

# Privacy-Preserving Aggregation of Time-Series Data with Public Verifiability from Simple Assumptions\*

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## Abstract

Aggregator oblivious encryption was proposed by Shi et al. (NDSS 2011), where an aggregator can compute an aggregated sum of data and is unable to learn anything else (aggregator obliviousness). Since the aggregator does not learn individual data that may reveal users' habits and behaviors, several applications, such as privacy-preserving smart metering, have been considered. In this paper, we propose aggregator oblivious encryption schemes with public verifiability where the aggregator is required to generate a proof of an aggregated sum and anyone can verify whether the aggregated sum has been correctly computed by the aggregator. Though Leontiadis et al. (CANS 2015) considered the verifiability, their scheme requires an interactive complexity assumption to provide the unforgeability of the proof. Our schemes are proven to be unforgeable under a static and simple assumption (a variant of the Computational Diffie-Hellman assumption). Moreover, our schemes inherit the tightness of the reduction of the Benhamouda et al. scheme (ACM TISSEC 2016) for proving aggregator obliviousness. This tight reduction allows us to employ elliptic curves of a smaller order and leads to efficient implementation.

## 1 Introduction

### 1.1 Aggregator Oblivious Encryption

Aggregator oblivious encryption was proposed by Shi et al. [52], where an aggregated sum of  $n$  users' data (such as energy consumption from smart meters) can be computed in a privacy-preserving manner. In brief, an honest dealer generates secret keys for users and an aggregator. A user  $i$  encrypts data  $x_{i,t}$  at time  $t$ , and sends the ciphertext  $c_{i,t}$  to the aggregator. The aggregator can compute the aggregated sum  $X_t = \sum_{i=1}^n x_{i,t}$  from  $\{c_{i,t}\}_{i \in [1,n]}$  and sends  $X_t$  to a data analyzer (such as an energy provider). It is particularly worth noting that the aggregator learns  $X_t$  and nothing else and this security notion has been formalized as aggregator obliviousness. Note that if homomorphic encryption [26, 49] is simply employed, then the aggregator has the capability to decrypt each  $c_{i,t}$  and can obtain  $x_{i,t}$ . Since  $x_{i,t}$  may reveal consumer habits and behaviors, e.g., when a certain consumer turns the air conditioner on, it may appear when the consumer returns home, aggregator oblivious encryption is better to preserve the privacy of users. Moreover, the aggregator is not required to be a fully trusted authority and is modeled as honest-but-curious. That is, the data analyzer can collect the aggregated sum of  $x_{i,t}$  via the aggregator in a privacy-preserving manner. In addition, only a unidirectional channel is required from each user to the aggregator. This could be an advantage compared to the schemes that require bidirectional channels between the smart meters and the aggregator [50, 25]. Though the Shi et al. scheme is not tolerant of user failures (i.e., if even a single user fails to respond in a certain aggregation round, the aggregation algorithm does not work), Chan et al. [15] proposed a fault-tolerant solution such that the aggregator can still compute the aggregated sum from the remaining users.

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The Shi et al. scheme is aggregator obliviousness under the Decisional Diffie-Hellman (DDH) assumption in the random oracle model. They employed the lifted ElGamal encryption approach [17] and therefore  $X_t = \sum_{i=1}^n x_{i,t}$  needs to be suitably small since the aggregator is required to solve the discrete logarithm  $g^{X_t}$  with respect to basis  $g$ . Later, Joye and Libert [34] proposed an aggregator oblivious encryption scheme with large plaintext spaces by employing the Paillier-type homomorphic operation [49]. The Joye-Libert scheme is aggregator obliviousness under the Decision Composite Residuosity (DCR) assumption in the random oracle model. Both schemes [34, 52] were generalized by Benhamouda, Joye, and Libert (BJL) [10]. They gave a generic construction of aggregator oblivious encryption from smooth projective hash functions [18] with an extra additively homomorphic property over the key space, with both DDH and DCR-based instantiations. An attractive point of the B JL construction is its tight reduction. Namely, the reduction loss is  $O(t_{\max})$  whereas that of the Shi et al. scheme [52] is  $O(t_{\max}n^3)$  where  $t_{\max}$  is the maximum time to be supported by the system and  $n$  is the number of users. If we consider the exact security [9, 45], then tight reduction is important. As in Benhamouda et al. [10], we set that  $n = t_{\max} = 2^{20} \approx 10^6$  which approximately allows the computation of an aggregation every 15 minutes for 30 years throughout a city like Paris. Then, the security loss of the Shi et al. scheme is approximately  $2^{80}$ . That is, for achieving 112-bit security, the Shi et al. scheme requires approximately 7,680-bit public key or elliptic curves with 384–511-bit order, which is recommended by NIST [5] for achieving 192-bit security. On the other hand, the security loss of the Benhamouda et al. scheme is approximately  $2^{20}$ , and to achieve 112-bit security, approximately 3,072-bit public key or elliptic curves with 256–383-bit order is required.<sup>1</sup>

## 1.2 Aggregator Oblivious Encryption with Public Verifiability

As mentioned above, the aggregator is modeled as honest-but-curious and is assumed to output  $X_t$  correctly. For stronger security, Leontiadis et al. [39] considered a new model: a user  $i$  produces a tag  $\sigma_{i,t}$  in addition to  $c_{i,t}$ , and sends  $(c_{i,t}, \sigma_{i,t})$  to the aggregator, and the aggregator is required to generate a publicly verifiable proof  $\sigma_t$  that proves the decryption result of  $\{c_{i,t}\}_{i \in [1,n]}$  is exactly  $X_t$ . Of course, it is required that the aggregator cannot produce a forged  $\sigma_t$  for some  $X_t \neq \sum_{i=1}^n x_{i,t}$ , and this security notion is formalized as aggregator unforgeability. Since the data analyzer can recognize whether the aggregator correctly computed the aggregated sum, this functionality can be seen as a kind of verifiable computation [3, 24].

Though the Leontiadis et al. approach is interesting, one drawback of their construction is the underlying complexity assumption. They introduced an interactive assumption called the LEOM assumption for proving aggregator unforgeability. The LEOM assumption is defined as follows.

**Definition 1 (LEOM Assumption [39])** *Let  $D = (p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$  be bilinear groups. Choose  $\alpha \xleftarrow{\$} \mathbb{G}_1$  and  $\delta, \gamma_1, \dots, \gamma_n \xleftarrow{\$} \mathbb{Z}_p$  and set  $\Gamma = g_2^\gamma$  and  $\Delta = g_2^{\sum_{i=1}^n \gamma_i}$ . The LEOM oracle  $\mathcal{O}_{\text{LEOM}}$  takes as input  $(t, \{x_{i,t}\}_{i=1}^n)$ , chooses  $\beta_t \xleftarrow{\$} \mathbb{G}_1$ , and returns  $(\alpha, \beta_t, \{\beta_t^{\gamma_i} \alpha^{\delta x_{i,t}}\}_{i=1}^n)$ . If a query at  $t$  contains  $i' \in [1, n]$  such that  $x_{i,t} \neq x'_{i,t}$ , then  $\mathcal{O}_{\text{LEOM}}$  returns  $\perp$ . Assume that  $\mathcal{O}_{\text{LEOM}}$  is called once at each  $t$ . We say that the LEOM assumption holds if for any probabilistic polynomial time (PPT) adversary  $\mathcal{A}$ , the advantage  $\text{Adv}_{\text{LEOM}}(\lambda) := \Pr[\mathcal{A}^{\mathcal{O}_{\text{LEOM}}(\cdot, \cdot)}(D, \Gamma, \Delta) \rightarrow (t, z, c)]$  is negligible where  $\mathcal{A}$  has queried  $(t, \{x_{i,t}\}_{i=1}^n)$  and  $z \neq \sum_{i=1}^n x_{i,t}$  and  $c = \beta_t^{\sum_{i=1}^n \gamma_i} \alpha^{z\delta}$  holds.*

However, as explained by Naor [46], it is better to avoid interactive assumptions as much as possible to prevent circular arguments. Making cryptographic primitives secure under weak assumptions is one of the important topics of cryptography. To name a few, verifiable random functions [30, 31], group signatures [41, 42], structure-preserving signatures [2], identity-based encryption [54], attribute-based encryption [48, 53], oblivious transfer [28] and so on, and constructing an aggregator oblivious encryption scheme with public verifiability from static and simple assumptions are still left as open problems.

<sup>1</sup>This key-length is recommended by NIST [5] for achieving 128-bit security. To be precise, the Benhamouda et al. scheme archives 108-bit security under this key length. Thus, a slightly longer key is required to achieve 112-bit security.

Table 1: Comparison of DL-based Aggregator Oblivious Encryption

Scheme	Ciphertext	Tag	Secret Key	Public Parameter
	Size ( $c_{i,t}$ )	Size ( $\sigma_{i,t}$ )	Size	Size (params + vk)
BJL (DDH) [10]	$ \mathbb{G}_1 $	-	$2 \mathbb{Z}_p $	$ \mathbb{G}_1  + 2$ hash
LEOM [39]	$ \mathbb{G}_1 $	$ \mathbb{G}_1 $	$2 \mathbb{Z}_p  +  \mathbb{G}_1 $	$ \mathbb{G}_1  +  \mathbb{G}_2  + 1$ hash
Ours 1	$ \mathbb{G}_1 $	$ \mathbb{G}_1 $	$3 \mathbb{Z}_p  +  \mathbb{G}_1 $	$ \mathbb{G}_1  + (1 + t_{\max}) \mathbb{G}_2  +  \mathbb{G}_T  + 6$ hash <sup>1</sup>
Ours 2	$ \mathbb{G}_1 $	$ \mathbb{G}_1 $	$2 \mathbb{Z}_p  +  \mathbb{G}_1 $	$ \mathbb{G}_1  +  \mathbb{G}_2  +  \mathbb{G}_T  + 5$ hash

  

Scheme	$ p ^\ddagger$	Reduction Loss	Encryption	Aggregator	Complexity Assumptions	Bulletin Board
		AO/AU <sup>2</sup>	Algorithm	Unforgeability	for proving AO/AU <sup>2</sup>	
BJL (DDH) [10]	256	$O(t_{\max})/-$	Deterministic	-	DDH/-	-
LEOM [39]	1031	$O(t_{\max}n^3)/O(1)$	Deterministic	Full	DDH/LEOM <sup>3</sup>	-
Ours 1	383	$O(t_{\max})/O(1)$	Deterministic	Weak	DDH/mCDH <sup>4</sup>	-
Ours 2	383–1031	$O(t_{\max})/O(t_{\max}^2)$	Probabilistic	Semi-Adaptive	DDH/DDH&mCDH <sup>4</sup>	Required

<sup>†</sup>  $\mathbb{Z}_p$ ,  $|\mathbb{G}_1|$ ,  $|\mathbb{G}_2|$ , and  $|\mathbb{G}_T|$  denote the bit-length of an element of  $\mathbb{Z}_p$ ,  $\mathbb{G}_1$ ,  $\mathbb{G}_2$ , and  $\mathbb{G}_T$ , respectively.

