FIN: Practical Signature-Free Asynchronous Common Subset in Constant Time

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ABSTRACT

Asynchronous common subset (ACS) is a powerful paradigm enabling applications such as Byzantine fault-tolerance (BFT) and multi-party computation (MPC). The most efficient ACS framework in the information-theoretic setting is due to Ben-Or, Kelmer, and Rabin (BKR, 1994). The BKR ACS protocol has been both theoretically and practically impactful. However, the BKR protocol has an $O(\log n)$ running time (where n is the number of replicas) due to the usage of n parallel asynchronous binary agreement (ABA) instances, impacting both performance and scalability. Indeed, for a network of 16~64 replicas, the parallel ABA phase occupies about 95%~97% of the total runtime in BKR. A long-standing open problem is whether we can build an ACS framework with $O(1)$ time while not increasing the message or communication complexity of the BKR protocol.

In this paper, we resolve the open problem, presenting the first constant-time ACS protocol with $O(n^3)$ messages in the information-theoretic and signature-free settings. Moreover, as a key ingredient of our new ACS framework and an interesting primitive in its own right, we provide the first information-theoretic multivalued validated Byzantine agreement (MVBA) protocol with $O(1)$ time and $O(n^3)$ messages. Both results can improve—asymptotically and concretely—various applications using ACS and MVBA in the information-theoretic, quantum-safe, or signature-free settings. As an example, we implement FIN, a BFT protocol instantiated using our framework. Via a 121-server deployment on Amazon EC2, we show FIN is significantly more efficient than PACE (CCS 2022), the state-of-the-art asynchronous BFT protocol of the same type. In particular, FIN reduces the overhead of the ABA phase to as low as 1.23% of the total runtime, and FIN achieves up to 3.41x the throughput of PACE. We also show that FIN outperforms other BFT protocols with the standard liveness property such as Dumbo and Speeding Dumbo.

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CCS CONCEPTS

• Security and privacy → Information-theoretic techniques; **Distributed systems security**; **• Computer systems organization → Dependable and fault-tolerant systems and networks**.

KEYWORDS

Asynchronous Common Subset, Blockchains, Byzantine Fault Tolerance

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1 INTRODUCTION

Overview. This paper is about resolving a long-standing open problem in fault-tolerant distributed computing and multi-party computation (MPC). We present the first practical $O(1)$-time and $O(n^3)$-message asynchronous common subset (ACS) protocol [6, 8], while prior constructions have either $O(\log n)$ time and $O(n^3)$ messages or have $O(1)$ time and $O(n^4)$ messages.

History. The concept of asynchronous common subset (ACS) is due to Ben-Or, Canetti, and Goldreich (BCG) in the context of asynchronous MPC—under a different name called *agreement on a core set* [6]. BCG proposed an ACS construction using two core building blocks in fault-tolerant distributed computing—reliable broadcast (RBC) and asynchronous binary agreement (ABA). Soon later, Ben-Or, Kelmer, and Rabin (BKR) presented a refined and practical ACS construction using n RBC and n ABA instances [8]. Meanwhile, BKR renamed "agreement on a core set" as "agreement on a common subset."

Information-theoretic and signature-free settings. The ACS notion has been historically associated with the information-theoretic setting: as emphasized by Cachin et al. [11, Section 4], "the primitive of agreement on a core set (which) is used in the information-theoretic model." Indeed, ACS constructions typically rely on *ideal* building primitives in distributed computing such as RBC, ABA, and common coins [10, 15].

An equivalent setting is the *signature-free setting* focusing on protocols assuming the existence of common coins.[1] In particular, the line of work for signature-free ABA—which is at the core of practical ACS constructions—begins with the seminal work by Rabin [51] and is followed by [23, 43, 46, 47, 54].

[1] Here we directly borrow the term from the line of work [46–48]; the setting may also be called *cryptography-free*.

The unique benefits of information-theoretic/signature-free ACS. The information-theoretic ACS paradigm (BCG and BKR) has been enormously impactful, empowering numerous applications as well as implementations, such as MPC [5, 8, 9, 14, 18–21, 41], Byzantine fault tolerence (BFT) [2, 9, 30, 40, 44, 56, 57], distributed key generation [25, 28, 38], and proactive secret sharing [26, 35, 36]. For instance, we illustrate of benefits of information-theoretic ACS using MPC applications:

- **MPC.** A more efficient information-theoretic ACS directly improves a large number of asynchronous MPC protocols with fairness and guaranteed output delivery that use information-theoretic ACS (BCG and BKR) [5, 8, 9, 14, 18–21, 41]. In this approach, one just replaces RBC with asynchronous complete secret sharing [8].[2] The only MPC system with fairness and guaranteed output delivery falls into this category [41].

Even just being used for the BFT purpose, information-theoretic ACS has unique benefits and features, when compared to the other two main BFT approaches—multivalued validated Byzantine agreement (MVBA) based BFT[11] and direct acyclic graph (DAG) based BFT [37]:

- **Quantum safety.** Information-theoretic ACS protocols and DAG-based protocols can achieve the desirable quantum safety property (but no quantum liveness) as defined in [37], where the safety of the system is always achieved even in the presence of a quantum adversary. In contrast, this is not the case for MVBA-based approach.
- **Weaker cryptographic assumptions.** Information-theoretic ACS and DAG-based BFT can be realized using the standard Computational Diffie-Hellman (CDH) assumption; others would need to use pairing assumptions to save communication costs.
- **Liveness.** Unlike information-theoretic ACS protocols that achieve standard liveness, some other BFT protocols (including DAG-based ones) either require unbounded memory for liveness (DAG-Rider [37], Dumbo-NG [31]) or achieve weak liveness (Bullshark [32], Tusk [24]).

The open problem. Almost all ACS implementations use the BKR ACS construction, which runs n parallel ABA instances, thereby achieving $O(n^3)$ messages and $O(\log n)$ time. The ABA phase involving n parallel ABA instances is the well-known performance and scalability bottleneck reported in various BFT implementations [30, 34, 54]. For example, the ABA phase in a network with 16∼64 replicas occupies about 95%∼97% of the total runtime in BKR [34]. Recently, Zhang and Duan proposed a refined PACE ACS framework [54] that offers up to 6x peak throughput of BEAT [30] (that improved upon HoneyBadgerBFT [44]). In PACE ACS, the n ABA instances are made fully parallelizable to gain in improved performance, but the running time remains $O(\log n)$—remaining the critical bottleneck.

A long-standing open problem, ever since the 1990s, in ACS—or generally in fault-tolerant distributed computing and cryptography—is:

Can we build ACS—in the information-theoretic setting or signature-free setting—with $O(n^3)$ messages and $O(1)$ time?

[2]In fact, the original BKR framework was presented as an asynchronous MPC framework using asynchronous complete secret sharing and ABA.

protocol	messages	time	# (R)ABA
BKR variants [30, 40, 44]	$O(n^3)$	$O(\log n)$	n
PACE [54]	$O(n^3)$	$O(\log n)$	n
FIN (§7; this paper)	$O(n^3)$	$O(1)$	$O(1)$

Table 1: Comparison among ACS implementations relying on common coins.

Note that the only known such ACS construction terminating in constant time is due to Ben-Or and El-Yaniv [7]. The construction uses n^2 RBC instances with $O(n^4)$ messages; earlier works such as HoneyBadger [44, Footnote 4] and Dumbo [34, Footnote 5] explained why [7] has lower throughput than BKR and that the communication of [7] bottlenecks the system.

Our results and our approach. In this work, we resolve the open problem, showing an ACS protocol with $O(n^3)$ messages and $O(1)$ time. At a high level, we begin with a new multivalued validated Byzantine agreement (MVBA) protocol [11] in the information-theoretic setting with $O(n^3)$ and $O(1)$ time. Then we reduce the ACS problem to MVBA and RBC.

Recall that the notion of MVBA is different from the conventional multivalued Byzantine agreement in that MVBA assumes the existence of a global predicate, and replicas only decide values satisfying the global predicate. To build an MVBA protocol towards our goal, we use RBC, common coins, and reproposable ABA (RABA)—a notion due to Zhang and Duan [54]. RABA, like ABA, can be readily built from common coins and authenticated channels and terminate in expected constant time. Our MVBA uses n parallel RBC instances and an expected constant number of RABA instances. Our MVBA protocol is also the first information-theoretically secure and signature-free MVBA protocol—by directly assuming common coins—with $O(n^3)$ messages and $O(1)$ time.

We also show that one can use a weaker RBC primitive to realize a computation and bandwidth more efficient MVBA protocol while still achieving $O(n^3)$ messages and $O(1)$ time.

Our transformation from MVBA to ACS is simple and efficient, consisting of n parallel RBC instances and a single MVBA instance.

Both our MVBA and ACS constructions are efficient, inheriting the fast path in RABA, enabling rapid termination.

Our contributions. We make the following contributions:

- We present the first information-theoretically secure and signature-free ACS protocol with $O(1)$ time and $O(n^3)$ messages, assuming ideal building blocks only. In contrast to prior constructions, our ACS protocol requires only an expected constant number of binary agreement instances. Our protocol directly improves various ACS-enabled applications such as asynchronous multi-party computation [8, 14, 21, 41].
- As a core ingredient of our ACS construction and a primitive that is interesting in its own right, we present the first signature-free multivalued validated Byzantine agreement protocol with $O(1)$ time and $O(n^3)$ messages while existing such MVBA protocols have $O(\log n)$ time and $O(n^3)$ messages. Moreover, our MVBA protocols lead to instantiations having lower communication than existing ones. We also show an efficient and tailored MVBA construction, optimizing both communication cost and computational efficiency.

- To demonstrate the efficiency of our ACS protocol, we instantiate it with a practical BFT system called FIN (see Table 1 for a comparison). We implement FIN and PACE, the state-of-the-art ACS protocol of the same kind. Additionally, we also assess Dumbo [34] and Speeding Dumbo [33], two asynchronous and computationally-secure BFT protocols with $O(1)$ time and the standard liveness guarantee. Via a 121-instance deployment on Amazon EC2, we show that for *all metrics*, FIN consistently and drastically outperforms PACE when $n > 16$ and all the above protocols when $n > 22$. Compared to PACE, FIN significantly reduces the overhead of the ABA phase to only 1.23%-5.22% of the total runtime, in contrast to the 15.10%-83.66% overhead in PACE. Moreover, the performance difference between FIN and the other three protocols becomes significantly higher as n increases; for instance, when $n = 121$ (tolerating 40 failures), the peak throughput of FIN is 3.41x that of PACE, 4.15x that of Speeding Dumbo, and 8.79x that of Dumbo.

2 RELATED WORK

Interactive consistency and vector consensus. The ACS problem can be viewed as an asynchronous version of the *interactive consistency* problem defined for synchronous systems by Pease, Shostak, and Lamport [50]. Replicas in asynchronous interactive consistency reach an agreement on a vector with the values proposed by *all* correct replicas.

In contrast, the ACS abstraction (also called asynchronous interactive consistency by Ben-Or and El-Yaniv [7]) naturally requires that the output of each correct replica contains $n-f$ values such that at least $n - 2f$ elements are proposed by correct replicas. Namely, ACS requires only that the majority of the values were proposed by correct replicas. Indeed, it is impossible to guarantee that the vector has the values of all correct replicas in asynchronous settings.

ACS is also called vector consensus in some literature [22, 29, 45, 49]. Note that ACS is different from set agreement [17] that provides only an approximation of agreement.

ACS constructions (in information-theoretic and signature-free settings). BKR ACS reduces asynchronous BFT to reliable broadcast (RBC) and asynchronous binary agreement (ABA) [8]. In BKR ACS, all replicas run an RBC phase to reliably broadcast their proposals. Then they run an ABA phase with n parallel instances. The i-th ABA instance agrees on if the proposal of p_i has been delivered in the RBC phase. Upon RBC delivery of a proposal from p_j, the replica proposes 1 to the j-th ABA instance. If a correct replica p_j decides 1 for the i-th ABA instance, the proposal from p_i is delivered. BKR ACS requires if a replica has not received some proposals during the RBC phase, the replica abstains from proposing 0 until $n-f$ ABA instances terminate with 1. In PACE ACS, ABA instances are replaced using RABA instances, and thus all RABA instances can be run in a fully parallelizable manner [54]. Both BKR and PACE ACS approaches require running n (R)ABA instances terminating in expected constant rounds, leading to expected $O(\log n)$ time in total.

The ACS construction by Ben-Or and El-Yaniv [7] terminates in expected constant time and uses n^2 RBC instances with $O(n^4)$ messages, which is prohibitively expensive.

Another line of ACS constructions reduces the ACS problem to RBC and multivalued Byzantine agreement (MBA) [22, 45]. The construction requires running $O(f)$ sequential MBA instances, resulting in $O(n)$ running time.

Our ACS approach fundamentally differs from existing ones, reducing the ACS problem to information-theoretically secure MVBA and then to RBC with a constant number of RABA instances.

DispersedLedger. DispersedLedger [52] provides two general techniques (allowing committing additional transactions from prior epochs; using asynchronous verifiable information dispersal) to improve the performance of the BKR framework. The techniques equally work for PACE and FIN. Therefore, the DispersedLedger techniques can be used to improve FIN, and FIN can be used in DispersedLedger as its consensus engine for higher performance.

Separating message transmission from consensus. Tusk [24], Bullshark [32], and Dumbo-NG [31] are BFT protocols that separate data transmission from consensus for higher throughput. FIN can use the technique to improve performance.

As mentioned in the introduction, Tusk, Bullshark, and Dumbo-NG do not achieve standard liveness. We comment the fact that they do not have standard liveness is not due to the framework that separates the transmission from consensus.

