designer, the attacker cannot compute the Fourier coefficients. Hence, when designing a new PUF, it is important to consider the noise sensitivity as an indicator of the robustness of PUF against LMN attacks. We believe that in addition to the proof of PAC learnability in the presence of noise, this paper provides several interesting insights into not only the assessment of the security of PUFs, but also the design of PUFs with better security-related characteristics.

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