Security Analysis of an Image Encryption Scheme Based on a New Secure Variant of Hill Cipher and 1D Chaotic Maps

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Abstract. In 2019, Essaid *et al.* introduced a chaotic map-based encryption scheme for color images. Their approach employs three improved chaotic maps to dynamically generate the key bytes and matrix required by the cryptosystem. It should be noted that these parameters are dependent on the size of the source image. According to the authors, their method offers adequate security (*i.e.* 279 bits) for transmitting color images over unsecured channels. However, we show in this paper that this is not the case. Specifically, we present two cryptanalytic attacks that undermine the security of Essaid *et al.*'s encryption scheme. In the case of the chosen plaintext attack, we require only two chosen plaintexts to completely break the scheme. The second attack is a a chosen ciphertext attack, which requires two chosen ciphertexts and compared to the first one has a rough complexity of 2^{24}. The attacks are feasible due to the fact that the key bits and matrix generated by the algorithm remain unaltered for distinct plaintext images.

Keywords: image encryption scheme, chaos based encryption, cryptanalysis

1 Introduction

The exponential increase in social media usage has led to a heightened concern for the security of digital images, particularly with regards to theft and unauthorized distribution. Consequently, this issue has gained significant attention, prompting numerous researchers to develop various image encryption techniques. Chaotic maps have emerged as a favored approach for encrypting images, largely due to their high sensitivity to previous states, initial conditions, or both. This desirable feature makes it challenging to anticipate their behavior or outputs, thus giving rise to numerous novel cryptographic algorithms based on chaos. We refer the reader to [8, 24, 26, 44] for some surveys of such proposals. Regrettably, due to inadequate security analysis and a lack of design guidelines, a significant number of image encryption schemes based on chaos have been found to contain critical

Scheme	[40]	[22]	[35]	[11]	[12]	[31]	[3]	[9]	[25]	[10]
Broken by	[18]	[34]	[2]	[38]	[1]	[37]	[9]	[15]	[14]	[42]
Scheme	[28]	[19]	[29]	[30]	[39]	[41]	[13]	[27]	[23]	[5]
Broken by	[33]	[21]	[36]	[43]	[4]	[20]	[7]	[16]	[17]	[32]

Table 1. Broken chaos based image encryption algorithms.

security vulnerabilities. To illustrate our point, we provide a list of compromised schemes in Table 1. Please be aware that the list is not exhaustive.

In [6] a chaos based encryption scheme is proposed. The authors use the Enhanced Logistic Map (ELM), Enhanced Chebyshev Map (ECM) and Enhanced Sine Map (ESM) as pseudorandom number generators (PRNGs). Using these three PRNGs, Essaid *et al.* randomly generate the necessary key bytes. Then, the ELM PRNG is used to generate a key matrix of size 2×2, such that the first element of the matrix is invertible modulo 256. Since ELM, ECM and ESM are simply used as PRNGs and the scheme's weakness is independent of the employed generators, we omit their description and simply consider the key bytes and matrix as being randomly generated.

This paper presents our security analysis of the Essaid *et al.* scheme. Specifically, we describe a chosen plaintext attack and a chosen ciphertext attack, which enables an attacker to decrypt all images of a particular size. To accomplish this, it is necessary to obtain the ciphertexts of two chosen plaintexts or the plaintexts of two chosen ciphertexts. Note that in the chosen plaintext scenario, we reduce the scheme's security from 279 bits to 0 bits, while in the chosen ciphertext scenario we reduce it to roughly 24 bits.

Structure of the paper. We provide the necessary preliminaries in Section 2. An alternative mathematical description of Essaid *et al.*'s scheme is outlined in Section 3. In Sections 4 and 5 we show how an attacker can recover the secret values in a chosen plaintext/ciphertext scenario. We conclude in Section 6.

2 Preliminaries

Notations. In this paper, the subset $\{1, \ldots, s-1\} \in \mathbb{N}$ is denoted by $[1, s)$. The action of selecting a random element x from a sample space X is represented by $x \xleftarrow{\$} X$, while $x \leftarrow y$ indicates the assignment of value y to variable x. By H and W we denote an image's height and width. Hexadecimal numbers will always contain the prefix `0x`.