RBC. We use RBC (abbreviated as BRB in some works) [10] to build our MVBA and ACS construction. For both our MVBA and ACS, RBC dominates their communication. One could use any RBC constructions to instantiate them. In this paper, we discuss constructions using CT RBC [13] (using hashes), EFBRB [3] (information-theoretically secure), and CCBRB [3] (using hashes and online error correction coding [6]).

For our ACS implementation, we use CT RBC [13]. Internally in our MVBA implementation, we show that one can use a tailored, weaker RBC construction (using collision-resistant hashes only) to build a concretely more efficient protocol.

RABA. The notion of reproposable ABA (RABA) is due to Zhang and Duan [54]. RABA was originally proposed to solve a BKR bottleneck to allow all ABA instances to run in parallel and remove the two-subphase bottleneck. Later, such a primitive was used to develop a quantum secure and adaptively secure asynchronous BFT protocol without trusted setup [56], and to build an asynchronous distributed key generation protocol without random oracles or PKI [55].

Zhang and Duan have argued that RABA could be useful as a general and "first-class distributed computing primitive" [54]. Our results bolster this point of view.

However, the way we use RABA in this paper is fundamentally different from all these works. Indeed, existing protocols that use RABA in the BKR framework need to run n parallel RABA instances; in contrast, this paper only needs to run an expected constant number of RABA instances.

MVBA. The notion of MVBA was introduced by Cachin, Kursawe, Petzold, and Shoup [11].

In the computational model (assuming—in addition to common coins—cryptographic tools such as threshold signatures), Abraham, Malkhi, and Spiegelman proposed an MVBA protocol [1] with $O(Ln^2 + \kappa n^2)$ communication, optimal word complexity, and the quality property, where L is the length of the input from each replica. Lu et al. [42] reduced the communication from $O(Ln^2+\kappa n^2)$ to $O(Ln+\kappa n^2)$ by additionally using constant-size vector commitments [16].

In the information-theoretic and signature-free setting, the asynchronous distributed key generation protocol by Das et al. [28] implies an MVBA, as demonstrated in a recent work [25]. The MVBA protocols [25, 28] are information-theoretically secure (assuming information-theoretically secure common coins or Rabin dealer [51]), terminating in $O(\log n)$ time. In contrast, the MVBA proposed in this paper has the same message complexity as those in [25, 28], but terminates in $O(1)$ time. Also, our MVBA protocol can lead to instantiations with lower communication than those from [25, 28].

Our MVBA protocol may also be used—possibly in a non-trivial manner—in certain (but not all) protocols in [26, 35, 36] without increasing the communication or making stronger assumptions.

From atomic broadcast to ACS. There are asynchronous atomic broadcast protocols without using ACS or MVBA (e.g., [37]). It is, however, unclear how to *efficiently* transform an atomic broadcast protocol to ACS, which has direct and practical applications beyond just BFT state machine replication (e.g., multi-party computation, asynchronous distributed key generation).

3 SYSTEM MODEL AND PROBLEM STATEMENT

We consider a system with n replicas, $\{p_1, \cdots, p_n\}$, where f out of them may fail arbitrarily (Byzantine failures). A non-Byzantine replica is called a correct replica. All protocols we consider assume that $f \leq \lfloor \frac{n-1}{3} \rfloor$, which is optimal. A (Byzantine) *quorum* is a set of $\lceil \frac{n+f+1}{2} \rceil$ replicas. Without loss of generality, this paper may assume $n = 3f+1$ and a quorum size of $2f+1$. We assume the existence of point-to-point authenticated channels between each pair of replicas. We consider asynchronous networks making no timing assumptions on message processing or transmission delays.

We consider both adaptive corruption and static corruption. In adaptive corruption, the adversary can choose its set of corrupted replicas at any moment during the execution of the protocol, based on the information it has accumulated thus far. In contrast, in the static adversary model, the adversary is restricted to choosing its set of corrupted replicas at the beginning of the protocol. If all building blocks satisfy adaptive security, then our protocol achieves adaptive security. The ACS protocol implemented in this paper achieves static security, just as in prior protocols [30, 41, 44, 54]; but it can be made adaptively secure if using an adaptively secure common coin protocol [4, 39].

Each protocol instance is associated with a unique tag id. We may omit the identifiers in the pseudocode when no ambiguity arises. We may use subscripts to denote the instance identifier; for instance, RBC_i denotes the reliable broadcast instance tagged with a unique identifier i and initiated by replica p_i.

We present in Appendix A the acronyms we use in this paper.

Asynchronous Common Subset (ACS). In ACS [6, 8], each replica holds an input, and correct replicas reach an agreement on a set of values. An ACS protocol is specified by *acs-propose* and *acs-decide* events. The value *acs-proposed* by a replica is called an input to the ACS protocol, and the value *acs-decided* by a replica is called an output of the protocol. ACS should satisfy the following properties:

- **Validity**: If a correct replica *acs-decides* a set $\mathbf{v}$, then $|\mathbf{v}| \geq n - f$ and $\mathbf{v}$ contains values *acs-proposed* by at least $n - 2f$ correct replicas.
- **Agreement**: If a correct replica *acs-decides* $\mathbf{v}$, then every correct replicas *acs-decides* $\mathbf{v}$.
- **Termination**: If all correct replicas *acs-propose*, then all correct replicas *acs-decide*.

In this paper, we use the conventional validity notion. Note that as argued in [54], for the validity property, the size of $\mathbf{v}$ (denoted $|\mathbf{v}|$) can be relaxed such that $|\mathbf{v}| \geq f+1$; namely, in many applications, it suffices to ensure a weaker validity notion by requiring that $\mathbf{v}$ contains values from at least one correct replica.

Multivalued validated Byzantine agreement (MVBA). MVBA allows each replica that has an input to agree on a value that satisfies a predicate Q known by all replicas [11]. An MVBA protocol satisfies the following properties:

- **External validity**: Every correct replica that terminates *mvba-decides* v such that $Q(v)$ holds.
- **Agreement**: If a correct replica *mvba-decides* v, then any correct replica that terminates *mvba-decides* v.
- **Integrity**: If all replicas follow the protocol, and if a correct replica *mvba-decides* v such that $Q(v)$ holds, then some replica *mvba-proposed* v such that $Q(v)$ holds.
- **Termination**: If all correct replicas are activated and all messages sent among correct replicas have been delivered, then all correct replicas *mvba-decide*.

The quality property was introduced by Abraham, Malkhi, and Spiegelman to bound the probability that the decided value was proposed by a correct replica [1]:

- **Quality**: The probability of *mvba-deciding* a value that was proposed by a correct replica is at least 1/2.

In this paper, we first present an MVBA protocol without the quality property and then show how to modify it to achieve the quality property additionally. For our ACS construction, we only need an MVBA protocol without the quality property though.

The way we present our MVBA protocol follows that of [25]. In particular, the MVBA formalization in [25] requires that the predicate additionally uses some variable depending on the state of each node, a property needed in [25] and our MVBA protocols.

Byzantine fault tolerance (BFT). In a BFT protocol, clients *submit* transactions (requests) and replicas *deliver* them. The client obtains a final response to the submitted transaction from the replica responses. In a BFT system with n replicas, it tolerates $f \leq \lfloor \frac{n-1}{3} \rfloor$ Byzantine failures. The correctness of a BFT protocol is specified as follows:

- **Safety**: If a correct replica *delivers* a transaction tx before *delivering* tx', then no correct replica *delivers* a transaction tx' without first *delivering* tx.
- **Liveness**: If a transaction tx is *submitted* to all correct replicas, then all correct replicas eventually *deliver* tx.

4 BUILDING BLOCKS

We review the building blocks for our systems. To help understand RABA [54], we first review the notion of ABA. If v is a binary value, we use $\bar{v}$ to denote $1 - v$.

Asynchronous binary Byzantine agreement (ABA). An ABA abstraction is specified by *aba-propose* and *aba-decide*. Each replica proposes a binary value (aka a vote) and correct replicas will decide on some value. ABA should satisfy the following properties:

- **Validity**: If all correct replicas *aba-propose* v, then any correct replica that terminates *aba-decides* v.
- **Agreement**: If a correct replica *aba-decides* v, then any correct replica that terminates *aba-decides* v.
- **Termination**: Every correct replica eventually *aba-decides* some value.
- **Integrity**: No correct replica *aba-decides* twice.

Reproposable Asynchronous Binary Agreement (RABA). RABA is a new primitive introduced by Zhang and Duan [54]. In contrast to conventional ABA protocols, where replicas can vote once only, RABA allows replicas to change their votes and vote twice. A RABA protocol is specified by *raba-propose*, *raba-repropose*, and *raba-decide*. For our purpose, RABA is "biased towards 1." A correct replica that proposed 0 is allowed to change its mind and repropose 1. A replica that proposed 1 is not allowed to repropose 0. If a replica reproposes 1, it does so at most once. RABA (biased towards 1) satisfies the following properties:

- **Validity**: If all correct replicas *raba-propose* v and never *raba-repropose* $\bar{v}$, then any correct replica that terminates *raba-decides* v.
- **Unanimous termination**: If all correct replicas *raba-propose* v and never *raba-repropose* $\bar{v}$, then all correct replicas eventually terminate.
- **Agreement**: If a correct replica *raba-decides* v, then any correct replica that terminates *raba-decides* v.
- **Biased validity**: If $f + 1$ correct replicas *raba-propose* 1, then any correct replica that terminates *raba-decides* 1.
- **Biased termination**: Let Q be the set of correct replicas. Let Q_1 be the set of correct replicas that *raba-propose* 1 and never *raba-repropose* 0. Let Q_2 be correct replicas that *raba-propose* 0 and later *raba-repropose* 1. If $Q_2 \neq \emptyset$ and $Q = Q_1 \cup Q_2$, then each correct replica eventually terminates.
- **Integrity**: No correct replica *raba-decides* twice.

We explain some differences between ABA and RABA. Validity in RABA is slightly different from that for ABA, as we need to modify it to accommodate the RABA syntax. Integrity in RABA is used to ensure that RABA decides only once (even though we have an additional *raba-repropose* event).

Biased validity in RABA requires that if $f + 1$ correct replicas, not all correct replicas, propose 1, then a correct replica that terminates decides 1. We emphasize that the biased validity property was initially defined to build the PACE ACS protocol [54] such that sufficient transactions are delivered (the ACS validity property); however, in this paper, biased validity is essentially to ensure constant-time termination for our ACS construction.

Unanimous termination and biased termination are defined to ensure RABA termination in two different scenarios. In formally, most RABA protocols terminate under three conditions, considering the protocol is biased towards 1: 1) all correct replicas *raba-propose* 0 and never *raba-repropose* 1; 2) at least $f + 1$ correct replicas *raba-propose* 1; 3) at least one correct replica *raba-propose* 1 and later one those correct replicas that *raba-propose* 0 change their mind and *raba-repropose* 1. In the first condition, unanimous termination is satisfied. In the third condition, the biased termination property is satisfied. Most RABA protocols known so far [54] can guarantee termination in the second condition even if some correct replicas do not *raba-repropose*. However, some protocols may terminate still due to the fact that those correct replicas that *raba-propose* 0 change their mind and *raba-repropose* 1.

For our implementation, we use the Pisa RABA protocol due to Zhang and Duan [54] assuming common coins and authenticated channels.

Byzantine reliable broadcast (RBC). The RBC abstraction allows a sender p_s to reliably broadcast a message to the replicas. An RBC protocol is specified by two events *r-broadcast* and *r-deliver* such that the following properties hold:

- **Validity**: If a correct replica p_s *r-broadcasts* a message m, then p_s eventually *r-delivers* m.
- **Agreement**: If some correct replica *r-delivers* a message m, then every correct replica eventually *r-delivers* m.
- **Integrity**: For any message m, every correct replica *r-delivers* m at most once. Moreover, if a replica *r-delivers* a message m with sender p_s, then m was previously broadcast by replica p_s.

For our ACS implementation, we use CT RBC due to Cachin and Tessaro [13] that uses hash functions (with output length κ) and has a communication of $O(n|m| + \kappa n^2 \log n)$.

Common coins. Following prior works [8, 23, 46, 47, 54], we assume our protocols are supplied by a common coin, an object that is introduced by Rabin [51], which delivers the same sequence of random coins to replicas. We use the common coin protocol for the underlying *random leader election* protocol (denoted by $\mathsf{Election}()$) and use it in the underlying RABA protocol.

For our implementation, we use the threshold PRF scheme by Cachin, Kursawe, and Shoup [12].

5 OUR MVBA APPROACH

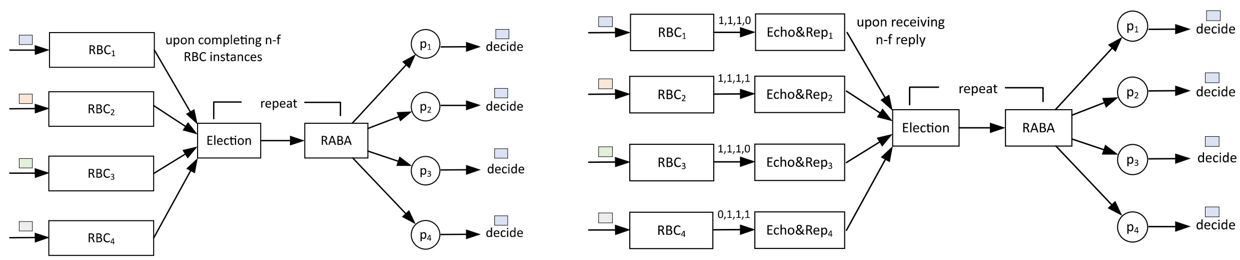
5.1 Overview

We present our signature-free MVBA protocol with $O(1)$ time and $O(n^3)$ messages. In particular, we reduce MVBA to reliable broadcast, random leader election (via common coins), and reproposable asynchronous binary agreement.