<sup>‡</sup>  $|p|$  denotes the bit-length of  $p$  for 112-bit security. Here, we set  $n = t_{\max} = 2^{20}$  [10]. For the B JL scheme, we refer the NIST recommendation [5] since the B JL scheme is pairing-free. For the LEOM scheme and ours, we refer the result by Menezes, Sarkar, and Singh [44] who re-evaluated parameters of pairing-friendly elliptic curves by considering the result by Kim and Barbulescu [37]. Since it is not clear how large  $p$  is required for 152-bit security, we denote  $|p|$ : 383–1031 for the second scheme.

<sup>1</sup> Remark that no user is required to have the large-size verification key.

<sup>2</sup> AO/AU: Aggregator Obliviousness/Aggregator Unforgeability

<sup>3</sup> LEOM: Leontiadis-Elkhiyaoui-Önen-Molva. An interactive complexity assumption.

<sup>4</sup> mCDH: modified Computational Diffie-Hellman. A static complexity assumption.

### 1.3 Our Contribution

In this paper, we propose two aggregator oblivious encryption schemes with public verifiability from static and simple assumptions (a variant of the Computational Diffie-Hellman (CDH) assumption). See Table 1 for detailed comparisons. For aggregator obliviousness, both schemes are tightly reduced to the B JL scheme. That is, our schemes inherit the tightness of the reduction of the Benhamouda et al. scheme. This tight reduction allows us to employ elliptic curves with a smaller order and leads to efficient implementation. On the other hand, the Leontiadis et al. scheme is reduced to the Shi et al. scheme and has a loose reduction. Remark that Benhamouda et al. [10] also show that a degradation factor of at least  $\Omega(n^2)$  is unavoidable in the Shi et al. scheme. They show that any blackbox nonrewinding reduction from the Shi et al. scheme to a noninteractive problem loses a factor of at least  $n^2$ . That is, this bound cannot be improved in the Shi et al. scheme, and the Leontiadis et al. scheme also.

The first scheme provides weak aggregator unforgeability, where an adversary can obtain ciphertexts and tags  $\{(c_{i,t}, \sigma_{i,t})\}_{i=1}^n$  of  $x_{i,t}$  chosen by the encryption oracle. Note that in the smart meter setting,  $x_{i,t}$  (such as power consumption) is measured by the meter. Thus, we believe that weak aggregator unforgeability is still meaningful in the actual usage. One drawback to the first scheme, beside weak aggregator unforgeability, is the large-size verification key  $\text{vk} = \{\text{vk}_t := g_2^{\sum_{i=1}^n v_{i,t}}\}_{t \in [1, t_{\max}]}$  where  $t_{\max}$  is the maximum time to be supported by the system. If we employ Barreto-Naehrig (BN) curves [6] with a 383-bit order, then approximately 100MByte-sized verification keys need to be published when  $t_{\max} = 2^{20} \approx 10^6$  [10]. Note that no user is required to have the large-size verification key. Moreover, verification keys for past times can be removed. In addition, if we can assume that these keys are updated by the dealer every time over a certain time period (i.e., periodic inspection of meters every one to two years), or if we can set a relatively small  $t_{\max}$ , then we can significantly reduce the size of the keys to be stored. Remark that, if a user manages all  $v_{i,t}$  as its secret key, then the secret key size also depends on  $t_{\max}$ . To avoid such a large-size secret key, we additionally introduce a hash function  $H$  and a time-independent secret key  $v_i$ , and we compute  $v_{i,t} = H(v_i, t)$ . This helps us to reduce the secret key size. For weak aggregator unforgeability, the first scheme provides a tight reduction loss from the advantage of the the mCDH problem.

Though we can reduce the verification key size according to the  $t_{\max}$  settings, it would be better to support constant-size keys. Our second scheme solves the large-size key problem by choosing  $v_{i,t}$  on the fly. That is, in the second scheme a user  $i$  chooses  $v_{i,t}$  in the encryption phase, whereas in the first

scheme all keys are generated by an honest dealer, as in previous works [52, 10, 34, 39]. Though the Enc algorithm becomes probabilistic, this strategy allows us to prove that the scheme provides aggregator unforgeability with semi-adaptive chosen message attack where an adversary can obtain ciphertexts and tags  $\{(c_{i,t}, \sigma_{i,t})\}_{i=1}^n$  of  $x_{i,t}$  chosen by the adversary. Here, semi-adaptive means that the adversary is required to send all  $\{x_{i,t}\}_{i=1}^n$ , and obtains the corresponding  $\{(c_{i,t}, \sigma_{i,t})\}_{i=1}^n$ . Though  $vk$  can be removed from the public value, a drawback of the second scheme is that a malicious aggregator could modify  $vk$ . Thus, we additionally need to introduce public channels equipped with memory, such as a bulletin board [29] that is publicly readable and that every user can write to, but nobody can delete from. See Section 4 for a more detailed explanation. Another drawback is its reduction loss. Though aggregator obliviousness of the second scheme is tightly reduced to the BJKL scheme (this requires  $O(t_{\max})$  reduction loss from the advantage of the DDH problem), semi-adaptive aggregator unforgeability of the second scheme requires additional reduction loss. Concretely,  $O(t_{\max}^2)$  reduction loss from the advantage of the DDH problem. Thus, we need to employ elliptic curves with a relatively large order.

## 1.4 Related Work

Aggregator oblivious encryption considers collecting the aggregated sum of users (e.g., the total consumption of customers) in a certain region for each time period. This could be employed for privacy-preserving energy management systems. On the other hand, collecting the aggregated sum of a particular user might be desired for a certain reason. For example, if an energy provider would like to send an invoice to a customer and would like to know the total amount of the consumption of the customer. This could be employed for privacy-preserving supplier billing systems [32, 51]. Some schemes support both billing and energy management functionality [7, 47, 20]. Ohara et al. [47] in particular proposed such a smart metering scheme with verifiability of the integrity of the total amount of consumption or the billing price.

In our setting (as in [10, 34, 39, 52]), the number of users  $n$  is selected and fixed during the setup phase. Some papers considered dynamic joins and leaves [40, 15, 33, 38]. Chan et al. [15] proposed a binary interval tree technique that reduces the communication cost for joins and leaves, and Jawurek et al. [33] further improved the communication overhead of the Chan et al. scheme. Although the Chan et al. and Jawurek et al. schemes require public key settings, Li and Cao [40] proposed a more efficient scheme that only requires symmetric key settings. Though these schemes assume an honest dealer that issues keys to the users and the aggregator via a secure channel, Leontiadis et al. [38] proposed a key update mechanism that does not require any trusted dealer. They introduced an additional semi-trusted party called the collector that collects partial key information from users via a secure channel.

Datta and Joye [21] showed that a protocol for computing an aggregate sum proposed by Jung, Li, and Wan [35] is universally breakable, where anyone can recover private data from ciphertexts.

Some schemes employ bilinear groups with composite order  $N = pq$  [43, 23]. This could be a bottleneck since we need to assume that  $N$  is difficult to be factorized and is selected as sufficiently large. In the meantime, our schemes are constructed over bilinear groups with a prime order.

Zhuo et al. [55] proposed a privacy-preserving verifiable data aggregation. They employed homomorphic encryption [14]. As in our scheme, the correctness of computation results can be verified. However, no formal security definition is given (especially unforgeability of the computation results), and thus its security is not analyzed in the sense of provable security.

Corrigan-Gibbs and Boneh [16] proposed a privacy-preserving system for computing aggregate statistics, which they call Prio. Though there is a single aggregator in aggregator oblivious encryption, whereas in the Prio system, a set of servers compute statistical functions over the values of all clients. It is assumed that at least one server is honest. Then, the servers learn nothing about the clients private data except information obtained from the aggregate statistics.

Benhamouda et al. [10] mentioned that multi-input functional encryption [27] implies aggregator oblivious encryption. Since Badrinarayanan et al. [4] proposed verifiable functional encryption and also considered its multi-input setting, we might be able to construct verifiable aggregator oblivious encryption from verifiable multi-input functional encryption. Though, as in Benhamouda et al., we leave this attempt in this paper due to the efficiency point of view.

Beimel et al. [8] proposed non-interactive secure multiparty computation (NIMPC), and they mentioned that NIMPC can be viewed as a simplified and restricted form of multi-input functional encryption. Moreover, they also gave a NIMPC protocol for summation (in a group  $G$ ). The construction idea is essentially the same as that of the Shi et al. aggregator oblivious encryption scheme. Briefly,  $R_1, \dots, R_n$  are randomly chosen from  $G$ , and set  $R_n = -\sum_{i=1}^{n-1} R_i$ . Each user encrypts a value  $x_i \in G$  such that  $M_i := x_i + R_i$ , and the summation can be computed by  $\sum_{i=1}^n M_i$ . In the Shi et al. scheme, the randomness is prepared by a hash function and a secret key, i.e.,  $R_i$  at time  $t$  can be seen as  $\log(H(t)^{\text{sk}_i})$ , and thus the randomness is not required to be distributed at each time.

## 1.5 Differences from the Proceedings Version

In the proceedings version [22], we claimed that the second scheme provides full aggregator unforgeability where an adversary is allowed to adaptively choose  $x_{i,t}$ , and can obtain the corresponding ciphertext and tag  $(c_{i,t}, \sigma_{i,t})$ . Intuitively, in the security proof, the simulator responds the encryption query  $(i, t, x_{i,t})$  for  $i \in [1, n-1]$  by preparing a ciphertext and tag of  $r_{i,t}$  for some random  $r_{i,t} \in \mathbb{Z}_p$  (regardless of  $x_{i,t}$ ), and for the encryption query  $(n, t, x_{n,t})$ , the simulator prepares a ciphertext and tag of  $\sum_{i=1}^n x_{i,t} - \sum_{i=1}^{n-1} r_{i,t}$ . Though the decryption result of these ciphertexts is exactly  $\sum_{i=1}^n x_{i,t}$  that the adversary queried, we need to show that ciphertexts and tags of  $r_{i,t}$  (resp.  $\sum_{i=1}^n x_{i,t} - \sum_{i=1}^{n-1} r_{i,t}$ ) and those of  $x_{i,t}$  (resp.  $x_{n,t}$ ) are indistinguishable. Though we can reduce this indistinguishability to aggregator obliviousness, for simulation, the adversary is required to send all  $\{x_{i,t}\}_{i=1}^n$ . Thus, we re-claim that the second scheme is aggregator unforgeability secure against semi-adaptive chosen message attack. Due to the additional reduction, the reduction loss of the second scheme becomes  $O(t_{\max}^2)$ . Thus, we need to reconsider the order of the underlying bilinear groups.

## 2 Preliminaries

Let  $p$  is a  $\lambda$ -bit prime,  $\mathbb{G}_1, \mathbb{G}_2$  and  $\mathbb{G}_T$  are groups of order  $p$ ,  $e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$  is a bilinear map, and  $g_1$  and  $g_2$  are generators of  $\mathbb{G}_1$  and  $\mathbb{G}_2$ , respectively. We use the (type 3) asymmetric setting, i.e.,  $\mathbb{G}_1 \neq \mathbb{G}_2$ , and no efficient isomorphism between  $\mathbb{G}_1$  and  $\mathbb{G}_2$  is known.

Next, we define the Decisional Diffie-Hellman (DDH) assumption on  $\mathbb{G}_1$  as follows.

