

Related-Key Impossible-Differential Attack on Reduced-Round SKINNY

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Abstract. At CRYPTO’16, Beierle et al. presented SKINNY, a family of lightweight tweakable block ciphers intended to compete with SIMON. SKINNY can be implemented efficiently in both soft- and hardware, possesses a Substitution-Permutation-Network structure, and supports block sizes of 64 and 128 bits as well as key and tweak sizes of 64, 128, 192, and 256 bits. This paper outlines a related-tweakey impossible-differential attack on 21 rounds of SKINNY-64/128 and two attacks on 22 and 23 rounds of SKINNY-64/128 under the assumption that 48 bits of the tweakey are public.

Keywords: Symmetric cryptography · cryptanalysis · tweakable block cipher · impossible differential · lightweight cryptography.

1 Introduction

SKINNY is a family of lightweight tweakable block ciphers recently proposed at CRYPTO 2016 by Beierle et al. [3]. Its goal was to design a cipher that could be implemented highly efficiently on both soft- and hardware platforms, with performance comparable or better than the SIMON and SPECK families of block ciphers [1]. Like the NSA designs SIMON and SPECK, SKINNY supports a wide range of block sizes and tweak/key sizes – however, in contrast to the And-RX and Add-RX based NSA proposals, SKINNY should base on the better understood Substitution-Permutation-Network approach.

SKINNY offers a large security margin within the number of rounds for each member of the SKINNY family. The designers show that the currently best known attacks approach close to half of the number of rounds of the cipher. To motivate third-party cryptanalysis, the designers of SKINNY recently announced a cryptanalysis competition [2] for SKINNY-64/128 and SKINNY-128/128 with the obvious challenge of attacking more rounds than the preliminary analysis, concerning both the single- and related-key models.

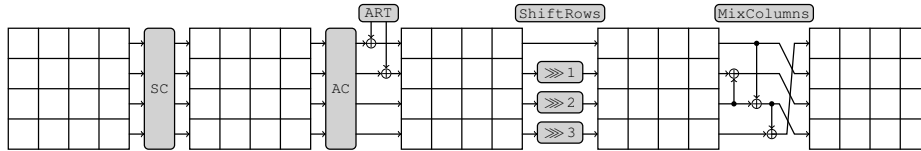


Fig. 1: Round function of SKINNY.

Related Work. Liu *et al.* [7] analyzed SKINNY in the related-tweakey model, showing impossible-differential and rectangle attacks on 18, 22, and 27 rounds of SKINNY- n/n , SKINNY- $n/2n$ and SKINNY- $n/3n$, respectively. Tolba *et al.* [9] showed impossible-differential attacks for 18, 20, 22 rounds of SKINNY- n/n , SKINNY- $n/2n$ and SKINNY- $n/3n$, respectively. Moreover, Sadeghi *et al.* [8] studied related-tweakey impossible-differential and zero-correlation linear characteristics. In comparison our proposed 22-round related-tweakey impossible-differential attack has the lowest time complexity so far.

Contributions and Outline. In this paper, we propose an impossible-differential attack on SKINNY-64/128 reduced to 21 rounds in the related-key model which we then extend to 22 rounds. The attack uses an 11-round impossible differential trail, to which six and four rounds can be added to the beginning and end, respectively, for obtaining a 21-round attack. Later, we show that another round can be appended in the end to give a 22-round attack, and even a 23-round attack.

The paper is organized in the following manner: In Section 2, we give a brief introduction to the SKINNY family of block ciphers. In Section 3, we detail the attack on SKINNY and provide time and memory complexities. Finally, Section 5 concludes the paper.

2 Description of SKINNY

Each round of SKINNY consists of the operations SUBCELLS, ADDROUNDCONSTANTS, ADDROUNDTWEAKEY, SHIFTRows, and MIXCOLUMNS. The round operations are schematically illustrated in Figure 1. A cell represents a 4-bit value in SKINNY-64/* and an 8-bit value in SKINNY-128/*.

We concentrate on SKINNY-64/128, which has a block size of 64 bits and a tweakey size of 128 bits. The data is arranged nibble-by-nibble in a row-wise fashion in a 4×4 -matrix.

SUBCELLS (SC) substitutes each nibble x by $S(x)$, which is given below.

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$S(x)$	c	6	9	0	1	a	2	b	3	8	5	d	4	e	7	f

ADDROUNDCONSTANTS (AC) adds LFSR-based round constants to Cells 0, 4, and 8 of the state.

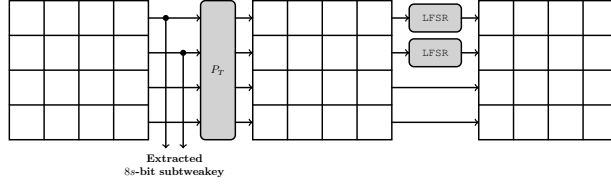


Fig. 2: Tweakkey schedule of SKINNY.

ADDRoundTweakey (ART) adds the round tweakkey to the first two state rows.

SHIFTRows (SR) rotates the i^{th} row, for $0 \leq i \leq 3$, by i positions to the right.

MIXColumns (MC) multiplies each column of the state by a matrix M :

$$M = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

SKINNY-64/128 recommends 36 rounds.

Tweakkey Schedule. The tweakkey schedule of SKINNY, as illustrated in Figure 2, follows the TWEAKEY framework [5]. As a major contrast to previous TWEAKEY designs DEOXYs-BC and JOLTIK-BC, SKINNY employs a significantly more lightweight strategy. In each round, only the two topmost rows of each tweakkey word are extracted and XORed to the state. An additional round-dependent constant is also XORed to the state to prevent attacks from symmetry, such as slide attacks, and complicate subspace cryptanalysis.

The 128-bit tweakkey is arranged in two 64-bit tweakkey words, represented by 4×4 matrices TK_1 and TK_2 . As mentioned, the arrangement is row-wise and nibble-by-nibble. In each round, the tweakkey words are updated by a cell permutation P_T that ensures that the two bottom rows of a tweakkey word in a certain round are exchanged with the two top rows in the tweakkey word in the subsequent round. The permutation is given as:

$$P_T = \{9, 15, 8, 13, 10, 14, 12, 11, 0, 1, 2, 3, 4, 5, 6, 7\}$$

The permutation P_T has a period of 16, as visualized in Fig. 7 in the appendix. Moreover, each individual cell in the two topmost rows of the tweakkey word TK_2 is transformed by a 4-bit LFSR to thwart iterative differentials; TK_1 employs no LFSR transformation. The LFSR based transformation L is given by

$$L(x_3, x_2, x_1, x_0) := (x_2, x_1, x_0, x_3 \oplus x_2),$$

where x_3, x_2, x_1, x_0 represent the individual bits (x_0 represents the LSB of the cell) of every tweakable nibble. To avoid confusion, the update equation for the tweak cells can be written explicitly as:

$$TK_1^{r+1}[i] = \begin{cases} TK_1^r[P[i]] & \text{for } 0 \leq i \leq 15, \\ L(TK_2^r[P[i]]) & \text{if } 0 \leq i \leq 7, \\ TK_2^r[P[i]] & \text{otherwise.} \end{cases}$$

where $TK_a^r[i]$ represents the i^{th} nibble of TK_a ($a = 1, 2$) in round r . Note that the r^{th} -round tweakable key is given by $K^r = TK_1^r[i] \oplus TK_2^r[i]$, for $0 \leq i \leq 7$.

3 Related-Key Impossible-Differential Attack

Impossible-differential attacks were introduced independently by Biham *et al.* [4] and Knudsen [6]. They are widely used as an important cryptanalytic technique. The attack starts with finding an input difference that can never result in an output difference, which makes up an impossible differential. By adding rounds before and/or after the impossible differential, one can collect pairs with certain plaintext and ciphertext differences. If there exists a pair that meets the input and output values of the impossible differential under some subkey, these subkeys must be wrong. In this way, we can filter as many wrong keys as possible and exhaustively search the rest of the keys.