At a high level, our MVBA protocol works as follows. First, each replica p_i runs RBC_i to disseminate its proposal, resulting in n parallel RBC instances. We aim to have replicas agree on the value *r-delivered* by *exactly one* of the n RBC instances. A crucial observation is that if $n - f$ correct replicas complete $n - f$ RBC instances, then at least $f + 1$ correct replicas have *r-delivered* some values for at least $f + 1$ RBC instances. If we know this set I of the $f + 1$ RBC instances, then we are almost done. In particular, all we need to do is to pick any of the RBC instances in I, say, RBC_k, and correspondingly, replicas run a RABA instance by proposing 1 once they *r-deliver* some value for the RBC_k instance. To see why this intuitively works, first note that the agreement property in RBC

protocol	communication (threshold PRF)	time	assumption
CKPS [11]	$O(Ln^2 + \kappa n^2 + n^3)$	$O(1)$	threshold sig
AMS [1]	$O(Ln^2 + \kappa n^2)$	$O(1)$	threshold sig
Dumbo-MVBA [42]	$O(Ln + \kappa n^2)$	$O(1)$	threshold sig; vc
DYX+ MVBA [28]	$O(Ln^2 + \kappa n^3 \log n)$	$O(\log n)$	hash
DXKR MVBA [25]	$O(Ln^2 + \kappa n^3 \log n)$	$O(\log n)$	hash
Our MVBA (Sec. 5.2) + CT RBC [13]	$O(Ln^2 + \kappa n^3 \log n)$	$O(1)$	hash
Our MVBA (Sec. 5.2) + EFBRB [3]	$O(Ln^2 + \kappa n^2 + n^3 \log n)$	$O(1)$	none
Our MVBA (Sec. 5.2)+ CCBRB [3]	$O(Ln^2 + \kappa n^3)$	$O(1)$	hash
Our tailored MVBA (Sec. 5.4)	$O(Ln^2 + \kappa n^3)$	$O(1)$	hash

Table 2: Comparison of the MVBA protocols using common coins. Here we examine the communication cost of protocols by using threshold PRF [12] to generate common coins. L is the length of the input from each replica and κ is a security parameter. The "assumption" column means the additional assumption besides common coins and authenticated channels. "vc" stands for a constant-size vector commitment which typically requires trusted setup and pairing assumptions. Our protocols lead to the first $O(1)$-time MVBA protocols without threshold signatures. In addition, instantiating our MVBA (Sec. 5.2) with EFBRB [3] leads to an MVBA protocol achieving lower communication than any instantiations from [25, 28], as our MVBA uses only $O(1)$ RABA and $O(1)$ common coin instances—$O(\kappa n^2)$ bits using threshold PRF.



(a) MVBA without the quality property. **(b) MVBA with the quality property.**

Figure 1: Our MVBA protocols.

and the biased termination property in RABA together guarantee termination, and meanwhile, the biased validity property ensures that even if only $f + 1$ correct replicas, not all correct replicas, propose 1, correct replicas that terminate will decide 1. Observing $|I| \geq f + 1$, if we take a random guess among all $3f + 1$ replicas, then with a probability of at least 1/3, we can hit a good $k \in I$. Thus, our MVBA protocol will terminate in expected constant time. Throughout the process, we use ideal building blocks—RBC, RABA, and common coins—no cryptographic primitives such as threshold signatures.

We comment that external validity can be trivially enforced, as long as the predicate is publicly verifiable. Namely, we have treated it as a general predicate. Note, however, that if removing external validity, our MVBA protocol does not directly lead to a multivalued Byzantine agreement (MBA) protocol (e.g., [48]); this is because it does not satisfy the validity property in MBA.[3]

In this section, we first build MVBA without the quality property, satisfying all the security properties defined in CKPS [11]. We will show this MVBA protocol suffices to build our ACS protocol with $O(1)$ time and $O(n^3)$ messages. Then we show that by including two additional communication rounds (using a variant of HotStuff technique [1, 53]), we can build MVBA with the quality property. Last, we demonstrate a highly efficient MVBA protocol that optimizes concrete communication cost and computational efficiency. Such an MVBA protocol benefits from a core observation that we may not necessarily need a fully-fledged RBC to construct MVBA.

[3]Validity in MBA requires that if *all* correct replicas propose 1, then all replicas that terminate decide 1.

5.2 Our MVBA Protocol

We present the pseudocode of our MVBA protocol in Figure 2. As illustrated in Figure 1a, the protocol has two phases: an RBC phase with n parallel RBC instances; an iteration phase with only one RABA instance for each iteration.

RBC phase (lines 01-02). In the RBC phase, each replica p_i holds an input v_i that it proposes for the MVBA protocol such that $Q(v_i)$ holds. Upon the event *mvba-propose*(v_i), replica p_i *r-broadcasts* v_i for an RBC instance RBC_i. Upon receiving value v_i *r-broadcast* by p_i in RBC_i, every replica waits until $Q(v_i)$ holds before participating in RBC_i.

Note that there are up to n parallel RBC instances, and every correct replica p_i verifies the predicate for all RBC instances running. As usual, we require that the predicate Q is verifiable across all correct replicas. Meanwhile, in certain applications, we may also require that the predicate Q depends on the internal state st of a particular replica. Namely, it is possible that $Q(v, st)$ for some v and

MVBA

Input: value v_i such that a global predicate $Q(v_i)$ holds
Output: value v_k (proposed by p_k)
Initialization: $r \leftarrow 0$

---

01 **upon event** *mvba-propose*(v_i)
02 *r-broadcast* v_i for RBC_i {▷ RBC phase}
{every replica verifies whether $Q(v_i)$ holds upon receiving v_i before participating in RBC_i }
03 **wait for** $n - f$ RBC instances to complete {▷ Iteration phase}
04 **repeat**
05 $k \leftarrow$ Election()
06 **if** some value is *r-delivered* in RBC_k
07 *raba-propose* 1 for RABA_r
08 **else**
09 *raba-propose* 0 for RABA_r
10 **if** later some value is *r-delivered* in RBC_k
11 *raba-repropose* 1 for RABA_r
12 **if** RABA_r outputs 1
13 **wait for** RBC_k to *r-deliver* value v_k
14 **terminate** the protocol **and** *mvba-decide*(v_k)
15 $r \leftarrow r + 1$

Figure 2: Our MVBA protocol with a predicate Q. Code for p_i.

some replica fails to hold at the beginning, but $Q(v, st)$ will hold at some point, all depending on *st* of the replica.

Iteration phase (lines 03-15). Each replica waits until it *r-delivers* $n - f$ RBC instances before it enters the iteration phase. In each iteration, replicas iterate the Election() function and a RABA instance until a RABA instance outputs 1.

Concretely, in each iteration r, replicas query the Election() function and obtain a random k such that $1 \le k \le n$ (line 05). At lines 06-07, if a replica has previously *r-delivered* some value in RBC_k, it *raba-proposes* 1 for the RABA instance denoted RABA_r[4] in iteration r. Otherwise, at lines 08-09, the replica *raba-proposes* 0. If a replica originally *raba-proposes* 0 but later *r-delivers* some value in RBC_k, p_i *raba-reproposes* 1 (at lines 10-11), ensuring protocol termination.

After each replica provides some input to RABA_r, it waits for the output of RABA_r. If p_i *raba-decides* 0, the replica continues to the next iteration $r + 1$ (line 15). If p_i *raba-decides* 1, it waits for the output of RBC_k denoted as v_k (line 13). Then it *mvba-decides* v_k and terminates the protocol (line 14).

Complexity and instantiations. To understand the time complexity of our MVBA protocol, we first observe that at the end of the RBC phase, for at least $f + 1$ instances, at least $f + 1$ correct replicas have *r-delivered* some values (see Lemma 5.4 in this section). Second, the biased validity property of RABA guarantees that for any of these $f + 1$ instances, replicas will *raba-decide* 1. As the Election() function selects any of the $f + 1$ instances with at least a probability of 1/3, the protocol terminates in expected $O(1)$ time.

[4]Note that if we assume the Election() function is a random permutation instead of a random function, then we can also use RABA_k to uniquely and unambiguously denote the RABA instance.

Unlike prior signature-free MVBA constructions [25, 28] that terminate in expected $O(\log n)$ time (summarized in Table 2), our MVBA protocols have expected $O(1)$ time.

Both the message complexity and the communication complexity of our MVBA are bounded by the n parallel RBC instances. As each RBC instance has $O(n^2)$ messages, our MVBA construction has $O(n^3)$ messages.

Table 2 summarizes the communication cost of our protocols assuming a threshold PRF [12]. For instance, if we use EFBRB [3] (an information-theoretically secure RBC), we obtain an MVBA protocol assuming common coins only. If we use CCBRB [3], we obtain a protocol using hashes and common coins and having lower communication than instantiations from [25, 28]. In particular, our MVBA protocol with EFBRB results in a protocol with $O(Ln^2 + \kappa n^2 + n^3 \log n)$ communication (the term $O(\kappa n^2)$ is due to the cost of common coins in $O(1)$ RABA and Election() instances); in contrast, [25, 28] with EFBRB lead to MVBA protocols with $O(Ln^2 + \kappa n^3 + n^3 \log n)$ communication.

We comment that if the underlying RBC and common coin protocols are instantiated using information-theoretically secure protocols, then our MVBA protocols are also information-theoretically secure.

Proof of our MVBA. We show that the MVBA protocol presented in Sec. 5.2 achieves external validity, agreement, integrity, and termination.

Theorem 5.1 (External validity). *Every correct replica that terminates mvba-decides v such that $Q(v)$ holds.*

Proof. If any correct replica *mvba-decides* v, it has *raba-decided* 1. Hence, according to the validity property of RABA, at least one correct replica has *raba-proposed* 1 or *raba-reproposed* 1. If the correct replica *raba-proposes* 1 or *raba-reproposes* 1, it has *r-delivered* v in the corresponding RBC instance. In the RBC instance, according to our specification of MVBA, every replica verifies whether $Q(v)$ holds upon receiving any value from another replica in the RBC phase. If a correct replica *r-delivered* v in an RBC instance, then at least one correct replica has previously received v and verified that $Q(v)$ holds. Hence, if one correct replica *mvba-decides* v, then at least one correct replica has verified that $Q(v)$ holds. As $Q(v)$ is verifiable across all correct replicas, every correct replica that *mvba-decides* v must have that $Q(v)$ holds. □

Theorem 5.2 (Agreement). *If a correct replica mvba-decides v, then any correct replica that terminates mvba-decides v.*

Proof. Assume a correct replica p_i *mvba-decides* v. Then p_i must have *raba-decided* 1 in some RABA_r for iteration $r > 0$, and for any iteration $\hat{r} < r$, it holds that $\text{RABA}_{\hat{r}}$ outputs 0.

Now we assume another correct p_j *mvba-decides* v'. We now prove by contradiction that $v' = v$. We distinguish two cases for p_j: p_j *mvba-decides* in iteration $r' = r$; p_j *mvba-decides* in iteration $r' \neq r$.

Case 1: p_j *mvba-decides* v' in round r. In this case, we assume that p_j obtains k' from the Election() function and $\text{RBC}_{k'}$ outputs v'. As Election() outputs a *common* coin for the same input r, it must hold that $k = k'$. Hence, if $v \neq v'$, the agreement property of RBC would be violated.

Case 2: If $r' > r$, then according to our protocol, p_j *raba-decides* 1 in $\text{RABA}_{r'}$ and *raba-decides* 0 for any lower iteration r', including r. This violates the agreement property of RABA_r. Similarly, the argument holds for the case $r' < r$. □

Theorem 5.3 (Integrity). *If all replicas follow the protocol, and if a correct replica mvba-decides v such that $Q(v)$ holds, then some replica mvba-proposed v such that $Q(v)$ holds.*

Proof. If a correct replica p_i *mvba-decides* v, it *raba-decides* 1 in some RABA_r and *r-delivers* v in RBC_k where k is the corresponding common coin. According to the integrity property of RBC, v was previously broadcast by replica p_k. □

Lemma 5.4. *If all correct replicas enter the iteration phase, then for at least $f+1$ RBC instances, at least $f+1$ correct replicas have r-delivered some values.*

Proof. Instead of *directly* bounding the number of correct replicas that have *r-delivered* some values, we bound the number of instances where fewer than $f+1$ correct replicas have *r-delivered* some values. First, we observe that all correct replicas *r-deliver* some values for $(2f+1)(2f+1)$ RBC instances in total. As there are at most $(3f+1)(2f+1)$ instances for correct replicas, the total number of instances where correct replicas do not *r-deliver* some values are upper bounded by $(3f+1)(2f+1)-(2f+1)(2f+1) = 2f^2+f$. Hence, the number of RBC instances where fewer than $f+1$ correct replicas have *r-delivered* some values is bounded by $\frac{2f^2+f}{f+1} < \frac{2f^2+2f}{f+1} = 2f$. That is, the number of RBC instances where at least $f+1$ correct replicas have *r-delivered* some values is at least $f+1$. □

We comment that PACE observed a similar claim that works in a context with a different goal.

