Trivial Nonce-Misusing Attack on Pure OMD

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Abstract. Pure OMD is an authenticated encryption mode that will be presented by Reyhanitabar et al. at FSE 2015. It is (among others) claimed to achieve authenticity against nonce-misusing adversaries. We show that this claim is incorrect, by presenting an adversary that makes 3 queries (including the forgery) of a total complexity 6.

1 Introduction

Offset Merkle-Damgård (OMD) is a submission to the CAESAR competition [1] by Cogliani et al. [2]. It is characterized by the usage of a full-fledged compression function, and in fact the CAESAR submission takes the sha256 compression function. OMD is proven to achieve birthday-bound security on the state against adversaries that are not allowed to re-use the nonce. At ProvSec 2014, Reyhanitabar et al. [3] showed how to generalize the scheme to achieve security against nonce-misusing adversaries. On the downside, these schemes are not online and are less efficient than OMD. At FSE 2015 Reyhanitabar et al. [4] will present pOMD (pure OMD). pOMD improves over classical OMD in that the associated data is processed almost for free. The authors prove that pOMD inherits all security features of OMD, particularly birthday-bound security against nonce-respecting adversaries. As a bonus, the authors claim authenticity against nonce-misusing adversaries.

Our Contribution

We show that pOMD does not achieve authenticity against misuse-resistant adversaries. In fact, we show an attack on pOMD that makes three evaluations (including the forgery) of total complexity 6 primitive calls and succeeds with probability 1.

We remark that the attack is specific for pOMD and relies on the right of the adversary to misuse the nonce. Consequently, the attack does not invalidate the main result of pOMD on nonce-respecting adversaries. It additionally does not apply to the OMD CAESAR submission and to the misuse-resistant variants of [3].

2 pOMD

Let $k, m, n, \tau \in \mathbb{N}$ such that $m \leq n$. Let $F : \{0, 1\}^k \times \{0, 1\}^n \times \{0, 1\}^m \rightarrow \{0, 1\}^n$ be a keyed compression function. pOMD is a mapping that takes as input a key $K \in \{0, 1\}^k$, a nonce $N \in \{0, 1\}^{n-1}$, an arbitrarily sized associated data $A \in \{0, 1\}^*$, and an arbitrarily sized message $M \in \{0, 1\}^*$, and it returns a ciphertext $C \in \{0, 1\}^{|M|}$ and tag $T \in \{0, 1\}^\tau$.

For our attack it suffices to describe pOMD for the specific case where $|A| = 2n$ and $|M| = m$ (or in other words, the associated data consists of two integral blocks and the
message of one integral block). It is depicted in Fig. 1 (and corresponds to Case A of [4]). Here,
\[
\begin{align*}
\Delta_{N,1,0} &= F_K(N \parallel 10^{n-1-|N|}, 0^m) \oplus 16F_K(0^n, 0^m), \\
\Delta_{N,2,4} &= F_K(N \parallel 10^{n-1-|N|}, 0^m) \oplus (32 \oplus 16 \oplus 4)F_K(0^n, 0^m),
\end{align*}
\]
but our attack will not effectively use these masking values.

### 3 Security of pOMD

Security of an authenticated encryption scheme \( \Pi = (\mathcal{E}, \mathcal{D}) \) is usually measured using privacy and authenticity. For privacy, we consider an adversary that tries to distinguish \( \mathcal{E}_K \) for uniformly random \( K \leftarrow \{0, 1\}^k \) from an ideal encryption scheme \( \mathcal{E} \) with the same interface. For integrity, we consider an adversary that tries to forge a ciphertext, which means that \( \mathcal{D}_K \) ever returns a valid message (other than \( \bot \)) on input of \( (N, A, C, T) \) and no previous encryption query \( \mathcal{E}_K(N, A, M) \) returned \( (C, T) \).

In these games, the adversary is limited in a certain sense. In more detail, we consider two types of adversaries: nonce-respecting, where no two encryption queries should be made with the same nonce, and nonce-misusing, where multiple encryption queries with the same nonce are allowed.

**Security Claim of pOMD**

Assuming that \( F \) underlying pOMD is sufficiently secure (as we assume sha256 to be), Reyhanitabar et al. [4] prove the following security levels:

<table>
<thead>
<tr>
<th></th>
<th>privacy</th>
<th>authenticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>nonce-respecting</td>
<td>(2^{n/2})</td>
<td>(2^{n/2})</td>
</tr>
<tr>
<td>nonce-misusing</td>
<td>0</td>
<td>(2^{n/4})</td>
</tr>
</tbody>
</table>

We refer to [4] for the technicalities regarding the bounds.

### 4 Nonce-Misusing Attack on pOMD

We consider a nonce-misusing adversary that operates as follows:

(i) Fix \( N = \varepsilon \) and choose arbitrary \( M \in \{0, 1\}^m \) and \( A_1, A_2, A'_1 \in \{0, 1\}^n \) such that \( A_1 \neq A'_1 \).
(ii) Query $pOMD^K(N, A_1 A_2, M) \rightarrow (C, T)$;
(iii) Query $pOMD^K(N, A_1' A_2, M) \rightarrow (C', T')$;
(iv) Set $A_2' = C \oplus C' \oplus A_2$;
(v) Query forgery $pOMD^{-1}_K(N, A_1' A_2', C', T)$.

For the first and second evaluation of $pOMD$, it holds that the state difference right before the second $F$-evaluation equals $C \oplus C'$. The forgery is formed simply by adding this value to $A_2$. Consequently, it holds that the first call to $pOMD$ and the forgery attempt have the exact same input to the second $F$-evaluation, and thus the same tag. Therefore, the forgery attempt succeeds as

$$pOMD^{-1}_K(N, A_1' A_2', C', T) = M$$

by construction.

**Where is the Flaw in the Proof**

Reyhanitabar et al. [4] claim that $pOMD$ achieves birthday-bound authenticity even against nonce-misusing adversaries, and our attack shows the existence of a flaw in their reasoning. To be precise, the flaw is located in Lemma 4 case 4, and more specifically the analysis of probability $Pr(intcol \mid E_4)$. The authors claim that an adversary can, indeed, find an internal collision, but that any such collision happens with a birthday bound only. This reasoning, however, assumes that the input to every $F$-call is random, which is not the case given that the adversary can re-use the nonce and thus observe and modify the state using encryption queries.

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**References**