**Definition 2 (DDH Assumption)** Let  $D := (p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$ ,  $g_1' \xleftarrow{\$} \mathbb{G}_1$  and  $r_1, r_2 \xleftarrow{\$} \mathbb{Z}_p^*$  where  $r_1 \neq r_2$ . We say that the DDH assumption holds on  $\mathbb{G}_1$  if for any PPT adversary  $\mathcal{A}$ , the advantage  $\text{Adv}_{DDH}(\lambda) := |\Pr[\mathcal{A}(D, g_1', g_1^{r_1}, g_1^{r_1}) \rightarrow \text{true}] - \Pr[\mathcal{A}(D, g_1', g_1^{r_1}, g_1^{r_2}) \rightarrow \text{true}]|$  is negligible.

Next, we define a new complexity assumption. This is a variant of the Computational Diffie-Hellman (CDH) assumption. We call this assumption the modified CDH (mCDH) assumption.<sup>2</sup>

**Definition 3 (Modified CDH Assumption)** Let  $D := (p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$ , and  $a, b \xleftarrow{\$} \mathbb{Z}_p^*$ . We say that the Modified CDH assumption holds if for any PPT adversary  $\mathcal{A}$ , the advantage  $\text{Adv}_{mCDH}(\lambda) := \Pr[\mathcal{A}(D, g_1^a, g_1^{1/a}, g_1^b, g_2^a) \rightarrow g_1^{ab}]$  is negligible.

We can check that the mCDH assumption holds in the generic bilinear group model by reducing the mCDH problem to the following problem: given  $(g_1, g_1^a, g_1^{a^2}, g_1^b, g_2, g_2^a) \in \mathbb{G}_1^4 \times \mathbb{G}_2^2$  for random  $a, b \in \mathbb{Z}_p$ , compute  $e(g_1, g_2)^{a^2 b}$ . We can assume that the problem is difficult to be solved since it belongs to the Uber assumption family [12]. This reduction can be easily done by setting  $g_1' := g_1^{1/a}$  and  $B := ab$ . Then, an instance of the mCDH problem  $(g_1, g_1^a, g_1^{1/a}, g_1^b, g_2, g_2^a)$  is represented as: given  $(g_1^a, g_1^{a^2}, g_1', g_1^B, g_2, g_2^a)$ , compute  $g_1^{ab} = g_1^{a^2 b} = g_1^{aB}$ . We rewrite it: given  $(g_1, g_1^a, g_1^{a^2}, g_1^b, g_2, g_2^a)$ , compute  $g_1^{ab}$ . That is, if the mCDH problem can be solved, then we can compute  $e(g_1^a, g_2^a) = e(g_1, g_2)^{a^2 b}$ .

<sup>2</sup>Kiltz and Vahlis [36] defined the modified Decisional Bilinear Diffie-Hellman (mDBDH) assumption where given  $(g, g^x, g^y, g^{y^2}, g^z, Z)$  decide whether  $Z = e(g, g)^{xyz}$  or not. That is, compared to the original DBDH assumption, the element  $g^{y^2}$  is additionally given to the adversary. In our assumption, if we set  $g_1^{1/a} := g_1'$  then  $(g_1^{1/a}, g_1, g_1^a)$  can be seen as  $(g_1', g_1^a, g_1^{a^2})$ . That is, the element  $g_1^{a^2}$  is added to an instance of the CDH assumption. Hence, we call the assumption mCDH.

### 3 Definitions of Verifiable Aggregator Oblivious Encryption

In this section, we give the syntax of verifiable aggregator oblivious encryption and its security definitions (aggregator obliviousness and aggregator unforgeability), and introduce the DDH-based BJJ scheme [10]. As in Shi et al. we consider encryption-once security where each user only encrypts once at each time  $t$ .

#### 3.1 Syntax of Verifiable Aggregator Oblivious Encryption

##### Definition 4 (Verifiable Aggregator Oblivious Encryption [39])

**Setup:** *The setup algorithm takes as input a security parameter  $\lambda$ , and outputs a public parameter  $\text{param}$  and a secret key of aggregator  $\text{sk}_A$ , a set of user secret keys  $\{\text{sk}_i\}_{i=1}^n$ , and the aggregate verification key  $\text{vk}$ . We assume that the maximum time  $t_{\max}$  is contained in  $\text{param}$ , and  $t_{\max}$  is a polynomial of the security parameter. We assume that  $t \in [1, t_{\max}]$  and the verification key at  $t$   $\text{vk}_t$  is contained in  $\text{vk}$ .*

**Enc:** *The encryption algorithm takes as input  $\text{param}$ ,  $t$ , a value  $x_{i,t} \in \mathbb{Z}_M$ , and  $\text{sk}_i$ , and outputs a ciphertext  $c_{i,t}$  and a tag  $\sigma_{i,t}$ . Here,  $M$  is some fixed integer contained in  $\text{param}$ .*

**AggrDec:** *The aggregation and decryption algorithm takes as input  $\text{param}$ ,  $t$ , and a set of ciphertexts and tags  $\{(c_{i,t}, \sigma_{i,t})\}_{i=1}^n$ , and  $\text{sk}_A$ , and outputs  $X_t := \sum_{i=1}^n x_{i,t} \bmod M$ , and the proof  $\sigma_t$ .*

**VerifySum:** *The verification of aggregation algorithm takes as input  $\text{param}$ ,  $t$ ,  $\text{vk}_t$ , and  $(X_t, \sigma_t)$ , and outputs 1 or 0.*

We require the following correctness. For all  $(\text{param}, \text{sk}_A, \{\text{sk}_i\}_{i=1}^n, \text{vk}) \leftarrow \text{Setup}(1^\lambda)$ , and  $(c_{i,t}, \sigma_{i,t}) \leftarrow \text{Enc}(\text{param}, t, x_{i,t}, \text{sk}_i)$ , and  $(X_t, \sigma_t) \leftarrow \text{AggrDec}(\text{param}, t, \{(c_{i,t}, \sigma_{i,t})\}_{i=1}^n, \text{sk}_A)$ ,  $\text{VerifySum}(\text{param}, t, X_t, \sigma_t, \text{vk}_t) = 1$ , and  $X_t = \sum_{i=1}^n x_{i,t} \bmod M$  hold.

Let us introduce the entities of the system and how to run the algorithms above as follows. We consider four entities, a trusted dealer, an aggregator, users, and a data analyzer. First, the dealer runs  $(\text{param}, \text{sk}_A, \{\text{sk}_i\}_{i=1}^n, \text{vk}) \leftarrow \text{Setup}(1^\lambda)$ , and issues  $\text{sk}_A$  to the aggregator and  $\text{sk}_i$  to the user  $i$ , respectively, and publishes  $(\text{param}, \text{vk})$ .<sup>3</sup> At time  $t$ , each user  $i$  encrypts  $x_{i,t}$  such that  $(c_{i,t}, \sigma_{i,t}) \leftarrow \text{Enc}(\text{param}, t, x_{i,t}, \text{sk}_i)$ , and sends  $(c_{i,t}, \sigma_{i,t})$  to the aggregator. The aggregator runs  $(X_t, \sigma_t) \leftarrow \text{AggrDec}(\text{param}, t, \{(c_{i,t}, \sigma_{i,t})\}_{i=1}^n, \text{sk}_A)$ , and sends  $(X_t, \sigma_t)$  to the data analyzer. The data analyzer checks whether the computed aggregated sum  $X_t$  is correct by running  $1/0 \leftarrow \text{VerifySum}(\text{param}, t, X_t, \sigma_t, \text{vk}_t)$ .

#### 3.2 Security Definitions

Next, we define aggregator obliviousness. This requires that the aggregator cannot learn anything more than the aggregate value  $X_t$  for each time  $t$ . We additionally require that tags  $\sigma_{i,t}$  do not affect the security. Let  $st$  be state information that  $\mathcal{A}$  can preserve any information, and  $st$  is used for transferring state information to the other stage. Let  $\mathbb{U}$  be the whole set of users for which, at the end of the game, no encryption queries have been made on  $t^*$  and no corruption queries have been made. The adversary indicates  $\mathbb{S}_{t^*} \subseteq \mathbb{U}$  and obtains  $(c_{i,t^*}, \sigma_{i,t^*})$  for all  $i \in \mathbb{S}_{t^*}$ . Remark that the **AggrDec** algorithm works only when all ciphertexts are collected. That is, if  $\mathbb{S}_{t^*}$  is a proper subset of  $\mathbb{U}$  ( $\mathbb{S}_{t^*} \subsetneq \mathbb{U}$ ), then there exist at least one ciphertext  $c_{i,t^*}$  such that  $i \in \mathbb{U} \setminus \mathbb{S}_{t^*}$ . In this case, the adversary cannot run the **AggrDec** algorithm. Thus, as in the definition of Benhamouda et al. [10] and Shi et al. [52], we require that  $\sum_{i \in \mathbb{S}_{t^*}} x_{i,t^*}^{(0)} \bmod M = \sum_{i \in \mathbb{S}_{t^*}} x_{i,t^*}^{(1)} \bmod M$  must be hold if  $\text{sk}_A$  is compromised by the adversary and  $\mathbb{S}_{t^*} = \mathbb{U}$ . Though in the definition of Leontiadis et al. [39],  $\text{sk}_A$  is always given to the adversary and always the condition  $\sum_{i \in \mathbb{S}_{t^*}} x_{i,t^*}^{(0)} \bmod M = \sum_{i \in \mathbb{S}_{t^*}} x_{i,t^*}^{(1)} \bmod M$  is required, we follow the definition given in [10, 52] where the adversary is allowed to select whether the adversary compromises  $\text{sk}_A$  or not.

<sup>3</sup>In the definition of Leontiadis et al. [39], each user  $i$  chooses a tag value  $\text{tk}_i$ , and sends its encoding value to the dealer in the **Setup** phase. The dealer computes  $\text{vk}$  from all  $\text{tk}_i$ . Here we simply assume that  $\text{vk}$  is generated by the dealer since the dealer is modeled as a trusted entity. Later, we consider the case that  $\text{vk}$  is generated by users in the encryption phase.

**Definition 5 (Aggregator Obliviousness [10, 39])** For any PPT adversary  $\mathcal{A}$  and a security parameter  $\lambda \in \mathbb{N}$ , we define the experiment  $\text{Exp}_{\mathcal{A}}^{AO}(\lambda)$  as follows. If  $\text{sk}_A$  is compromised at the end of the game and  $\mathbb{S}_{t^*} = \mathbb{U}$ , then it is required that  $\sum_{i \in \mathbb{S}_{t^*}} x_{i,t^*}^{(0)} \bmod M = \sum_{i \in \mathbb{S}_{t^*}} x_{i,t^*}^{(1)} \bmod M$ .