Lemma 5.5. *After the first correct replica enters the election phase, an adversary (i.e., network scheduler) can schedule the messages received by correct replicas for at most a constant number of rounds, after which Lemma 5.4 holds.*

Proof. After one correct replica enters the election phase, the network scheduler that corrupts f replicas can learn the value of k_0. To ensure that RABA_0 terminates and outputs 0, at least $f+1$ correct replicas need to *raba-propose* 0, as otherwise the biased validity property of RABA is violated. Starting from the second iteration, the number of RBC instances completed by each of these $f+1$ correct replicas will only grow but the instances already completed cannot be manipulated any more. We now show that, within constant rounds and for each round $r \geq 0$, if RABA_r outputs 0, another $O(f)$ correct replicas will enter the election phase, so Lemma 5.4 holds afterwards.

Let S_r denote the set of correct replicas that have already started the iteration phase. According to the discussion above, $|S_r| \geq f+1$. Let c_r denote a value such that $|S_r| = c_r f$. Obviously, $c_r \in [1+\frac{1}{f}, 2+\frac{1}{f}]$. Assume that RABA_r outputs 0, then we know that at least $f+1$ correct replicas *raba-propose* 0 for $\textit{RABA}_r$ as otherwise the biased validity property or RABA is violated. If these correct replicas belong to S_r, we need to ensure that regardless of the value of k_r selected by the Election() function, with overwhelming probability, $O(f)$ replicas in S_r will *raba-propose* 0.

Instead of directly showing the number replicas in S_r that will *raba-propose* 0, we show that if RABA_r outputs 0, with a probability of at least 1/9, more than $\frac{f}{4}$ new correct replicas will enter the iteration phase so the size of S_r grows by $O(f)$, i.e., $c_{r+1} - c_r > \frac{f}{4}$ where $|S_{r+1}| = c_{r+1}f$. We calculate the probability that $c_{r+1} - c_r \leq \frac{f}{4}$ as follows. As $c_{r+1} - c_r \leq \frac{f}{4}$, fewer than $\frac{f}{4}$ new correct replicas enter the election phase in iteration $r+1$. Therefore, for RABA_{r+1} to output 0, at least $\frac{3f}{4}+1$ correct replicas in S_r *raba-propose* 0 for RABA_{r+1}. This is because $f+1$ correct replicas *raba-propose* 0 in RABA_{r+1}, as otherwise the biased validity property of RABA is violated.

Now consider the status of replicas in S_r in iteration $r+1$. As every correct replica complete $2f+1$ RBC instances before entering the election phase, replicas in S_r completes $|S_r|(2f+1)$ RBC instances in total. As there are $3f+1$ replicas in total, the upper bound of RBC instances where replicas in S_r have not *r-delivered* any value is bounded by $|S_r|f$. As every correct replica *raba-proposes* 0 in RABA_{r+1} only if it has not *r-delivered* any value, the number of instances (let the set of instances be L) that $\frac{3f}{4}+1$ correct replicas in S_r could *raba-propose* 0 in RABA_{r+1} is bounded by the following:

$$|L| = \frac{|S_r|f}{\frac{3f}{4}+1} = \frac{c_r f \cdot f}{\frac{3f}{4}+1} \leq \frac{c_r f^2}{\frac{3f}{4}} = \frac{4c_r f}{3}. \tag{1}$$

Then according to our assumption, $k_{r+1} \in L$ with a probability of at most $\frac{8}{9}$ as shown below:

$$\Pr[k_{r+1} \in L] = \frac{|L|}{n} \leq \frac{\frac{4c_r f}{3}}{3f+1} \leq \frac{\frac{4f\cdot(2+\frac{1}{f})}{3}}{3f+1} \tag{2}$$

$$\lim_{f\to\infty} \Pr[k_{r+1} \in L] \leq \lim_{f\to\infty} \frac{\frac{4f\cdot(2+\frac{1}{f})}{3}}{3f+1} = \frac{8}{9}$$

Accordingly, at least $\frac{f}{4}$ will join the election phase with a probability of at least 1/9. It is then not difficult to see that after a constant number of rounds, $|S_r| \approx 2f+1$. After that, Lemma 5.4 holds. □

Theorem 5.6 (Termination). *If all correct replicas are activated and all messages sent among correct replicas have been delivered, then all correct replicas mvba-decide.*

Proof. If all correct replicas start the protocol, then according to the validity property of RBC, every correct replica completes at least $n-f$ RBC instances. During the iteration phase, we first show that any iteration r will complete and then show that eventually, some RABA_r will output 1. For each iteration r, we assume that k is returned by the Election() function in iteration r.

We first show that every iteration r will complete. For each iteration r, we distinguish three cases: 1) all correct replicas have *r-delivered* some value in RBC_k; 2) at least one correct replica has *r-delivered* some value in RBC_k, and at least one correct replica has not *r-delivered* any value in RBC_k; 3) none of the correct replicas have *r-delivered* any value in RBC_k.

Case 1: Due to the unanimous termination property, it holds that RABA_r terminates.

Case 2: If at least one correct replica has *r-delivered* some value in RBC_k, then from the agreement property of RBC, any correct replica eventually *r-delivers* some value. According to our protocol, any correct replica that provides 0 as RABA input (in which case it has not *r-delivered* any value in RBC_k when the iteration begins) will eventually *raba-repropose* 1. Thus, the biased termination condition of RABA will eventually be satisfied. Hence, RABA_r will terminate, and iteration r will eventually complete.

Case 3: If none of the correct replicas *r-deliver* any value in RBC_k, iteration r will complete due to the unanimous termination property of RABA. Otherwise, if at least one correct replica later *r-delivers* some value in RBC_k, then according to case 2, iteration r will complete (due to the biased termination of RABA).

We now prove that eventually, in some iteration r, RABA_r outputs 1, so the protocol will terminate. From Lemma 5.4, for at least $f+1$ RBC instances, at least $f+1$ correct replicas have *r-delivered* some value after they enter the iteration phase. Let I be the set of the $f+1$ RBC instances. Due to the biased validity property of RABA, RABA_r outputs 1. As Election() outputs a uniformly random coin for each iteration, we have that with probability $\frac{f+1}{3f+1} \approx \frac{1}{3}$, it holds that $k \in I$.

After RABA_r outputs 1, every correct replica waits for the output of RBC_k. If RABA_r outputs 1, at least one correct replica has *raba-proposed* 1 or *raba-reproposed* 1. (Otherwise, the unanimous termination property of RABA would be violated.) Therefore, at least one correct replica has *r-delivered* some value in RBC_k. From the agreement property of RBC, every correct replica eventually *r-delivers* some value in RBC_k and then *mvba-decides.* □

THEOREM 5.7. *Our MVBA protocol has expected $O(1)$ running time.*

PROOF. From Lemma 5.5, we know that if at least one correct replica enters the election phase, Lemma 5.4 eventually holds after a constant number of rounds. From Lemma 5.4, for at least $f+1$ RBC instances, at least $f+1$ correct replicas have *r-delivered* some value after they enter the iteration phase. Due to the biased validity property of RABA, RABA_r outputs 1. Since Election() outputs a uniformly distributed random coin for each iteration, it holds that with probability $\frac{f+1}{3f+1} \approx \frac{1}{3}$, we have $k \in I$. Therefore, the protocol has expected $O(1)$ running time. □

5.3 MVBA with the Quality Property

We show that by adding two additional communication steps, we can build MVBA with the quality property. As illustrated in Figure 1b, we introduce an echo and reply procedure between the RBC phase and the iteration phase. In the pseudocode, all we need to do is to replace line 03 in Figure 2 using the lines of code in Figure 3. Each replica p_i now additionally maintains one vector W_i to track the set of completed RBC instances. After p_i completes RBC_j, it sets $W_i[j]$ as 1 (lines 01-02). When p_i completes $n-f$ RBC instances, it broadcasts the W_i vector to all replicas (03-04). It then expects to receive $n-f$ (REP) messages from the replicas, representing that $n-f$ replicas have also *r-delivered* some values for the same $n-f$ RBC instances. To achieve this goal, for each replica p_i, upon receiving W_j from p_j, p_i first checks whether the vector contains $n-f$ 1's. Then for each $W_j[l]=1$, p_i waits until some value is *r-delivered* in RBC_l (lines 06-07). After that, p_i sends a (REP) message to p_j (line 08). Upon receiving $n-f$ (REP) messages, p_i enters the iteration phase.

MVBA with the quality property

Initialization: $W_i \leftarrow [0]^n$

replace line 03 in Figure 2 using the following lines:
01 **upon** *r-delivering* v_j for RBC_j
02 $\quad W_i[j] \leftarrow 1$
03 **wait for** $n-f$ RBC instances to complete
04 $\quad$ send (ECHO, W_i) to all replicas
05 **upon** (ECHO, W_j) from p_j such that there are $n-f$ 1's in W_j
06 $\quad$ **if** for any $W_j[l]=1$, RBC_l outputs some value
07 $\quad\quad$ send (REP, i) to p_j
08 **wait for** $n-f$ (REP) messages

Figure 3: Our MVBA protocol with the quality property. The code for p_i.

The protocol achieves quality, mainly because upon receiving a vector W_j, each replica verifies whether it has *r-delivered* some value in each RBC_l instance for $W_j[l]=1$. Hence, if a correct replica p_j receives $n-f$ (REP) messages, at least $n-f$ replicas must have *r-delivered* the same values in the same set of $n-f$ RBC instances due to the agreement property of RBC. Hence, with probability $\frac{2f+1}{3f+1} \approx \frac{2}{3}$, replicas *mvba-decide* a value from a correct replica. The protocol thus achieves quality.

The way of achieving quality can be viewed as using the technique from [1, 53] and using the agreement property in RBC. We show the correctness of the MVBA in this subsection in Appendix C.

5.4 Tailored MVBA from Weak RBC

While we can use EFBRB and CCBRB for low communication cost in our MVBA protocol, both of them rely on online error correcting (OEC) code and may suffer some degraded performance during failures (due to the "trial-and-error" OEC pattern). Additionally, EFBRB has significantly more steps than the classic RBC protocols [10, 13].

In this section, we provide a more practical MVBA construction that achieves $O(Ln^2 + \kappa n^3)$ communication. While its communication is the same as that of using CCBRB, our protocol in this subsection outperforms the CCBRB instantiation (in Sec. 5.2) in terms of both concrete communication cost and computational efficiency. First, the construction in this subsection does not use erasure coding or online error correcting code. Hence, the hidden constant in the bulk data term Ln^2 is 1 (namely $1Ln^2$) instead of 3 (at least $3Ln^2$ or more if using erasure coding or error correcting code). Namely, for a large L, the communication cost of this construction is about 1/3 of that of CCBRB-based MVBA. Second, as the construction in this subsection uses hashes only and does not use online error correction, it is computationally more efficient in both gracious and uncivil executions.

We show our tailored MVBA construction in Figure 4. The protocol relies on a weak RBC primitive, which we call WRBC. The workflow of WRBC (lines 21-37) is similar to the 3-phase RBC (e.g.,

```
A Practical MVBA Construction
Input: Value v_i such that a global predicate Q(v_i) holds
Output: Value v_k proposed by p_k
Initialization: r ← 0, T_i ← [⊥]^n
-----------------------------------------------------------------
01 upon event mvba-propose(v_i)
02   wr-broadcast(v_i) for WRBC_i                  {▷ WRBC phase}
03   wait for n − f WRBC instances to complete  {▷ Iteration phase}
04   repeat
05     k ← Election()
06     if some value is wr-delivered in WRBC_k
07       raba-propose 1 for RABA_r
08     else
09       raba-propose 0 for RABA_r
10       if later some value is r-delivered in WRBC_k
11         raba-repropose 1 for RABA_r
12     if RABA_r outputs 1
13       wait for WRBC_k to wr-deliver value h_k
14       if T_i[k] ≠ ⊥
15         broadcast (VALUE, T_i[k])
16       else
17         wait for (VALUE, v_k) such that Hash(v_k) = h_k
18         T_i[k] ← v_k
19       terminate the protocol and mvba-decide(T_i[k])
20     r ← r + 1
-----------------------------------------------------------------
21 upon event wr-broadcast(v_j) for instance WRBC_j
22   {replica p_j broadcasts (SEND, v_j)}
23   upon receiving (SEND, v_j) from p_j
24     if Q(v_j) holds
25       T_i[j] ← v
26       broadcast (ECHO, Hash(v_j))
28     else
29       store the message until Q(v_j) holds
30   upon receiving n − f matching (ECHO, h)
31     broadcast (READY, h)
32   upon receiving f + 1 matching (READY, h) and (READY) message
has not been sent yet
33     broadcast (READY, h)
34   upon receiving n − f matching (READY, h)
35     if Hash(T_i[j]) ≠ h
36       T_i[j] ← ⊥
37     wr-deliver h
```

Figure 4: A Practical MVBA protocol. Code is for p_i.

Bracha's broadcast [10]), but we only use hashes in the second and the third phases. As a result, when a replica successfully terminates WRBC, all correct replicas will eventually obtain the hash of the WRBC input message; when needed (later), all correct replicas can retrieve the message based on the hash. (WRBC appears implicitly and partly used in, e.g., [27], and we claim no novelty about WRBC itself.)

Concretely, at lines 21-37, the sender p_j in each WRBC$_j$ first broadcasts its input v_j in (SEND) messages. Upon receiving the value v_j from p_j, each replica p_i verifies whether the predicate $Q(v_j)$ holds. If so, p_i updates $T_i[j]$ as v_j and then broadcasts a (ECHO, $Hash(v_j)$) message (lines 25-26). Otherwise, p_i stores the (SEND) message and processes it until $Q(v_j)$ holds (lines 28-29). Upon receiving $n - f$ (ECHO, h) messages with the same hash value h, each replica broadcasts a (READY, h) message (lines 30-31). If a replica p_i receives $f + 1$ (READY, h) messages but has not sent a (READY) message, p_i also broadcasts a (READY, h) message (lines 32-33). Upon receiving $2f + 1$ (READY, h) messages, p_i *wr-delivers* h (line 37). If some h is *wr-delivered* but the hash of $T_i[j]$ is not h, p_j sets $T_i[j]$ as $\perp$ (lines 35-36).