Notations. Before proceeding, let us state a few notations that we will use in the attack description:

K^r represents the r^{th} round key. This is equal to $TK_1^r \oplus TK_2^r$, the first and second tweakable blocks. Similarly, $k^r[i] = tk_1^r[i] \oplus tk_2^r[i]$ represents the individual i^{th} tweakable nibble in round r .

A^r represents the internal state before SC in round r , and $A^r[i]$ represents the i^{th} nibble of A^r .

B^r represents the internal state after SC in round r , and $B^r[i]$ represents the i^{th} nibble of B^r .

C^r represents the internal state after AT in round r , and $C^r[i]$ represents the i^{th} nibble of C^r .

D^r represents the internal state after SR in round r , and $D^r[i]$ represents the i^{th} nibble of D^r .

E^r represents the internal state after MC in round r , and $E^r[i]$ represents the i^{th} nibble of E^r . Incidentally, we have $E^r = A^{r+1}$.

L^t represents the t -times composition of LFSR function L .

\overline{X} represents the corresponding variable X under the related-key encryption flow.

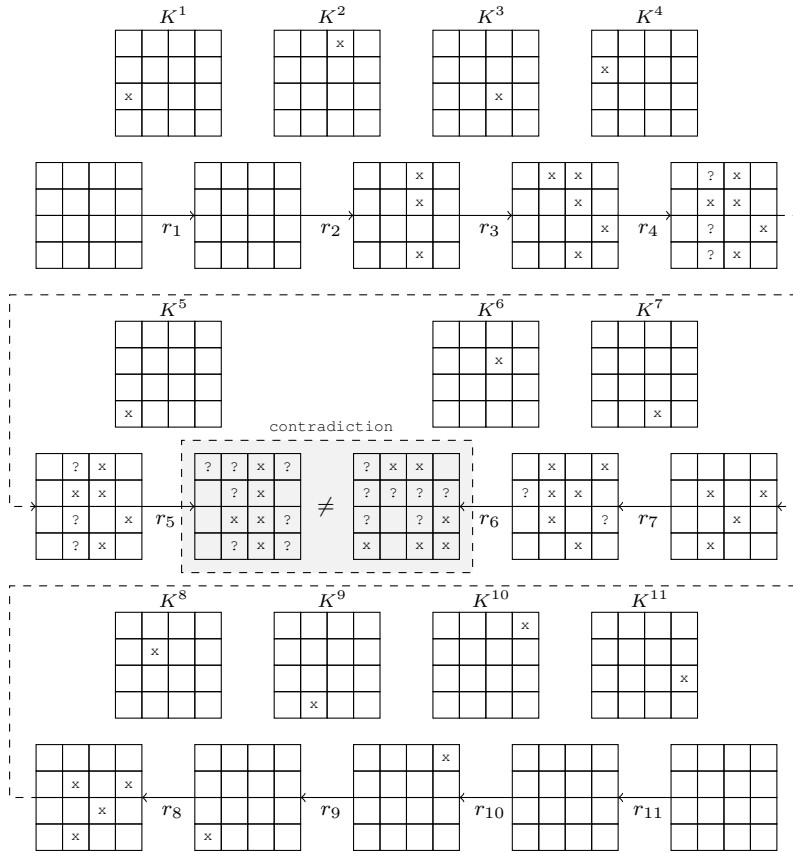


Fig. 3: Related-key impossible-differential trail over 11 rounds of SKINNY-64/128.

Impossible-Differential Trail. Fig. 3 presents the 11-round related-key differential trail that we use in this paper. We introduce a nibble difference in Cell 8 of the combined tweakey. Since the initial difference is in Cell 8, *i.e.* in one of the bottom two rows in the tweakey, it does not affect the state in the first round, and will be introduced in the state from the second round onwards. Similarly in the backward trail, the difference in the 11th round tweakey appears in Cell 11 (also situated in one of the bottom two rows), due to which we get an extra round in the backward direction too.

Lemma 1. *The equation $S(x + \Delta_i) + S(x) = \Delta_o$ has one solution x on average for $\Delta_i, \Delta_o \neq 0$. Similar result holds for the inverse S-Box S^{-1} .*

Proof. The above fact can be deduced by analyzing the Differential-Distribution Table (DDT) of the S-box S as illustrated in Table 1 in the appendix. The

average can be calculated as $\frac{1}{225} \cdot \sum_{\Delta_i, \Delta_o \neq 0} DDT(\Delta_i, \Delta_o) \approx 1$. A similar exercise can be done for the inverse S-box yielding the same result.

Lemma 2. *For random values of x and $\Delta_i, \Delta_o \neq 0$, the equation $S(x + \Delta_i) + S(x) = \Delta_o$ holds with probability around 2^{-4} .*

Proof. The above fact can also be deduced by analyzing the Differential-Distribution Table (DDT) of the S-box S as illustrated in Table 1 in the appendix. The probability can be calculated as (let $\Pr[(x, \delta_i, \delta_o)]$ denote the probability that the equation is satisfied for the triplet x, δ_i, δ_o)

$$\begin{aligned} \Pr[(x, \Delta_i, \Delta_o)] &= \sum_{\delta_i, \delta_o \neq 0} \Pr[(x, \delta_i, \delta_o) | \Delta_i = \delta_i, \Delta_o = \delta_o] \Pr[\Delta_i = \delta_i, \Delta_o = \delta_o] \\ &= \frac{1}{225} \cdot \sum_{\Delta_i, \Delta_o \neq 0} DDT(\Delta_i, \Delta_o) \cdot 2^{-4} \approx 2^{-4} \end{aligned}$$

Attack on 21 Rounds. The impossible differential trail described in Fig. 3 can be extended by six and four rounds in backward and forward direction as will be explained in the following two lemmas.

Lemma 3. *It is possible to find plaintext pairs P, \bar{P} and related-tweakey pairs K, \bar{K} such that if the tweakey pairs differ only in nibble position 11, then there is no difference in the internal state after executing six rounds of SKINNY-64/128 with the plaintext-tweakey pairs (P, K) and (\bar{P}, \bar{K}) .*

Proof. We will proceed to demonstrate how the required plaintext and tweakey pairs are generated. We choose the nibble at Position 11 to introduce the initial difference because after completing six rounds, the difference is shuffled to Cell 8 of the round key, which coincides with the beginning of the impossible-differential trail, shown in Fig. 3. To begin, it can be seen that the ADDROUNDTWEAKEY in the first round can be pushed behind the MIXCOLUMNS operation by changing the first round key to $\text{Lin}(K_1)$ where $\text{Lin} = \text{MC} \circ \text{SR}$ represents the linear layer (please refer to Fig. 4).

$$\text{Lin}(K^1) = \begin{bmatrix} k^1[0] & k^1[1] & k^1[2] & k^1[3] \\ k^1[0] & k^1[1] & k^1[2] & k^1[3] \\ k^1[7] & k^1[4] & k^1[5] & k^1[6] \\ k^1[0] & k^1[1] & k^1[2] & k^1[3] \end{bmatrix}$$

Furthermore, the initial difference between $K = TK_1^1 + TK_2^1$ and $\bar{K} = \overline{TK_1^1} + \overline{TK_2^1}$ can be selected in a specific form, so that in Round 6, the tweakey difference is zero. Let us denote $\delta_1 = tk_1^1[11] + \overline{tk_1^1}[11]$ and $\delta_2 = tk_2^1[11] + \overline{tk_2^1}[11]$. In Round 6, the difference will appear in Cell 0 of the round key and so we want:

$$\begin{aligned} k^6[0] + \overline{k^6}[0] &= tk_1^6[0] + \overline{tk_1^6}[0] + tk_2^6[0] + \overline{tk_2^6}[0] \\ &= tk_1^1[11] + \overline{tk_1^1}[11] + L^3(tk_2^1[11]) + L^3(\overline{tk_2^1}[11]) \\ &= \delta_1 + L^3(\delta_2) = 0 \end{aligned}$$