2.1 Essaid *et al.* Image Encryption Scheme

In this section we present Essaid *et al.*'s encryption (Algorithm 1) and decryption (Algorithm 2) algorithms as described in [6]. Before the encryption/decryption process starts, the image is always converted into a vector of size $H \cdot W$. At the

Algorithm 1: Encryption algorithm.

Input: A plaintext P, three secret keys k_1, k_2 and k_3, and a secret matrix h
Output: A ciphertext C

for $i \in [0, HW)$ **do**
  if $i = 0$ **then**
    $\begin{pmatrix} C_0 \\ T_0 \end{pmatrix} \leftarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} P_0 \\ k_{1,0} \end{pmatrix} + \begin{pmatrix} k_{2,0} \\ k_{3,0} \end{pmatrix} \bmod 256$
    $S_1 \leftarrow T_0 + P_1 \bmod 256$
  else
    $\begin{pmatrix} C_i \\ T_i \end{pmatrix} \leftarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} S_i \\ k_{1,i} \end{pmatrix} + \begin{pmatrix} k_{2,i} \\ k_{3,i} \end{pmatrix} \bmod 256$
    $S_{i+1} \leftarrow T_i + P_{i+1} \bmod 256$
return C

Algorithm 2: Decryption algorithm.

Input: A ciphertext C, three secret keys k_1, k_2 and k_3, and a secret matrix h
Output: A plaintext P

for $i \in [0, HW)$ **do**
  if $i = 0$ **then**
    $P_0 \leftarrow a^{-1} \cdot (C_0 - b \cdot k_{1,0} - k_{2,0}) \bmod 256$
    $tmp \leftarrow c \cdot P_0 + d \cdot k_{1,0} + k_{3,0} \bmod 256$
  else
    $P_i \leftarrow a^{-1} \cdot (C_i - b \cdot k_{1,i} - k_{2,i}) - tmp \bmod 256$
    $tmp \leftarrow c \cdot a^{-1} \cdot (C_i - b \cdot k_{1,i} - k_{2,i}) + d \cdot k_{1,i} + k_{3,i} \bmod 256$
return P

end, the resulting vector is translated back into an image of size $H \times W$. Please note that both the key bytes $k_{1,i}$, $k_{2,i}$, and $k_{2,i}$, and the matrix h values a, b, c, and d are generated randomly. Also, a is always invertible modulo 256.

3 A New Look at Essaid *et al.*'s Scheme

In this section we provide an equivalent description of the scheme presented in [6]. We first start with studying Algorithm 1.

Lemma 1. *Let C_i and T_i be the variables from Algorithm 1. Then we can rewrite them as follows*

$$C_i \equiv a \sum_{j=0}^{i} c^{i-j} P_j + \alpha_i \bmod 256,$$

$$T_i \equiv c \sum_{j=0}^{i} c^{i-j} P_j + \beta_i \bmod 256,$$

where $\beta_{-1} = 0$ *and*

$$\alpha_i \equiv a\beta_{i-1} + bk_{1,i} + k_{2,i} \bmod 256,$$
$$\beta_i \equiv c\beta_{i-1} + dk_{1,i} + k_{3,i} \bmod 256.$$

Proof. We will prove our assertion using induction. When $i = 0$ we have that

$$C_0 \equiv aP_0 + bk_{1,0} + k_{2,0} = aP_0 + \alpha_0 \bmod 256,$$
$$T_0 \equiv cP_0 + dk_{1,0} + k_{3,0} = cP_0 + \beta_0 \bmod 256.$$

We assume that the assertion is true for i and we prove it for $i + 1$. Therefore, we have

$$\begin{aligned} C_{i+1} &\equiv aS_{i+1} + bk_{1,i+1} + k_{2,i+1} \\ &\equiv a(T_i + P_{i+1}) + bk_{1,i+1} + k_{2,i+1} \\ &\equiv ac\sum_{j=0}^{i} c^{i-j}P_j + aP_{i+1} + a\beta_i + bk_{1,i+1} + k_{2,i+1} \\ &\equiv a\sum_{j=0}^{i+1} c^{(i+1)-j}P_j + \alpha_{i+1} \bmod 256 \end{aligned}$$