$\text{Exp}_{\mathcal{A}}^{AO}(\lambda) :$

$(\text{param}, \text{sk}_A, \{\text{sk}_i\}_{i=1}^n, \text{vk}) \leftarrow \text{Setup}(1^\lambda)$

$(\mathbb{S}_{t^*}, t^*, \{(x_{i,t^*}^{(0)}, x_{i,t^*}^{(1)})\}_{i \in \mathbb{S}_{t^*}}) \leftarrow \mathcal{A}^{\mathcal{O}_{\text{enc}}, \mathcal{O}_{\text{corrupt}}}(\text{param}, \text{vk}, st); \mathbb{S}_{t^*} \subseteq \mathbb{U}; b \xleftarrow{\$} \{0, 1\}$

For all  $i \in \mathbb{S}_{t^*}$

$(c_{i,t^*}, \sigma_{i,t^*}) \leftarrow \text{Enc}(\text{param}, t, x_{i,t^*}^{(b)}, \text{sk}_i)$

$b' \leftarrow \mathcal{A}^{\mathcal{O}_{\text{enc}}, \mathcal{O}_{\text{corrupt}}}(\{(c_{i,t^*}, \sigma_{i,t^*})\}_{i \in \mathbb{S}_{t^*}}, st)$

If  $b = b'$ , then return 1 and 0 otherwise

- $\mathcal{O}_{\text{enc}}$ : This encryption oracle takes as input a tuple  $(i, t, x_{i,t})$ , and returns  $(c_{i,t}, \sigma_{i,t}) \leftarrow \text{Enc}(\text{param}, t, x_{i,t}, \text{sk}_i)$ . Note that  $\mathcal{A}$  is not allowed to input  $(i, t^*, \cdot)$  where  $i \in \mathbb{S}_{t^*}$  to this oracle.
- $\mathcal{O}_{\text{corrupt}}$ : This corruption oracle takes as input  $i \in [0, n]$ , and returns  $\text{sk}_i$ . If  $i = 0$ , then the oracle returns  $\text{sk}_A$ . Note that  $\mathcal{A}$  is not allowed to input  $i \in \mathbb{S}_{t^*}$  to this oracle.

We say that an encryption scheme is aggregator obliviousness if the advantage  $\text{Adv}_{\mathcal{A}}^{AO}(\lambda) := 2|\Pr[\text{Exp}_{\mathcal{A}}^{AO}(\lambda) = 1] - 1/2|$  is negligible for any PPT adversary  $\mathcal{A}$ .

Next, we define aggregator unforgeability. This requires that an adversary (modeled as the malicious aggregator) cannot produce a forged tag  $\sigma_t$  that is accepted by the `VerifySum` algorithm. As in the definition of unforgeability given by Leontiadis et al. [39], we consider two cases: an adversary is required either the adversary does not obtain ciphertexts and tags at the challenge time  $t^*$  (type I forgery) or the adversary has obtained all ciphertexts and tags  $\{(c_{i,t^*}, \sigma_{i,t^*})\}_{i=1}^n$  (type II forgery). In the type II forgery case, it is assumed that ciphertexts and tags are honestly generated, and  $\mathcal{A}$  obtains ciphertexts and tags of all users in the system. Type I adversary captures the case that the aggregator tries to generate a forged tag  $\sigma_t$  at a future time  $t$  (i.e., users have not generated  $(c_{i,t}, \sigma_{i,t})$ ). Type II adversary captures the case that the aggregator tries to generate a forged tag  $\sigma_t$  at a past/current time  $t$  (i.e., users have generated  $(c_{i,t}, \sigma_{i,t})$ ).

**Definition 6 (Aggregator Unforgeability [39])** For any PPT adversary  $\mathcal{A}$  and a security parameter  $\lambda \in \mathbb{N}$ , we define the experiment  $\text{Exp}_{\mathcal{A}}^{AU}(\lambda)$  as follows.

$\text{Exp}_{\mathcal{A}}^{AU}(\lambda) :$

$(\text{param}, \text{sk}_A, \{\text{sk}_i\}_{i=1}^n, \text{vk}) \leftarrow \text{Setup}(1^\lambda)$

$(t^*, X_{t^*}, \sigma_{t^*}) \leftarrow \mathcal{A}^{\mathcal{O}_{\text{enc}}}(\text{param}, \text{sk}_A, \text{vk})$

If one of the followings hold, then return 1 and 0 otherwise

(Type I):  $\text{VerifySum}(\text{param}, t^*, X_{t^*}, \sigma_{t^*}, \text{vk}_{t^*}) = 1$

$\wedge$  No encryption oracle is called at  $t^*$

(Type II):  $\text{VerifySum}(\text{param}, t^*, X_{t^*}, \sigma_{t^*}, \text{vk}_{t^*}) = 1$

$\wedge X_{t^*} \neq \sum_{i=1}^n x_{i,t^*} \bmod M$

- $\mathcal{O}_{\text{enc}}$ : This encryption oracle takes as input a tuple  $(i, t, x_{i,t})$ , and returns  $(c_{i,t}, \sigma_{i,t}) \leftarrow \text{Enc}(\text{param}, t, x_{i,t}, \text{sk}_i)$ .

We say that an encryption scheme is aggregator unforgeable if the advantage  $\text{Adv}_{\mathcal{A}}^{AU}(\lambda) := \Pr[\text{Exp}_{\mathcal{A}}^{AU}(\lambda) = 1]$  is negligible for any PPT adversary  $\mathcal{A}$ .

Next, we slightly weaken the definition of Leontiadis et al. in the following. In their definition, the adversary (modeled as the malicious aggregator) can adaptively choose  $x_{i,t}$  and can obtain the corresponding  $(c_{i,t}, \sigma_{i,t})$  from the encryption oracle. This definition is an analogy of Existential Unforgeability against Chosen Message Attack (EUF-CMA) in the signature context where an adversary is allowed to obtain signatures on messages which are (adaptively) chosen by the adversary. However, in the actual situation, the aggregator does not decide  $x_{i,t}$ , and just receives  $c_{i,t}$  sent from users. Actually, in the smart meter setting,  $x_{i,t}$  (such as power consumption) is measured by the meter. Thus, it seems reasonable to propose that the adversary just queries  $(i, t)$  to the encryption oracle, and the oracle chooses  $x_{i,t}$  and returns the corresponding  $(c_{i,t}, \sigma_{i,t})$  to the adversary. Our definition is an analogy of Existential Unforgeability against Random Message Attack (EUF-RMA) in the signature context where an adversary is given signatures on randomly chosen messages.

**Definition 7 (Weak Aggregator Unforgeability)** For any PPT adversary  $\mathcal{A}$  and a security parameter  $\lambda \in \mathbb{N}$ , the experiment  $\text{Exp}_{\mathcal{A}}^{wAU}(\lambda)$  is the same as  $\text{Exp}_{\mathcal{A}}^{AU}(\lambda)$  except  $\mathcal{O}_{\text{enc}}$ .

- $\mathcal{O}_{\text{enc}}$ : This encryption oracle takes as input a tuple  $(i, t)$ . The oracle chooses  $x_{i,t}$  and returns  $(c_{i,t}, \sigma_{i,t}) \leftarrow \text{Enc}(\text{param}, t, x_{i,t}, \text{sk}_i)$ .

We say that an encryption scheme is weakly aggregator unforgeable if the advantage  $\text{Adv}_{\mathcal{A}}^{wAU}(\lambda) := \Pr[\text{Exp}_{\mathcal{A}}^{wAU}(\lambda) = 1]$  is negligible for any PPT adversary  $\mathcal{A}$ .

Next, we introduce semi-adaptive aggregator unforgeability which is stronger than the weak one but is weaker than the full aggregator unforgeability.

**Definition 8 (Semi-Adaptive Aggregator Unforgeability)** For any PPT adversary  $\mathcal{A}$  and a security parameter  $\lambda \in \mathbb{N}$ , the experiment  $\text{Exp}_{\mathcal{A}}^{saAU}(\lambda)$  is the same as  $\text{Exp}_{\mathcal{A}}^{AU}(\lambda)$  except  $\mathcal{O}_{\text{enc}}$ .

- $\mathcal{O}_{\text{enc}}$ : This encryption oracle takes as input tuples  $\{(i, t, x_{i,t})\}_{i=1}^n$ . The oracle computes  $(c_{i,t}, \sigma_{i,t}) \leftarrow \text{Enc}(\text{param}, t, x_{i,t}, \text{sk}_i)$  for all  $i \in [1, n]$ , and returns  $\{(c_{i,t}, \sigma_{i,t})\}_{i=1}^n$ .

We say that an encryption scheme is semi-adaptive aggregator unforgeable if the advantage  $\text{Adv}_{\mathcal{A}}^{saAU}(\lambda) := \Pr[\text{Exp}_{\mathcal{A}}^{saAU}(\lambda) = 1]$  is negligible for any PPT adversary  $\mathcal{A}$ .

### 3.3 The DDH-based BJJ Scheme

Benhamouda, Joye, and Libert (BJL) [10] gave a generic construction of aggregator oblivious encryption from smooth projective hash functions [18]. Here, we introduce its DDH instantiation. The underlying idea is essentially the same as that of the She et al. aggregator oblivious encryption. The aggregator has keys  $(s_0, t_0)$  where  $s_0 + \sum_{i=1}^n s_i = 0$  and  $t_0 + \sum_{i=1}^n t_i = 0$ , and this structure allows the aggregator to cancel out a part of ciphertext  $H_1(t)^{\sum_{i=1}^n s_i}$  and  $H_2(t)^{\sum_{i=1}^n t_i}$ .

**Setup:** Let  $\mathbb{G}_1$  be a DDH-hard group with  $\lambda$ -bit prime order  $p = M$  and  $g_1$  be a generator of  $\mathbb{G}_1$ .

Let  $H_i : \mathbb{Z} \rightarrow \mathbb{G}_1$  ( $i = 1, 2$ ) be hash functions. Choose  $s_1, \dots, s_n, t_1, \dots, t_n \xleftarrow{\$} \mathbb{Z}_p$ , set  $s_0 = -\sum_{i=1}^n s_i$  and  $t_0 = -\sum_{i=1}^n t_i$ . Output  $\text{param} = ((p, g_1, \mathbb{G}_1), H_1, H_2)$ ,  $\text{sk}_A = (s_0, t_0)$  and  $\text{sk}_i = (s_i, t_i)$ .

**Enc:** Parse  $\text{sk}_i = (s_i, t_i)$ . For  $x_{i,t} \in \mathbb{Z}_p$ , compute  $c_{i,t} = g_1^{x_{i,t}} H_1(t)^{s_i} H_2(t)^{t_i}$  and output  $c_{i,t}$ .

**AggrDec:** Parse  $\text{sk}_A = (s_0, t_0)$ . Compute  $V_t = H_1(t)^{s_0} H_2(t)^{t_0} \prod_{i=1}^n c_{i,t} = g_1^{X_t}$  where  $X_t = \sum_{i=1}^n x_{i,t}$ , and solve the discrete logarithm  $V_t$  with respect to basis  $g_1$ . Output  $X_t$ .

## 4 Proposed Constructions

In this section, we propose two schemes. For aggregator obliviousness, both schemes are tightly reduced to the DDH-based BJJ scheme. The first scheme only provides weak aggregator unforgeability, whereas the second scheme provides semi-adaptive aggregator unforgeability. The unforgeability of both schemes relies on the mCDH assumption and the second scheme additionally requires public channels with memory, such as a bulletin board [29] (which is publicly readable, and every user can write to, but nobody can delete from). Moreover, users are required to generate random numbers in the Enc algorithm. Thus, the Enc algorithm in the second scheme is probabilistic whereas that of the first scheme is deterministic.