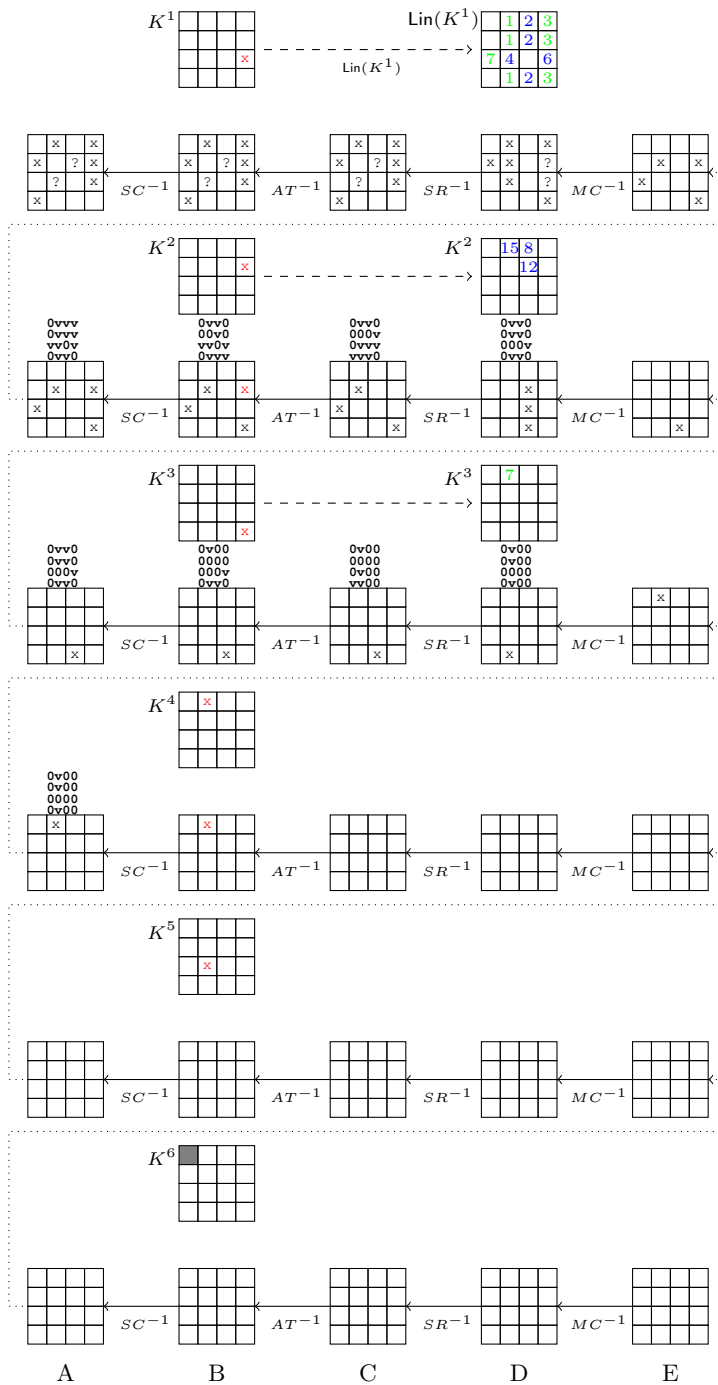


Fig. 4: Trail for the six forward rounds (the values of active nibbles in red are functions of δ_1, δ_2 , the dark gray cell visualises the tweakey cancellation).

So, if the attacker chooses δ_1, δ_2 satisfying the equation $\delta_1 + L^3(\delta_2) = 0$, then there is no difference introduced via the round-key addition in Round 6. The attacker should therefore follow the steps:

1. Take any Plaintext P and compute the state after the first round MIX-COLUMNS, *i.e.* E^1 .
2. Take any three-nibble difference $\Delta_1, \Delta_3, \Delta_4$ to construct $\overline{E^1}$ such that

$$E^1 \oplus \overline{E^1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta_1 & 0 & \Delta_2 \\ \Delta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_4 \end{bmatrix}$$

The value of Δ_2 will be determined shortly. The attacker can recover \overline{P} by inverting the MC, SR, AC and SC layers on $\overline{E^1}$.

3. The attacker chooses the difference α in Cell 14 of E^2 . She calculates then $k^1[1], k^1[3], k^1[7]$ so that

$$B^2 \oplus \overline{B^2} = \text{Lin}^{-1}(E^2) \oplus \text{Lin}^{-1}(\overline{E^2}) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & \beta \\ \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}.$$

For example, $k^1[1]$ is a solution of the equation:

$$S(E^1[5] \oplus k^1[1]) \oplus S(E^1[5] \oplus \Delta_1 \oplus k^1[1]) = \alpha.$$

Note that according to Lemma 1, the equation above has one solution on average.

4. β needs to be equal to $k^2[7] \oplus \overline{k^2[7]} = tk_1^2[7] + tk_2^2[7] + \overline{tk_1^2[7]} + \overline{tk_2^2[7]}$. This is equal to $tk_1^1[11] + L(tk_2^2[11]) + \overline{tk_1^1[11]} + L(\overline{tk_2^2[7]}) = \delta_1 \oplus L(\delta_2)$. So, the attacker chooses δ_1 and δ_2 satisfying $\delta_1 + L^3(\delta_2) = 0$ and calculates $\beta = \delta_1 \oplus L(\delta_2)$. Δ_2 can then be determined as a solution of the equation:

$$S(E^1[7] \oplus k^1[3]) \oplus S(E^1[7] \oplus \Delta_2 \oplus k^1[3]) = \beta \quad (1)$$

Again by Lemma 1, there exists on average one solution of the above equation. The attacker now has the values of $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ and so, he can compute $E^1, \overline{E^1}$ and hence P, \overline{P} .

5. However, the attacker still needs that in Round 4, the active nibble in $B^4[1]$ is equal to $\delta_1 \oplus L^2(\delta_2)$ to make all the state cells inactive in C^4, D^4 , and E^4 .
6. The attacker needs to guess three additional key values in Round 1 (*i.e.* $k^1[2], k^1[4], k^1[6]$) and three additional key values in Round 2 (*i.e.* $k^2[1] = tk_1^1[15] + L(tk_2^1[15]), k^2[2] = tk_1^1[8] + L(tk_2^1[8]), k^2[6] = tk_1^1[12] + L(tk_2^1[12])$).

If the attacker can guess these values, then he knows the actual values (marked with \vee) of the state cells for the plaintext pair P, \bar{P} as opposed to only differences (marked by 0) in both Fig. 4 and Fig. 5.

7. Guessing the tweak nibbles mentioned above enables the attacker to calculate the value of $B^3[1]$. Then, she calculates $k^3[1] = tk_1^1[7] \oplus L(tk_2^1[7])$ as follows. Since $D^3[1] = B^3[1] \oplus k^3[1]$ holds, we have:

$$S(D^3[1] \oplus D^3[9] \oplus D^3[13]) \oplus S(D^3[1] \oplus D^3[9] \oplus \overline{D^3[13]}) = \delta_1 \oplus L^2(\delta_2).$$

Since the knowledge of the guessed key nibbles already allows the attacker to calculate $D^3[9]$, $D^3[13]$, and $\overline{D^3[13]}$, $k^3[1] = tk_1^1[7] \oplus L(tk_2^1[7])$ is the solution to the equation above. Again, Lemma 1 guarantees one solution on average. Since the attacker has already determined $k^1[7] = tk_1^1[7] \oplus tk_2^1[7]$, this also determines the values of $tk_1^1[7]$ and $tk_2^1[7]$.

8. This guarantees that there are no more active nibbles after Round 4. The key difference does not add to the state in Round 5, and due to the fact that $\delta_1 + L^3(\delta_2) = 0$, the tweak difference becomes 0 in Round 6.