and

$$\begin{aligned} T_{i+1} &\equiv cS_{i+1} + dk_{1,i+1} + k_{3,i+1} \\ &\equiv c(T_i + P_{i+1}) + dk_{1,i+1} + k_{2,i+1} \\ &\equiv c^2\sum_{j=0}^{i} c^{i-j}P_j + cP_{i+1} + c\beta_i + dk_{1,i+1} + k_{3,i+1} \\ &\equiv c\sum_{j=0}^{i+1} c^{(i+1)-j}P_j + \beta_{i+1} \bmod 256, \end{aligned}$$

as desired. □

According to Lemma 1, in order to encrypt an image using Essaid *et al.*'s scheme is enough to know the secret values a, c and α_i, for $i \in [0, HW)$. As a consequence, we can also decrypt using these values.

Corollary 1. *We can recover* P_i *using*

$$P_i \equiv a^{-1}(C_i - a\sum_{j=0}^{i-1} c^{i-j}P_j - \alpha_i) \bmod 256.$$

A more efficient method for decrypting is given in the following lemma.

Corollary 2. *We can recover* P_i *using*

$$P_i \equiv a^{-1}(C_i - \gamma_i - \alpha_i) \bmod 256,$$

where $\gamma_0 = 0$ *and*

$$\gamma_i \equiv acP_{i-1} + c\gamma_{i-1} \bmod 256.$$

4 Chosen Plaintext Attack

A chosen plaintext attack (CPA) is a scenario in which the attacker A briefly gains access to the encryption machine $\mathcal{O}_{enc}$ and is permitted to query it with various inputs. In this way, A generates specific plaintexts that can facilitate his attack and uses $\mathcal{O}_{enc}$ to obtain the corresponding ciphertexts. We demonstrate in this paper that Essaid *et al.*'s image encryption scheme is vulnerable to such attacks.

Lets assume that we query $\mathcal{O}_{enc}$ with two plaintexts P and P' and receive C and C', respectively. According to Lemma 1 we have

$$C_0 \equiv aP_0 + \alpha_0 \bmod 256,$$
$$C'_0 \equiv aP'_0 + \alpha_0 \bmod 256.$$

Therefore, if $\gcd(P_0 - P'_0, 256) = 1$ then we can recover a using

$$a \equiv (C_0 - C'_0)(P_0 - P'_0)^{-1} \bmod 256,$$

and α_0 from

$$\alpha_0 \equiv C'_0 - aP'_0 \bmod 256. \tag{1}$$

Using Lemma 1 we also obtain

$$C_1 \equiv aP_1 + acP_0 + \alpha_1 \bmod 256,$$
$$C'_1 \equiv aP'_1 + acP'_0 + \alpha_1 \bmod 256,$$

and since we already computed a we can rewrite the equations as

$$C_1 - aP_1 \equiv acP_0 + \alpha_1 \bmod 256,$$
$$C'_1 - aP'_1 \equiv acP'_0 + \alpha_1 \bmod 256.$$

Therefore, we can recover c using

$$c \equiv (C_1 - aP_1 - C'_1 + aP'_1) \cdot a^{-1}(P_0 - P'_0)^{-1} \bmod 256,$$

since $\gcd(a, 256) = \gcd(P_0 - P'_0, 256) = 1$. Also, α_1 is computed as follows

$$\alpha_1 \equiv C'_1 - aP'_1 - acP'_0 \bmod 256. \tag{2}$$

Once a and c are computed, the remaining α_i are computed from

$$\alpha_i \equiv C'_i - a\sum_{j=0}^{i} c^{i-j}P'_j \bmod 256. \tag{3}$$

In order to optimize the recovery of the secret values, we choose two plaintexts such that $P_0 = 1$ and $P_1 = \ldots = P_{HW-1} = P'_0 = \ldots = P'_{HW-1} = 0$. Therefore, we obtain the following relations

$$a \equiv C_0 - C'_0 \bmod 256,$$
$$c \equiv a^{-1}(C_1 - C'_1) \bmod 256,$$
$$\alpha_i \equiv C'_i \bmod 256, \text{ for } i \in [0, HW).$$

We can easily see that the complexity of our attack is constant and is dominated by computing an inverse and a multiplication modulo 256. Therefore, it is very efficient.