We now describe our tailored MVBA protocol. There are two major changes on top of our MVBA protocol in Sec. 5.2. First, in the RBC phase, we use WRBC instead of the standard RBC, where for each WRBC$_j$, the sender p_j *wr-broadcasts* a value v_j and correct replicas *wr-deliver* $h_j = Hash(v_j)$. Second, as each WRBC instance outputs the hash of the value instead of the original value broadcast by the sender, we need to retrieve the value after replicas reach an agreement in the iteration phase. In particular, in some iteration r where k is the output of the Election() function, after RABA$_r$ outputs 1, each replica first waits for WRBC$_k$ to output some value h_k and then starts the retrieval (lines 13-18). If p_i has some value in $T_i[k]$, it broadcasts a (VALUE, $T_i[k]$) to all replicas. If p_i does not hold a $T_i[k]$ value, it waits to receive a (VALUE, v_k) such that $Hash(v_k) = h_k$ and then sets $T_i[k]$ as v_k. After $Hash(T_i[k]) = h_k$, p_i terminates the protocol and *mvba-decides* $T_i[k]$ (line 19).

The communication bottleneck of this protocol is the WRBC phase, as the communication cost for other steps (including the retrieval step) is $O(\kappa n^2)$. For each WRBC instance, the sender broadcasts a message (length L), and replicas exchange hashes of values in the second phase and the third phase, so each instance has $O(n^2)$ messages and $O(Ln + \kappa n^2)$ communication. As there are n parallel WRBC instances, our practical MVBA construction has $O(Ln^2 + \kappa n^3)$ communication. The correctness of our protocol is similar to that of our MVBA in Sec. 5.2. In particular, each WRBC protocol WRBC$_j$ can guarantee that if a correct replica *wr-delivers* some value h, every correct replica eventually *wr-delivers* h. Furthermore, at least $f+1$ correct replicas must set their $T_i[j]$ as value v such that $Hash(v) = h$. Hence, the value v can be retrieved by any correct replicas. We prove the correctness of this tailored MVBA construction in Appendix D.

5.5 Tailored MVBA from Weak RBC with Fewer Expected Rounds

Our MVBA protocol (without the quality property) presented in Sec. 5.2 and the tailored MVBA presented in Sec. 5.4 require an involved proof for constant time complexity, i.e., Lemma 5.5 is required to show that if at least one correct replica enters the iteration phase, the protocol terminates in constant time. As a result, the protocols are expected to terminate in a larger number of expected rounds. In this section, we further revise our tailored MVBA protocol and build one with lower expected number of rounds. The proof for the time complexity can also be simplified accordingly. Our standard MVBA protocol can be revised in a similar way to enjoy lower expected number of rounds.

The new MVBA protocol is shown in Figure 5. Compared to the protocol shown in Figure 5.4, this protocol has only one change: After each replica p_i complete WRBC$_j$, it sends a (REP, i) message

```
A Practical MVBA Construction with Fewer Expected Rounds
Input: Value v_i such that a global predicate Q(v_i) holds
Output: Value v_k proposed by p_k
Initialization: r ← 0, T_i ← [⊥]^n
-----------------------------------------------------------
upon event mvba-propose(v_i)
  wr-broadcast(v_i) for WRBC_i                    {▷ WRBC phase}
  upon event wr-delivering some value in WRBC_j
    send (REP, i) to p_j
  wait for n − f WRBC instances to complete and n − f (REP)
messages                                       {▷ Iteration phase}
  repeat
    k ← Election()
    if some value is wr-delivered in WRBC_k
      raba-propose 1 for RABA_r
    else
      raba-propose 0 for RABA_r
      if later some value is r-delivered in WRBC_k
        raba-repropose 1 for RABA_r
    if RABA_r outputs 1
      wait for WRBC_k to wr-deliver value h_k
      if T_i[k] ≠ ⊥
        broadcast (VALUE, T_i[k])
      else
        wait for (VALUE, v_k) such that Hash(v_k) = h_k
        T_i[k] ← v_k
      terminate the protocol and mvba-decide(T_i[k])
    r ← r + 1
-----------------------------------------------------------
upon event wr-broadcast(v_j) for instance WRBC_j
  {replica p_j broadcasts (SEND, v_j)}
  upon receiving (SEND, v_j) from p_j
    if Q(v_j) holds
      T_i[j] ← v
      broadcast (ECHO, Hash(v_j))
    else
      store the message until Q(v_j) holds
  upon receiving n − f matching (ECHO, h)
    broadcast (READY, h)
  upon receiving f + 1 matching (READY, h) and (READY) message
has not been sent yet
    broadcast (READY, h)
  upon receiving n − f matching (READY, h)
    if Hash(T_i[j]) ≠ h
      T_i[j] ← ⊥
    wr-deliver h
```

Figure 5: A Practical MVBA protocol with fewer rounds. Code is for p_i.

to p_j. Replica p_i enters the iteration phase after it completes $n - f$ WRBC instances and receives $n - f$ (Rep) messages.

We now briefly discuss why this new MVBA protocol can simplify the proof and reduce the expected number of rounds. First, at least one correct replica enters the iteration phase to guarantee the termination of the protocol. If the adversary learns the value of the k where k is the output of the Election() function in iteration 0, it can manipulate the messages received by correct replicas and force RABA_0 ot output 0. However, to make RABA_0 output 0, at least $f + 1$ correct replicas must *raba-propose* 0 as otherwise the biased validity of RABA is violated. Let the identities of these $f + 1$ correct replicas be I. For any $i \in I$, p_i receives $n - f$ (Rep) messages before it enters the iteration phase. Among the $n - f$ (Rep) messages, at least $f + 1$ are sent by correct replicas and each of them has completed WRBC_i. If i is selected by the Election() function in iteration r, at least $f + 1$ correct replicas will *raba-propose* 1 so the biased validity property of RABA ensures that RABA outputs 1. Therefore, with $\frac{f+1}{3f+1}$ probability, RABA outputs 1 and the protocol terminates.

6 OUR ACS APPROACH

6.1 Overview

We now present our ACS protocol with $O(1)$ time and $O(n^3)$ messages. At the core of our ACS protocol is the reduction of ACS to our MVBA construction with a specific predicate. In particular, we use n parallel RBC instances for replicas to disseminate their *acs-proposed* values. Then each replica *mvba-proposes* a vector of $n - f$ bits, representing the $n - f$ completed RBC instances. Crucially, we define the global predicate of MVBA as the following: given a proposal with $n - f$ bits, each replica considers the proposal valid only if it has completed the same $n - f$ RBC instances. (As we commented earlier, the predicate depends on the state of a particular replica: it is possible that the predicate does not hold at the beginning but will hold at some point.) In this way, we can guarantee that the output of ACS consists of at least $n - f$ *acs-proposed* values, satisfying the validity property of ACS. As our MVBA component completes in expected $O(1)$ time, our ACS protocol also terminates in expected constant time.

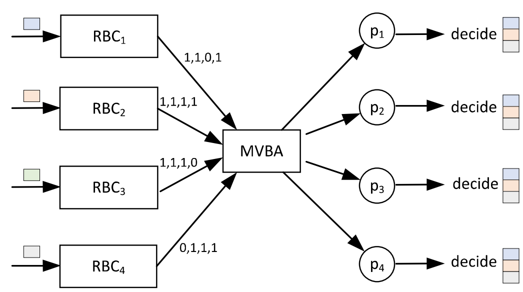


Figure 6: Our ACS protocol.

6.2 The ACS Protocol

We describe our ACS protocol in Figure 6 and the pseudocode in Figure 7. Our ACS protocol has two phases: an RBC phase and an MVBA phase.

RBC phase (lines 02-04). Each replica p_i holds an input v_i. Upon the event *acs-propose*(v_i), p_i *r-broadcasts* v_i. Upon completing an RBC instance RBC_j, p_i sets $W[j]$ as 1 (lines 03-04), where W is a global map used to track the status of the RBC instances.

MVBA phase (lines 05-12). Replica p_i enters the MVBA phase after completing $n - f$ RBC instances. Each replica p_i sets W_i as W and uses W_i as input for MVBA with a predicate $Q(W_j, W)$ (lines

ACS

Input: value v_i
Output: $n-f$ values $\mathbf{v}$, among which at least $n-2f$ are proposed by correct replicas
Initialization: $r \leftarrow 0, W \leftarrow [0]^n, \mathbf{v} \leftarrow \perp$
Let Q be the following predicate for MVBA:
Given value W_j *mvba-proposed* by some replica p_j, $Q(W_j, W) \equiv$ ($|W_j| = n$ **and** for at least $n-f$ l such that $1 \leq l \leq n$, $W_j[l] = 1$ **and** $W_j \subseteq W$)

01 **upon event** *acs-propose*(v_i)
02 *r-broadcast* v_i for RBC_i {▷ RBC phase}
03 **upon** *r-delivering* v_j for RBC_j
04 $W[j] \leftarrow 1$
05 **wait for** $n-f$ RBC instances to complete {▷ MVBA phase}
06 $W_i \leftarrow W$
07 *mvba-propose*(W_i) with predicate $Q(W_j, W)$
08 **upon event** *mvba-decide*(W_k)
09 for every $W_k[j] = 1$
10 **wait for** RBC_j to *r-deliver* value v_j
11 $\mathbf{v} \leftarrow \mathbf{v} \cup v_j$
12 **terminate** the protocol and *acs-decide*($\mathbf{v}$)

Figure 7: The ACS protocol. Code is for p_i.

06-07). Note that the 1's in W for each p_i continue to grow as more RBC instances complete. The value *mvba-proposed* by each p_i is a snapshot of W_i with at least $n-f$ RBC instances completed.

We define a global predicate $Q(W_j, W)$ for each value W_j *mvba-proposed* by p_j as follows. First, each W_j is a n-bit vector. Second, W_j consists of at least $n-f$ 1's, representing the $n-f$ RBC instances that replica p_j has completed. Third, $W_j \subseteq W$. Namely, for each $W_j[l] = 1$, replica p_i must wait until it has *r-delivered* some value in RBC_l. Hence, in MVBA, every replica may need to wait until the global predicate is satisfied for each *mvba-proposed* message. As discussed previously in Sec. 5, the predicate depends on the internal state W of each replica p_i; it is possible that $Q(W_j, W)$ fails to hold when p_i receives the *mvba-proposed* value by p_j, but $Q(W_j, W)$ will hold at some point as p_i *r-delivers* more RBC instances.

After p_i provides some input to MVBA, it waits for the output of MVBA (line 08). According to the integrity property of MVBA, the output of MVBA W_k must be proposed by some replica such that $Q(W_k, W)$ is satisfied. At lines 09-11, for each $W_k[j] = 1$, p_i waits for the output of RBC_j, v_j. Then v_j is added to a set $\mathbf{v}$. After all such RBC instances are complete, p_i *acs-decides* $\mathbf{v}$ (line 12).

Complexity and discussion. Our ACS protocol terminates in expected constant time, as the underlying MVBA protocol runs in expected $O(1)$ time. Moreover, our ACS protocol clearly has $O(n^3)$ messages. The communication cost of our ACS protocol depends on the underlying RBC protocol, as the input length to MVBA is only n. For instance, our ACS protocol with CT RBC has $O(Ln^2 + \kappa n^3 \log n)$ communication.

In contrast to BKR ACS and the state-of-the-art PACE ACS that terminate in $O(\log n)$ time and need n (R)ABA instances, our ACS protocol has $O(1)$ time and uses only $O(1)$ RABA instances.

Note that as the input of MVBA indicates that at least $n-2f$ *acs-proposed* values from correct replicas will eventually be *acs-decided*, we do not need the quality property for the underlying MVBA protocol.

6.3 Proof of our ACS Protocol

Theorem 6.1 (Validity). *If a correct replica acs-decides a set* $\mathbf{v}$, *then* $|\mathbf{v}| \geq n-f$ *and* $\mathbf{v}$ *contains values acs-proposed by at least* $n-2f$ *correct replicas.*

Proof. According to the protocol, every correct replica p_i first *mvba-decide*(W_k) and then obtains $\mathbf{v}$. The predicate of $Q(W_k, W)$ specifies that there are at least $n-f$ 1's in the W_j vector. Then p_i waits for each RBC_j to output some v_j for each $W_k[j] = 1$ and includes v_j in $\mathbf{v}$. As there are $n-f$ values in $\mathbf{v}$, corresponding to $n-f$ RBC instances, it holds that $|\mathbf{v}| \geq n-f$. Since there are at most f faulty replicas, at least $n-2f$ values are *acs-proposed* by correct replicas. □

Theorem 6.2 (Agreement). *If a correct replica acs-decides* $\mathbf{v}$, *then every correct replicas outputs* $\mathbf{v}$.

Proof. If a correct replica p_i *acs-decides* $\mathbf{v}$, it first *mvba-decides* W_k. Then for each $W_k[j] = 1$, v_j is *r-delivered* by RBC_j and v_j is included in $\mathbf{v}$. We assume that another correct replica p_j *acs-decides* $\mathbf{v}' \neq \mathbf{v}$ and then prove the theorem by contradiction.