Thus, by guessing six and calculating three key nibbles, we can construct P, \bar{P} and K, \bar{K} so that the internal state after six rounds has no active nibbles.

Lemma 4. *Given C, \bar{C} as the two output ciphertexts after querying plaintext-tweakey pairs (P, K) and \bar{P}, \bar{K} as described above, to a 21-round SKINNY-64/128 encryption oracle. Then for a fraction 2^{-40} of the ciphertext pairs, it is possible to construct a backward trail for round 21 to round 18 by guessing intermediate tweak nibbles so that there are no active nibbles in the internal state at the end of round 17.*

Proof. The attacker starts working backward from the ciphertext pairs C, \bar{C} and proceeds as follows (illustrated in Fig. 5):

1. The attacker rejects ciphertext pairs which do not have seven inactive cells in Cells 3, 4, 5, 8, 9, 11, and 14) after peeling off the final MIXCOLUMNS layer (*i.e.* D^{21}). Thus, a fraction of 2^{-28} pairs are filtered after this stage.
2. Furthermore, the attacker rejects ciphertext pairs which do not have the difference $\delta_1 \oplus L^{10}(\delta_2)$ in Cell 13 of A^{21} , *i.e.* reject if $A^{21}[13] \oplus \overline{A^{21}[13]} \neq \delta_1 \oplus L^{10}(\delta_2)$. Since calculating this cell does not require any key guess, the attacker can do this filtering instantly. So, a fraction of 2^{-4} pairs remain after this stage.
3. Since the two bottommost rows of the state are not affected by the tweak addition, and since $tk_1^1[7], tk_2^1[7]$ are already known, the attacker can calculate the actual values in Cells 0, 8, and 12 in A^{21} for the ciphertext pairs. These have to be equal since they are the output of the 20th-round MIXCOLUMNS operation on the leftmost column which had only one active nibble in its input. If the active Cells 8 and 12 are different, the attacker can reject the pair. This adds another filter with probability 2^{-4} .

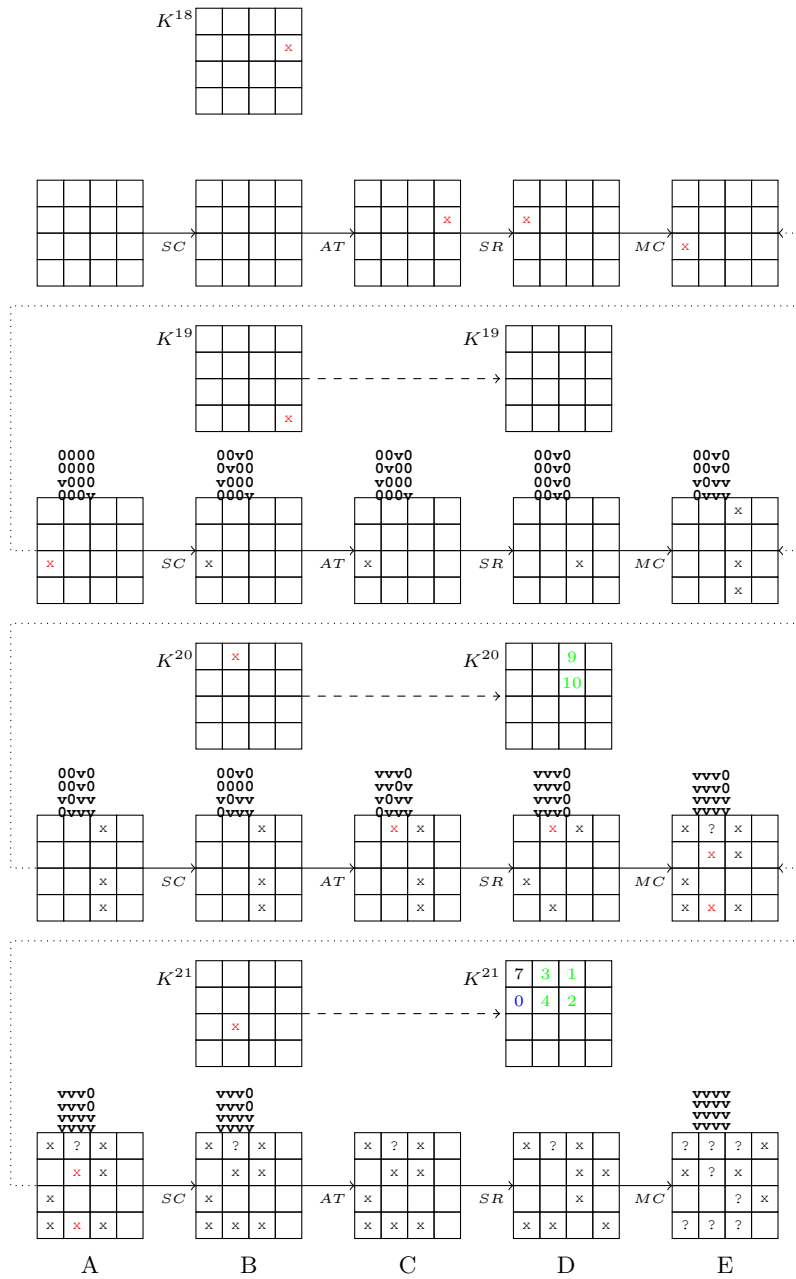


Fig. 5: Trail for the four backward rounds (the values of active nibbles in red are functions of δ_1 and δ_2).

4. Since the actual values in Cell 0 in A^{21} for the ciphertext pairs were already calculated in the previous step, the attacker checks if the value of the active Cell 0 is equal to that of Cells 8 and 12, and rejects the pair otherwise. This adds another filter of probability 2^{-4} .
5. The attacker determines $k^{21}[5] = tk_1^1[4] \oplus L^{10}(tk_2^1[4])$ so that the active nibble in Cell 5 of A^{21} is $\delta_1 \oplus L^{10}(\delta_2)$. Since $A^{21}[5] = S^{-1}(k^{21}[5] \oplus C^{21}[5])$, $k^{21}[5]$ is a solution to the equation below:

$$S^{-1}(k^{21}[5] \oplus C^{21}[5]) \oplus S^{-1}(k^{21}[5] \oplus \overline{C^{21}}[5]) = \delta_1 \oplus L^{10}(\delta_2).$$

6. The attacker determines $k^{21}[2] = tk_1^1[1] \oplus L^{10}(tk_2^1[1])$ and $k^{21}[6] = tk_1^1[2] \oplus L^{10}(tk_2^1[2])$ so that the active nibble in Cell 2 and 6 of A^{21} are equal to the active nibble in Cell 14. Again, this works since those cells are output of the 20th-round MIXCOLUMNS operation on Column 2 which had only one active nibble in its input.
7. Additionally, the attacker guesses $k^{21}[4] = tk_1^1[0] \oplus L^{10}(tk_2^1[0])$. This enables the attacker to compute the actual values for the entire leftmost column of A^{21} and hence to compute D^{20} after applying the inverse MIXCOLUMNS operation.
8. The value of the active nibble in cell 10 of A^{20} is given as:

$$\begin{aligned} A^{20}[10] \oplus \overline{A^{20}}[10] &= S^{-1}(B^{20}[10]) \oplus S^{-1}(\overline{B^{20}}[10]) \\ &= S^{-1}(D^{20}[8]) \oplus S^{-1}(\overline{D^{20}}[8]) = \eta. \end{aligned} \tag{2}$$

Since the leftmost column of D^{20} is known, the attacker can calculate η , which must be equal to Cell 14 of A^{20} since they are output of the 19th-round MIXCOLUMNS operation with one active input nibble. This is given as:

$$\begin{aligned} A^{20}[14] \oplus \overline{A^{20}}[14] &= S^{-1}(D^{20}[13]) \oplus S^{-1}(\overline{D^{20}}[13]) \\ &= S^{-1}(A^{21}[1] \oplus A^{21}[13]) \oplus S^{-1}(\overline{A^{21}}[1] \oplus \overline{A^{21}}[13]). \end{aligned} \tag{3}$$

It holds that $A^{21}[1] = S^{-1}(C^{21}[1] \oplus k^{21}[1])$. Similarly, it holds that $\overline{A^{21}}[1] = S^{-1}(\overline{C^{21}}[1] \oplus k^{21}[1])$. By calculating Equations (2) and (3), the attacker can solve for $k^{21}[1] = tk_1^1[3] \oplus L^{10}(tk_2^1[3])$. One solution on average is guaranteed by Lemma 1.