## 4.1 High-level Description

**Aggregator Obliviousness:** We employ (type 3) elliptic curves where  $\mathbb{G}_1 \neq \mathbb{G}_2$  and no efficient isomorphism between  $\mathbb{G}_1$  and  $\mathbb{G}_2$  is known. Then, we run the B JL scheme [10] over the DDH-hard group  $\mathbb{G}_1$ , and borrow the ciphertext form  $c_{i,t}$  and secret keys  $\text{sk}_A$  and  $\text{sk}_i$ . Since the B JL scheme is aggregator obliviousness under the DDH assumption, we can expect that our scheme is also aggregator obliviousness. In order to directly reduce the aggregator obliviousness of our scheme to that of the B JL scheme, we independently prepare the verification part. That is, we introduce  $v_{i,t}$  for each user  $i$  and in the security proof,  $v_{i,t}$  can be chosen independently from the B JL scheme. This setting allows us to compute the tag  $\sigma_{i,t}$  from  $c_{i,t}$  and  $v_{i,t}$  in the security proof. More precisely, the challenge ciphertexts and tags of our scheme  $\{(c_{i,t^*}, \sigma_{i,t^*})\}_{i \in \mathbb{S}_{t^*}}$  can be constructed from the challenge ciphertext of the B JL scheme  $\{c_{i,t^*}\}_{i \in \mathbb{S}_{t^*}}$  and the corresponding  $v_{i,t}$ . Thus, we can construct an algorithm that breaks the aggregator obliviousness of the B JL scheme by using an adversary of our scheme. Remark that  $\sigma_{i,t}$  has the similar form of  $c_{i,t}$  in our scheme due to this reason. This strategy has been considered by Leontiadis et al. [39]. They provided a reduction of their scheme to the Shi et al. scheme [52]. However, as mentioned by Benhamouda et al. [10], the security loss is  $O(t_{\max} n^3)$  in the Shi et al. scheme, whereas it is  $O(t_{\max})$  in the B JL scheme. Thus, we have chosen the B JL scheme as the underlying scheme in this paper.

**Aggregator Unforgeability:** For public verification, we pay attention to that the form of the ciphertext  $c_{i,t}$  of the B JL scheme is similar to a decryption key of the Boneh-Boyen identity-based encryption (IBE) scheme [11].<sup>4</sup> Due to the above reason, the tag  $\sigma_{i,t}$  has the similar form of  $c_{i,t}$  in our schemes. Since secure IBE implies a signature [19] (informally, ID is regarded as a message to be signed, and its decryption key is regarded as a signature), we can expect that  $\sigma_{i,t}$  is unforgeable. However, to utilize the Boneh-Boyen technique,  $X_t$  needs to be embedded into  $\text{vk}$  in the security proof. Here, we have two choices: whether  $\text{vk}$  is fixed in the setup phase or not. If  $\text{vk}$  is chosen by the honest dealer and is fixed in the setup phase,  $X_t$  is also required to be fixed in the setup phase (to utilize the security proof technique of selective-ID security of Boneh-Boyen IBE), and therefore only weak aggregator unforgeability is provided. Moreover, since one  $X_t$  is embedded with one  $\text{vk}$ , long verification keys is also required where the size linearly depends on  $t_{\max}$ . We set  $\text{vk} = \{\text{vk}_t\}_{t \in [1, t_{\max}]}$  and  $\text{vk}_t := g_2^{\sum_{i=1}^n v_{i,t}}$  for  $t \in [1, t_{\max}]$ . We remark that no user is required to have the large-size verification key. Moreover, if a user  $i$  manages all  $v_{i,t}$  for  $t \in [1, t_{\max}]$  as its secret key  $\text{sk}_i$ , the secret key size also depends on  $t_{\max}$ . To avoid such a large-size secret key, we additionally introduce a hash function  $H$  and a time-independent secret key  $v_i$ , and we compute  $v_{i,t} = H(v_i, t)$ . That is, in the scheme  $v_{i,t}$  is computed by  $H(v_i, t)$  whereas in the security proof,  $v_{i,t}$  is selected so as to utilize the Boneh-Boyen technique, and set  $H(v_i, t) := v_{i,t}$ . This helps us to reduce the secret key size.

## 4.2 The Proposed Scheme 1: Providing Weak Aggregator Unforgeability

We give the first scheme as follows. As mentioned above,  $\text{vk}$  is chosen in the setup phase.

**Setup( $1^\lambda$ ):** Choose  $(p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$  where  $\mathbb{G}_1, \mathbb{G}_2$  and  $\mathbb{G}_T$  are groups of  $\lambda$ -bit prime order  $p = M$ ,  $g_1 \in \mathbb{G}_1$  and  $g_2 \in \mathbb{G}_2$  are generators, and  $e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$  is a bilinear map. Let  $H : \mathbb{Z}_p \times [1, t_{\max}] \rightarrow \mathbb{Z}_p$  and  $H_i : \mathbb{Z} \rightarrow \mathbb{G}_1$  ( $i = 1, 2, 3, 4, 5$ ) be hash functions. Choose  $\gamma, s_1, \dots, s_n, t_1, \dots, t_n, v_1, \dots, v_n \xleftarrow{\$} \mathbb{Z}_p$ , compute  $v_{i,t} = H(v_i, t)$  for all  $i \in [1, n]$  and  $t \in [1, t_{\max}]$  and set  $s_0 = -\sum_{i=1}^n s_i$ ,  $t_0 = -\sum_{i=1}^n t_i$ ,  $h = g_1^\gamma$ , and  $Z = e(h, g_2)$ . Output  $\text{param} = ((p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T), Z, H, H_1, H_2, H_3, H_4, H_5)$ ,  $\text{sk}_A = (s_0, t_0)$ ,  $\text{sk}_i = (s_i, t_i, v_i, h)$ , and  $\text{vk} = \{\text{vk}_t\}_{t \in [1, t_{\max}]}$  where  $\text{vk}_t = g_2^{\sum_{i=1}^n v_{i,t}}$ .

**Enc( $\text{param}, t, x_{i,t}, \text{sk}_i$ ):** Parse  $\text{sk}_i = (s_i, t_i, v_i, h)$ . Compute

$$v_{i,t} = H(v_i, t), \quad c_{i,t} = g_1^{x_{i,t}} H_1(t)^{s_i} H_2(t)^{t_i}, \quad \text{and} \quad \sigma_{i,t} = h^{x_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} H_5(t)^{v_{i,t}}$$

and output  $(c_{i,t}, \sigma_{i,t})$ .

<sup>4</sup>A decryption key of the Boneh-Boyen IBE scheme is informally described as  $(g^\alpha H_{\text{BB}}(ID)^r, g^r)$  for a master key  $\alpha$  and a random  $r$ , the Boneh-Boyen hash  $H_{\text{BB}}$ . In our first construction,  $\alpha, ID$ , and  $r$  are regarded as  $x_{i,t}, t$ , and  $v_{i,t}$  respectively. Thus, the number of verification keys depends on  $t_{\max}$ .

AggrDec(param,  $t$ ,  $\{(c_{i,t}, \sigma_{i,t})\}_{i=1}^n$ ,  $\text{sk}_A$ ): Parse  $\text{sk}_A = (s_0, t_0)$ . Compute

$$V_t = H_1(t)^{s_0} H_2(t)^{t_0} \prod_{i=1}^n c_{i,t} = g_1^{X_t}$$

where  $X_t = \sum_{i=1}^n x_{i,t}$ , and solve the discrete logarithm  $V_t$  with respect to basis  $g_1$ . Moreover, compute

$$\sigma_t = H_3(t)^{s_0} H_4(t)^{t_0} \prod_{i=1}^n \sigma_{i,t}$$

Output  $(X_t, \sigma_t)$ .

VerifySum(param,  $t$ ,  $X_t, \sigma_t, \text{vk}_t$ ): Output 1 if

$$\frac{e(\sigma_t, g_2)}{e(H_5(t), \text{vk}_t)} = Z^{X_t}$$

holds. Otherwise, output 0.

The correctness clearly holds from the following equations.

$$\begin{aligned} H_1(t)^{s_0} H_2(t)^{t_0} \prod_{i=1}^n c_{i,t} &= H_1(t)^{s_0} H_2(t)^{t_0} \prod_{i=1}^n g_1^{x_{i,t}} H_1(t)^{s_i} H_2(t)^{t_i} \\ &= H_1(t)^{s_0 - \sum_{i=1}^n s_i} H_2(t)^{t_0 - \sum_{i=1}^n t_i} g_1^{\sum_{i=1}^n x_{i,t}} \\ &= g_1^{X_t} \\ \sigma_t &= H_3(t)^{s_0} H_4(t)^{t_0} \prod_{i=1}^n \sigma_{i,t} \\ &= H_3(t)^{s_0} H_4(t)^{t_0} \prod_{i=1}^n h^{x_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} H_5(t)^{v_{i,t}} \\ &= h^{X_t} H_5(t)^{\sum_{i=1}^n v_{i,t}} \\ e(\sigma_t, g_2) &= e(h^{X_t} H_5(t)^{\sum_{i=1}^n v_{i,t}}, g_2) = e(h, g_2)^{X_t} e(H_5(t), g_2^{\sum_{i=1}^n v_{i,t}}) \\ &= Z^{X_t} e(H_5(t), \text{vk}_t) \end{aligned}$$

**Theorem 4.1** *Our scheme 1 is aggregator obliviousness under the DDH assumption on  $\mathbb{G}_1$  in the random oracle model.*

We consider the following two games. Game 0 is the original game. Game 1 is the same as Game 0 except that  $H_3$  and  $H_4$  are computed as  $H_3(t) = H_1(t)^\gamma$  and  $H_4(t) = H_2(t)^\gamma$  for some  $\gamma \in \mathbb{Z}_p$ . Since  $(H_1(t), H_2(t), H_3(t), H_4(t))$  is a DDH tuple, this modification does not affect the security under the DDH assumption on  $\mathbb{G}_1$ . Briefly, let  $(g_1, g'_1, g_1^{r_1}, g_1^{r_2}) \in \mathbb{G}_1^4$  be an DDH instance on  $\mathbb{G}_1$ . For  $t \in [1, t_{\max}]$ , choose  $\tilde{t}_1, \tilde{t}_2 \xleftarrow{\$} \mathbb{Z}_p$ , and set  $H_1(t) := g_1^{\tilde{t}_1}$ ,  $H_2(t) := g_1^{\tilde{t}_2}$ ,  $H_3(t) := (g_1^{r_1})^{\tilde{t}_1}$ , and  $H_4(t) := (g_1^{r_2})^{\tilde{t}_2}$ . Clearly, if the instance is not a DDH tuple, i.e.,  $r_1 \neq r_2$ , then we simulate Game 0, and if the instance is a DDH tuple, i.e.,  $r_1 = r_2$ , then we simulate Game 1. In Game 1, we construct an algorithm  $\mathcal{B}$  that breaks aggregator obliviousness of the BJJL scheme as follows.