If p_j *acs-decides* $\mathbf{v}'$, it *mvba-decides* W'_k. According to the agreement property of MVBA, it must hold that $W_k = W'_k$. Then for each $W'_k[j] = 1$, p_j obtains the output of RBC_j v'_j and includes v'_j in $\mathbf{v}'$. If for any j, $v_j \in \mathbf{v}$ and $v'_j \in \mathbf{v}'$ and $v_j \neq v'_j$, the agreement property of RBC is violated. Hence, we have $\mathbf{v} = \mathbf{v}'$. □

Theorem 6.3 (Termination). *If all correct replicas acs-propose, then all correct replicas acs-decide.*

Proof. If all correct replicas *acs-propose*, every correct replica p_i starts RBC_i. Due to the validity property of RBC, every correct replica *r-delivers* in RBC_i. Therefore, every correct replica eventually completes $n-f$ RBC instances. Then each correct replica p_i *mvba-proposes* W_i. For each $W_i[j] = 1$, we know that p_i *r-delivers* some value in RBC_j. From the agreement property of RBC, every correct replica eventually *r-delivers* some value in RBC_j. Thus, the predicate $Q(W_i, W)$ eventually holds at every correct replica. Namely, for each $W_i[l] = 1$, each correct replica p_j eventually *r-delivers* some value in RBC_l, so $Q(W_i, W)$ eventually holds at p_j.

According to the termination property of MVBA, every correct replica eventually *mvba-decides* some value W_k. Furthermore, from the integrity property and external validity property of MVBA, $Q(W_k, W)$ eventually holds, so for each $W_k[j] = 1$, at least one correct replica *r-delivers* some value in RBC_j. Due to the agreement property of RBC, every correct replica eventually *r-delivers* some value v_j for each RBC_j. Thus, every correct replica includes each v_j in its output. □

7 A PRACTICAL ACS INSTANTIATION

We use CT RBC as the underlying RBC protocol in ACS. We use our tailored MVBA protocol in Sec. 5.4 as our MVBA protocol. Our tailored MVBA protocol internally uses the hash-based WRBC protocol in Sec. 5.4 and Pisa RABA protocol [54]. We call the resulting instantiation FIN.

8 IMPLEMENTATION AND EVALUATION

We implemented FIN in Golang. In the same library, we implemented PACE [54], the most efficient ACS construction of the same type. Our implementation involves around 9,000 LOC for the two protocols and about 1,000 LOC for evaluation. Additionally, we also assess Dumbo [34][5] and Speeding Dumbo (denoted as sDumbo) [33][6]. Note Dumbo and sDumbo rely on threshold signatures and use stronger pairing assumptions. We do not compare FIN with BFT protocols with unbounded memory (e.g., Dumbo-NG [31]) or weaker liveness properties (e.g., Bullshark [32] and Tusk [24]).[7]

In our implementation, we use gRPC as the communication library. We use HMAC to realize the authenticated channel and use SHA256 as the underlying hash function. We implement threshold PRF [12] to realize common coins for RABA and the random leader election protocol. We use a Golang-based erasure coding library[8] to implement CT RBC.

We evaluate the performance of our protocols on Amazon EC2 using up to 121 virtual machines (VMs). We use *m5.xlarge* instances for our evaluation. The m5.xlarge instance has four virtual CPUs and 16GB memory. We deploy our protocols in the WAN setting, where replicas are evenly distributed across different regions: us-west-2 (Oregon, US), us-east-2 (Ohio, US), ap-southeast-1 (Singapore), and eu-west-1 (Ireland).

We conduct the experiments under different network sizes and batch sizes. We use f to denote the network size; in each experiment, we use $n = 3f + 1$ replicas in total. We use b to denote the batch size, where each replica proposes b transactions in each epoch (i.e., one ACS instance). For each experiment, we run five epochs and report the average performance (for both throughput and latency). The default transaction size is 250 bytes. We also additionally evaluate the performance using a transaction size of 100 bytes and report the performance in Appendix B.

We summarize our main evaluation results as follows.

- We evaluate latency vs. throughput and the peak throughput varying f from 1 to 40. We demonstrate that for *both metrics*, FIN consistently and drastically outperforms PACE when $n > 16$ and all the protocols we evaluated (PACE, Dumbo, sDumbo) when $n > 22$. The performance difference between FIN and the other protocols drastically increases as n grows. For instance, when $n = 121$ and $f = 40$, the peak throughput of FIN is 3.41x that of PACE, 4.15x that of sDumbo, and 8.79x that of Dumbo.
- We also assess the latency breakdown to analyze the improvement of FIN over PACE. The experiments we carefully designed explain well why FIN outperforms PACE. In particular, we show that the RABA phase for FIN (with constant RABA instances) occupies only 1.23%-5.22% of the overall latency, in sharp contrast to PACE, where its RABA phase (with n parallel RABA instances) occupies 15.10%-83.66% of the total runtime.
- We evaluate the performance of the protocols under different failure scenarios. Our results show that FIN is highly robust against various failures.
- We additionally evaluate the performance with a smaller transaction size of 100 bytes, where we find that FIN outperforms PACE in a more significant manner. For instance, when $f = 20$, the throughput of FIN with 100-byte transactions is 2.14x that of PACE.

Latency vs. throughput. We report latency vs. throughput and peak throughput as f increases. We first show the latency vs. throughput of FIN, PACE, Dumbo, and sDumbo for $f = 1, 6, 8, 10, 20, 30, 40$ in Figure 8a-8g. Each point represents one experiment for a particular batch size. In each experiment, we begin with a small batch size and assess its latency and throughput for the batch size. We repeat the process until the throughput does not increase but the latency keeps growing (i.e., when the throughput reaches its peak—peak throughput).

For $f = 1$, the performance of FIN is lower than sDumbo, but only slightly lower than PACE; it is consistently higher than Dumbo. The reason why PACE is slightly more efficient than FIN is that FIN involves two phases of RBC (one for ACS and one inside MVBA), while the parallel RABA instances in PACE do not bottleneck the performance for such small f's. Meanwhile, the reason why FIN achieves lower performance than sDumbo is that sDumbo optimizes the failure-free case where the expensive erasure-coded recovery phase is not triggered; both PACE and FIN can use the same technique for higher failure-free performance. The drawback of using the technique is that it will cause significant performance degradation under failures and attacks.

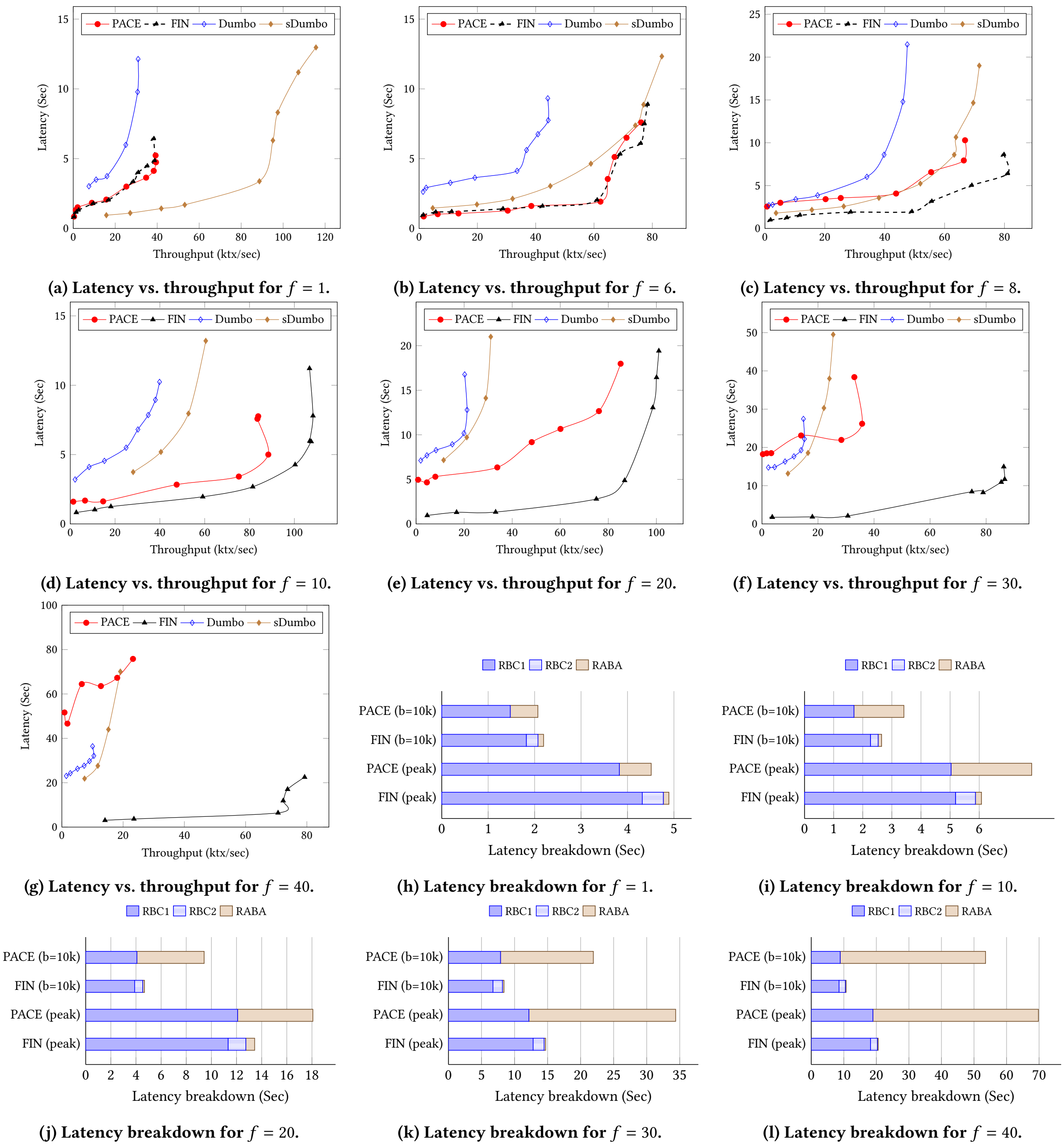
When $f = 6$, FIN outpaces sDumbo for all points except the ones where they attain their peaks; FIN starts to experience higher performance than PACE from this point on. When $f \geq 7$, FIN consistently outperforms all the other protocols we assessed in terms of latency vs. throughput.

The performance gain for FIN over PACE is clearly due to the constant-time termination and the constant number of RABA instances.

Peak throughput. We report the peak throughput of all the four protocols we evaluate in Figure 9 for $f = 1, 5, 6, 7, 8, 10, 20, 30, 40$. As shown in Figure 9, FIN outperforms Dumbo for all cases. When $f \leq 5$, FIN achieves a lower peak throughput than PACE and sDumbo. When $f = 6$, FIN starts to outperform PACE, but the peak performance of FIN is slightly lower than sDumbo. For $f \geq 8$, FIN consistently outperforms all the other three protocols in terms of peak throughput. The performance difference between FIN and the other protocols becomes increasingly significant as f increases. For instance, when $f = 30$, the peak throughput of FIN is 2.42x that of PACE, 3.40x that of sDumbo, and 6.20x that of Dumbo. When $f = 40$, the peak throughput of FIN is 3.41x that of PACE, 4.15x that of sDumbo, and 8.79x that of Dumbo.

Note that there are minor "inconsistencies" between the latency vs. throughput evaluation and the peak throughput evaluation. For example, when $f = 6$, FIN outperforms sDumbo in terms of latency vs. throughput for almost all points (experiments) but has

[5]https://github.com/yylluu/dumbo
[6]https://github.com/xygdys/Consensus
[7]Dumbo-NG, Tusk, and Bullshark are BFT protocols that separate data transmission from consensus for higher throughput, and FIN can also use the technique to improve the performance.
[8]https://github.com/klauspost/reedsolomon



(a) Latency vs. throughput for $f = 1$.
(b) Latency vs. throughput for $f = 6$.
(c) Latency vs. throughput for $f = 8$.
(d) Latency vs. throughput for $f = 10$.
(e) Latency vs. throughput for $f = 20$.
(f) Latency vs. throughput for $f = 30$.
(g) Latency vs. throughput for $f = 40$.
(h) Latency breakdown for $f = 1$.
(i) Latency breakdown for $f = 10$.
(j) Latency breakdown for $f = 20$.
(k) Latency breakdown for $f = 30$.
(l) Latency breakdown for $f = 40$.

Figure 8: Latency vs. throughput and latency breakdown.

slightly lower peak throughput. We comment that these minor inconsistencies are completely normal, and users should choose their protocols according to their priorities.