9. The values $tk_1^1[i] \oplus tk_2^1[i]$, for $i = 1, 2, 3, 4$, were already determined during the calculation of the forward trail. So, using their values, the attacker can determine the actual values $tk_1^1[i]$, $tk_2^1[i]$ for $i = 1, 2, 3, 4$.
10. The attacker calculates $k^{20}[2] = tk_1^1[9] \oplus L^{10}(tk_2^1[9])$ so that the active nibble in Cell 2 in A^{20} is equal to the active value η in Cells 10 and 14 since they

are output of the 19^{th} -round MIXCOLUMNS operation with one active input nibble. This is done by solving

$$\eta = A^{20}[2] \oplus \overline{A^{20}}[2] = S^{-1} (C^{20}[2] \oplus k^{20}[2]) \oplus S^{-1} (\overline{C^{20}}[2] \oplus k^{20}[2]). \quad (4)$$

11. The final condition to be satisfied is that the active nibble in Cell 8 of A^{19} has to be equal to $\delta_1 \oplus L^9(\delta_2) = \gamma$.

$$\begin{aligned} \gamma &= S^{-1} (D^{19}[10]) \oplus S^{-1} (\overline{D^{19}}[10]) \\ &= S^{-1} (A^{20}[6] \oplus A^{20}[14]) \oplus S^{-1} (\overline{A^{20}}[6] \oplus \overline{A^{20}}[14]). \end{aligned} \quad (5)$$

Note that $A^{20}[6] = S^{-1}(C^{20}[6] \oplus k^{20}[6])$. And since $\overline{A^{20}}[6] = A^{20}[6]$, solving Equation (5) helps to determine $k^{20}[6] = tk_1^1[10] \oplus L^{10}(tk_2^1[10])$.

The result follows since in the Steps 1-4, a total of $2^{-28-4-4-4} = 2^{-40}$ ciphertext pairs are filtered.

3.1 Attack Algorithm

Now, we put together the findings of Lemma 3 and 4 into an attack procedure:

1. The adversary chooses a random base plaintext P and requests the corresponding ciphertext C for (P, K) .
2. She chooses fixed differences δ_1 and δ_2 such that $\delta_1 = L^3(\delta_2)$.
3. For each nonzero difference $(\Delta_1, \Delta_3, \Delta_4)$ ($(2^4 - 1)^3$ choices):
 - Choose α ($2^4 - 1$ choices) and determine Δ_2 .
 - With the value of $(\Delta_1, \Delta_2, \Delta_3, \Delta_4)$, compute \overline{P}
 - Get the ciphertext \overline{C} for $(\overline{P}, \overline{K})$.
 - If $C \oplus \overline{C}$ does not pass the 2^{-36} filter (Step 1, 2, 3 in Lemma 4), then abort and start again.
 - If they pass the filter: the adversary can guess seven tweak cells (2^{28} guesses) and calculate 17 key/tweak cells as follows:

#	Guessed	Rnd	Calculated	Rnd
1	$tk_1^1[i] \oplus tk_2^1[i]$ for $i = 2, 4, 6$	1		
2	$tk_1^1[i] \oplus L(tk_2^1[i])$ for $i = 8, 12, 15$	2		
3	$tk_1^1[i] \oplus L^{10}(tk_2^1[i])$ for $i = 0$	21		
4			$tk_1^1[i], tk_2^1[i]$ for $i = 7$	3
5			$tk_1^1[i], tk_2^1[i]$ for $i = 1, 2, 3, 4$	21
6			$tk_1^1[i] \oplus L^{10}(tk_2^1[i])$ for $i = 9, 10$	20

The 17 tweakkey nibbles used for elimination are therefore:

- (a) $tk_1^1[i], tk_2^1[i]$ for $i = 1, 2, 3, 4, 7$
- (b) $tk_1^1[i] \oplus L^{10}(tk_2^1[i])$ for $i = 9, 10$

- (c) $tk_1^1[i] \oplus L^{10}(tk_2^1[i])$ for $i = 0$
- (d) $tk_1^1[i] \oplus L(tk_2^1[i])$ for $i = 8, 12, 15$
- (e) $tk_1^1[i] \oplus tk_2^1[i]$ for $i = 6$
- A fraction of 2^{-4} tweakeys fails the condition in Step 4 of Lemma 4.
- Therefore, the adversary has a set of $2^{28-4} = 2^{24}$ wrong key candidates.

The procedure above is repeated with 2^x chosen plaintexts until a single key solution remains for the 17 nibbles of the tweakey.

Complexity. For every plaintext, the adversary has $(2^4 - 1)^3$ choices of differences, and for each α she has on average one value of Δ_2 . Since there are $2^4 - 1$ choices of α , there are $2^4 - 1$ choices of Δ_2 on average. This makes a total of $(2^4 - 1)^4 \approx 2^{16}$ encryption calls. With 2^x such base plaintexts, she has 2^{x+16} encryption calls. With probability 2^{-36} , the adversary obtains a workable ciphertext difference to process.

Each such instance generates $2^{28-4} = 2^{24}$ key candidates (in 17 nibbles) for elimination. On average after $2^{x+16-36} = 2^{x-20}$ times, she gets to guess a set of 2^{24} tweakey candidates to eliminate.

$$\text{Time complexity} = \max \{2^{x+16}, 2^{x-20+24}\} = 2^{x+16}.$$

The attacker gets wrong solutions for $2^{x-20+24} = 2^{x+4}$ incorrect solutions for 17 nibbles. To reduce the keyspace to a single surviving key, we need:

$$2^{17 \times 4} \cdot (1 - 2^{-17 \times 4})^{2^{x+4}} \approx 2^{17 \times 4} \cdot e^{-2^{x-64}} = 1.$$

For this, we need $x = 70$. So, the total number of encryption calls to 21-round SKINNY-64/128 is $2^{70+16} = 2^{86}$.

3.2 Second Attack

This section presents a variant of the attack procedure that changes the way the related plaintext/tweakey pairs are constructed:

1. The attacker chooses the nibble values of the random base variable E^1 in all locations except Cells 5, 7, 8, and 15.
2. She chooses fixed differences δ_1, δ_2 satisfying $\delta_1 = L^3(\delta_2)$.
3. For each choice of $(E^1[5], E^1[7], E^1[8], E^1[15])$ (2^{16} choices):
 - Calculate P by inverting the first round.
 - Query the 21-round encryption oracle for P, K and P, \bar{K} .

So, for every choice of the base variable E^1 , we have 2^{17} encryption calls. We can pair related plaintext and tweakey pairs in the following way: For every plaintext P_i , choose a plaintext P_j so that E^1 for P_i and P_j have a non-zero difference in all Cells 5, 7, 8, and 15. For every P_i , there exist $(2^4 - 1)^4 \approx 2^{15.6}$ such values of P_j , and so $2^{16+15.6} = 2^{31.6}$ pairs to work with. The attack now proceeds as follows.