**Proof:** Let  $\mathcal{A}$  be the adversary of our scheme, and  $\mathcal{C}$  be the challenger of the BJJL scheme. We construct an algorithm  $\mathcal{B}$  that breaks aggregator obliviousness of the BJJL scheme as follows. First,  $\mathcal{C}$  prepares  $(p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, H_1, H_2)$  and sends it to  $\mathcal{B}$ .  $\mathcal{B}$  chooses  $\gamma, v_1, \dots, v_n, v_{1,1}, \dots, v_{n, t_{\max}} \xleftarrow{\$} \mathbb{Z}_p$ .  $\mathcal{B}$  computes  $h = g_1^\gamma$ ,  $Z = e(h, g_2)$ ,  $\text{vk}_{i,t} = g_2^{v_{i,t}}$  for  $i \in [1, n]$  and  $t \in [1, t_{\max}]$ , and  $\text{vk}_t = g_2^{\sum_{i=1}^n v_{i,t}}$ .  $\mathcal{B}$  sets  $H(v_i, t) := v_{i,t}$  for  $i \in [1, n]$  and  $t \in [1, t_{\max}]$ . Remark that if  $\mathcal{A}$  sends a hash query  $t$ , then  $\mathcal{B}$  forwards it to  $\mathcal{C}$  when  $\mathcal{A}$  requests  $H_1(t)$  or  $H_2(t)$ . For  $H_3$  and  $H_4$ ,  $\mathcal{B}$  sets  $H_3(t) = H_1(t)^\gamma$  and

$H_4(t) = H_2(t)^\gamma$ , and returns the hash values. For  $H_5$ ,  $\mathcal{B}$  just returns a random value.  $\mathcal{B}$  sends  $\text{param} = ((p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T), Z, H, H_1, H_2, H_3, H_4, H_5), \{\text{vk}_t\}_{t \in [1, t_{\max}]}$ , and  $\text{vk}$  to  $\mathcal{A}$ .

If  $\mathcal{A}$  sends an encryption query  $(i, t, x_{i,t})$  to  $\mathcal{B}$ , then  $\mathcal{B}$  forwards it to  $\mathcal{C}$  as an encryption oracle, and obtains  $c_{i,t}$ .  $\mathcal{B}$  computes  $c_{i,t}^\gamma H_5(t)^{v_{i,t}} = h^{x_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} H_5(t)^{v_{i,t}}$ , and returns  $(c_{i,t}, \sigma_{i,t})$  to  $\mathcal{A}$ . If  $\mathcal{A}$  sends a corruption query  $i \in [0, n]$  to  $\mathcal{B}$ ,  $\mathcal{B}$  forwards it to  $\mathcal{C}$  as a corruption query, and obtains  $\text{sk}_A$  (if  $i = 0$ ) or  $(s_i, t_i)$  (if  $i \in [1, n]$ ). If  $i = 0$ , then  $\mathcal{B}$  returns  $\text{sk}_A$  to  $\mathcal{A}$ . If  $i \in [1, n]$ , then  $\mathcal{B}$  sets  $\text{sk}_i = (s_i, t_i, v_i, h)$ , and returns  $\text{sk}_i$  to  $\mathcal{A}$ . We remark that if  $\mathcal{A}$  sends a hash query  $(v_i, t)$ , then  $\mathcal{B}$  responds  $v_{i,t}$  to  $\mathcal{A}$ .

In the challenge phase,  $\mathcal{A}$  sends  $(\mathbb{S}_{t^*}, t^*, \{(x_{i,t^*}^{(0)}, x_{i,t^*}^{(1)})\}_{i \in \mathbb{S}_{t^*}})$  to  $\mathcal{B}$ . Then,  $\mathcal{B}$  forwards it to  $\mathcal{C}$  as the challenge, and obtains  $\{c_{i,t^*}\}_{i \in \mathbb{S}_{t^*}}$ . As in the response of encryption queries,  $\mathcal{B}$  computes  $\sigma_{i,t^*} = c_{i,t^*}^\gamma H_5(t)^{v_{i,t^*}}$  for  $i \in \mathbb{S}_{t^*}$ , and returns  $\{(c_{i,t^*}, \sigma_{i,t^*})\}_{i \in \mathbb{S}_{t^*}}$  to  $\mathcal{A}$ .

$\mathcal{B}$  responds queries sent from  $\mathcal{A}$  as in the previous phase. Finally,  $\mathcal{A}$  outputs a bit  $b'$ .  $\mathcal{B}$  outputs  $b'$  and then  $\mathcal{B}$  can break aggregator obliviousness of the B JL scheme with the same advantage of  $\mathcal{A}$ . This concludes the proof since the B JL scheme is aggregator obliviousness under the DDH assumption on  $\mathbb{G}_1$  in the random oracle model.  $\square$

**Theorem 4.2** *Our scheme 1 is weakly aggregator unforgeable under the mCDH assumption in the random oracle model.*

For the proof of Type I forgery, we employ the following assumption: given  $(g_1^a, g_1^b, g_2^a)$  compute  $g_1^{ab}$ . Since this is equivalent to the CDH assumption if the symmetric pairing setting is employed, we simply call the assumption the CDH assumption in this paper. Remark that this is weaker than mCDH since  $g_1^{1/a}$  is not contained in the instance. Since no encryption oracle is called at  $t^*$ , the proof is relatively easy. We embed the instance  $g_1^a$  to  $v_{i,t}$  and  $g_1^b$  to the response of the random oracle  $H_5$  respectively. At time  $t^*$ ,  $\mathcal{A}$  outputs  $(\sigma_{t^*}, X_{t^*})$ . From the verification equation,  $(\sigma_{t^*}, X_{t^*})$  must satisfy  $\sigma_{t^*} = H_5(t^*)^{\sum_{i=1}^n v_{i,t^*}} h^{X_{t^*}}$ . Since  $H_5(t^*)^{\sum_{i=1}^n v_{i,t^*}}$  contains  $g_1^{ab}$ , we can solve the CDH problem. Remark that this proof strategy requires  $O(t_{\max})$  reduction loss from the advantage of the CDH problem. However, we can achieve a tight reduction (i.e.,  $O(1)$  reduction loss) from the advantage of the mCDH problem (see below).

For the proof of Type II forgery, our proof strategy is explained as follows. Again,  $(\sigma_{t^*}, X_{t^*})$  must satisfy  $\sigma_{t^*} = H_5(t^*)^{\sum_{i=1}^n v_{i,t^*}} h^{X_{t^*}}$ . Though  $Z = e(h, g_2)$  is published,  $h$  itself is not published (contained in  $\text{sk}_i$ ). Thus, we set  $h = g_1^{ab}$  and simulate the encryption oracle by using the Boneh-Boyen technique. We embed  $1/a$  to  $x_{i,t}$  such that  $x_{i,t} := x'_{i,t}/a$  for  $x'_{i,t} \in \mathbb{Z}_p$ . This setting helps us to compute  $h^{x_{i,t}} = (g_1^{ab})^{x'_{i,t}/a} = (g_1^b)^{x'_{i,t}}$  without knowing  $h = g_1^{ab}$ . Remark that ciphertexts  $\{c_{i,t}\}$  must be decryptable by the adversary, i.e., the discrete logarithm  $\log_{g_1} V_t$  must be sufficiently small. If all  $x_{i,t}$  are related to  $1/a$  as above, then  $\log_{g_1} V_{t^*} = (\sum_{i=1}^n x'_{i,t^*})/a$  is not computable. Thus, for relatively small  $X'_t$ , we set  $x_{i,t} := x'_{i,t}/a$  for  $i \in [1, n-1]$  and set  $x_{n,t} := X'_t - \sum_{i=1}^{n-1} x'_{i,t}/a$ . Then,  $\sum_{i=1}^n x_{i,t} = X'_t$  holds and  $\log_{g_1} V_t = X'_t$  is computable by the adversary as in the scheme. For simulation, we need to decide each  $X'_t$  in the setup phase, and embed it to  $v_{n,t}$  for utilizing the Boneh-Boyen technique. This is the reason why our scheme is weak aggregator unforgeable ( $x_{i,t}$  is chosen by the oracle), and the size of verification keys linearly depend on  $t_{\max}$ . Remark that we can achieve a tight reduction (i.e.,  $O(1)$  reduction loss) from the advantage of the mCDH problem, and this proof also works well for Type I forgery (simply we assume that the encryption oracle at  $t^*$  is not sent from  $\mathcal{A}$ , choose  $X'_{t^*}$  randomly, and  $X_{t^*} \neq X'_{t^*}$  holds with overwhelming probability  $1 - 1/p$ ).

**Proof:**

**Type I Forgery :** Let  $(p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, (g_1^a, g_1^b, g_2^a))$  be an instance of the CDH problem. We construct an algorithm  $\mathcal{B}$  that computes  $g_1^{ab}$  by using an adversary  $\mathcal{A}$  that breaks weak aggregator unforgeability of our scheme as follows.  $\mathcal{B}$  sets  $\text{param} = (p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$ , chooses  $\gamma, s_i, t_i$ , and  $\text{sk}_A$  as usual, chooses  $v'_{i,t} \xleftarrow{\$} \mathbb{Z}_p$  for  $i \in [1, n]$  such that  $\sum_{i=1}^n v'_{i,t} \neq 0$ , and chooses  $t \in [1, t_{\max}]$ , and implicitly sets  $v_{i,t} := v'_{i,t} a$ .  $\mathcal{B}$  computes  $\text{vk}_t = (g_2^a)^{\sum_{i=1}^n v'_{i,t}}$ .  $\mathcal{B}$  sends  $(\text{params}, \text{sk}_A, \text{vk} = \{\text{vk}_t\}_{t \in [1, t_{\max}]})$  to  $\mathcal{A}$ .

Moreover,  $\mathcal{B}$  guesses  $t^*$  (with success probability  $1/t_{\max}$ ). For a time  $t$ ,  $\mathcal{B}$  chooses  $\tilde{t} \xleftarrow{\$} \mathbb{Z}_p$  and sets  $H_5(t)$  as

$$H_5(t) = \begin{cases} g_1^{\tilde{t}} & (t \neq t^*) \\ (g_1^b)^{\tilde{t}^*} & (t = t^*) \end{cases}$$

For other hash functions,  $\mathcal{B}$  just returns a random value. For responding an encryption query  $(i, t)$  where  $t \neq t^*$ ,  $\mathcal{B}$  chooses  $x_{i,t}$  and computes  $c_{i,t}$  as usual, and computes  $\sigma_{i,t} = h^{x_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} (g_1^a)^{v'_{i,t} \tilde{t}} = h^{x_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} (g_1^{\tilde{t}})^{av'_{i,t}} = h^{x_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} H_5(t)^{v_{i,t}}$ . Remark that  $\mathcal{A}$  does not send an encryption query at time  $t^*$  in this type.

Finally, at time  $t^*$ ,  $\mathcal{A}$  outputs  $(\sigma_{t^*}, X_{t^*})$ . From the verification equation,  $(\sigma_{t^*}, X_{t^*})$  must satisfy  $\sigma_{t^*} = H_5(t^*)^{\sum_{i=1}^n v_{i,t^*}} h^{X_{t^*}}$ . That is,

$$\sigma_{t^*} h^{-X_{t^*}} = H_5(t^*)^{\sum_{i=1}^n v_{i,t^*}} = ((g_1^b)^{\tilde{t}^*})^a \sum_{i=1}^n v'_{i,t^*}$$

holds.  $\mathcal{B}$  solves the CDH problem by computing  $(\sigma_{t^*} h^{-X_{t^*}})^{1/\tilde{t}^* \sum_{i=1}^n v'_{i,t^*}} = g_1^{ab}$ .