We also observe that as f increases, the peak throughput of protocols of PACE and FIN first increases and decreases. Indeed, as f increases, the number of transactions delivered for both protocols increases, but when f further increases, the network bandwidth consumption dominates the performance. In our experiments, FIN achieves the highest peak throughput when $f = 10$ and PACE

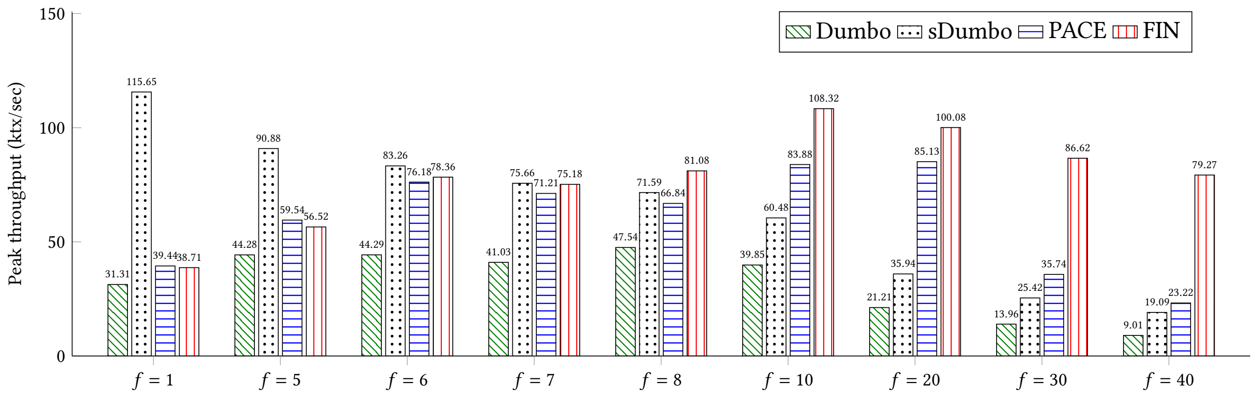
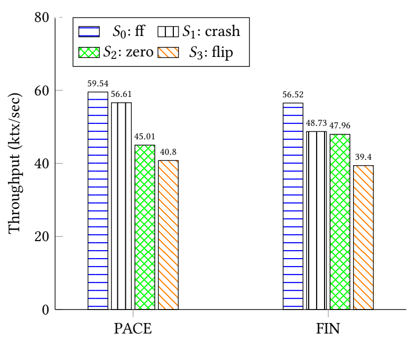
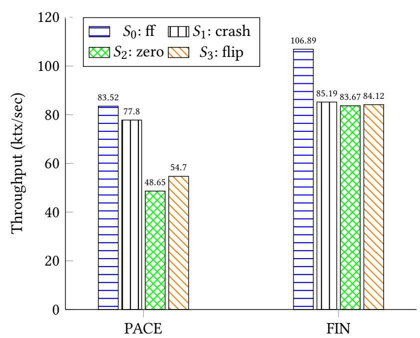


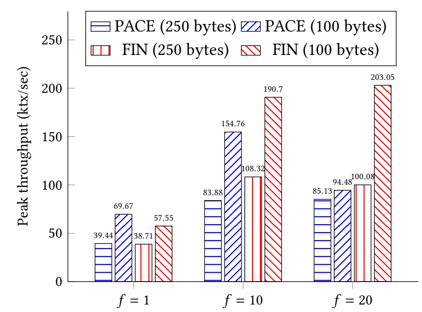
Figure 9: Peak throughput of all the four protocols as f grows.



(a) Failure scenarios for $f = 5$.



(b) Failure scenarios for $f = 10$.



(c) Performance with different transaction sizes.

Figure 10: Evaluation results of FIN and PACE under failures and using different transaction sizes.

achieves its highest peak throughput when $f = 20$. In contrast, the performance of Dumbo and sDumbo degrades as f increases.

Latency breakdown. To help understand why FIN outperforms PACE, we report the latency breakdown for the experiments. In FIN, there are three phases: the RBC phase for ACS (denoted as RBC1), the RBC phase inside MVBA (denoted as RBC2), and the iteration phase with a random leader election and one RABA instance at a time (denoted as RABA). In contrast, PACE has an RBC phase (the same as that in FIN, denoted as RBC1) and a RABA phase with n parallel RABA instances (denoted as RABA). Here the latency of the RBC phase is measured from the beginning of the first RBC instance to the completion of the $(n - f)$-th RBC instance. Moreover, the latency of the RABA phase for PACE is measured from the beginning of the first RABA instance to the time ACS completes. Finally, the latency of the RABA phase for FIN is measured from the beginning of the iteration phase to the time when ACS completes. The latency breakdown experiments can explain why FIN outperforms PACE, help identify the bottleneck of the two protocols, and assist in understanding the scalability results.

Figure 8h-8l report the latency breakdown for FIN and PACE. We test two settings: a fixed one with $b = 10{,}000$, and the smallest batch size where both protocols achieve their peak throughput. We first observe that the RBC1 phases in the two protocols share almost the same latency. This is not surprising, as the RBC1 phase is the only phase that carries bulk data for both protocols, and we use CT RBC for both of them. Additionally, the latency percentage for the RBC2 phase in FIN is comparatively very low. This is because the RBC2 phase does not have any bulk data, and its input size is small (n bits). Hence, the RBC2 phase in FIN does not incur much overhead to the protocol.

In all cases, the RABA latency in PACE is much higher than that for FIN. For f =1 to 40, the latency of the RABA phase in PACE is 15.10%-83.66% of the overall latency. In contrast, the RABA phase in FIN occupies only 1.23%-5.22% of the total runtime.

Moreover, the latency percentage of the RABA phase within the overall PACE latency becomes increasingly larger as f increases, but the RABA latency percentage in FIN remains steady despite an increasing f. Indeed, when $f = 40$, the RABA phase occupies 83.66% of the overall consensus latency for PACE. In contrast, the latency of the RABA phase in FIN is only 1.23% of the overall latency. In fact, even if we consider the latency caused by RBC2 and RABA (i.e., MVBA) in FIN, it only occupies 11.45% of the overall latency. This observation explains well why the performance difference between FIN and PACE becomes increasingly larger as f increases. Indeed, FIN only needs an expected constant number of RABA instances.

Performance under failures. To assess the robustness of FIN, we report the performance of FIN and PACE under various failure scenarios. Following prior works [54, 56], we consider the following scenarios, where the f faulty replicas are evenly distributed in the EC2 regions we use.

- S_0**: (failure-free)** In this scenario, all replicas are correct.
- S_1**: (crash)** In this scenario, we let f replicas crash by not participating in the protocols.
- S_2**: (Byzantine; keep voting 0)** In this scenario, we fail f replicas and let them keep voting for 0 in each step of RABA.
- S_3**: (Byzantine; flipping the RABA input)** In this scenario, we fail f replicas and ask them to always vote for a flipped value in RABA.

We fix b to 30,000 and present the throughput for $f = 5$ in Figure 10a and $f = 10$ in Figure 10b. We choose these two settings because they are the settings where the two protocols share similar performance. First, for the crash failure scenario, the performance of both protocols degrades only slightly. For both Byzantine scenarios, the percentage for the performance degradation of FIN is lower than that of PACE. The performance of FIN in Byzantine scenarios degrades by 15.1%-30.2% compared to the failure-free scenario. In contrast, the performance of PACE degrades by 24.39%-41.74%. Notably, for $f = 10$, FIN under all failure scenarios outperforms PACE in its failure-free scenario.

Performance with different transaction sizes. We additionally evaluate the performance of FIN and PACE with a smaller transaction size of 100 bytes as shown in Figure 10c. In this setting, FIN outperforms PACE more significantly. For instance, when $f = 20$, the throughput of FIN with 100-byte transactions is 2.14x that of PACE, while for 250-byte transactions, the throughput of FIN is only 17.56% higher. We also find that the throughput of FIN with 100-byte transactions is roughly 2x that with 250-byte transactions.

9 CONCLUSION

We present the first signature-free ACS protocol with $O(1)$ time and $O(n^3)$ messages, resolving a long-standing open problem in fault-tolerant distributed computing and cryptography. As a core ingredient in our ACS construction and a primitive of independent interests, we present the first signature-free MVBA protocols with $O(1)$ time and $O(n^3)$ messages. In contrast, existing signature-free MVBA protocols have $O(\log n)$ time and $O(n^3)$ messages. From the practical side, we implement a practical ACS protocol called FIN. We demonstrate that FIN significantly outperforms the state-of-the-art BFT protocol of the same kind—PACE and outperforms other BFT protocols with standard liveness guarantees.

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A ACRONYMS

We explain the acronyms used in this work in Table 3.

BFT	Byzantine fault tolerance
ACS	asynchronous common subset
MPC	multi-party computation
MVBA	multivalued validated Byzantine agreement
DAG	direct acyclic graph
RBC	reliable broadcast
WRBC	weak reliable broadcast
ABA	asynchronous binary agreement
RABA	reproposable asynchronous binary agreement

Table 3: Acronyms

B ADDITIONAL EVALUATION RESULTS

In Figure 11, we present the throughput vs. latency results for the transaction size of 100 bytes for $f = 1, 10, 20$. While the trend in this setting is similar to that with the transaction size of 250 bytes, FIN outperforms PACE in a more drastic way. Despite the case for $f = 1$ where the peak throughput of FIN is 17.4% lower than that of PACE, FIN achieves 1.23x and 2.14x the peak throughput of PACE, for $f = 10$ and $f = 20$, respectively.

C PROOF OF OUR MVBA PROTOCOL WITH QUALITY

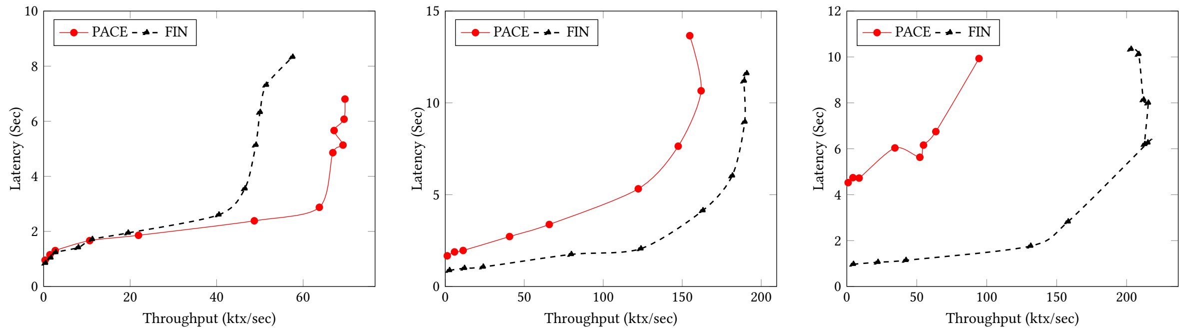
We show that the MVBA protocol presented in Sec. 5.3 additionally achieves the quality property. All the other properties except termination follow from our MVBA without the quality property. Therefore, we prove quality and termination in this section.

Lemma C.1. *If a correct replica enters the iteration phase, then for at least $2f + 1$ RBC instances, at least $f + 1$ correct replicas have r-delivered some values.*

Proof. If a correct replica p_i enters the iteration phase, it receives $n - f$ (Rep) message. Prior to that, p_i has broadcast (Echo, W_i) where W_i consists of at least $n-f$ 1's, i.e., p_i has completed $n-f$ RBC instances. Each replica p_j replies with a (Rep) message only if for any $W_i[l] = 1$, p_j has also *r-delivered* some value in RBC$_l$. Hence, for any $W_i[l] = 1$, at least $f + 1$ correct replicas have *r-delivered* some value. □

Theorem C.2 (Quality). *The probability of mvba-deciding a value that was proposed by a correct replica is at least 1/2.*

Proof. According to Lemma C.1, for at least $2f+1$ RBC instances, at least $f + 1$ correct replicas have *r-delivered* some value. From the biased validity property of RABA, if any of the $2f + 1$ RBC instances is selected by the Election() function, RABA will output 1. So every correct replica then *mvba-decides*. Therefore, the probability that the decided value was proposed by an adversary is bounded by $\frac{f+1}{3f+1}$. As the probability of deciding a value proposed by a faulty replica is at most $\Sigma_{k=1}^{\infty}(1/3)^k = 1/2$, the probability of deciding a value that was proposed by a correct replica is at least $1/2$. □



(a) Latency vs. throughput for $f = 1$. (b) Latency vs. throughput for $f = 10$. (c) Latency vs. throughput for $f = 20$.

Figure 11: Evaluation results for transaction size of 100 bytes.

Theorem C.3 (Termination). *If all correct replicas are activated and all messages sent among correct replicas have been delivered, then all correct replicas mvba-decide.*

Proof. If all correct replicas are activated, each correct replica starts one RBC instance. According to the validity property of RBC, at least $n-f$ RBC instances started by the $n-f$ correct replicas will eventually complete. Then, each correct replica p_i sends a (Echo, W_i) message, every replica replies only if it has *r-delivered* some value in RBC_l for any $W_i[l] = 1$. According to the agreement property of RBC, we know that for the message sent by p_i, every correct replica eventually replies with a (Rep) message. Accordingly, every correct replica eventually enters the iteration phase.

During the iteration phase, we first prove that every iteration r completes and then show that eventually some RABA_r outputs 1. For each iteration r, k is returned by the Election() function.

The proof that every iteration r completes is similar to that for the protocol without quality. We include one additional echo-and-reply procedure, where every replica sends its W_i to all replicas and proceeds to the next phase if it receives $n - f$ replies. According to the agreement property of RBC, we know that every correct replicas eventually *r-delivers* some values in the same set of RBC instances in W_i. Thus, every correct replica completes each epoch.

We now prove that eventually, in some iteration r, RABA_r outputs 1, so the protocol terminates. From Lemma C.1, for at least $2f + 1$ RBC instances, at least $f + 1$ correct replicas have *r-delivered* some value after they enter the iteration phase. Let I be the set of the $2f + 1$ RBC instances. Due to the biased validity property of RABA, RABA_r outputs 1. So with probability $\frac{2f+1}{3f+1} \approx \frac{2}{3}$, it holds that $k \in I$.

After RABA_r outputs 1, every correct replica waits for the output of RBC_k. Note that if RABA_r outputs 1, at least one correct replica has *raba-proposeed* 1 or *raba-reproposed* 1. This is due to the unanimous termination property of RABA. Therefore, at least one correct replica has *r-delivered* some value in RBC_k. Due to the agreement property of RBC, every correct replica eventually *r-delivers* some value in RBC_k and then *mvba-decides.* □

D PROOF OF OUR TAILORED MVBA

In this section, we prove the correctness of our tailored MVBA protocol. We first show a few lemmas about WRBC and then show the correctness of our tailored MVBA construction. As external validity is the same as the MVBA protocol presented in Sec. 5.2, we focus on agreement, integrity, and termination in this section.