1. For each choice of P_i, P_j ($2^{31.6}$ choices):
 - Denote $P = P_i$ and $\overline{P} = P_j$.
 - The attacker can choose α and proceed with the steps of the above attack with one exception: She can no longer choose Δ_2 as in Step 4 of Lemma 3 since she has already chosen $P, \overline{P}, K, \overline{K}$.
 - With probability 2^{-4} (as per Lemma 2), the plaintext pair satisfies Equation (1) in Step 4 of Lemma 3 and proceeds; otherwise, she aborts.
 - Request the ciphertext \overline{C} for $(\overline{P}, \overline{K})$ and the ciphertext C for (P, K) .
 - If $C \oplus \overline{C}$ does not pass the 2^{-36} filter (Steps 1, 2, and 3 in Lemma 4), then abort and start again.
 - If they pass the filter, the attacker can guess seven tweak cells (2^{28} guesses) and calculate 17 key/tweak cells as in previous attack.
 - A fraction of 2^{-4} tweakeys will fail the condition required in Step 4 of Lemma 4.
 - Therefore, the attacker has a set of $2^{28-4} = 2^{24}$ wrong key candidates.

The above procedure is repeated with 2^x chosen plaintexts until a single key solution remains for the 17 nibbles of the tweak.

Complexity. With 2^x such base plaintexts, the attacker has 2^{x+17} encryption calls but $2^{x+31.6}$ plaintext and hence ciphertext pairs. With probability 2^{-36} the attacker gets a workable ciphertext difference to process. Each such instance generates $2^{28-4} = 2^{24}$ key candidates (in 17 nibbles) for elimination. On average, after $2^{x+31.6-36} = 2^{x-4.4}$ times, she gets to guess a set of 2^{24} tweak candidates to eliminate.

So, the calculation of the time complexity is similar as it is for the previous attack. The major difference is, that for $N \geq 2^{50}$ and $N = 2^{x-4.4}$, we need only $x = 2^{54.4}$ structures, and thus, $2 \cdot 2^{54.4} \cdot 2^{16} = 2^{71.4}$ chosen plaintexts. The time complexity is given then by approximately

$$2^{x+17} + 2^{x+5.6} + 2^{x-9.4} + 2^{60} \approx 2^{71.4} \text{ encryptions}$$

plus memory accesses for denoting keys as invalid. The memory complexity is equal to that before.

3.3 Attacking 22-Round SKINNY-64/128 under Partially Known Tweak

The attack above can be extended to 22-round SKINNY-64/128 under the assumption that 48 of the 128 bits in the tweak are publicly known tweak. In particular, we assume that $tk_1^1[i], tk_2^1[i]$ for $i = 8, 11, 12, 13, 14, 15$ are reserved for the tweak. The remaining 80 bit constitute the secret key.

In this case, the attacker can add a round at the end (see Figure 6 for details). Knowing six out of eight cells in the lower half of the tweak blocks helps in

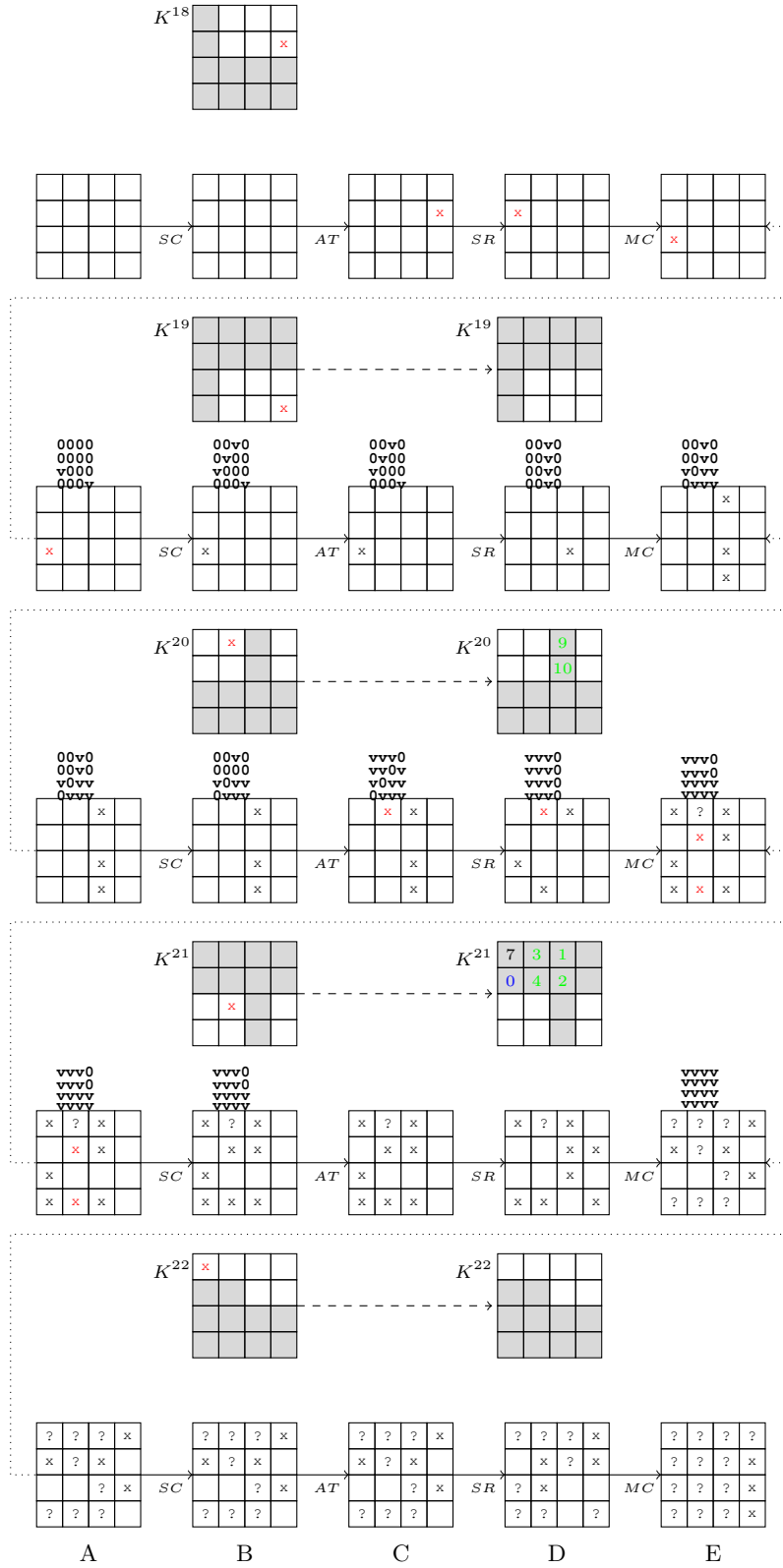


Fig. 6: Trail for the five backward rounds (the values of active nibbles in red are functions of δ_1, δ_2 , grey cells are the key, white cells are the tweak).

the following way. From the ciphertext (i.e., E^{22}), one can revert the final round to compute E^{21} if we guess $k^{22}[4, 5]$, i.e., $tk_1^1[9, 10] \oplus L^{11}(tk_2^1[9, 10])$. Thereupon, the attack is almost the same as the previous attack except that the tweakey indices $i = 8, 11, 12, 13, 14, 15$ and their functions are known and need not be guessed.