**Type II Forgery** : Let  $(p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, (g_1^a, g_1^b, g_1^{1/a}, g_2^a))$  be an instance of the Modified CDH problem. We construct an algorithm  $\mathcal{B}$  that computes  $g_1^{ab}$  by using an adversary  $\mathcal{A}$  that breaks weak aggregator unforgeability of our scheme as follows.  $\mathcal{B}$  sets  $\text{param} = (p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$ , chooses  $\gamma, s_i, t_i, \text{sk}_A$ , and  $v_{i,t}$  for  $i = [1, n-1]$  and  $t \in [1, t_{\max}]$  as usual. For  $t \in [1, t_{\max}]$ ,  $\mathcal{B}$  chooses  $v'_{n,t} \xleftarrow{\$} \mathbb{Z}_p$ , and also chooses  $X'_t \xleftarrow{\$} \mathbb{Z}_p$  such that the size of  $X'_t$  is sufficiently small where the discrete logarithm problem  $g_1^{X'_t}$  with respect to basis  $g_1$  can be solved. This is the necessary condition that ciphertexts can be decrypted by the adversary as in the scheme. For  $t \in [1, t_{\max}]$ ,  $\mathcal{B}$  chooses  $\tilde{t} \xleftarrow{\$} \mathbb{Z}_p$  and sets  $H_5(t)$  as  $(g_1^b)^{\tilde{t}}$ .  $\mathcal{B}$  implicitly sets  $v_{n,t} = v'_{n,t} + (-aX'_t)/\tilde{t}$ .  $\mathcal{B}$  computes  $\text{vk}_t = (g_2^a)^{-X'_t/\tilde{t}} g_2^{v'_{n,t} + \sum_{i=1}^{n-1} v_{i,t}}$ .  $\mathcal{B}$  implicitly sets  $h = g_1^{ab}$  and computes  $Z = e(g_1^b, g_2^a) = e(h, g_2)$ .  $\mathcal{B}$  sends  $(\text{params}, \text{sk}_A, \text{vk} = \{\text{vk}_t\}_{t \in [1, t_{\max}]})$  to  $\mathcal{A}$ .

For responding an encryption query  $(i, t)$ ,  $\mathcal{B}$  computes  $(c_{i,t}, \sigma_{i,t})$  as follows.  $\mathcal{B}$  chooses  $x'_{i,t} \xleftarrow{\$} \mathbb{Z}_p$  for  $i \in [1, n-1]$  and implicitly sets  $x_{i,t}$  as

$$x_{i,t} = \begin{cases} x'_{i,t}/a & (i \in [1, n-1]) \\ X'_t - \sum_{i=1}^{n-1} x'_{i,t}/a & (i = n) \end{cases}$$

and computes

$$\begin{aligned} i \in [1, n-1] : c_{i,t} &= (g_1^{1/a})^{x'_{i,t}} H_1(t)^{s_i} H_2(t)^{t_i} \\ &= g_1^{x'_{i,t}/a} H_1(t)^{s_i} H_2(t)^{t_i} = g_1^{x_{i,t}} H_1(t)^{s_i} H_2(t)^{t_i} \\ i = n : c_{i,t} &= g_1^{X'_t} (g_1^{1/a})^{-\sum_{i=1}^{n-1} x'_{i,t}} H_1(t)^{s_i} H_2(t)^{t_i} \\ &= g_1^{X'_t - \sum_{i=1}^{n-1} x'_{i,t}/a} H_1(t)^{s_i} H_2(t)^{t_i} \\ &= g_1^{x_{i,t}} H_1(t)^{s_i} H_2(t)^{t_i} \end{aligned}$$

and

$$\begin{aligned} i \in [1, n-1] : \sigma_{i,t} &= (g_1^b)^{x'_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} H_5(t)^{v_{i,t}} \\ &= (g_1^{ab})^{x'_{i,t}/a} H_3(t)^{s_i} H_4(t)^{t_i} H_5(t)^{v_{i,t}} \\ &= h^{x_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} H_5(t)^{v_{i,t}} \\ i = n : \sigma_{i,t} &= (g_1^b)^{-\sum_{i=1}^{n-1} x'_{i,t} + \tilde{t} v'_{n,t}} H_3(t)^{s_i} H_4(t)^{t_i} \\ &= (g_1^{ab})^{X'_t} (g_1^b)^{-\sum_{i=1}^{n-1} x'_{i,t}} (g_1^{-ab})^{X'_t} (g_1^b)^{\tilde{t} v'_{n,t}} H_3(t)^{s_i} H_4(t)^{t_i} \\ &= (g_1^{ab})^{X'_t} (g_1^{ab})^{-\sum_{i=1}^{n-1} x'_{i,t}/a} (g_1^{-ab})^{X'_t} (g_1^b)^{\tilde{t} v'_{n,t}} H_3(t)^{s_i} H_4(t)^{t_i} \\ &= (g_1^{ab})^{X'_t - \sum_{i=1}^{n-1} x'_{i,t}/a} H_3(t)^{s_i} H_4(t)^{t_i} ((g_1^b)^{\tilde{t}})^{v'_{n,t} + (-aX'_t)/\tilde{t}} \\ &= h^{x_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} H_5(t)^{v_{i,t}} \end{aligned}$$

Remark that  $\sum_{i=1}^n x_{i,t} = X'_t$  and  $\{c_{i,t}\}_{i \in [1, n]}$  can be decrypted by the adversary who has  $\text{sk}_A$ .

Finally,  $\mathcal{A}$  outputs  $(t^*, X_{t^*}, \sigma_{t^*})$  where  $t^* \in [1, t_{\max}]$  and  $X_{t^*} \neq X'_{t^*}$ . From the verification equation,  $(\sigma_{t^*}, X_{t^*})$  must satisfy  $\sigma_{t^*} = H_5(t^*)^{\sum_{i=1}^n v_{i,t^*}} h^{X_{t^*}}$ . Here,  $\sigma_{t^*} = H_5(t^*)^{\sum_{i=1}^n v_{i,t^*}} h^{X_{t^*}} = ((g_1^b)^{\tilde{t}^*})^{v'_{n,t^*} + (-aX'_{t^*}/\tilde{t}^*) + \sum_{i=1}^{n-1} v_{i,t^*}} (g_1^{ab})^{X_{t^*}} = (g_1^{ab})^{X_{t^*} - X'_{t^*}} (g_1^b)^{\tilde{t}^* (v'_{n,t^*} + \sum_{i=1}^{n-1} v_{i,t^*})}$  holds.  $\mathcal{B}$  computes

$$(\sigma_{t^*} / (g_1^b)^{\tilde{t}^* (v'_{n,t^*} + \sum_{i=1}^{n-1} v_{i,t^*})})^{1/(X_{t^*} - X'_{t^*})} = g_1^{ab}$$

and solves the mCDH problem.  $\square$

### 4.3 The Proposed Scheme 2: Providing Semi-Adaptive Aggregator Unforgeability

In the first scheme,  $v_{i,t}$  is chosen in the setup phase. This leads to large-size verification keys, and is the reason why the first scheme provides weak aggregator unforgeability. As mentioned before, as another choice, a user  $i$  chooses  $v_{i,t} \xleftarrow{\$} \mathbb{Z}_p$  at time  $t$  on the fly (i.e., in the encryption phase), computes  $\text{vk}_{i,t} := g_1^{v_{i,t}}$ , and sends  $\text{vk}_{i,t}$  to the aggregator together with  $(c_{i,t}, \sigma_{i,t})$ . Then  $\text{vk}_t = \prod_{i=1}^n \text{vk}_{i,t}$  is used in the `VerifySum` algorithm. In this case,  $X_t$ , chosen by the adversary in the security proof, can be embedded to  $\text{vk}_t$  on the fly in the encryption oracle. Moreover, one hash function  $H$  and  $\text{vk}$  can be removed from the public value, and  $v_i$  can also be removed from  $\text{sk}_i$ .

One problem with this strategy is that the `Enc` algorithm becomes probabilistic. That is, a user is required to generate a random number  $v_{i,t}$  for each time  $t$ . This could be problematic if users have limited computational power. Another problem is that the aggregator (which is an adversary of the aggregator unforgeability game) could modify  $\text{vk}_t$ , and the `VerifySum` algorithm is run by a maliciously generated  $\text{vk}_t$ . Then, no security is guaranteed. One solution is to use a bulletin board [29] which is publicly readable and every user can write to, but nobody can delete from. The bulletin board can be considered a public channel with memory. That is, a user  $i$  writes  $\text{vk}_{i,t}$  to the bulletin board `BB`. Remark that the computation cost of  $\text{vk}_t = \prod_{i=1}^n \text{vk}_{i,t}$  is almost similar to that of  $V_t = H_1(t)^{s_0} H_2(t)^{t_0} \prod_{i=1}^n c_{i,t}$ . That is, if a data analyzer who runs the `VerifySum` algorithm computes  $\text{vk}_t$ , then the data analyzer does not need to delegate the computation of the aggregated sum to the aggregator, and this leads to a wag-the-dog situation. So, we assume that the aggregator computes  $\text{vk}_t$ , and  $\text{vk}_{i,t}$  written in `BB` acts as a deterrent against the aggregator that modifies  $\text{vk}_t$ , since the data analyzer can check anytime whether  $\text{vk}_t$  provided by the aggregator is computed by  $\{\text{vk}_{i,t}\}_{i=1}^n$  or not.<sup>5</sup> In summary, we slightly modify the syntax such that the bulletin board `BB` is added as an input of the `Enc` algorithm, and the `AggrDec` algorithm outputs  $\text{vk}_t$  together with  $(X_t, \sigma_t)$ .

We give the second scheme as follows.

**Setup( $1^\lambda$ ):** Choose  $(p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$  where  $\mathbb{G}_1, \mathbb{G}_2$  and  $\mathbb{G}_T$  are groups of  $\lambda$ -bit prime order  $p = M$ ,  $g_1 \in \mathbb{G}_1$  and  $g_2 \in \mathbb{G}_2$  are generators, and  $e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$  is a bilinear map. Let  $H_i : \mathbb{Z} \rightarrow \mathbb{G}_1$  ( $i = 1, 2, 3, 4, 5$ ) be hash functions. Choose  $\gamma, s_1, \dots, s_n, t_1, \dots, t_n \xleftarrow{\$} \mathbb{Z}_p$ , set  $s_0 = -\sum_{i=1}^n s_i$ ,  $t_0 = -\sum_{i=1}^n t_i$ ,  $h = g_1^\gamma$ , and  $Z = e(h, g_2)$ . Output `param` =  $((p, e, g_1, g_2, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T), Z, H_1, H_2, H_3, H_4, H_5)$ , `skA` =  $(s_0, t_0)$ , `ski` =  $(s_i, t_i, h)$ , and `vk` =  $\emptyset$ .

**Enc(`param`,  $t, x_{i,t}, \text{sk}_i, \text{BB}):$**  Parse `ski` =  $(s_i, t_i, h)$ . Choose  $v_{i,t} \xleftarrow{\$} \mathbb{Z}_p$ , compute  $\text{vk}_{i,t} := g_1^{v_{i,t}}$ , and compute

$$c_{i,t} = g_1^{x_{i,t}} H_1(t)^{s_i} H_2(t)^{t_i} \text{ and } \sigma_{i,t} = h^{x_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} H_5(t)^{v_{i,t}}$$

and output  $(c_{i,t}, \sigma_{i,t}, \text{vk}_{i,t})$ . Moreover, write  $\text{vk}_{i,t}$  to the bulletin board `BB`.