5 Chosen Ciphertext Attack

In contrast to a chosen plaintext attack, a chosen ciphertext attack (CCA) assumes that the attacker A briefly gains access to the decryption machine $\mathcal{O}_{dec}$. A then generates specific ciphertexts that can assist his attack and uses $\mathcal{O}_{dec}$ to obtain the corresponding plaintexts. In this scenario, we describe an attack on Essaid *et al.*'s cryptosystem.

Lets assume that we query $\mathcal{O}_{dec}$ with two ciphertexts C and C' and receive P and P', respectively. Using Corollary 1 we obtain

$$\begin{aligned} P_0 &\equiv a^{-1}(C_0 - \alpha_0) \bmod 256, \\ P'_0 &\equiv a^{-1}(C'_0 - \alpha_0) \bmod 256. \end{aligned}$$

Therefore, if $\gcd(C_0 - C'_0, 256) = 1$ then we can recover a^{-1} using

$$a^{-1} \equiv (P_0 - P'_0)(C_0 - C'_0)^{-1} \bmod 256.$$

Applying Corollary 1 to the second byte we obtain

$$\begin{aligned} P_1 &\equiv a^{-1}(C_1 - acP_0 - \alpha_1) \bmod 256, \\ P'_1 &\equiv a^{-1}(C'_1 - acP'_0 - \alpha_1) \bmod 256, \end{aligned}$$

and since we already computed a^{-1} we can rewrite the equations as

$$\begin{aligned} a^{-1}C_1 - P_1 &\equiv cP_0 + a^{-1}\alpha_1 \bmod 256, \\ a^{-1}C'_1 - P'_1 &\equiv cP'_0 + a^{-1}\alpha_1 \bmod 256. \end{aligned}$$

Note that since $\gcd(a, 256) = \gcd(C_0 - C'_0, 256) = 1$, we obtain that $\gcd(P_0 - P'_0, 256) = 1$. Therefore, we can recover c using

$$c \equiv (a^{-1}C_1 - P_1 - a^{-1}C'_1 + P'_1) \cdot (P_0 - P'_0)^{-1} \bmod 256.$$

Once a and c are computed, the α_i values are computed using Equations (1) to (3).

In order to optimize the recovery of the secret values, we choose two ciphertexts such that $C_0 = 1$ and $C_1 = \ldots = C_{HW-1} = C'_0 = \ldots = C'_{HW-1} = 0$. Therefore, we obtain the following relations

$$\begin{aligned} a &\equiv (P_0 - P'_0)^{-1} \bmod 256, \\ c &\equiv a(P'_1 - P_1) \bmod 256, \\ \alpha_i &\equiv -a \sum_{j=0}^{i} c^{i-j} P'_j \bmod 256, \text{ for } i \in [0, HW). \end{aligned} \tag{4}$$

Note that Equation (4) can be rewritten as

$$\alpha_0 \equiv -aP'_0 \bmod 256,$$
$$\alpha_i \equiv -aP'_i + c\alpha_{i-1} \bmod 256, \text{ for } i \in [1, HW).$$

The complexity of our attack dominated by two inverses and $2HW$ multiplications modulo 256. Using the fact that an inverse and a multiplication modulo 256 has constant complexity $\mathcal{O}(1)$, we obtain that our attack has a complexity of $\mathcal{O}(2HW)$. For example, if we encrypt 2 megapixels[3] images we obtain a complexity of $\mathcal{O}(2^{21.87})$. In the case of 12 megapixels[4], we obtain $\mathcal{O}(2^{24.51})$.

6 Conclusions

The authors of [6] presented an image encryption scheme that they claimed to have a security strength of 279 bits. However, our research in this paper demonstrated that the actual security strength of Essaid *et al.*'s scheme is essentially 0 bits. To establish our security bound, we designed a chosen plaintext attack that requires only 2 queries to the encryption oracle. Furthermore, we outline a chosen ciphertext attack that requires 2 queries to the decryption oracle and has a complexity of roughly $\mathcal{O}(2^{24})$.

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[3] $W \times H = 1600 \times 1200$
[4] $W \times H = 4000 \times 3000$

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