1. Generate $2^{31.6}$ plaintext/ciphertext pairs from every base choice of E^1 and 2^{17} encryption calls.
2. For each choice of P_i, P_j ($2^{31.6}$ choices):
 - Denote $P = P_i$ and $\overline{P} = P_j$.
 - The attacker can choose α and calculate $k^1[1]$, $k^1[3]$, and $k^1[7]$ as per Step 3 of Lemma 3.
 - She can no longer choose Δ_2 as in Step 4 of Lemma 3 since she has already chosen $P, \overline{P}, K, \overline{K}$.
 - With probability 2^{-4} , the plaintext pair satisfies Equation (1) in Step 4 of Lemma 3 and proceeds; otherwise, she aborts.
 - As already outlined, the attacker need not guess the Round 2 tweakey nibbles in Step 6 of Lemma 3: i.e. functions of $k^1[8, 12, 15]$ since these are in the lower half of the tweakey blocks and therefore known.
 - Retrieve the ciphertext \overline{C} for $(\overline{P}, \overline{K})$ and the ciphertext C for (P, K) .
 - Guess $k^{22}[4, 5]$ which is $tk_1^1[9, 10] \oplus L^{11}(tk_2^1[9, 10])$ to invert the final round and get E_{21} .
 - If $E_{21} \oplus \overline{E_{21}}$ does not pass the 2^{-36} filter (Steps 1, 2, 3 in Lemma 4), then abort and start again.
 - After determining $k^{20}[2] = tk_1^1[9] \oplus L^{10}(tk_2^1[9])$ and $k^{20}[6] = tk_1^1[10] \oplus L^{10}(tk_2^1[10])$ in Steps 10 and 11 of Lemma 4, the attacker can uniquely determine $tk_1^1[9, 10]$ since $tk_1^1[9, 10] \oplus L^{11}(tk_2^1[9, 10])$ is already guessed.
 - If they pass the filter, the attacker can guess six tweakey cells (2^{24} guesses) and calculate 16 key cells as follows:

#	Guessed	Rnd	Calculated	Rnd
1	$tk_1^1[i] \oplus tk_2^1[i]$ for $i = 2, 4, 6$	1		
2	$tk_1^1[i] \oplus L^{10}(tk_2^1[i])$ for $i = 0$	21		
3	$tk_1^1[i] \oplus L^{11}(tk_2^1[i])$ for $i = 9, 10$	22		
4			$tk_1^1[i], tk_2^1[i]$ for $i = 7$	3
5			$tk_1^1[i], tk_2^1[i]$ for $i = 1, 2, 3, 4$	21
6			$tk_1^1[i], tk_2^1[i]$ for $i = 9, 10$	20

The 16 tweakey nibbles used for elimination are therefore:

- (a) $tk_1^1[i], tk_2^1[i]$ for $i = 1, 2, 3, 4, 7, 9, 10$.
 - (b) $tk_1^1[i] \oplus L^{10}(tk_2^1[i])$ for $i = 0$.
 - (c) $tk_1^1[i] \oplus tk_2^1[i]$ for $i = 6$.
- A fraction of 2^{-4} tweakeys fails the condition in Step 4 of Lemma 4.

- Therefore, the attacker has a set of $2^{24-4} = 2^{20}$ wrong key candidates.

The procedure above is repeated with 2^x chosen plaintexts until a single key solution remains for the 12 nibbles of the tweak.

Complexity. With 2^x such base plaintexts, she has 2^{x+17} encryption calls but $2^{x+31.6}$ plaintext and hence ciphertext pairs. With probability 2^{-36} the attacker obtains a workable ciphertext difference to process. Each such instance generates $2^{24-4} = 2^{20}$ key candidates (in 16 nibbles) for elimination. On average, after $2^{x+31.6-36} = 2^{x-4.4}$ times, she gets to guess a set of 2^{20} tweak candidates to eliminate. Again, we need $N \geq 2^{50}$ and therefore $x = 54.4$ structures, which gives $2 \cdot 2^{x+16} = 2^{71.4}$ chosen plaintexts.

The time complexity results from:

- The attacker requests the full encryption plus the inverse first round of 2^{x+17} chosen plaintexts, or $2^{x+17} \cdot 22/21 \approx 2^{x+17.05}$ plaintexts.
- For 2^{24} key guesses, it computes for the $2^{x+31.6-36} = 2^{x-4.4}$ workable pairs which pass the 36-bit filter at most four rounds in forward direction. We only proceed if the first 2^{-4} filter from Lemma 2 at plaintext side holds. Thus, we have $2 \cdot 2^{24-4} \cdot 2^{x-4.4} \cdot 4/22 \approx 2^{x+14.2}$ 22-round encryptions. Since we have a 16-bit filter in total at the plaintext side, there are $2^{24} \cdot 2^{x-4.4} \cdot 2^{-16} = 2^{x+3.6}$ pairs remaining.
- For those remaining pairs, it inverts at most the final five rounds in backward direction, which yields $2 \cdot 2^{x+3.6} \cdot 5/22 \approx 2^{x+2.5}$ 22-round encryptions. Since we have a 28-bit filter there, we expect about $2^{x+3.6} \cdot 2^{-28} = 2^{x-24.4}$ pairs to remain.
- Each such instance generates 2^{20} key candidates (in 16 nibbles) for elimination for which we have at most $2^{x-24.4+20} + 2^{64} \approx 2^{64}$ memory accesses.
- Since the attacker can recover 16 key nibbles or 64 bit of the key, it finally needs 2^{64} 22-round encryptions to successfully recover the full key.

Summing up, the time complexity for $x = 54.4$ is approximately

$$2^{x+17.05} + 2^{x+14.2} + 2^{x+2.5} + 2^{64} \approx 2^{71.6} \text{ encryptions.}$$

plus 2^{64} memory accesses, which are negligible in the total complexity. The memory complexity is upper bounded by storing one bit per candidate or $2^{64} \cdot 1/64 = 2^{58}$ 64-bit states of SKINNY-64/* . The memory for storing the approximately $2 \cdot 2^{17}$ plaintexts and corresponding ciphertexts of a structure at each time is negligible.

4 Attacking 23-Round SKINNY-64/128 under Partially Known Tweak

In this section, we extend the attack above to 23 rounds in the following manner: we will prepend one round at the beginning of the basic 22-round attack described in the previous section. In order to not disturb the notation, we denote

Algorithm 1 The 23-round attack.

for all guesses of $k^0[9, 10]$ (2^8 guesses) **do**
 The attacker computes P, \bar{P} from E^1, \bar{E}^1 .
 The attacker runs the 22-round attack.

the additional round prepended at the beginning as the 0-th round. That is, the 23 rounds are labelled as rounds 0 to 22, and the variables A^0, B^0 etc. are defined as above. The plaintext is denoted by A^0 and the ciphertext by E^{22} . Note that, from the base value of E^1 , the plaintext can be calculated if we guess $k^0[9, 10]$. Therefore, the attack we define is as given in Algorithm 1.

There are two principal differences to the 22-round attack:

1. When the attacker guesses $k^{22}[4, 5]$ which is $tk_1^1[9, 10] \oplus L^{11}(tk_2^1[9, 10])$ to invert the final round to get E_{21} , he uniquely determines $tk_1^1[9, 10]$ and $tk_2^1[9, 10]$. This is because at the beginning of the outer loop $k^0[9, 10]$ has already been guessed by the attacker to invert the initial round.
2. So, the attacker can no longer determine $k^{20}[2] = tk_1^1[9] \oplus L^{10}(tk_2^1[9])$ and $k^{20}[6] = tk_1^1[10] \oplus L^{10}(tk_2^1[10])$ using Steps 10 and 11 of Lemma 4. The probability that the with the given values of $tk_1^1[9, 10]$ and $tk_2^1[9, 10]$, Equations (4) and (5) are satisfied is 2^{-8} . This decreases the probability of ciphertext filter from 2^{-36} to 2^{-44} .

For each initial guess of $k^0[9, 10]$, the guessed and calculated key bytes are the following:

#	Guessed	Rnd	Calculated	Rnd
1	$tk_1^1[i] \oplus tk_2^1[i]$ for $i = 2, 4, 6$	1		
2	$tk_1^1[i] \oplus L^{10}(tk_2^1[i])$ for $i = 0$	21		
3	$tk_1^1[i] \oplus L^{11}(tk_2^1[i])$ for $i = 9, 10$	22		
4			$tk_1^1[i], tk_2^1[i]$ for $i = 7$	3
5			$tk_1^1[i], tk_2^1[i]$ for $i = 1, 2, 3, 4$	21

The 14 tweaky nibbles used for elimination are therefore:

- (a) $tk_1^1[i], tk_2^1[i]$ for $i = 1, 2, 3, 4, 7$.
- (b) $tk_1^1[i] \oplus L^{10}(tk_2^1[i])$ for $i = 0$.
- (c) $tk_1^1[i] \oplus tk_2^1[i]$ for $i = 6$.
- (d) $tk_1^1[i] \oplus L^{11}(tk_2^1[i])$ for $i = 9, 10$

As before, a fraction of 2^{-4} tweakeys fails the condition in Step 4 of Lemma 4. Therefore, the attacker has a set of $2^{24-4} = 2^{20}$ wrong key candidates.