**AggrDec(`param`,  $t, \{(c_{i,t}, \sigma_{i,t}, \text{vk}_{i,t})\}_{i=1}^n, \text{sk}_A):$**  Parse `skA` =  $(s_0, t_0)$ . Compute

$$V_t = H_1(t)^{s_0} H_2(t)^{t_0} \prod_{i=1}^n c_{i,t} = g_1^{X_t}$$

<sup>5</sup>Under this assumption, it may be enough to add ciphertexts of the BJJ scheme to `BB` since the data analyzer, who is also assumed to have `skA`, can check anytime whether the summation provided by the aggregator is correctly computed or not by running the `AggrDec` algorithm by myself. Then, no tag is required for verification. However, in this case, the data analyzer is required to solve the discrete logarithm problem which is not required in the second scheme. Thus, for reducing the computational cost of the data analyzer, we choose the current setting but there is room for argument on this point.

where  $X_t = \sum_{i=1}^n x_{i,t}$ , and solve the discrete logarithm  $V_t$  with respect to basis  $g_1$ . Moreover, compute

$$\sigma_t = H_3(t)^{s_0} H_4(t)^{t_0} \prod_{i=1}^n \sigma_{i,t} \text{ and } \mathbf{vk}_t = \prod_{i=1}^n \mathbf{vk}_{i,t}$$

Output  $(X_t, \sigma_t, \mathbf{vk}_t)$ .

**VerifySum(param, t, X\_t, \sigma\_t, \mathbf{vk}\_t)**: Output 1 if

$$\frac{e(\sigma_t, g_2)}{e(H_5(t), \mathbf{vk}_t)} = Z^{X_t}$$

holds. Otherwise, output 0.

**Theorem 4.3** *Our scheme 2 is aggregator obliviousness under the DDH assumption on  $\mathbb{G}_1$  in the random oracle model.*

This is essentially the same as that of the first scheme. We omit it.

**Theorem 4.4** *Our scheme 2 is semi-adaptively aggregator unforgeable under the DDH and mCDH assumptions in the random oracle model.*

**Proof:** The simulation is almost similar to that of the first scheme. Remark that  $c_{i,t}$  is not a ciphertext of  $x_{i,t}$  in the simulation. We can regard  $c_{i,t}$  is a ciphertext of  $r_{i,t}$  for some random  $r_{i,t} \in \mathbb{Z}_p$ , and  $c_{n,t}$  is a ciphertext of  $\sum_{i=1}^n x_{i,t} - \sum_{i=1}^{n-1} r_{i,t}$ . Thus, we need to show that these modifications do not affect the security. We reduce the indistinguishability to aggregator obliviousness as follows. We define sequential of games. Let  $\text{Game}_0$  be the original game, and  $\text{Game}_1$  be the same as  $\text{Game}_0$  except that ciphertexts and tags are computed as above. We define subgames  $\text{Game}_j$  for  $j \in [1, t_{\max} + 1]$  where  $\text{Game}_{t_{\max}} := \text{Game}_1$ . Let  $\mathcal{C}$  be the challenger of aggregator obliviousness that prepares  $(\text{param}, \text{sk}_A, \{\text{sk}_i\}_{i=1}^n)$ , and  $\mathcal{B}$  be the simulator.  $\mathcal{B}$  requests  $\text{sk}_A$  to  $\mathcal{C}$ , and sends  $(\text{param}, \text{sk}_A)$  to the adversary  $\mathcal{A}$ . Let  $\{(i, j, x_{i,j})\}_{i=1}^n$  be the  $j$ -th encryption query.  $\mathcal{B}$  randomly chooses  $r_{i,j} \xleftarrow{\$} \mathbb{Z}_p$  for  $i \in [1, n-1]$ , and sets the challenge message  $(\mathbb{U}, j := t^*, \{(x_{i,j}^{(0)}, x_{i,j}^{(1)})\}_{i \in \mathbb{U}})$  where  $x_{i,j}^{(0)} = x_{i,j}$  for  $i \in [1, n]$ ,  $x_{i,j}^{(1)} = r_{i,j}$  for  $i \in [1, n-1]$ , and  $x_{n,j}^{(1)} = \sum_{i=1}^n x_{i,j} - \sum_{i=1}^{n-1} r_{i,j}$ . Remark that  $\sum_{i=1}^n x_{i,j}^{(0)} = \sum_{i=1}^n x_{i,j}^{(1)}$  holds. If  $b = 0$ , then it simulates  $\text{Game}_{j-1}$ , and if  $b = 1$ , then it simulates  $\text{Game}_j$ . Thus, two games are indistinguishable due to aggregator obliviousness. This modifications require  $O(t_{\max})$  reduction loss, and require  $O(t_{\max}^2)$  reduction loss from the advantage of the DDH problem.

In  $\text{Game}_{t_{\max}} := \text{Game}_1$ , for each encryption query  $\{(i, t, x_{i,t})\}_{i=1}^n$ , choose  $x'_{i,t} \xleftarrow{\$} \mathbb{Z}_p$  (regardless of  $x_{i,t}$ ) and  $v_{i,t} \xleftarrow{\$} \mathbb{Z}_p$  for  $i \in [1, n-1]$ , compute

$$c_{i,t} = (g_1^{1/a})^{x'_{i,t}} H_1(t)^{s_i} H_2(t)^{t_i} \text{ and } \sigma_{i,t} = (g_1^b)^{x'_{i,t}} H_3(t)^{s_i} H_4(t)^{t_i} H_5(t)^{v_{i,t}}$$

For  $i = n$ , choose  $v'_{i,t} \xleftarrow{\$} \mathbb{Z}_p$ , compute  $X'_t = \sum_{i=1}^n x_{i,t}$  from queries  $\{(i, t, x_{i,t})\}_{i \in [1, n]}$ , and compute

$$c_{i,t} = g_1^{X'_t} (g_1^{1/a})^{-\sum_{i=1}^{n-1} x'_{i,t}} H_1(t)^{s_i} H_2(t)^{t_i} \text{ and } \sigma_{i,t} = (g_1^b)^{-\sum_{i=1}^{n-1} x'_{i,t} + \tilde{t} v'_{n,t}} H_3(t)^{s_i} H_4(t)^{t_i}$$

Here,  $H_5(t)$  is set as  $(g_1^b)^{\tilde{t}}$  as in the proof of the first scheme. Return  $\{(c_{i,t}, \sigma_{i,t}, \mathbf{vk}_{i,t})\}_{i=1}^n$  to  $\mathcal{A}$ , and write  $\mathbf{vk}_{i,t} = g_2^{v_{i,t}}$  for  $i \in [1, n-1]$  and  $\mathbf{vk}_{i,t} = (g_2^a)^{-X'_t/\tilde{t}} g_2^{v'_{n,t}}$  to  $\text{BB}$ . We note that  $\{c_{i,t}\}_{i \in [1, n]}$  can be decrypted by the adversary, and the decryption result is exactly  $\sum_{i=1}^n x_{i,t}$  that the adversary queried.

Finally,  $\mathcal{A}$  outputs  $(t^*, X_{t^*}, \sigma_{t^*})$  where  $t^* \in [1, t_{\max}]$  and  $X_{t^*} \neq X'_{t^*}$ . From the verification equation,  $(\sigma_{t^*}, X_{t^*})$  must satisfy  $\sigma_{t^*} = H_5(t^*)^{\sum_{i=1}^n v_{i,t^*}} h^{X_{t^*}}$ .  $\mathcal{B}$  computes

$$(\sigma_{t^*} / (g_1^b)^{\tilde{t}^* (v'_{n,t^*} + \sum_{i=1}^{n-1} v_{i,t^*})})^{1/(X_{t^*} - X'_{t^*})} = g_1^{ab}$$

and solves the mCDH problem.  $\square$

## 5 Regarding Message Space

As in the definition of previous works [10, 52, 39], for some fixed integer  $M$ , we assume that  $x_{i,t} \in \mathbb{Z}_M$  and aggregator obliviousness requires the condition  $\sum_{i \in \mathbb{S}_{t^*}} x_{i,t^*}^{(0)} \bmod M = \sum_{i \in \mathbb{S}_{t^*}} x_{i,t^*}^{(1)} \bmod M$  (when  $\mathbb{S}_{t^*} = \mathbb{U}$  and  $\text{sk}_A$  is compromised). Since  $M = p$ ,  $x_{i,t}$  might be a large value even its summation is required to be sufficiently small. In the definitions of aggregator obliviousness and full/semi-adaptive aggregator unforgeability, an adversary chooses  $x_{i,t}$ , and thus selecting such a large  $x_{i,t}$  is acceptable (since this is just a strategy of the adversary). On the other hand, in the definition of weak aggregator unforgeability, the encryption oracle *randomly* chooses  $x_{i,t}$  from  $\mathbb{Z}_M$ . If it is desirable to restrict  $x_{i,t}$  to be small, then we can modify the definition of the encryption oracle such that the encryption oracle randomly chooses  $x_{i,t}$  from a small message space. Then, each  $x_{i,t}$  is not rounded up when its summation is computed by modulo  $p$ . Remark that this modification requires an additional reduction loss  $O(t_{\max})$ . In the security proof of the weak aggregator unforgeability, a part of mCDH instance  $a$  is embedded into  $x_{i,t}$ , and thus  $x_{i,t}$  is a random value of  $\mathbb{Z}_p$ . So, as in the security proof of the second scheme, we need to define sequential of games, and replace ciphertexts and tags of  $x_{i,t}$  to those of  $r_{i,t} \in \mathbb{Z}_p$  or  $\sum_{i=1}^n x_{i,t} - \sum_{i=1}^{n-1} r_{i,t}$ . This requires  $O(t_{\max})$  reduction loss, and requires  $O(t_{\max}^2)$  reduction loss from the advantage of the DDH problem in total.

## 6 Conclusion and Open Problem

In this paper, we propose two aggregator oblivious encryption schemes with public verifiability from static and simple assumptions. The first scheme just provides weak aggregator unforgeability, and it seems still meaningful in the smart meter settings since power consumption is measured by the meter. Though the scheme requires  $O(t_{\max})$ -size verification keys, and it could be a bottleneck for supporting long-term period, the scheme still efficiently works for a relatively short-term period. The second scheme provides semi-adaptive aggregator unforgeability and constant-size verification keys, whereas we need to additionally assume the existence of public channels with memory, such as bulletin board [29]. Thus, proposing aggregator oblivious encryption scheme providing full aggregator unforgeability from simple and static assumptions is still open problem. In our schemes, a value  $h$  is shared by all users as their secret key. The value has a crucial role for providing unforgeability. Obviously, if  $h$  is revealed, then anyone can easily produce a forged tag. That is, our scheme is vulnerable against the corruption attack where an adversary (modeled as a malicious aggregator) obtains secret keys of corrupted users. So, it is desirable to provide unforgeability with the collusion resistance. Moreover, as in [10], proposing a generic construction of aggregator oblivious encryption with public verifiability (containing a Paillier-type instantiation) also could be an interesting open problem. Since we consider summation as the aggregation function, constructing (verifiable) aggregator oblivious encryption with rich aggregation functions from (verifiable) multi-input functional encryption [1, 4, 27, 13] also could be an interesting open problem.

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