Lemma D.1. *If a correct replica wr-delivers h and another correct replica wr-delivers h', then $h = h'$.*

Proof. If a correct replica p_i *wr-delivers* h, it receives $n - f$ (Ready, h). If another correct replica p_j *wr-delivers* h', it receives $n - f$ (Ready, h'). Therefore, at least one correct replica has sent both (Ready, h) and (Ready, h'), a contradiction to the fact that every correct replica only sends a (Ready) message once. □

Lemma D.2. *If a correct replica wr-broadcasts a value v, every correct replica eventually wr-delivers h such that $h = Hash(v)$.*

Proof. For any WRBC instance, if the sender is correct, it is straightforward to see that every correct replica receives the same (Send, v) message, broadcasts a (Echo, m)essage, and receives $n - f$ (Echo, $Hash(v)$) messages. Then every correct replica eventually sends (Ready, $Hash(v)$) and will never receive $f+1$ (Ready) messages with a value different from $Hash(v)$. Hence, every correct replica eventually *wr-delivers* $h = Hash(v)$. □

Lemma D.3. *For any WRBC instance, if a correct replica wr-delivers some value h, any correct replica eventually wr-delivers some value.*

Proof. If a correct replica *wr-delivers* h, it receives $n-f$ (Ready, h) messages, among which are least $f + 1$ are sent by correct replicas. Thus, any correct replica that receives $f + 1$ (Ready, h) messages but has not sent any (Ready) message will also send a (Ready, h). Therefore, every correct replica eventually receives $n - f$ (Ready, h) messages and *wr-delivers.* □

Lemma D.4. *If a correct replica wr-delivers some value h in $WRBC_j$, at least $f + 1$ correct replica receives (Send, v) such that $Hash(v)$.*

Proof. If a correct replica *wr-delivers* h, it receives $n-f$ (Ready, h), among which are least $f + 1$ are sent by correct replicas. Any of the correct replicas receive $n - f$ (Echo, h), among which at least

$f+1$ are sent by correct replicas. The correct replicas must have received (SEND, v) from p_j such that $Hash(v) = h$. □

For completeness, below, we provide self-contained proof for agreement, integrity, and termination. Note that the proof is very similar to that for our MVBA protocol in Sec. 5.2.

THEOREM D.5 (AGREEMENT). *If a correct replica mvba-decides v, then any correct replica that terminates mvba-decides v.*

PROOF. If a correct replica p_i *mvba-decides* v, we assume it has *raba-decided* 1 in RABA$_r$ for some iteration $r > 0$ and for any iteration $\hat{r} < r$, RABA$_{\hat{r}}$ outputs 0. Furthermore, if k is returned by the Election() function in iteration r, WRBC$_k$ outputs h and p_i receives some v (from p_k or from a (VALUE) message) such that $H(v) = h$.

We assume that another correct replica p_j *mvba-decides* $v' \neq v$ and prove the theorem by contradiction. We consider two cases: p_j *mvba-decides* in iteration r; p_j *mvba-decides* in iteration $r' \neq r$.

Case 1: If p_j *mvba-decides* v' in round r, it obtains k' from Election() function and WRBC$_{k'}$ outputs h'. Then p_j either receives value v' from $p_{k'}$ such that $Hash(v') = h'$ or receives v' from another replica in a (VALUE) message. As Election() outputs a common coin, it must hold that $k = k'$. Thus, if $Hash(v) \neq h'$, p_j *wr-delivers* $h' \neq h$, a violation of Lemma D.1. Furthermore, if $v \neq v'$, the collision resistance property of the hash function is violated.

Case 2: Without loss of generality, we assume $r' > r$. According to our protocol, p_j *raba-decides* 1 in RABA$_{r'}$ and *raba-decides* 0 for any iteration lower than r', including r. This would violate the agreement property of RABA. □

THEOREM D.6 (INTEGRITY). *If all replicas follow the protocol, and if a correct replica mvba-decides such that $Q(v)$ holds, then some replica mvba-proposed such that $Q(v)$ holds.*

PROOF. If a correct replica p_i *mvba-decides* v, it *raba-decides* 1 in some RABA$_r$ and *wr-delivers* $h = H(v)$ in WRBC$_k$ where k is the output of the Election() function. According to our protocol, at least $n - f$ replicas have sent (READY, h) messages, among which at least one is sent by a correct replica. The correct replica has received $n - f$ (ECHO, h) messages, where at least $f + 1$ message are sent by correct replicas. The correct replicas must have received a (SEND, v) message from p_i such that $Hash(v) = h$. □

LEMMA D.7. *If all correct replica enter the iteration phase, then for at least $f + 1$ WRBC instances, at least $f + 1$ correct replicas have wr-delivered some values.*

PROOF. Correctness of the lemma is the same as that in Lemma 5.4, except that RBC is now WRBC. □

THEOREM D.8 (TERMINATION). *If all correct replicas are activated and all messages sent among correct replicas have been delivered, then all correct replicas mvba-decide.*

PROOF. If all correct replicas start the protocol, each correct replica starts one WRBC instance. According to Lemma D.2, any correct replica completes a WRBC instance started by a correct replica. Thus, every correct replica completes at least $n - f$ WRBC instances.

During the iteration phase, we consider each iteration r where k is the corresponding output of the Election() function. We first show that every iteration r completes and then eventually some RABA$_r$ outputs 1.

We first show that every iteration r completes. For each iteration r, there are three cases: 1) all correct replicas have *wr-delivered* some value in WRBC$_k$; 2) at least one correct replica has *wr-delivered* some value in WRBC$_k$ and at least one correct replica has not *wr-delivered* any value in WRBC$_k$; 3) none of the correct replicas have *r-delivered* any value in WRBC$_k$.

Case 1: According to the unanimous termination property, RABA$_r$ terminates.

Case 2: If at least one correct replica has *wr-delivered* some value in WRBC$_k$, then according to Lemma D.3, any correct replica eventually *wr-delivers* some value. Note that any correct replica that provides 0 as the RABA input (it has not *wr-delivered* any value in WRBC$_k$ when the iteration begins) will eventually *raba-repropose* 1. Thus, the biased termination condition of RABA is satisfied. RABA$_r$ will terminate and iteration r will complete.

Case 3: If none of the correct replicas *wr-deliver* any value in WRBC$_k$, iteration r completes due to the unanimous termination property of RABA. Otherwise, if at least one correct replica later *wr-delivers* some value in WRBC$_k$, then according to case 2, iteration r completes due to the biased termination property of RABA.

We now prove that eventually, in some iteration r, RABA$_r$ outputs 1 so the protocol terminates. From Lemma D.7, for at least $f + 1$ WRBC instances, at least $f + 1$ correct replicas have *wr-delivered* some value after they enter the iteration phase. Let I be the $f+1$ WRBC instances. According to the biased validity property of RABA, RABA$_r$ outputs 1. So with probability $\frac{f+1}{3f+1} \approx \frac{1}{3}$, we have $k \in I$.

After RABA$_r$ outputs 1, every correct replica waits for the output of WRBC$_k$. We also know that if RABA$_r$ outputs 1, then at least one correct replica has *raba-proposed* 1 or *raba-reproposed* 1. Otherwise, the unanimous termination of RABA is violated. Therefore, at least one correct replica has *wr-delivered* some value h in WRBC$_k$. From Lemma D.3 and Lemma D.1, every correct replica eventually *r-delivers* h in WRBC$_k$. From Lemma D.4, at least $f+1$ correct replicas receive (SEND, v) from p_i and set their $T_i[k]$ as v such that $Hash(v) = h$. The correct replicas will send (VALUE, v) to all replicas. Therefore, any correct eventually receives v and then *mvba-decides.* □

E PROOF OF MVBA WITH FEWER ROUNDS

In this section, we prove the correctness of our MVBA protocol presented in Sec. 5.5. The protocol largely follows the tailored MVBA. We thus reuse some proofs in Sec. D in this section.

THEOREM E.1 (AGREEMENT). *If a correct replica mvba-decides v, then any correct replica that terminates mvba-decides v.*

PROOF. Proof for this theorem is the same as that for Theorem D.5. □

THEOREM E.2 (INTEGRITY). *If all replicas follow the protocol, and if a correct replica mvba-decides such that $Q(v)$ holds, then some replica mvba-proposed such that $Q(v)$ holds.*

Proof. Proof for this theorem is the same as that for Theorem D.6. □

Lemma E.3. *If the adversary makes $RABA_0$ output 0, at least $f+1$ correct replicas have entered the iteration phase.*

Proof. To make $RABA_0$ output 0, at least $f+1$ correct replica must *raba-propose* 0 in $RABA_0$, as otherwise the biased validity property of RABA is violated. The lemma thus holds. □

Lemma E.4. *If at least $f+1$ correct replicas enter the iteration phase, there exist at least $f+1$ WRBC instances I, for any $j \in I$, at least $f+1$ correct replicas have completed $WRBC_j$.*

Proof. Let the identities of the $f+1$ correct replicas be I. Any of the correct replicas p_i enters the iteration phase after it receives $n-f$ (Rep) message. As every correct replica p_j sends a (Rep, j) message to p_i after p_j has *wr-delivered* some value in WRBC_i, at least $f+1$ correct replicas have completed WRBC_i. Therefore, for any $j \in I$, at least $f+1$ correct replicas have completed WRBC_j, after p_j enters the iteration phase. □

Theorem E.5 (Termination). *If all correct replicas are activated and all messages sent among correct replicas have been delivered, then all correct replicas mvba-decide.*

Proof. If all correct replicas start the protocol, each correct replica starts one WRBC instance. According to Lemma D.2, any correct replica completes a WRBC instance started by a correct replica. Thus, every correct replica completes at least $n-f$ WRBC instances. For each WRBC instance WRBC_i started by a correct replica p_i, all correct replicas will send a (Rep) message to p_i, so every correct replica eventually enters the iteration phase.

During the iteration phase, we consider each iteration r where k is the corresponding output of the $\mathsf{Election}()$ function. We first show that every iteration r completes and then eventually some $RABA_r$ outputs 1.

We first show that every iteration r completes. Consider the status of correct replicas when they enter the iteration, there are three cases: 1) all correct replicas have *wr-delivered* some value in WRBC_k; 2) at least one correct replica has *wr-delivered* some value in WRBC_k and at least one correct replica has not *wr-delivered* any value in WRBC_k; 3) none of the correct replicas have *r-delivered* any value in WRBC_k.

Case 1: Every correct replica *raba-proposes* 1 for $RABA_r$. According to the unanimous termination property, $RABA_r$ terminates.

Case 2: If at least one correct replica has *wr-delivered* some value in WRBC_k, then according to Lemma D.3, any correct replica eventually *wr-delivers* some value. Note that any correct replica that provides 0 as the RABA input (it has not *wr-delivered* any value in WRBC_k when the iteration begins) will eventually *raba-repropose* 1. Thus, the biased termination condition of RABA is satisfied. $RABA_r$ will terminate and iteration r will complete.

Case 3: If none of the correct replicas *wr-deliver* any value in WRBC_k, If none of the correct replicas *wr-deliver* any value in WRBC_k, iteration r completes due to the unanimous termination property of RABA. Otherwise, if at least one correct replica later *wr-delivers* some value in WRBC_k, then according to case 2, iteration r completes due to the biased termination property of RABA.

We now prove that eventually, in some iteration r, $RABA_r$ outputs 1 so the protocol terminates. From Lemma E.3, we know that at least $f+1$ correct replicas have entered the iteration phase if the adversary makes $RABA_0$ output 0. From Lemma E.4, after $f+1$ correct replicas enter the iteration phase, there exist at least $f+1$ WRBC instances I such that for each $j \in I$, at least $f+1$ correct replicas have completed WRBC_j. According to the biased validity property of RABA, $RABA_r$ outputs 1. So with probability $\frac{f+1}{3f+1} \approx \frac{1}{3}$, we have $k \in I$.

After $RABA_r$ outputs 1, every correct replica waits for the output of WRBC_k. We also know that if $RABA_r$ outputs 1, then at least one correct replica has *raba-proposed* 1 or *raba-reproposed* 1. Otherwise, the unanimous termination of RABA is violated. Therefore, at least one correct replica has *wr-delivered* some value h in WRBC_k. From Lemma D.3 and Lemma D.1, every correct replica eventually *r-delivers* h in WRBC_k. From Lemma D.4, at least $f+1$ correct replicas receive (Send, v) from p_i and set their $T_i[k]$ as v such that $Hash(v) = h$. The correct replicas will send (Value, v) to all replicas. Therefore, any correct eventually receives v and then *mvba-decides*. □

Theorem E.6. *Our MVBA protocol (with fewer rounds) has expected $O(1)$ running time.*

Proof. From Lemma E.3, we know that at least $f+1$ correct replicas have entered the iteration phase if the adversary makes $RABA_0$ output 0. From Lemma E.4, after $f+1$ correct replicas enter the iteration phase, there exist at least $f+1$ WRBC instances I such that for each $j \in I$, at least $f+1$ correct replicas have completed WRBC_j. If the output of the $\mathsf{Election}()$ function $k \in I$, RABA will output 1 according to the biased validity property of RABA. Since $\mathsf{Election}()$ outputs a uniformly distributed random coin for each iteration, it holds that with probability $\frac{f+1}{3f+1} \approx \frac{1}{3}$, we have $k \in I$. Therefore, the protocol has expected $O(1)$ running time. □