Complexity. For each iteration of the outer loop, the complexity is calculated as follows: For every base value of E^1 , the attacker makes 2^{17} encryption calls.

Out of those, he has $2^{31.6}$ pairs to work with. For each pair, the attacker can choose then α in $2^4 - 1$ ways, which gives her around $2^{35.6}$ initial guesses for the forward key nibbles $k^1[1], k^1[3], k^1[7]$, of which only a fraction of 2^{-4} passes the filter in Equation (1). So, the attacker has $2^{31.6}$ pairs to work with. In effect, for every pair (P_i, P_j) there is only once choice of α going forward on average.

$$\text{Time complexity} = \max \{ 2^{x+17} \text{ encryptions}, 2^{x+31.6-44+20} \text{ guesses} \} = 2^{x+17}.$$

The attacker gets wrong solutions for $2^{x+31.6-44+20} = 2^{x+7.6}$ incorrect solutions for 14 nibbles. To reduce the keyspace to 1 we need:

$$2^{14 \times 4} \cdot (1 - 2^{-14 \times 4})^{2^{x+7.6}} \approx 2^{14 \times 4} e^{-2^{x-48.4}} = 1.$$

For this, we need $x = 54$. So, the total number of encryption calls to 22-round SKINNY-64/128 is $2^{54+17} = 2^{71}$. Multiplying this by 2^8 for the outer loop gives us the total complexity $2^{71+8} = 2^{79}$ which is just short of exhaustive search for the 80-bit key.

5 Conclusion

In this paper, we outline related-key impossible-differential attacks against 21-round SKINNY-64/128 as well as attacks on 22 and 23 rounds under the assumption of having 48 of the 128-bit tweak as public tweak. Our attacks are based on an 11-round impossible differential trail, to which we prepend six and append five rounds before and after the trail, respectively, to obtain an attack on 22 rounds. Finally, we show that we can prepend a 23-rd round under similar assumptions.

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Supporting Materials

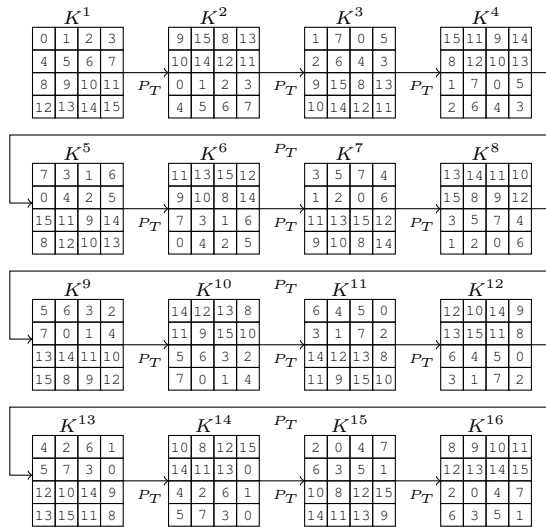


Fig. 7: The permutation P_T in the tweak schedule has a period of 16.

Table 1: Difference-Distribution Table

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16
1	4	4	4	4
2	.	4	.	4	.	4	4
3	2	2	2	2	2	2	2	2	2
4	.	.	4	.	.	.	2	2	.	.	4	2	2	.	.	.
5	.	.	4	.	.	2	2	.	4	.	2	2
6	.	2	.	2	2	.	2	2	.	2	.	2	.	2	2	.
7	.	2	.	2	2	.	2	.	2	.	2	2	.	2	.	2
8	.	.	.	4	4	2	2	2	2	2	2
9	.	.	.	4	4	2	2	2	2	2	2
a	4	4	.	2	2	2	2
b	.	4	.	4	2	2	2	2	.	.
c	.	.	4	.	.	2	2	4	2	2	.	.
d	.	.	4	.	.	2	2	.	4	.	.	.	2	2	.	.
e	.	2	.	2	2	.	2	.	2	.	2	.	2	2	.	2
f	.	2	.	2	2	.	2	2	.	2	.	2	.	2	.	2

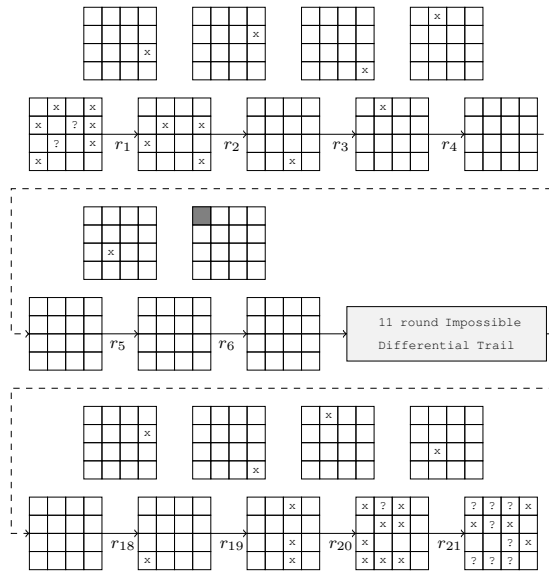


Fig. 8: Related-Key Impossible Differential Attack on 21 round SKINNY 64/128 (the dark gray cell visualises the cancelation of the tweakeys)

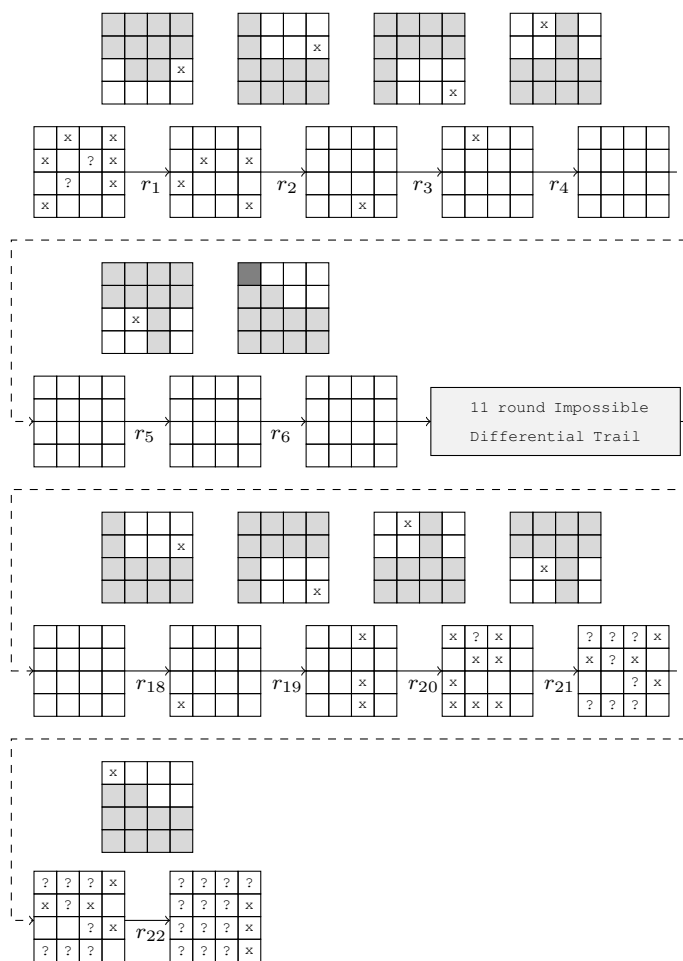


Fig. 9: Related-Key Impossible Differential Attack on 22 round SKINNY 64/128 (grey cells are the key, white cells are the tweak, the dark gray cell visualises the cancellation of the tweakeys)